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Physics Beyond the Standard Model

Part I: Theoretical Introduction

From an experimentalist perspective

Apologies for not acknowledging the authors of the illustrations

Short Bio

Steve Muanza, PhD & Habilitation Theses
Senior staff in experimental HEP with CNRS-IN2P3 & Aix-Marseille U., France

- Descent: I'm french, both my parents come from DR Congo
- School: I started my studies up to the 10^{th} grade in Belgium
- High school: 11-12^{th} grade in Boston US, graduated from high school
- College: 1^{st} year in college in Libreville, Gabon
- Undergraduate: rest of undergraduate studies at U. of Lille FR, B.Sc. in 1990
- Graduate: M.Sc. and Ph.D. in ATLAS in U. of Clermont-Fd FR in 1992 & 1996
- Permanent position (junior staff): with CNRS in 1996 at Institute for Nuclear Physics in Lyon till 2008, worked mainly on Higgs and SUSY searches in CMS, L3 and D0 experiments
- Permanent position (senior staff): moved to CPPM Marseille, finished my D0 activities, rejoined ATLAS, also head of activities on future colliders
- Initiator & co-founder of the African School of Physics

My parents

Birth place, school

11^{th}-12^{th} grade

College

Higher Ed. & Job

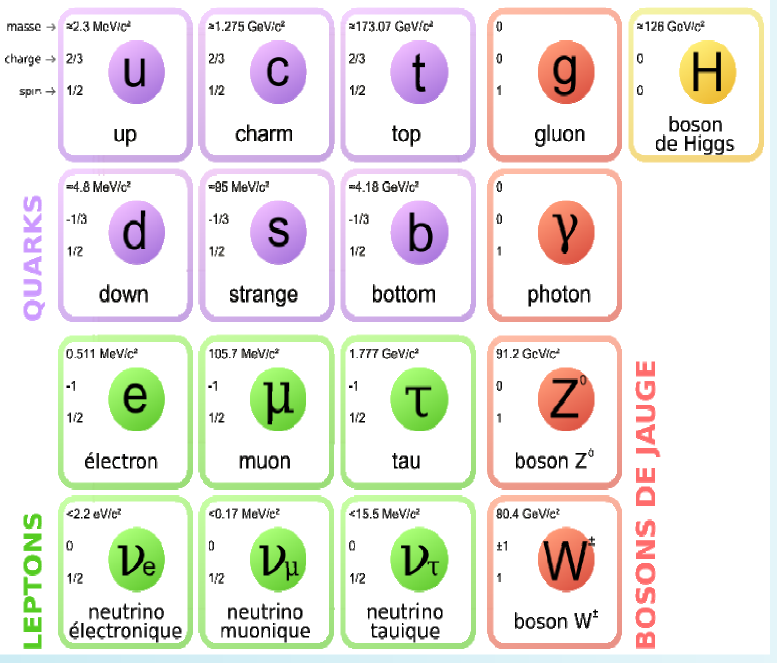
Outline

PART I: The SM of Particle Physics

SM: Summary

- As all QFT, the SM is defined by its PARTICLE CONTENT and its SYMMETRIES

- PARTICLE CONTENT:



$$Q_{\text{electric}} = T_3 + \frac{Y}{2}$$

- Renormalizable theory (anomaly free)

- SYMMETRIES:
  - Poincaré group:
    - Lorentz group (rotation, boost)
    - Translation
  - Discrete: invariance under

$$\boxed{C \cdot P \cdot T}$$

    - C: charge conjugation
    - P: space reversal
    - T: time reversal
  - Gauge: invariance under

$$\boxed{G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$$

- Fundamental Interactions:
  - Strong
  - Weak
  - Electromagnetic (EM)

Two Pilars of Modern Physics

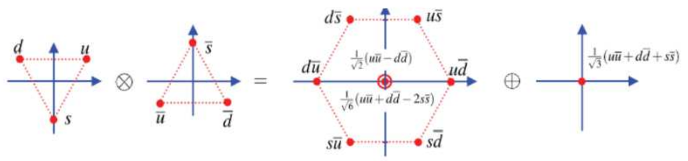
- The Standard Model (SM) of Particle Physics is a theory that describes all known elementary particles and explain their interactions (except for gravity)

- Formally it's a Relativistic Quantum Field Theory (QFT)
  thus it complies simultaneously with Special Relativity (SR) and Quantum Mechanics (QM)

- Symmetries play a crucial role, not only for the classification of the diverse particles, but also to understand their interactions. Formally Group Theory, (especially Lie groups) is used to this end.

Example: « Eightfold Way »

- Classify hadrons based upon their quark content, using SU(3) flavour symmetries



$$\boxed{M(q\bar{q}) = 3 \otimes \bar{3} = 8 \oplus 1}$$

Mesons

$$\boxed{B(qqq) = 3 \otimes 3 \otimes 3 = (3 \otimes 6) \oplus (3 \otimes \bar{3}) = 10_S \oplus 8_M \oplus 8_M \oplus 1_A}$$

Baryons

Dynamics

- Link (symmetry-dynamics) ensured by Noether's theorem: « to any continuous symmetry corresponds a conserved charge/current »

Symmetry Law	Conserved Quantity
Translation in space	**Linear Momentum**
Translation in time	**Energy**
Rotation in space	**Angular Momentum**
Gauge transformations	**Electric, weak, color charges**

Spin

- Symmetries of SR (Poincaré Group) show that elementary particles can be labelled by their mass and spin (or just helicity for massless particles)

- Depending on their spin, particles are divided into 2 classes:

Fermions

- Half-integer spin (S=1/2,3/2)
- Obey « Fermi-Dirac » statistics: individualistic behaviour (Ex: Pauli exclusion principle « 2 electrons cannot occupy the same quantum state » => electrons w/ opposite spins in each atomic orbital)
- Matter particles with S=1/2 (i.e. electrons, quarks,…)

Bosons

- Integer spin (S=0,1,2)
- Obey « Bose-Einstein » statistics: gregarious behaviour (Ex: lasers, lots of photons in the same quantum state)
- Interaction particles, vector bosons with S=1, i.e. gluons, W, Z, γ
- Scalar boson with S=0, i.e. Higgs boson

Interlude #1: QM « Lenses »

- Exploring matter at very small scales requires to use 3 separate magnifying lenses
- Theses lenses are direct consequences of QM:

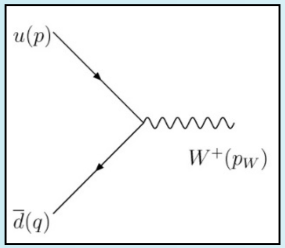
1. Since QM is intrinsically probablistic one needs a STATISTICAL LENS: repeat measurements MANY TIMES so as to measure the central value (statistical moments) with sufficient precision

2. Wave mechanics: all particles have both corpuscle and wave properties. Associated wave has de Broglie's wavelength: $\boxed{\lambda = \dfrac{\mathbf{h}}{\mathbf{p}}}$

   To probe matter at small scales, one needs an HIGH ENERGY LENS.

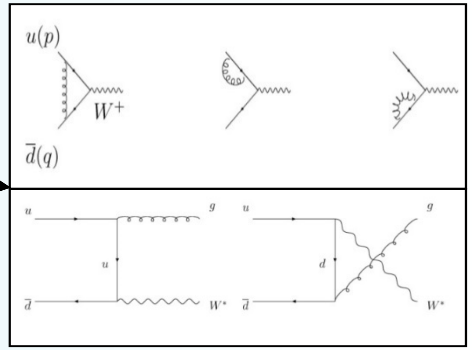
3. The 3rd lens required for accurate theory prediction is the QUANTUM LENS. In QM seldom physics cases have an exact solution! Instead one uses approximate methods, most often a perturbative expansion:

$$\boxed{\mathcal{M} = \mathcal{M}^{(0)} + \alpha \cdot \mathcal{M}^{(1)} + \alpha^{2} \cdot \mathcal{M}^{(2)} + \ldots}$$



LO

NLO



Interlude #2: Space-Time Symmetries

- The full space-time symmetries of SR constitue the Poincaré Group:
  - It contains the Lorentz Group formed of
    - Boosts in space-time
    - Rotations in space-time
  - In addition to
    - Translations in space-time
  - Its Lie algebra writes:

$$\boxed{\begin{gathered}[P_\mu, P_\nu] = 0\\ [M_{\mu\nu}, P_\rho] = g_{\mu\rho} P_\nu - g_{\nu\rho} P_\mu\\ [M_{\mu\nu}, M_{\rho\sigma}] = g_{\mu\rho} M_{\nu\sigma} - g_{\mu\sigma} M_{\nu\rho} - g_{\nu\rho} M_{\mu\sigma} + g_{\nu\sigma} M_{\mu\rho}\end{gathered}}$$

$$\begin{cases} P_\mu & \text{: 4-momentum, generator of translations} \\ M_{\mu\nu} & \text{: generators of boosts \& rotations} \end{cases}$$

- Casimir operators:

$$P^2 = m^2 \Rightarrow \begin{cases} [P^2, P_\mu] = 0 \\ [P^2, M_{\mu\nu}] = 0 \end{cases} \qquad W^2 = m^2 \cdot s(s+1) \Rightarrow \begin{cases} [W^2, P_\mu] = 0 \\ [W^2, M_{\mu\nu}] = 0 \end{cases}$$

- Invariants:

$$\boxed{P \leftrightarrow m} \qquad \boxed{W_\mu = \frac{\varepsilon_{\mu\nu\rho\sigma}}{2} P^\nu M^{\rho\sigma} \leftrightarrow S}$$

- $M_{\mu\nu}$ Examples:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad T = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{Boost} = \begin{pmatrix} \cosh\eta & \sinh\eta & 0 & 0 \\ \sinh\eta & \cosh\eta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{Rotation} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\psi & -\sin\psi \\ 0 & 0 & \sin\psi & \cos\psi \end{pmatrix}$$

$$\eta = \mathrm{artanh}(\beta)$$

Fermions: Dirac Equation

- Dirac equation: for a single free particle with S=1/2 (say an electron) denoted ψ, the dynamical equation that replaces Schrödinger equation in QFT is the Dirac equation
- Dirac started from the following expression of matter-energy equivalence:

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

And he linearized this equation by using α and β matrices:

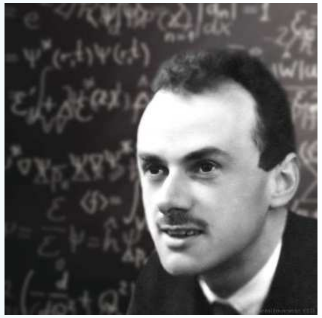
$$\boxed{\left[\beta mc^2 + \left(\sum_{j=1}^{3}\alpha_j \cdot p_j\right)\right]\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

- Using natural units $\boxed{\hbar = c = 1}$ and defining gamma matrices from α and β:

$$\boxed{\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0}$$

- Corresponding to the lagrangian (density):

$$\boxed{\mathcal{L} = \overline{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi}$$



P. Dirac, Nobel 1933

Fermions Properties

- Ordinary matter is made just of the 1st generation of quarks and leptons
- For some unknown reason there are 2 heavier replicates of the 1st generation

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor		Mass GeV/c^2	Electric charge
ν_L	lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e	electron	0.000511	–1
ν_M	middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ	muon	0.106	–1
ν_H	heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ	tau	1.777	–1

Quarks spin =1/2

Flavor		Approx. Mass GeV/c^2	Electric charge
u	up	0.002	2/3
d	down	0.005	–1/3
c	charm	1.3	2/3
s	strange	0.1	–1/3
t	top	173	2/3
b	bottom	4.2	–1/3

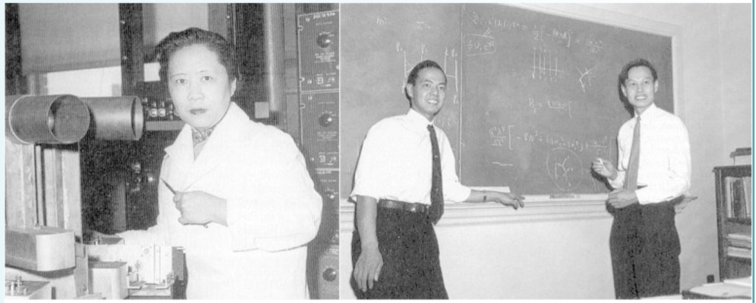
Fermions: Chirality

- Under weak interactions, fermions are chiral, i.e. not the same as their mirror image!

- This was introduced to account for parity violation in weak interactions:

$$\boxed{P\,\psi(\vec{x},t) = \psi(-\vec{x},t)}$$

  - Predicted by T.D. Lee and C.N. Yang in 1956
    - Nobel laureates 1957
  - Discovered by C.S. Wu et al. in 1957



- Right-handed and left-handed fermions have different weak interactions:

$$\psi = \psi_R + \psi_L$$

$$\mathrm{Proj}_R\,\psi = \frac{(1+\gamma_5)}{2}\psi = \psi_R \longrightarrow SU(2)_L \text{ singlet}$$

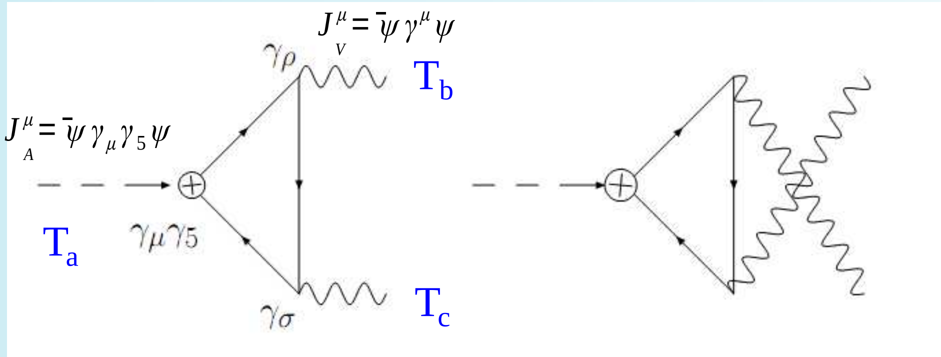
$$\mathrm{Proj}_L\,\psi = \frac{(1-\gamma_5)}{2}\psi = \psi_L \longrightarrow SU(2)_L \text{ doublet}$$

Dirac γ matrices

- In particular W bosons do not couple to right-handed fermions (or left-handed anti-fermions)

Fermions: Anomalies

- All chiral QFT have:
  - Interactions absent at tree level and only present at quantum level through fermion loops where:
    - 1 vertex has an axial coupling, and
    - 2 other vertices have vector couplings
  - They are called anomalies (aka axial / triangle / Adler-Jackiw-Bell anomalies)
  - These graphs are linearly divergent and non-renormalizable
  - Each fermion contribute, irrespective of its mass
  - The only way to cirumvent the problem is to ensure cancellations within generations



T_i: gauge group generator
J_V: vector current
J_A: axial current

- Condition for anomalies cancellation in the SM: $g_A^u = g_A^c = g_A^t = -g_A^d = -g_A^s = -g_A^b$

Refs:

- **S.L. Adler, Phys Rev 177 (1969) 2426**
- **J.S. Bell and R.W. Jackiw, Nuov. Cimento 60A (1969) 47**
- **C. Bouchiat, J. Iliopoulos and P. Meyer, Phys.Lett. B38 (1972) 519**

Fermions: Flavour Mixings

• Quarks mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V^{\dagger}V = 1_3$$

Unitary matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}$$

Ref: PDG 2019

• V is a complex unitary matrix, called the CKM matrix
• Its phase is the only known source of CP violation in the SM

$$\boxed{C\psi = \overline{\psi}}$$

Ref:
- **N. Cabibbo, Phys Rev Lett 10 (1963) 531**
- **S.L. Glashow, J. Iliopoulos, L. Maiani, Phys Rev D2 (1970) 1285** → Strong supression of FCNCs
- **M. Kobayashi and T. Maskawa, Prog Theor Phys 49 (1973) 652-657** → Nobel 2008

• Neutrinos mixing

- An analogous mixing exists for the flavours of neutrinos
- Parametrized by the Pontecorvo–Maki–Nakagawa–Sakata unitary matrix
- Oscillations between neutrino flavours impose non-zero neutrino masses

Gauge Symmetries in QFT

- Elementary particles are the quanta of fields φ
- Physical system is represented by a scalar function, the lagrangian $L(\varphi,\partial_\mu\varphi)$
- Action:
  - expressed using its lagrangian density: $S(\varphi,\partial_\mu\varphi)=\int d^4x\cdot\mathcal{L}(\varphi,\partial_\mu\varphi)$
  - dynamics infers from least action principle: $\dfrac{\delta S}{\delta\varphi_i}=0$
- Gauge transformation:

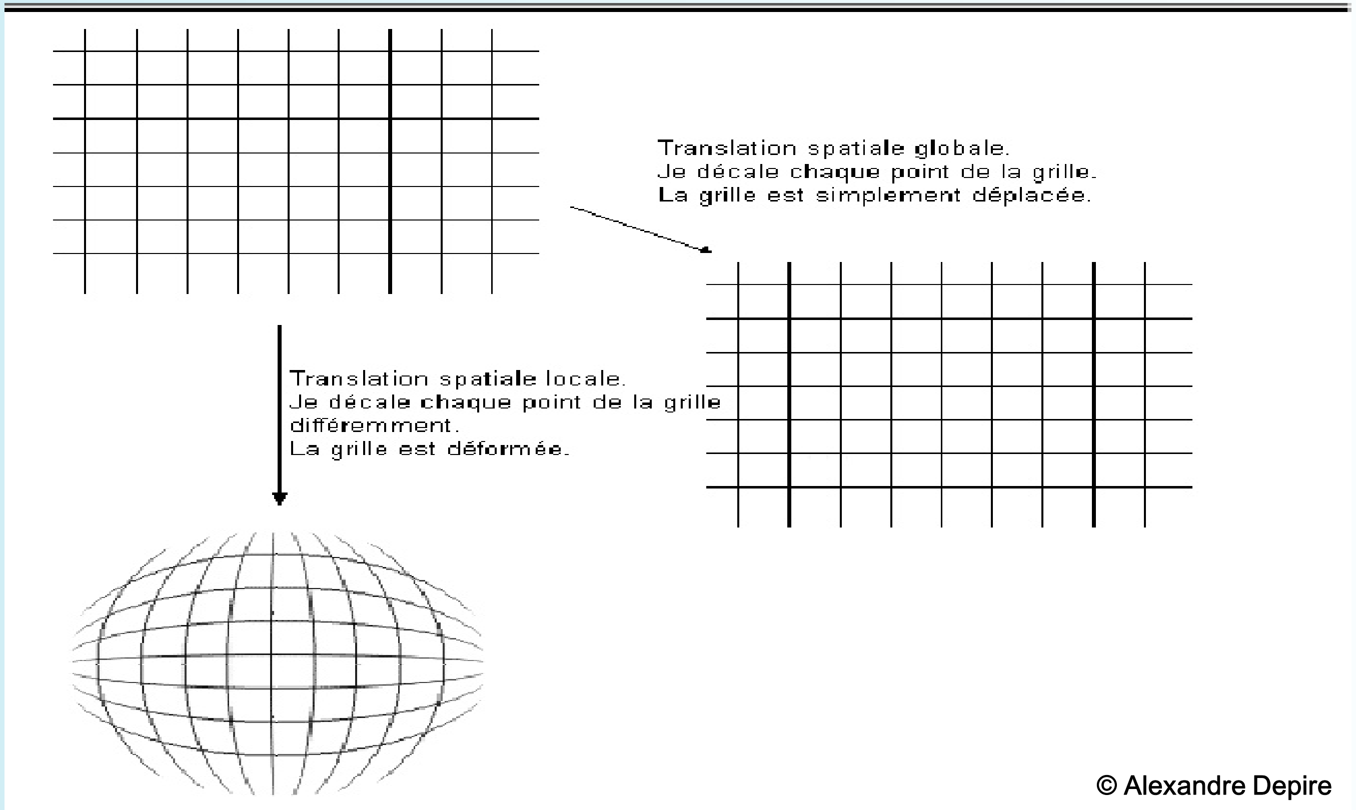
  where $U(\theta^a)=e^{i\theta^a T_a}$: unitary transformation matrix (→ group)
  T_a: transformation generators
  θ^a: transformation parameter


E. Wigner
Nobel 1963

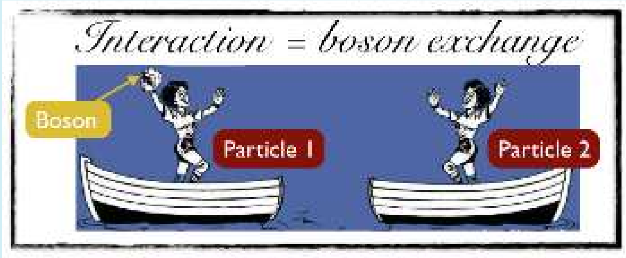
- Gauge invariance: $\mathcal{L}(\phi',\partial_\mu\phi')=\mathcal{L}(U\phi,U(\partial_\mu\phi))=\mathcal{L}(\phi,\partial_\mu\phi)$
  - Global: θ^a do not depend on x_μ (simply denoted x)
  - Local: θ^a depends on x => introduces gauge interactions via covariant derivatives, e.g.: $\partial_\mu\to D_\mu=\left(\partial_\mu-ig_2W^a_\mu\dfrac{\sigma_a}{2}\right)$
- Renormalizability: in natural units $(\boxed{\hbar=c=1})$, $[\mathcal{L}]=m^r$ with $r\leq 4$

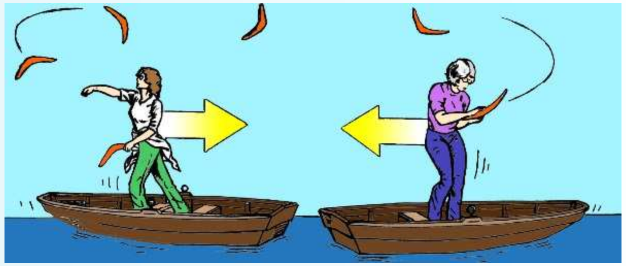
Global and Local Symmetries

- A global symmetry (e.g. 180° rotation) leaves the system invariant
- A local symmetry:
  - first deforms the system,
  - to leave it invariant one needs something that exactly compensate this deformation: it's the gauge interaction (owing to covariant derivative)



Fundamental Interactions



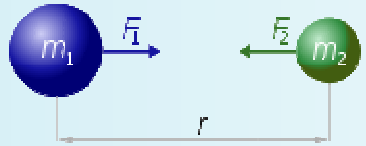


- Electromagnetism (EM):
  - QFT: Quantum Electro Dynamics (QED)
  - Abelian Symmetry group: $U(1)_{EM}$, 1 generator, the photon (γ), S=1
  - Photon couples to electric charge
  - Range: infinite; Relative Intensity: 10^{-3}
  - Cohesion of matter at atomic and molecular level
- Strong Interaction:
  - QFT: Quantum Chromo Dynamics (QCD)
  - Non-Abelian Symmetry group: $SU(3)_C$, 8 generators, the gluons (g), S=1
  - Gluons couple to colour charge (R,B,G) and have self-couplings
  - Range: 10^{-15} m; Relative Intensity: 1
  - Cohesion of matter at nucleon and nucleus level
- Weak Interaction:
  - QFT: Electroweak Theory
  - Non-Abelian Symmetry group: $SU(2)_L$, 3 generators, W^+, W^-, Z^0, S=1
  - Weak gauge bosons couple to weak hypercharge and have self-couplings
  - W & Z bosons couple differently to L-handed or R-handed fermions (P violation)
  - Range: 10^{-18} m; Relative intensity: 10^{-14}
  - Responsible of beta decays

Fundamental Interactions

- Gravitation:
  - Negligible strength for particles at low energy => not in the SM
  - No satisfactory quantum theory of gravity yet!
  - Hypothesized carrier, the graviton, S=2
  - Range: infinite; Relative intensity (@ low energy): 10^{-39}
  - Cohesion of matter at large scales (solar system, galaxies, galaxy clusters,…)

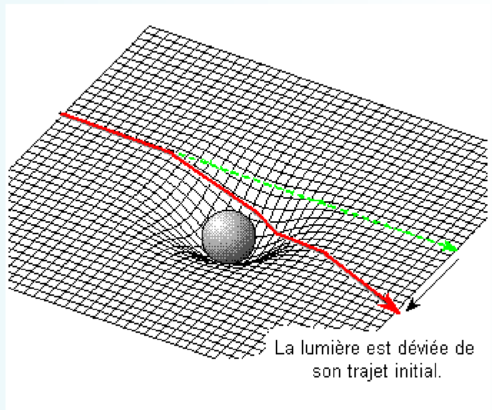
- Newton theory:



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

  - force is exerted on instantaneously (irrespective of the distance r)
  - this conflicts with Special Relativity!

- Einstein Theory: General Relativity (GR)
  - local matter-energy density deforms space-time locally
  - body trajectories are geodesic of this space-time
  - invariance wrt to a local change of coordinates
  - theory not quantized, a geometrical gauge theory



Gauge Bosons

- Abelian case: photon has no self-interactions, because it bares no electric charge
  - Maxwell equations:

$$\boxed{\partial_\mu F^{\mu\nu} = j^\nu}$$

where

$$\begin{cases} A_\mu = (\phi, \vec{A}) & \text{EM 4-vector potential} \\ j_\mu = (\rho, \vec{j}) & \text{EM 4-current} \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu & \text{EM field strength tensor} \end{cases}$$

  - Lagrangian: $$\boxed{\mathcal{L}_{EM} = \frac{-1}{4} F^{\mu\nu} F_{\mu\nu} - j_\mu A^\mu}$$

- Non-abelian cases: W/Z or g have self-interactions, because they bare weak or strong charges
  - Yang-Mills equations:

$$\boxed{\begin{cases} D_\mu F^{\mu\nu} = 0 \\ D^*_\mu F^{\mu\nu} = 0 \end{cases}}$$

where

$$\begin{cases} T_a & \text{generator} \\ A_\mu = A_{\mu a} \cdot T^a & \text{gauge bosons} \\ D_\mu = \partial_\mu - i A^a_\mu T_a & \text{covariant derivative} \end{cases}$$

  - Lagrangian: $$\boxed{\mathcal{L}_{Y-M} = \frac{-1}{4\tau_R} \cdot Tr\left(F^{\mu\nu} F_{\mu\nu}\right)}$$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$

Gauge Bosons

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Exact Gauge Symmetries

- In the SM, the masses of most particles are protected (from large quantum corrections) by some symmetries
- $SU(3)_C$: gauge invariance forbids introducing a mass term for the gluons => gluons are massless
- $U(1)_{EM}$: gauge invariance forbids introducing a mass term for the photon => photon is massless
- $SU(2)_L$: chirality/gauge invariance forbids introducing mass terms for the charged fermions / W,Z => charged fermions / weak gauge bosons are massless
- All particles are massless: this CANNOT be!

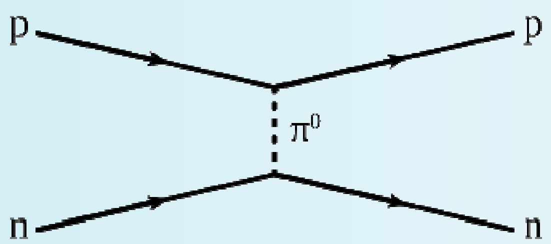
- Mixing SR and QM enables to estimate the range of the interactions:
  - Start from Heisenberg inequality: $$\boxed{\Delta E \cdot \Delta t \geq \frac{\hbar}{2}}$$
  - Plug-in matter-energy equivalence: $$\boxed{\mathbf{E_{mass} = m \cdot c^2}}$$
…
  - Reduced Compton wavelength: $$\boxed{\lambda_C = \frac{\hbar}{m \cdot c}}$$

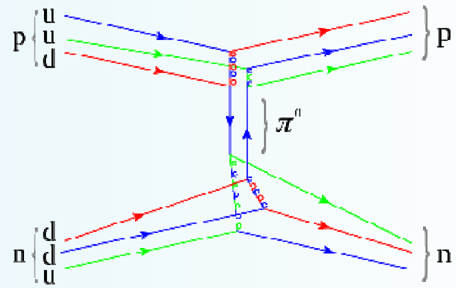
Range of Fundamental Interactions

$$\lambda_C = \frac{\hbar}{m \cdot c}$$

- $U(1)_{EM}$: fine, massless photon compatible with gauge invariance and with infinite range interaction

- $SU(3)_C$: strong interactions are observed to be short-ranged. But this can be explained by the large value of α_S. Historically that's how H. Yukawa (1934) made the first model of strong interaction via the exchange of pion, a particle he predicted to be 200 heavier than the electron. This prediction was confirmed with the discovery of $\pi^{\pm}$ in cosmic rays by C. Powell et al. (1947). The mass of the pion is compatible with the range of strong interactions.



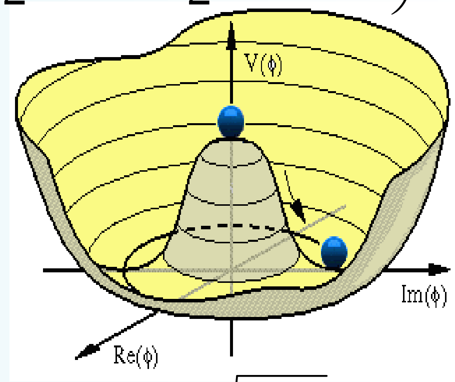


- $SU(2)_L$: however, for weak interactions, there is no such thing as strong coupling and confinement! So here, we cannot solved the contradiction between observed short range and gauge invariance which implies infinite range!
  The reason is that the exact gauge symmetry mentioned before are too high compared to Nature! For weak interaction, one needs a spontaneous symmetry breaking!

Higgs Mechanism

- Spontaneous symmetry breaking: equations have a symmetry, but the actual solution does not
- Goldstone theorem: in a global symmetry, each broken generator introduces a massless scalar boson, called the Nambu-Goldstone boson
- Introduce an $SU(2)_L$ doublet $\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1^+ + i\varphi_1^+ \\ \varphi_2^0 + i\varphi_2^0 \end{pmatrix}$, i.e. 4 d.o.f.
- Higgs lagrangian: $\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^2 - V(\Phi)$ with $D_\mu = \left(\partial_\mu + \frac{i}{2} g_1 B_\mu + \frac{i}{2} g_2 W_\mu^a \sigma_a \right)$
- Higgs potential: $V = \mu^2 \Phi^2 + \lambda \Phi^4$

with: $\begin{cases} \mu^2 < 0 \\ \lambda > 0 \end{cases}$ this triggers a spontaneous breaking of the EW symmetry

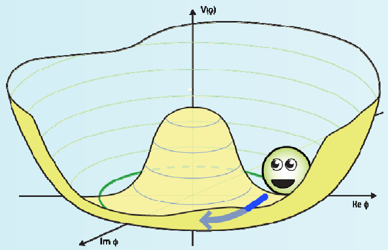


- Indeed $\mathcal{L}$ still conserves its rotational symmetry along the vertical axis whereas the field (lower position of the blue marble) breaks it
- Higgs vacuum expectation value (vev): $\Phi_{\min} = \langle 0 | \Phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ with $v = \sqrt{\frac{-\mu^2}{\lambda}}$

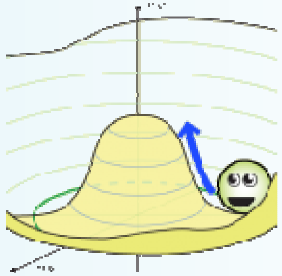
Refs:

- **J. Goldstone, Nuovo Cimento 19 (1961) 154; J. Goldstone, A. Salam and S. Weinberg, Phys Rev 127 (1962) 965**
- **P.W. Higgs, Phys Rev Lett 12 (1964) 132; P.W. Higgs, Phys Rev 145 (1966) 1156**
- **F. Englert and R. Brout, Phys Rev Lett 13 (1964) 321**
- **G.S. Guralnik, C.R. Hagen, T.W. Kibble, Phys Rev Lett 13 (1964) 585; T.W. Kibble, Phys Rev 155 (1967) 1554**

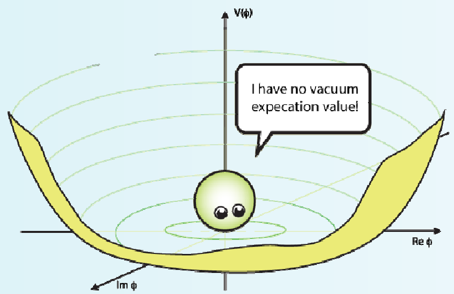
Goldstone and Higgs Bosons



• The 3 quanta of rotations of the smiley along the circle at the minimum of the potential are the Nambu-Goldstone bosons (eaten by the W_L^+, W_L^-, Z_L^0). Since it stays at V_{min} there's no energy to pay to move along the circle: these modes are massless



• The quantum of the smiley oscillating up and down the potential is the Higgs boson. There's some energy to pay to climb the potential: this mode is massive



• At high energy (or temperature) the potential becomes trivial and the EW symmetry is restored (ie: it is apparent, no longer hidden)

Mass Generation

- Before the spontaneous symmetry breaking of the electroweak symmetry (EWSB), SM particles are massless
- After the EWSB:
  - particles get their mass via their interaction with the Higgs field, in other words the Higgs boson couple to the particles mass

Type of Interaction	Mass
Gauge (weak) interaction	$m_W = g_2 \frac{v}{\sqrt{2}}$ $m_Z = g_2 \frac{v}{\sqrt{2}\cos\theta_W}$
Null Interaction	$m_\gamma = 0$
Self-Interaction	$m_H = \sqrt{-2\mu^2}$
No Interaction	$m_g = 0$
Yukawa Interaction	$m_f = y_f \frac{v}{\sqrt{2}}$

  - 3 d.o.f. of the Higgs filed are « eaten » by the W^+, W^- and Z bosons: 1 d.o.f. remains which corresponds to the physical Higgs boson

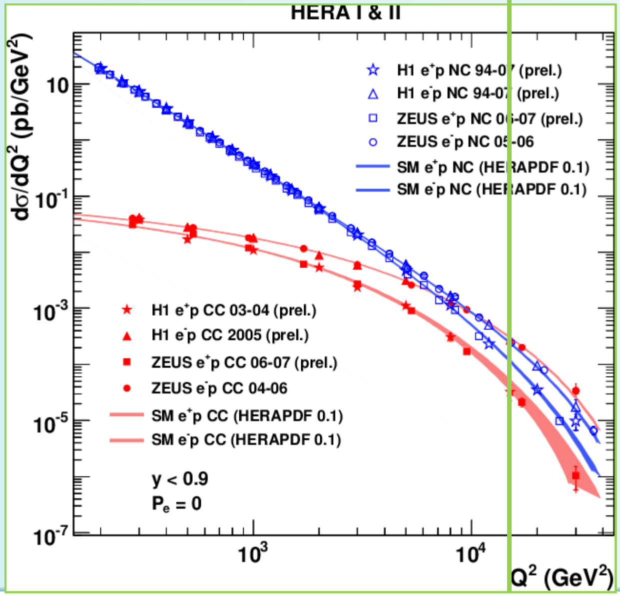
Electroweak Unification

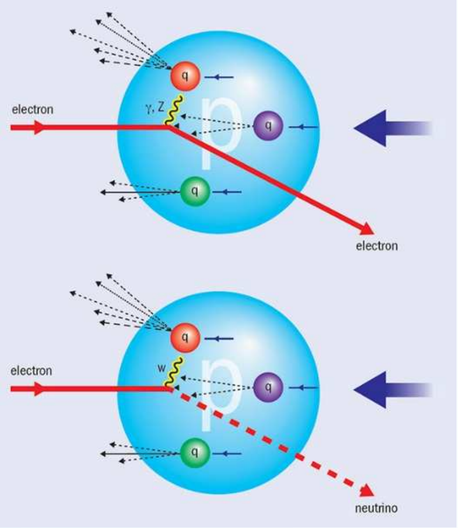
$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$G_{SM} = SU(3)_C \otimes U(1)_{EM}$$

Fermi scale

E [GeV]





Scalars: Klein-Gordon Equation

- Klein-Gordon equation: for a single free particle with S=0 (a Higgs boson) denoted ϕ, the dynamical equation that replaces Schrödinger equation in relativistic QFT is the Klein-Gordon equation
- It starts from the following expression of matter-energy equivalence:

$$\boxed{\mathrm{E}^2 = p^2 \cdot \mathrm{c}^2 + m^2 \cdot c^4}$$

and through the correspondence principle:

$$\boxed{P_\mu \rightarrow i\hbar\partial_\mu}$$

one gets:

$$\boxed{\Delta\phi(x,t) - \frac{1}{c^2} \cdot \frac{\partial^2\phi(x,t)}{\partial t^2} = \frac{m^2c^2}{\hbar^2} \cdot \frac{\partial\phi(x,t)}{\partial t}}$$

- Higgs Boson Identikit:
  - Color Singlet
  - Spin: 0
  - Electric Charge: 0
  - Mass:
    - not predicted by the SM
    - some theoretical constraints exist however

SM: Successes

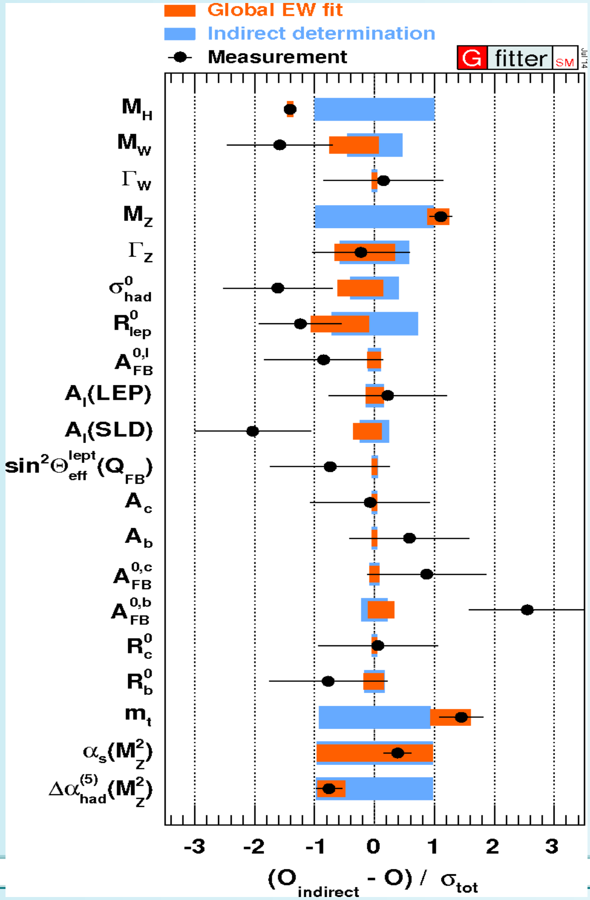
4th of July 2012: The Revolution

• CERN announced the discovery of a neutral boson which properties are compatible with that of the SM Higgs boson! Its mass is ~126 GeV.
• It is a narrow resonance observed in the $\gamma\gamma$ and 4l channels by both the ATLAS and the CMS experiments
• CDF and D0 collaborations report a 2.9σ excess in the mass range of: 110-140 GeV

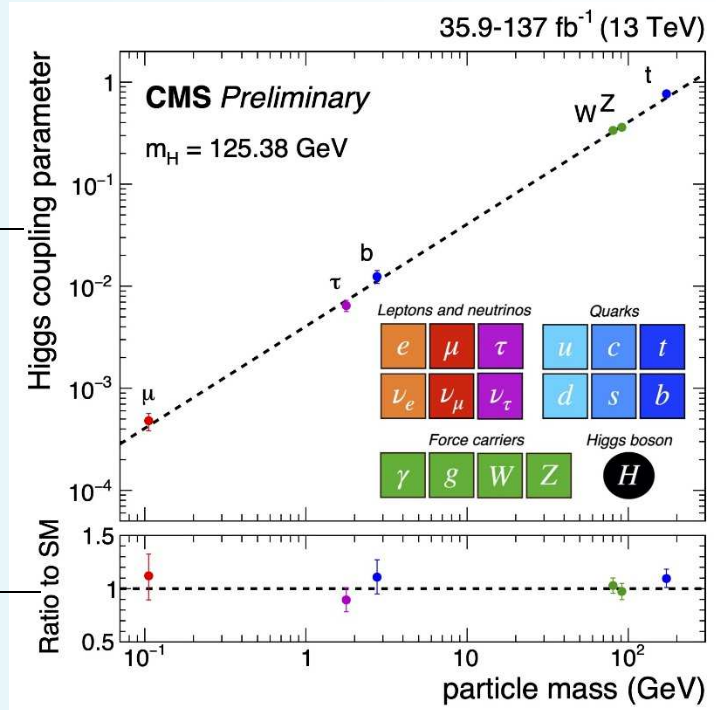
SM: Successes

Global EWK Fit

- Theory:
  - NNLO EW and NNLO EW-QCD
- Experiment:
  - EW Precision measurements
- χ^2 Global Fit of Data to SM:
  - χ^2_{min} / Nd.o.f. = 17.8 / 14, P(χ^2) = 22%



Higgs Boson Couplings



$$g_{H-f} = k_f \cdot \frac{m_f}{v} \qquad g_{H-V} = \sqrt{k_V} \cdot \frac{m_V}{v}$$

$$\frac{g_{H-f}}{g^{SM}_{H-f}} = k_f \qquad \frac{g_{H-V}}{g^{SM}_{H-V}} = \sqrt{k_V}$$

SM: Limitations

- It has 19 unpredicted parameters
  - 3 gauge coupling constants: g_1, g_2, g_3 (or g_2, g_3 and $\sin^2\theta_W$)
  - 9 Yukawa couplings (e,μ,τ,u,d,s,c,b,t)
  - μ and λ from Higgs potential (or λ and m_H)
  - 3 elements of the CKM matrix + 1 phase
  - 1 parameter θ_{PC} causing CP violation in the QCD lagrangian: the so-called « strong CP problem »
  - plus 7 parameters to parametrize the neutrino masses

- This number of free parameters
  - limits the model intrinsic predictivity
  - illustrates the limitations of our understanding

SM: Limitations

One can regroup the open problems into 3 categories:

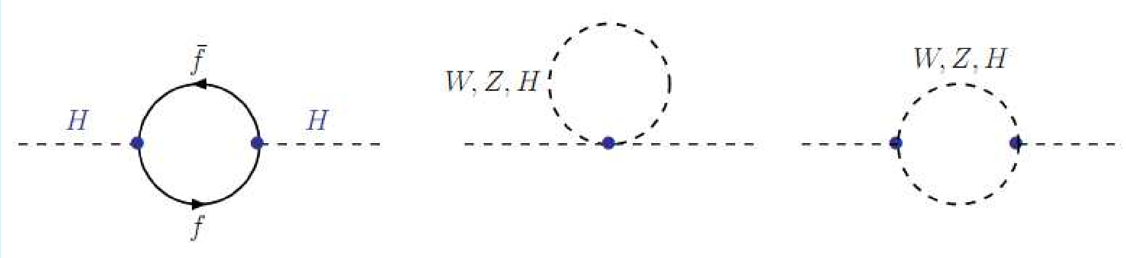
- Flavor problem:
  - Why are there 3 generations? Are quarks and leptons elementary?,…
  - What's the origin of the CKM angles?
  - What's the origin of CP violation? What are the new sources of CP violation necessary to explain the baryon asymmetry in today's universe?
  - What's the origin of the neutrino masses?

- Unification problem:
  - How to stabilize the Higgs boson mass in presence of H.E. scales (GUT, Planck)?
  - Is Grand Unification realized?
  - What's the fate of α_{EM} running at H.E.?
  - Is there a theory for quantum gravity?
  - Is there a unified theory of the 4 fundamental interactions?
  - Do this theory require extra spatial dimensions?
  - Do elementary particles have spatial extensions?
  - Why is electric charge quantized?,…

- Dark Universe Matter problem:
  - What is Dark Matter made of?
  - What is the nature of Dark Energy?

SM: Limitations

- Naturalness (2):



f = t (neglect lighter fermions)

$$m_H^2 = (m_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2\right)$$

=> m_H diverges quadratically wrt Λ ! (Note the signs)

- Hierarchy problem:
  - In presence of any large Λ, m_H is destabilized by this quadratic divergence
  - There's no guarantee to keep it within the EW range O(0.1-1 TeV)
  - Ex:
    - GU: $\Lambda=10^{16}$ GeV

    => 16 orders of magnitude fine-tuning, at each P.O. !

Avenues Beyond the SM

How to extend the SM so as to resolve part of its open questions?

Extended Gauge Group:
$SU(4) \times SU(2)_L \times SU(2)_R$, $SU(5)$, $SO(10)$, E_6, E_8,…

Supersymmetry

Extra-Dimensions

Strong Electroweak Symmetry Breaking

4^{th} generation

Compositeness

Or a combination of these hypotheses

PART II: Avenues Beyond the SM of Particle Physics

Extra Dimensions: History

- First attempt to introduce extra space dimensions by G. Nordstorm (1914): it was an attempt to unify newtonian gravity and EM. It did not attract much attention.

- General Relativity (1915): Einstein extended his SR to relative movements of accelerated frames. He promoted mass equivalence to a principle and finally came-up with a theory in which the local density of matter-E deforms space-time locally and explains gravity by the deformed space-time geometry: trajectories are still geodesic by in non-euclidean geometry.

- Kaluza-Klein Theory: in 1919, T. Kaluza attempted to unify GR and EM by extending the metric tensor of the 4-D Minkowski space-time:

$$0 \le m,n \le 4 \qquad g_{mn} = \begin{pmatrix} g_{\mu\nu} & A_\mu = g_{\mu 4} \\ A_\mu = g_{\mu 4} & \varphi = g_{44} \end{pmatrix} \quad \begin{cases} g_{\mu\nu} : \text{graviton } (S=2) \\ A_\mu : \text{photon } (S=1) \\ \varphi : \text{radion } (S=0) \end{cases}$$

In this model, 5-D gravity is seen as gravity+EM in 4-D.
In 1926, O. Klein in a quantum version of Kaluza's theory found a good justification for the extreme smallness of the 5th D. Had we a quantum theory of gravity, dimensional analysis would give us its only dimensionful quantity, its energy scale:

  - Mass scale (aka « Planck scale »): $M_{Pl} = \sqrt{\frac{\hbar c}{G_N}} = 1.22 \times 10^{19}$ GeV

From which one could draw:

  - Length (aka « Planck length »): $L_{Pl} = \sqrt{\frac{\hbar G_N}{c}} = 1.62 \times 10^{-35}$ GeV

Main problem: no massive fermions or bosons

- In 1995, E. Witten showed that 11-D supergravity is the low E limit of the 10-D type IIA superstring theory. This triggered a renewed interest for KK theories.

ED: Constraints from Newtonian Gravity

- Modified Newtonian Gravity:

Modifications, at distances $r \leq \mathrm{R}$ of the classical potential by EDs is often parametrized as:

$$V_{\text{mod}}(r) = -G_N \frac{m_1 m_2}{r}(1+\alpha \cdot e^{-r/\lambda})$$

For a compactification on a 2-torus: $\alpha = \frac{16}{3}$ and $\lambda = \mathrm{R}$

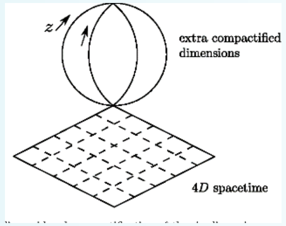
- Vailidty:
  - If EDs are too large they conflict with measurements of gravitation at small distances

δ	1	2	3	…	6
R	10^{12} m	5×10^{-4} m	5×10^{-9} m	…	2×10^{-13} m

  - Currently, for $\delta = 2$, R < 37 μm at 95% CL => $\delta < 3$ with larger EDs are ruled out

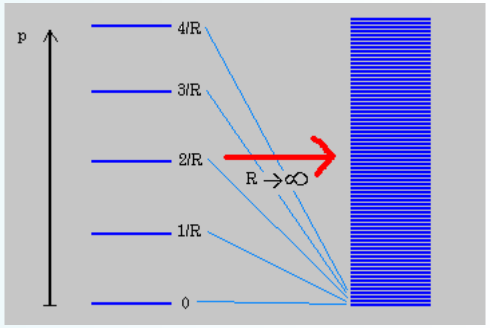
Large ED: ADD Model

- In 1998 N. Arkani-Hamed, S. Dimopoulos and G. Dvali (ADD) proposed a way to avoid the hierarchy problem:
  - they introduce $\delta > 1$ large EDs, where only gravity is allowed to propagate
  - geometry: $$R^4 \otimes S^1 \otimes \ldots \otimes S^1$$



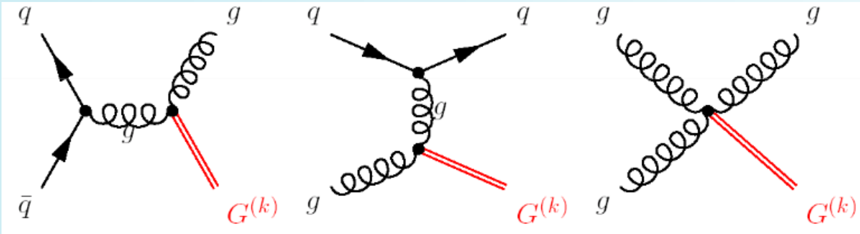
  - from Gauss law: $$\boxed{M_{Pl}^2 = R^{\delta} \cdot M_{Pl(4+\delta)}^{(2+\delta)}}$$
  - to get the apparent Planck mass in 4-D: $M_{Pl(4+\delta)}$ at the EWK scale, they need numerous and/or large enough EDs
  - since apparent Planck mass is taken downto the EWK scale, there's no longer any hierarchy problem
  - gravity appears relatively weak in 4-D because part of its flux goes into the EDs

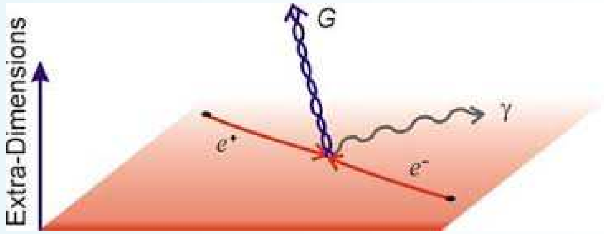
  - Due to compactification EDs are periodic
  - Example with δ=1: $$y \equiv y + 2\pi R$$ $$\varphi(y) \equiv \varphi(y + 2\pi R)$$
  - Fourier expansion: $$\varphi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \varphi^{(n)}(x) \cdot e^{\frac{i \cdot n \cdot y}{R}}$$
  - In 4-D: infinite tower of KK modes, with $$m_n^2 \equiv M^2 + \frac{n^2}{R^2}$$ (M: compactification scale)



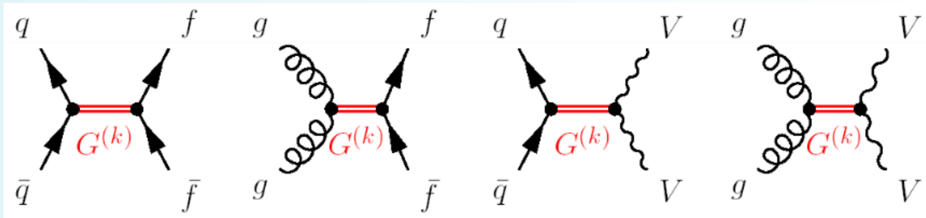
Large ED: ADD Signatures

- Real gravitons production:
  - e.g. graphs of monojet production at the LHC or monophoton production at e^+e^- collider

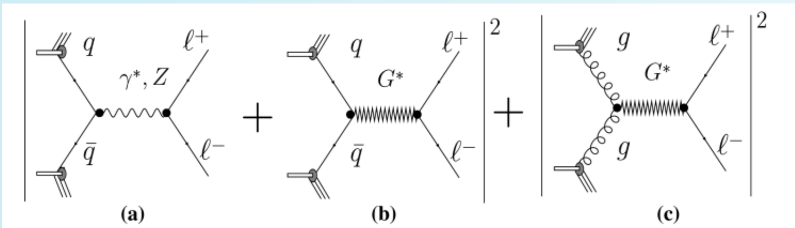




- Virtual gravitons exchange:
  - Mod's of di-fermions & di-bosons mass or angular distributions



  - Interference and increase



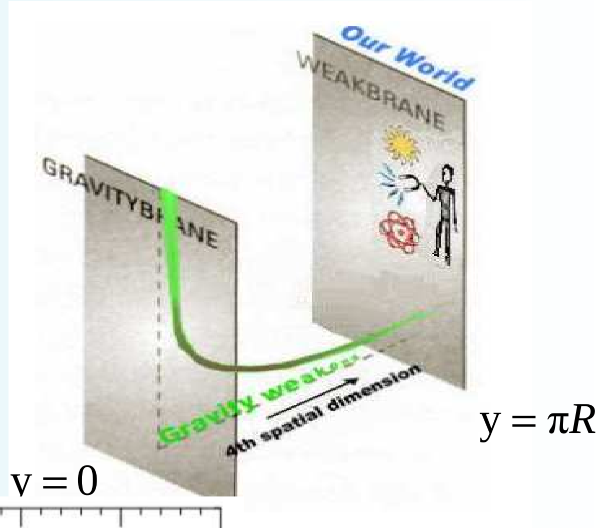
$$\frac{d^2\sigma}{d(\cos\theta^*)dm_{\ell^+\ell^-/\gamma\gamma}} = \frac{d^2\left(\sigma_{SM} + \eta\cdot\sigma_4 + \eta^2\cdot\sigma_8\right)}{d(\cos\theta^*)dm_{\ell^+\ell^-/\gamma\gamma}}$$

$$\begin{cases} \eta = \dfrac{F}{M_D^4} \text{(different conventions : GRW, HLZ, H)} \\ \sigma_4 : \text{ interference term} \\ \sigma_8 : \text{KK term} \end{cases}$$

Warped ED: RS Model

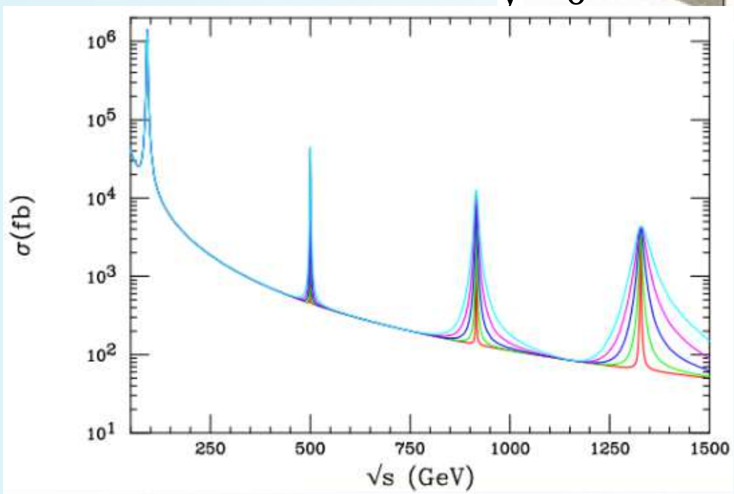
- Small, highly curved 5th dimension, introduced through a warped geometry
- Geometry:
  - Space: 5-D Anti-de-Sitter (AdS_5)
  - Metric: $ds^2 = e^{-2ky} \cdot g_{\mu\nu} \cdot dx^\mu dx^\nu - dy^2$

$$\begin{cases} g_{\mu\nu} : usual\ Minkowski\ metric \\ k : parameter\ of\ curvature\ scale \end{cases}$$



- Only gravity can propagate into the bulk

- Signatures:
  - Sharp S=2 resonances



- Ref: L. Randall, R. Sundrum (1999). "Large Mass Hierarchy from a Small Extra Dimension". Physical Review Letters. **83** (17): 3370–3373, arXiv:hep-ph/9905221

ED: Summary

Which open problems it could solve?

- Hierarchy Problem: i.e. orders of magnitude between EWK and Planck scale
  - Effective Planck scale moved down to the TeV

- Differenciation between gravity with EWK and strong interactions based upon geometry

Which new phenomena it predicts?

- Single Productions:
  - Monophoton,
  - Monojet,
  - Mono-W/Z

- Modified Mass or Angular Spectra due to Graviton Exchange:
  - Diphoton, Dilepton
  - Dijet, Di-W/Z

- Scalar Radion (to stabilize geometry)

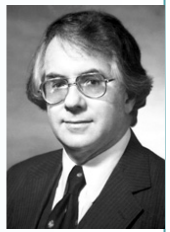
- Production and Evaporation of mini-Black Holes

Grand Unified Theory

H. Georgi

- H. Georgi and S.L. Glashow SU(5) Model in 1974
- Following the success of the unification of EM and weak interactions in the SM
- They looked for a larger gauge group to embed G_{SM}
  - Rank: nber of generators that can be diagonalized simultaneously
  - Note: Rank{SU(N)}=N-1 and Rank{U(N)}=N
  - Rank{G_{SM}}=4: Rank{$SU(3)_C$}=2 + Rank{$SU(2)_L$}=1 + Rank{$U(1)_Y$ }=1

    => Rank{G_{GUT}} >= 4
- They picked the simplest rank=4 group with complex REPs (chirality)



S.L. Glashow
Nobel L. (1979)

- Gauge group: SU(5), which has 5^2-1=24 generators

- SM Quarks and Leptons sit in 5-D fundamental representations of SU(5):

$$\bar{5}_F = (1,\bar{2}) \oplus (\bar{3},1) \qquad 10_F = (1,1) \oplus (\bar{3},1) \oplus (3,2)$$

$$\Psi_R = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ -\bar{\nu}_e \end{pmatrix} \qquad \Psi_L = \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & u_1 & -d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u_2 & -d_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}$$

- SU(5) is anomaly-free!

Ref: H. Georgi, S.L. Glashow, « Unity of All Elementary-Particle Forces », Phys Rev Lett 32 (1974) 438-441

SU(5) GUT

- Gauge bosons:
  - Sit in adjoint representations of SU(5) of dimension $(N^2-1)=24$
  - 12 LeptoQuark bosons X, Y
    - charged under $SU(2)_L$ and $SU(3)_C$
    - have fractional electric/colour charges Q(X)=+/-4/3, Q(Y)=+/-1/3
    - new interactions do not conserve B nor L, however all interactions conserve (B-L)

$$24 = \underbrace{(8,1)}_{\text{Gluons}} \oplus \underbrace{(1,3)}_{W^+,W^-,Z} \oplus \underbrace{(1,1)}_{\gamma} \oplus \underbrace{(3,2) \oplus (\bar{3},2)}_{\text{X,Y Bosons}}$$

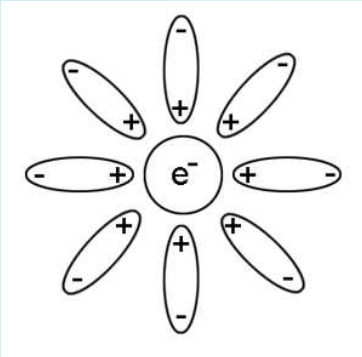
$$G = \begin{pmatrix} G_3 + \frac{G_8}{\sqrt{3}} - \sqrt{\frac{2}{5}}B & G_1 - iG_2 & G_4 - iG_5 & X_1 & Y_1 \\ G_1 + iG_2 & -G_3 + \frac{G_8}{\sqrt{3}} - \sqrt{\frac{2}{5}}B & \bar{u}_1 & X_2 & Y_2 \\ G_4 + iG_5 & G_6 + iG_7 & -\frac{2}{\sqrt{3}} - \sqrt{\frac{2}{5}}B & X_3 & Y_3 \\ \bar{X}_1 & \bar{X}_2 & \bar{X}_3 & \frac{W_3}{\sqrt{2}} + \sqrt{\frac{3}{10}}B & W^+ \\ \bar{Y}_1 & \bar{Y}_2 & \bar{Y}_3 & W^- & \frac{-W_3}{\sqrt{2}} + \sqrt{\frac{3}{10}}B \end{pmatrix}$$

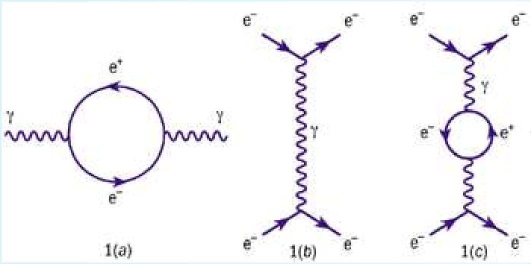
- Charge quantization:
  - Using SU(5) generators (5x5 matrices incl. Gell-Mann/Pauli), express $Q_{el} = T_3 + \frac{Y}{2}$
  - Applied to rep. 5_F: $\mathrm{Tr}(Q_{el}) = 0 \quad \Rightarrow \quad \boxed{Q_{el}(d) = \frac{-1}{3} Q_{el}(e^+)}$

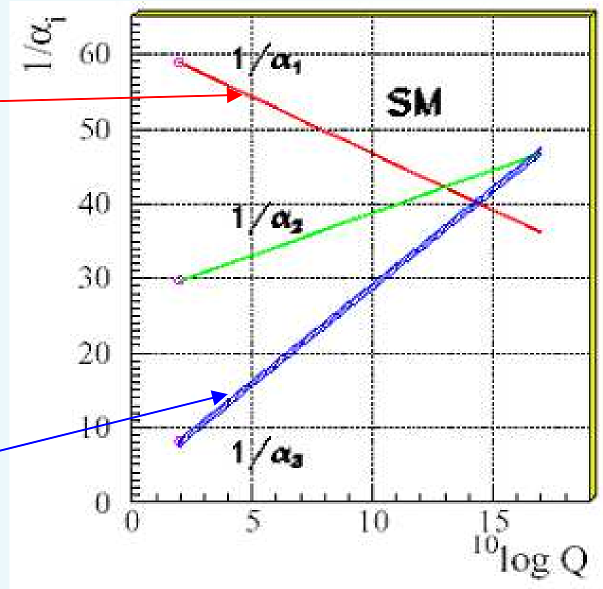
SU(5): Evolution of Gauge Couplings

- The evolution of the interactions coupling constants with energy depends on the way these interactions polarize the quantum vacuum

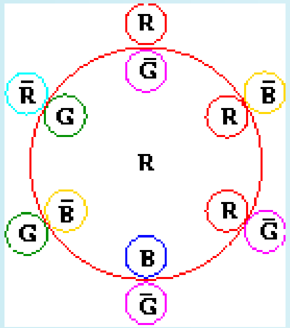
- Electromagnetism (EM): screening effect

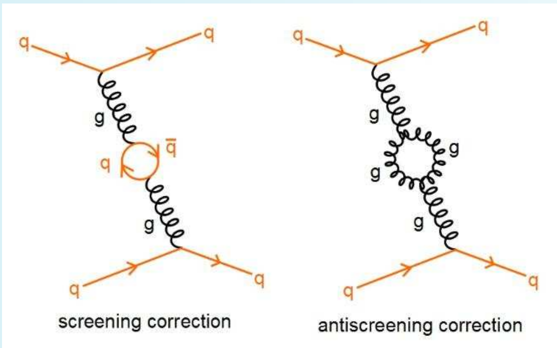






- Strong Interaction: both screening and anti-screening effects





SU(5): Evolution of Coupling Constants

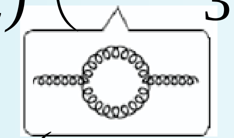
- Gauge coupling evolutions at 1-loop in the SM:

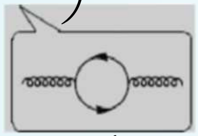
gauge bosons loops

- SU(N): $$\beta_N(g_N) = \frac{g_N^3}{(4\pi)^2}\left(\frac{-11}{3}N + \frac{4}{3}N_{\text{gen}}\right)$$ where $\beta(g) = \frac{\partial g}{\partial(\mathrm{Log}\,\mu_R)}$

fermions loops

- SU(3)$_C$: $$\beta_3(g_3) = \frac{g_3^3}{(4\pi)^2}\left(-11 + \frac{4}{3}N_{\text{gen}}\right)$$





- SU(2)$_L$: $$\beta_2(g_2) = \frac{g_2^3}{(4\pi)^2}\left(\frac{-22}{3} + \frac{4}{3}N_{\text{gen}} + \frac{1}{6}N_H\right)$$

Higgs loops

- U(1)$_Y$: $$\beta_1(g_1) = \frac{g_1^3}{(4\pi)^2}\left(\frac{4}{3}N_{\text{gen}} + \frac{1}{10}N_H\right)$$

$$g_i^2(\Lambda_{GUT}) = g_{GUT}^2(\Lambda_{GUT}), \frac{1}{g_i^2(\mu_0)} = \frac{1}{g_{GUT}^2} + \frac{\beta_i(g_i)}{2\pi}\cdot \mathrm{Log}\left(\frac{\mu_R = \Lambda_{GUT}}{\mu_0}\right) \Rightarrow \Lambda_{GUT} \approx 2\times 10^{16}\ \mathrm{GeV}$$

- GUT: $$\sin^2\theta_W = \frac{g_1^2(Q)}{g_1^2(Q) + C^2 g_2^2(Q)} \qquad C^2 = \frac{5}{3} \qquad \Rightarrow \sin^2\theta_W(\Lambda_{GUT}) = \frac{3}{8}$$

SU(5) EW CGC

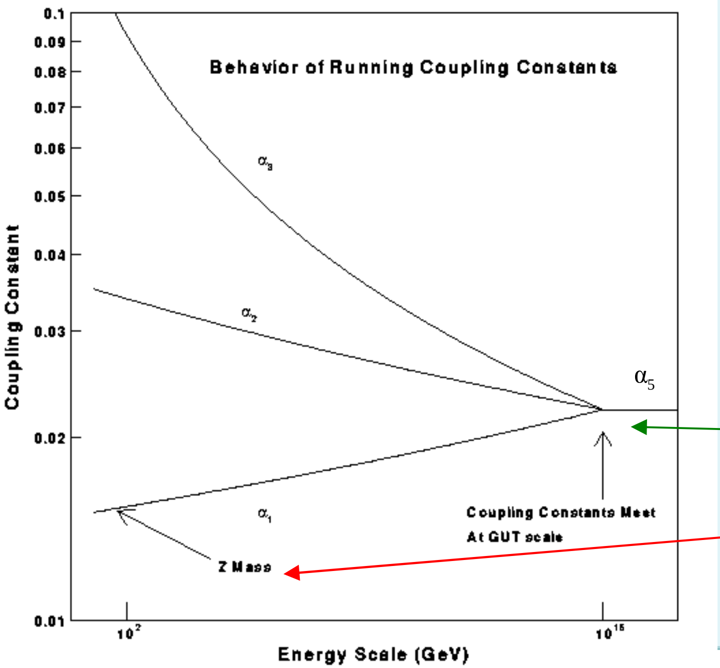
SU(5): Spontaneous Symmetry Breaking

- Breaking SU(5):
  - a 24-D Higgs multiplet 24_H spontaneously breaks SU(5) gives mass to X and Y of O(Λ_{GUT}):

$$M_X^2 = M_Y^2 = \frac{25}{8} \cdot g_5^2 \cdot \langle 0 | 24_H | 0 \rangle^2$$

- Reproducing $SU(2)_L x U(1)_Y$ Breaking:
  - a 5-D Higgs multiplet 5_H or $\bar{5}_H$, containing a colour triplet and the usual SM Higgs doublet is responsible for the EWSB

$$5_H = \begin{pmatrix} T \\ \Phi \end{pmatrix} \quad M_W^2 \approx \frac{g_2^2}{4} \cdot \langle 0 | 5_H | 0 \rangle^2 \qquad M_Z^2 \approx \frac{g_2^2}{4 \cdot \cos^2\theta_W} \cdot \langle 0 | 5_H | 0 \rangle^2$$



$\boxed{\langle 0 | 5_H | 0 \rangle}$ $\qquad$ $\boxed{\langle 0 | 24_H | 0 \rangle}$

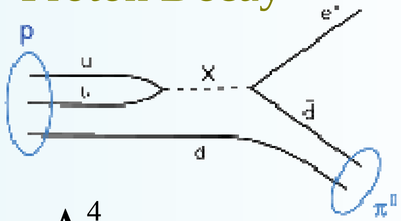
$SU(3)_C$ x $U(1)_{EM}$ ← $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ ← SU(5)

SU(5): Confronting Experiments

• Theoretical Predictions:

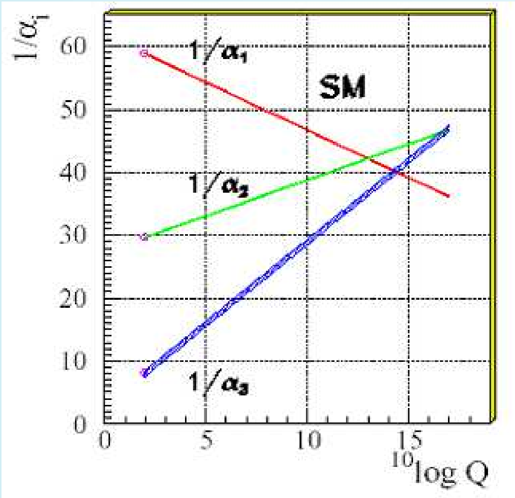
Unified Coupling Constant for Strong and EWK Intractions

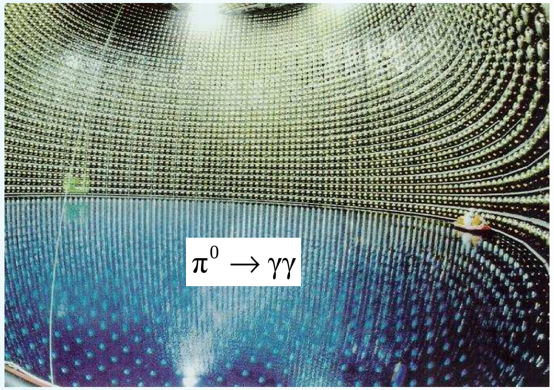
Proton Decay



$$\tau_p^{Th} \approx \frac{\Lambda_{GUT}^4}{m_p^5} \approx 10^{28-30}\ years$$

• Experimental Tests:





$$\tau_p^{Ex} > 8.2 \times 10^{33}\ years$$

Experiment Super-Kamiokande (2005)

• Conclusion:

- Model is « dead », but not burried!
- Considered very elegant and as a « near shot »…

GUT: Summary

Which open problems it could solve?

- Unification of all known QFT for elementary particles: EWK and Strong
- Unification of quarks and leptons
- Quantization of electric charge
- Solve the Landau pole in g_1 evolution
- Neutrino mass (SO(10),…)

Which new phenomena it predicts?

- Leptoquarks (heavy)
- Magnetic monopoles

Supersymmetry

- Supersymmetry (SUSY) is a symmetry between bosons and fermions:

$$\boxed{Q|S\rangle = |S \pm 1/2\rangle} \quad \Rightarrow \quad \begin{cases} Q|B\rangle = |F\rangle \\ Q|F\rangle = |B\rangle \end{cases}$$

  - Q is a fermionic (S=1/2) operator, called a generator of SUSY

- Theories with global / local SUSY may have up to $\mathcal{N}$= 4 / $\mathcal{N}$= 8 different generators

- Realistic TeV-scale SUSY models have just $\mathcal{N}$= 1 generator, because only those can have chiral representations

- Coleman-Mandula « no-go » theorem:
  - Any non-trivial union of Poincaré and internal groups yields an S-matrix equal to identity (this holds for Lie algebras only)

- Haag-Sohnius-Lopuzanski theorem:
  - Within an algebra containing both commutation and anti-commutation relations, it's possible to combine non-trivially the Poincaré and internal symmetries

Ref:

• S.R. Coleman, J. Mandula, « All Possible Symmetries Of The S Matrix », Phys. Rev. 159 (1967) 1251-1256
• R. Haag, J.T. Lopuszanski, M. Sohnius, « All Possible Generators of Supersymmetries of the s-Matrix », Nucl. Phys. B88 (1975) 257

Super-Poincaré Algebra

- Super-Poincaré algebra:

$$\begin{cases} \left[Q_\alpha, P^\mu\right] = 0 \\ \left[Q_\alpha, M_{\mu\nu}\right] \propto Q_\beta \end{cases}$$

$$\begin{cases} \left\{Q_\alpha, \overline{Q}_\beta\right\} \propto P_\mu \\ \left\{Q_\alpha, Q_\beta\right\} = 0 \end{cases}$$

$$\left[Q_\alpha, R\right] = Q_\alpha$$

- Casimir operator: $P^2 = m^2 \Rightarrow \left[Q_\alpha, P^2\right] = 0$ $\quad\left[W^2, Q_\alpha\right] \neq 0$
- Invariant: $P \leftrightarrow m$ $\quad W_\mu \not\leftrightarrow S, S \pm 1/2$
- Consequences:
  - super-multiplets (particles and sparticles): same m, but different spin (exact SUSY)
  - local SUSY is called supergravity since it contains GR: $\left\{Q_\alpha, \overline{Q}_\beta\right\} \propto P_\mu$
- Question: is SUSY realized in the SM? No, N(F) >> N(B)

Therefore to apply SUSY one needs to double the SM mass spectrum

- Chiral supermultiplet:
  - Fermions, Higgsinos (S=1/2) & Sfermions, Higgs (S=0)
- Vector supermultiplet:
  - Gauge Bosons (S=1) & Gauginos (S=1/2)
- Gravity supermultiplet: in local SUSY only
  - Graviton (S=2) & Gravitino (S=3/2)

SUSY: Breaking

- Broken SUSY:
  - If SUSY is realized in nature it is necessarily broken, otherwise we would have found selectrons with the same mass as electrons but obeying Bose-Einstein statistics!

- SUSY extension of the SM: $\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{Soft-X}}$

Remnants of SUSY X:
- explicitely violation of SUSY
- no quadratic divergences
- masses of SUSY particles

(Lots of new parameters)

- Includes $\mathcal{L}_{\text{SM}}$ (invariant under G_{SM})
- Global SUSY
- SM particles + superpartners
- Generates mass for SM particles
- Renormalizable
- Interactions (between SM & SUSY):
  - gauge interactions
  - Yukawa interactions

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} = & -\frac{1}{2}\left(M_3 \tilde{g}^a \tilde{g}^a + M_2 \widetilde{W}^i \widetilde{W}^i + M_1 \widetilde{B}\widetilde{B} + \text{h.c.}\right) \\
& - (m^2_{\tilde{q}})_{ij} \tilde{q}^*_{L,i} \tilde{q}_{L,j} - (m^2_{\tilde{u}})_{ij} \tilde{u}^*_{R,i} \tilde{u}_{R,j} - (m^2_{\tilde{d}})_{ij} \tilde{d}^*_{R,i} \tilde{d}_{R,j} \\
& - (m^2_{\tilde{l}_L})_{ij} \tilde{l}^*_{L,i} \tilde{l}_{L,j} - (m^2_{\tilde{l}_R})_{ij} \tilde{l}^*_{R,i} \tilde{l}_{R,j} \\
& - \widetilde{m}^2_1 H^\dagger_1 H_1 - \widetilde{m}^2_2 H^\dagger_2 H_2 - \left(m^2_{12} H_1 \cdot H_2 + \text{h.c.}\right) \\
& - \left[(\mathbf{h}_u \mathbf{A}_u)_{ij} \left(\tilde{q}_{L,i} \cdot H_2\right) \tilde{u}^*_{R,j} + (\mathbf{h}_d \mathbf{A}_d)_{ij} \left(H_1 \cdot \tilde{q}_{L,i}\right) \tilde{d}^*_{R,j} \right. \\
& \left. + (\mathbf{h}_l \mathbf{A}_l)_{ij} \left(H_1 \cdot \tilde{l}_{L,i}\right) \tilde{l}^*_{R,j} + \text{h.c.}\right] .
\end{aligned}
$$

Minimal Supersymmetric Standard Model

- MSSM:
  - $\mathcal{N}$= 1 SUSY
  - Same gauge group as the SM: $\boxed{G_{MSSM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$
  - Need 2 Higgs $SU(2)_L$ doublets: $\Phi_u = \begin{pmatrix} \varphi_u^+ \\ \varphi_u^0 \end{pmatrix}$ $\Phi_d = \begin{pmatrix} \varphi_d^0 \\ \varphi_d^- \end{pmatrix}$
    - Cancel anomalies: $Y(\Phi_u) = -Y(\Phi_d)$
    - Before EWSB: $N_{dof} = 8$
    - After EWSB: $N_{dof} = 5$, e.g. 5 physical Higgs bosons

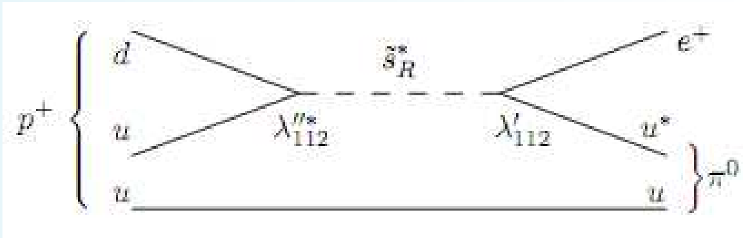
Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

R-Parity

- R-Parity: $$\boxed{R_P = (-1)^{L+2S+3B}}$$

$\rightarrow$ +1 for SM particles

$\rightarrow$ -1 for SUSY particles



- if conserved (MSSM)
  - => No rapid proton decay
  - => pair production of SUSY particles
  - => a LSP (Lightest SUSY Particle) appears at the end of each SUSY decay chain
  - $\Rightarrow$ the LSP is stable

- Often the LSP is the lightest neutralino: $\boxed{\tilde{\chi}^0_1}$
  - It has a mass from SUSY breaking (plus small EW contribution)
  - It's stable, provided R_P is conserved
  - It has no electric, nor color charge
  - It interacts only via weak and gravitational interactions

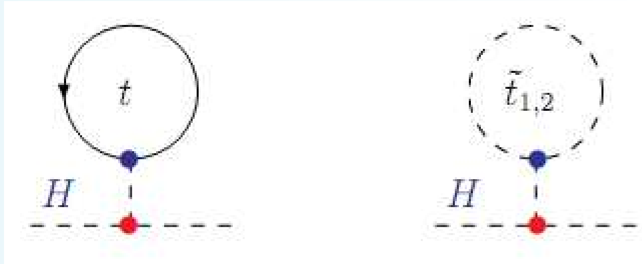
$\Rightarrow$ It's an excellent candidate for Dark Matter

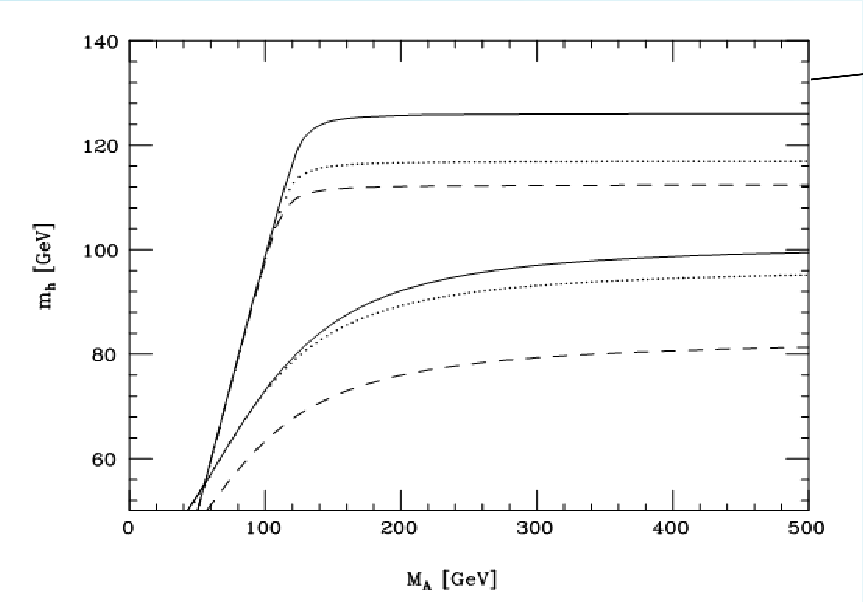
Lightest Higgs Boson Mass

- At tree level: $0 < M_{h^0} \leq M_{Z^0} \cdot |\cos(2\beta)|$
- It is strongly modified by radiative corrections

$$\delta M^2_{h^0} = \frac{3}{2^{3/4} v \pi^2} \cdot M_t^4 \cdot \mathrm{Log}\left(\frac{M_S^2}{M_t^2}\right)$$

$$M_S^2 \approx M_{\tilde{t}_1} M_{\tilde{t}_2}$$





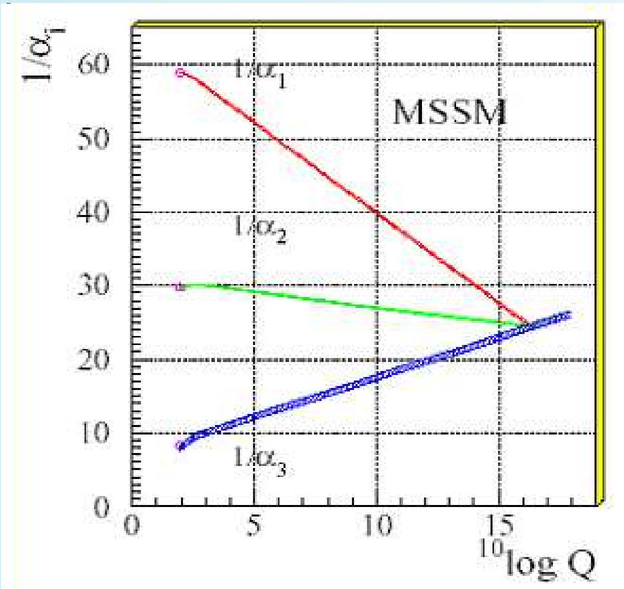
$$M_{h^0} \leq 136 \text{ GeV}$$

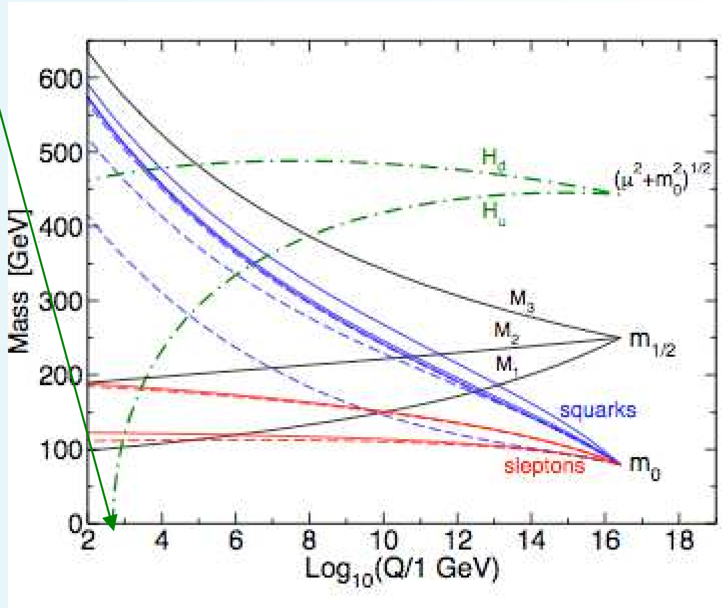
The existence of a light Higgs boson is a model indenpendent prediction of SUSY models

SUSY Response to SM Open Questions

• Non-trivial extension of the Poincaré group
• Candidate for Cold Dark Matter particle
• Grand Unification
• Prospect for a quantum theory of gravity and for its unification with the other fundamental interactions
• Radiative Breaking of the EWK Symmetry

$$\left\{Q, \overline{Q}\right\} \propto P_\mu$$



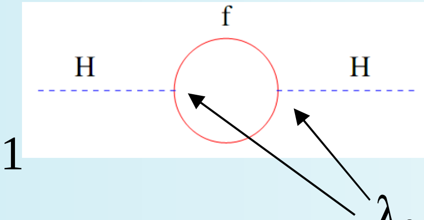


SUSY Solving the Hierarchy Problem

- Taming the Higgs Radiative Corrections:

SM:

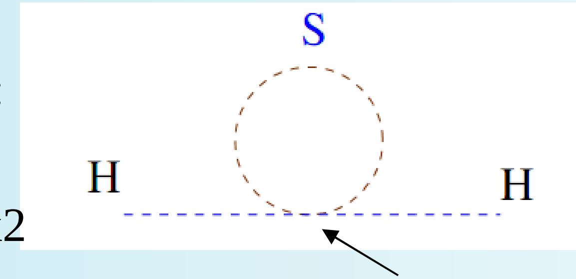
x1



$$\delta M_H^2 = \frac{|\lambda_f|^2}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_f^2 \mathrm{Log}\left(\frac{\Lambda_{UV}}{m_f}\right) + \mathrm{H.O.}\right]$$

SUSY:

x2



$$\delta M_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{UV}^2 - 2m_S^2 \mathrm{Log}\left(\frac{\Lambda_{UV}}{m_S}\right) + \mathrm{H.O.}\right]$$

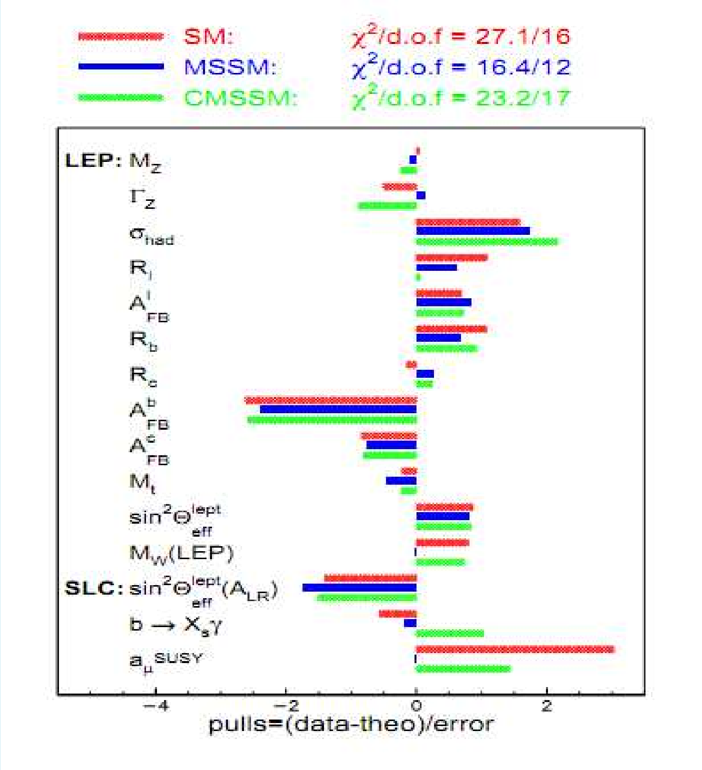
Exact cancellation, if and only if:
- $M_f = M_S$

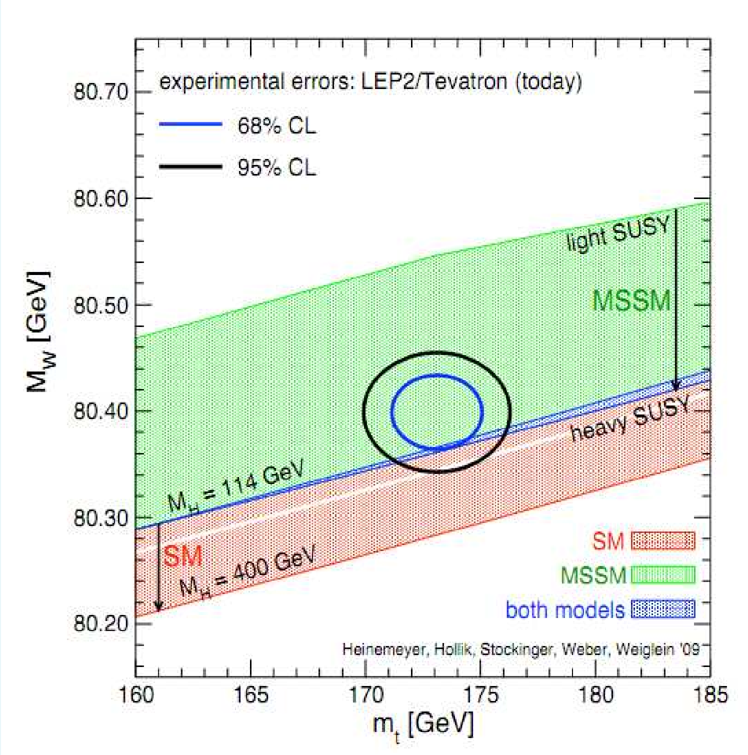
and
- $|\lambda_f|^2 = \lambda_S$

$$\boxed{M_H = O(M_W) \Rightarrow |M_f - M_S| < O(1\ \mathrm{TeV})}$$

SUSY vs Indirect EWK Constraints

- The MSSM is fully compatible w/ all current data and even fits the data better than the SM





SuperSymmetry: Summary

Which open problems it could solve?

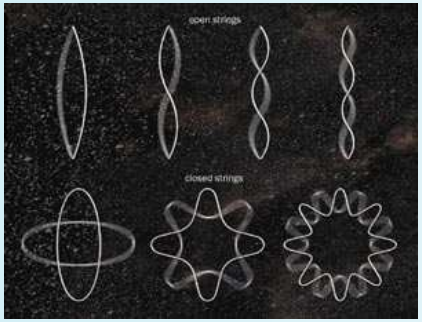
- Hierarchy Problem:
  - $|M(sp)-M(p)|^2 < O(1\ TeV^2)$
- Possible SUSY-GUT (SU(5), SO(10), E6, E8)
- (Almost) stable proton
- Candidates for Cold Dark Matter:
  - LSP: lightest neutralino, gravitino,…
- Stabilization of the Higgs boson mass in presence of H.E. fundamental scales
- Unification between fermions and bosons
- Finite QFTs, e.g. $\mathcal{N} = 4$
- Possible framework for Quantum Theory of Gravity (SuperGravity)

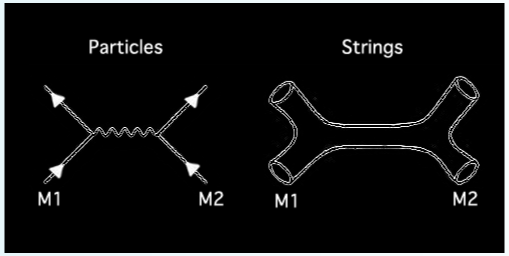
Which new phenomena it predicts?

- Supersymmetry:
  - at TeV and/or at Planck scale
- At least 5 Higgs bosons:
  - h^0, A^0, H^0, H^+, H^-
- Doubling of the SM particles
  - Lots of new Sparticles
  - Candidate for Dark Matter
- Modified quantum corrections
  - Search in B-physics, $(g\text{-}2)_\mu$, …

SuperStrings

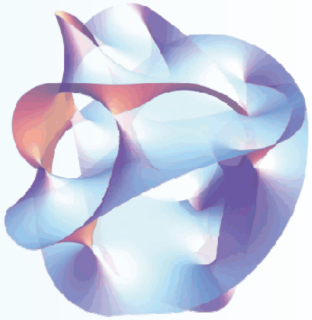
- It's not yet a finalized theory, but rather a somewhat promising theoretical framework
- The actual fundamental physics objects are extremely tiny strings (open or closed) whose proper vibrating modes correspond to elementary particles





- To avoid tachyons superstring theory requires SUSY
- It is the only known theory with finite quantum gravity

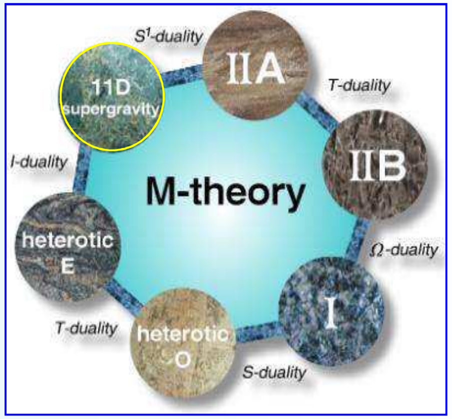
- Their quantification requires 6 additional space dimensions
- Retrieving a low E 4-D theory requires to compactify these EDs on Calabi-Yau 6-Dmanifolds



- Main problems:
  - Unsanely large number of C-Y manifolds, no known criteria to choose amongst them
  - Difficulties to interpret Dark Energy

SuperStrings

- Over the 1980's-1990's several superstrings theories were discovered
- E. Witten found they are all special limits of a common M-theory (vibrating membranes)
- They are related by dualities:
  - S-duality: strong vs weak couplings between theories
  - T-duality: string along R ⟷ string along 1/R
  - Ω-deformation: related to warped geometries



Type	Open/Closed	$\mathcal{N}_{SUSY}$	D	Comments
Bosonic	Both	0	26	No fermions! Tachyons!
Type I	Both	1	10	SO(32)
Type IIA	Closed	2	10	U(1), no chirality
Type IIB	Closed	2	10	No gauge group
Heterotic O	Closed	1	10	SO(32)
Heterotic E	Closed	1	10	E_8 x E_8 , Best hope!

- Type I/II and heterotic strings all have a graviton and no tachyons

SuperStrings: Summary

Which open problems
it could POSSIBLY solve?

- Unification of all types of particles: fondamental objects being just vibrating strings (of membranes)
- No free parameter at fundamental level
- Finite Quantum Theory of Gravity
- Unification of GUT with Gravity

Which new phenomena
it predicts?

- Supersymmetry:
  - at the Planck scale for sure
  - possibly at the TeV scale
- Extra Dimensions of Space:
  - probably not testable (very small)
- Axions testable in astrophysics

BACK-UP

SR: Minkowski Metric

Minkowski Metric

- Minkowski squared distance between 2 points P_1 and P_2 in space-time:

$$s^2 = g(\overrightarrow{P_1 P_2}) = g_{\mu\nu}(x_2^\mu - x_1^\mu)(x_2^\nu - x_1^\nu)$$

where

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

is the metric tensor

- Minkowski squared distance between 2 points P_1 and P_2 infinitely close in space-time:

$$ds^2 = g_{\mu\nu} x^\mu x^\nu$$