"What are the atoms of a BH?"

[Bekenstein-Hawking '70s]

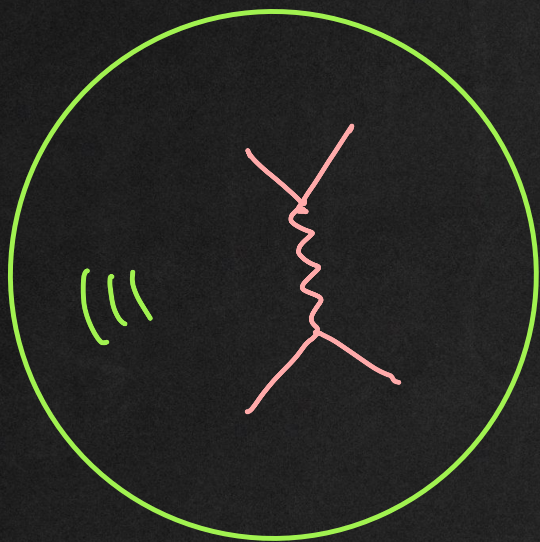
Black Hole

$$\boxed{S_{BH} = \frac{1}{4}\,\frac{A_H}{l_p^2}}$$

$$\left(l_p = \sqrt{\frac{\hbar G_N}{c^3}} \right)$$

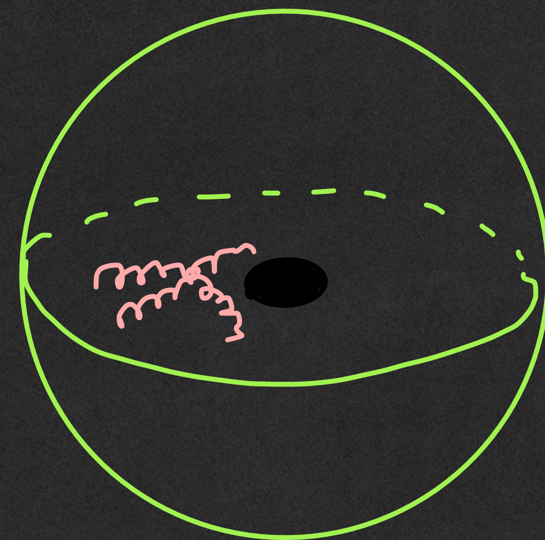
$$\boxed{S_{BH} \overset{?}{=} \log d_{\text{micro}}}$$

AdS/CFT $\Longrightarrow$ definition of quantum gravity



S^3

$=$



S^3

CFT_4 on $S^3 \times \mathbb{R}_t$

(gauge theory)

AdS_5 Quantum gravity

$\partial(AdS_5) = S^3 \times \mathbb{R}_t$

$\lceil S^3 \rightsquigarrow$ 3-manifold $\rfloor$

$\lceil \partial \equiv$ conformal bdry $\rfloor$

CFT

$\mathcal{N}=4$ SYM $U(N)$

$(N, \lambda \equiv g_{YM}^2 N)$

AdS

IIB / $AdS_5 \times S^5$

$(g_s, \ l = l_{AdS}/l_s)$

Gauge theory $\xleftarrow[\lambda \ll 1]{}$ $\boxed{\lambda = l^4}$ $\xrightarrow[\lambda \gg 1]{}$ Gravity

('t Hooft limit) (semi classical)

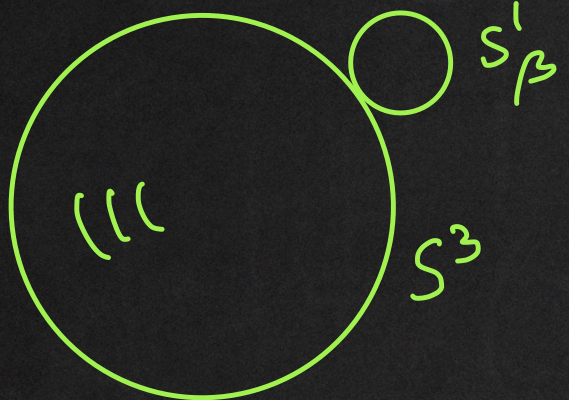
$$\boxed{\begin{array}{c} 1/N = g_s \\ \Downarrow \\ 1/N^2 = G_N \end{array}}$$

Gauge theory ('t Hooft limit) $\xrightarrow{1/N}$ Non-planar

Gravity (semi classical) $\xrightarrow{g_s}$ Quantum gravity

BH Thermodynamics in AdS/CFT

[Witten '98]



$=$

(Euclidean)

$$Z_{CFT}(\lambda, N; \beta) = Z^{AdS}_{grav}(\ell, g_s; \beta)$$

$$= \operatorname{Tr}_{\mathcal{H}} e^{-\beta H}$$

$$= \sum_E d_{micro}(E)\, e^{-\beta E}$$

$$= \sum_{\alpha \in \text{Saddles}} \exp(-I^\alpha)$$

$$= e^{-I^{AdS}} + e^{-I^{BH}} + \dots$$

E.g. Pure AdS_5 gravity

$$I[g] = -\frac{1}{16\pi G_N}\int d^5x\sqrt{g}\left(R + 6/\ell^2\right)$$

Ansatz: $\left(it = t_E \sim t_E + \beta\right)$

$$ds^2 = g(r)\,dt_E^2 + \frac{dr^2}{g(r)} + r^2 d\Omega_3^2$$

- AdS_5 : $g(r) = 1 + r^2/l^2$

- (AdS_5) BH : $g(r) = 1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2}$

(Schwarzschild)

$$\left(r_0^2 = \frac{8}{3\pi} G_N M \right)$$

Horizon : $g(r_+) = 0$

$$T_{BH} = \frac{1}{\beta} = \frac{1}{4\pi} g'(r_+)$$

Note : "Small" & "Large" BHs

$$T_{BH} = \frac{1}{2\pi l}\left(\frac{2r_+}{l} + \frac{l}{r_+}\right) \Rightarrow \boxed{T_{min} \text{ at } \frac{r_+}{l} = \frac{1}{\sqrt{2}}}$$

Exercise ↗

- $\frac{r_+}{l} << 1 \Rightarrow T_{BH} \to \frac{1}{2\pi r_+}$ (specific heat ↓ $C_v < 0$)

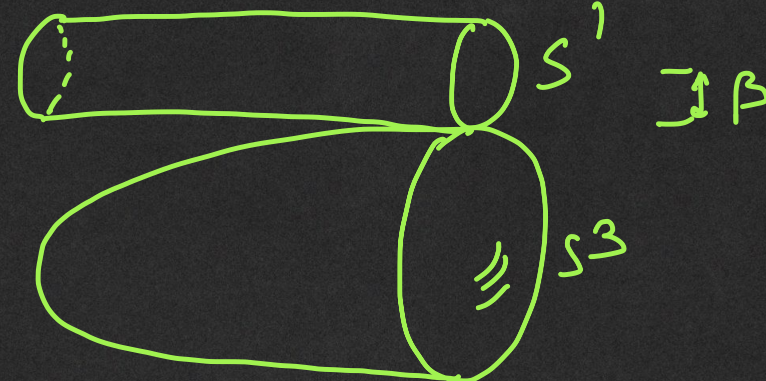
  (cf flat space)

- $\frac{r_+}{l} >> 1 \Rightarrow T_{BH} \to \frac{r_+}{\pi l^2}$ $\quad C_v > 0$ ⇓ stable

What happens below T_{min}?

(E)BH

(E)AdS



$$I[g]\big|_{\text{sol}} = \frac{1}{2\pi G_N}\int d^5x\sqrt{g}$$

$$= \text{"Volume of spacetime"}$$

Hawking - Page transition

[H-P '87]

$$I^{BH} - I^{AdS} = \frac{\pi^2}{4G_N} \frac{r_+^3 (l^2 - r_+^2)}{(2r_+^2 + l^2)}$$

Exercise

$$\Rightarrow \boxed{\begin{array}{lll} \text{High } T & : & \text{BH dominates} \\ \text{Low } T & : & \text{AdS} \quad '' \end{array}}$$

Free energy = on-shell action

[Gibbons-Hawking '77]

$$S_{BH} = \frac{A_H}{4G_N} = \frac{\pi l^2}{2}\frac{r_+^3}{G_N}$$

$$I := I^{AdS\,BH} - I^{AdS}$$

(= reg. on-shell action of BH)

$$E = \partial I/\partial\beta\,, \qquad S_{BH} = \beta E - \underset{\substack{\uparrow\\ F}}{I}$$

True for more general ensembles

Rotating BH: Horizon generated by

$$V = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}$$

angular velocity at horizon.

Charged BH: $g_{\mu\nu}$, A_μ

Electrostatic potential

$$\Phi = V \cdot A \Big|_{r_+} - V \cdot A \Big|_{\infty}$$

"Quantum statistical Relation" [Gibbons-Hawking]

$$I = -S_{BH} + \beta E - \beta \Omega J - \beta \Phi Q$$

$$E = \frac{\partial I}{\partial \beta}, \quad J = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega}, \quad Q = -\frac{1}{\beta}\frac{\partial I}{\partial \Phi}$$

AdS gravity $\Rightarrow$ Thermodynamics
canonical ensemble

What happens in $CFT_4 / S^3 \times S^1_\beta$?

CFT_4 d.o.f

$\Rightarrow E \sim 1/\beta^4$ as $\underline{\beta \to 0}$

$\Rightarrow S \sim 1/\beta^3$ " $\underline{\qquad}$ "

$U(N)$ gauge theory $\Rightarrow$ $S_{CFT} \sim \dfrac{N^2}{\beta^3}$ $\quad \beta \to 0$, $N \to \infty$

$S_{BH} \sim \dfrac{1}{G_N} r_+^3$ ✓

Basic picture of BH thermodynamics in AdS/CFT

	Gravity	Gauge theory	Entropy
Low temp $\beta \to \infty$	AdS (+ small fluc.)	Confined glueballs (singlets)	$1 + \ldots$
High temp $\beta \to 0$	AdS BH (+ small fluc.)	Deconfined quark-gluon plasma	$\dfrac{N^2}{\beta^3} + \ldots$

Supersymmetric BHs in AdS space

Minimal sugra (gauged) 8 supercharges

$$\mathcal{L} = R + 12/\ell^2 - \frac{2}{3}\ell^2 F^2 - \frac{1}{e}\frac{8}{27} A \wedge F \wedge F$$

$$AdS_5 \supset \underset{J}{SU(2)} \times \underset{\tilde{J}}{\widetilde{SU(2)}} \qquad \underset{R}{U(1) \text{ gauge}}$$

$$\boxed{\begin{aligned} J_1 &= J + \tilde{J} \\ J_2 &= J - \tilde{J} \end{aligned}}$$

- BH charges: (R, J_1, J_2)

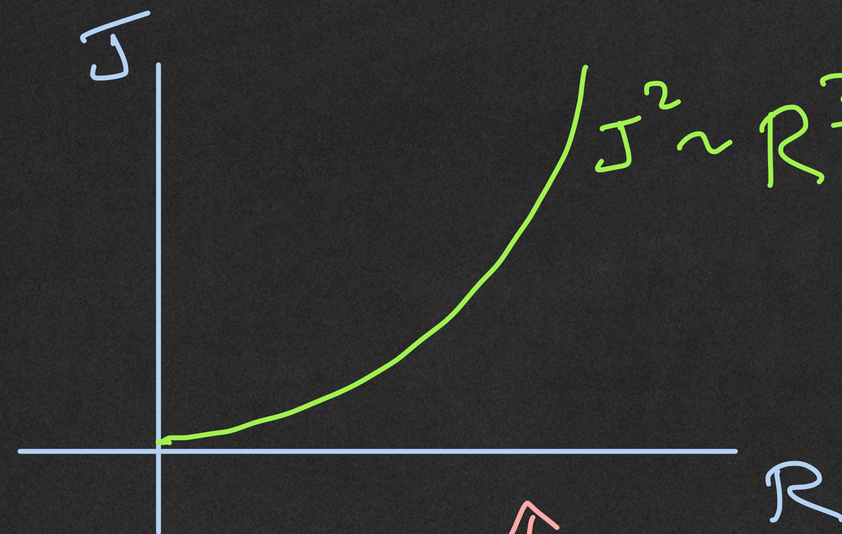
- $\{Q, Q^\dagger\} = E - J_1 - J_2 - \frac{3}{2} R$

$(J_1 = J_2 = J)$ $\qquad (0 < a < 1)$ $\qquad \boxed{\ell = 1}$

$$J_* = \frac{\pi}{2} \frac{a^2(a+3)}{(1-a)^3}$$

susy

$$R_* = \frac{\pi a}{(1-a)^2}$$



Exercise

$$S_{BH}^* = \frac{\pi^2 a}{(1-a)^2} \sqrt{a(a+2)}$$

Note: $J = 0 \Rightarrow a = 0$
$\Rightarrow R = 0, \; S_{BH} = 0$ $\Rightarrow$ SUSY BH must rotate!

Thermodynamics?

$$\beta \to \infty, \quad \Omega_1^* = \Omega_2^* = 1, \quad \Phi^* = 3/2$$

- Frozen! Nothing to vary!

  cf $\Omega(J, Q)$, $\Phi(J, Q)$

- Infinite throat in the interior

$$\Longrightarrow \quad I^{BH} \to \infty \qquad \left(e^{-I^{BH}} = 0\right)$$

Extremal BH = lim (non-extremal BH)

4-parameter family of BHs

[Chong-Cvetic-Lu-Pope '05]

$$(J_1, J_2, E, R) \Longleftrightarrow (a, b, m, q)$$

Horizon $g(r) = 0$

$$\Rightarrow (r^2 - r_+^2)(r^2 - r_0^2)(r^2 - r_-^2) = 0$$

$(r_+ \geq r_0 \geq r_-$: fns of $(a, b, m, q))$

Thermodynamic relations (QSR) hold as expected

$$I = \beta E - S_{BH} - \beta \Omega_i J_i - \beta \Phi R$$

$$\boxed{E = \frac{\partial I}{\partial \beta}, \quad J_i = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_i}, \quad R = -\frac{1}{\beta}\frac{\partial I}{\partial \Phi}}$$

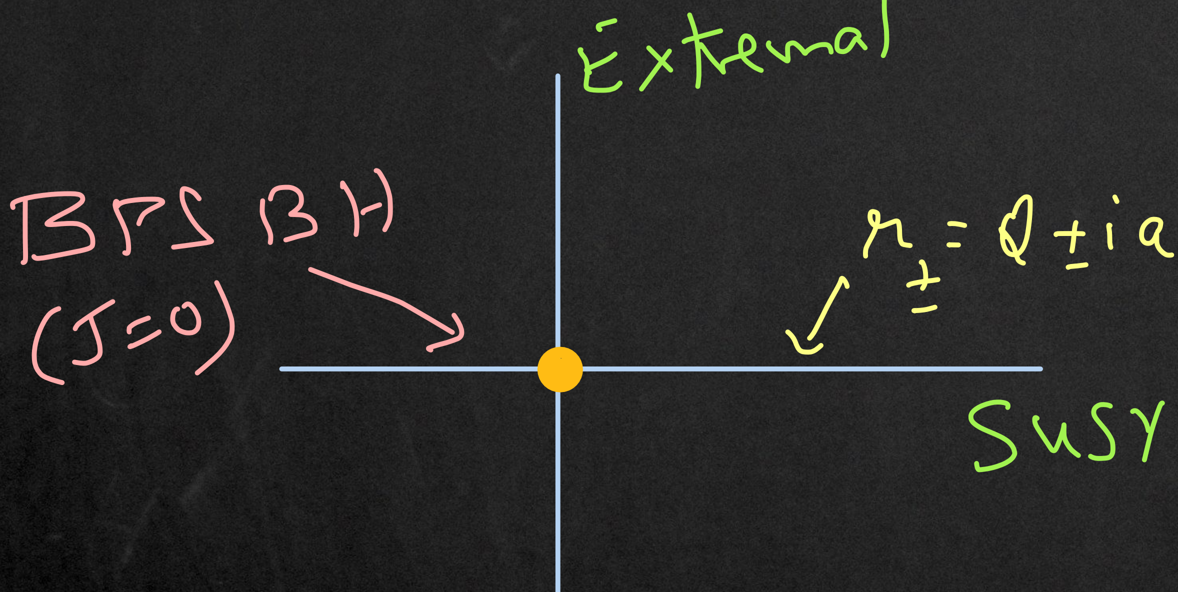
SUSY v/s Extremality (Reminder) [h/t J. Lucietti]

4d Kerr-Newman solution (M, Q, J)

Regularity $\Rightarrow f(M,Q,J) = M^2 - Q^2 - J^2/M^2 \geq 0$

<u>Extremal</u> : $f(M,Q,J) = 0$ $\quad (r_+ \to r_-)$

<u>SUSY</u> : $M = Q$ + regular $\Rightarrow J = 0$.



2-parameter reduction

$\downarrow$

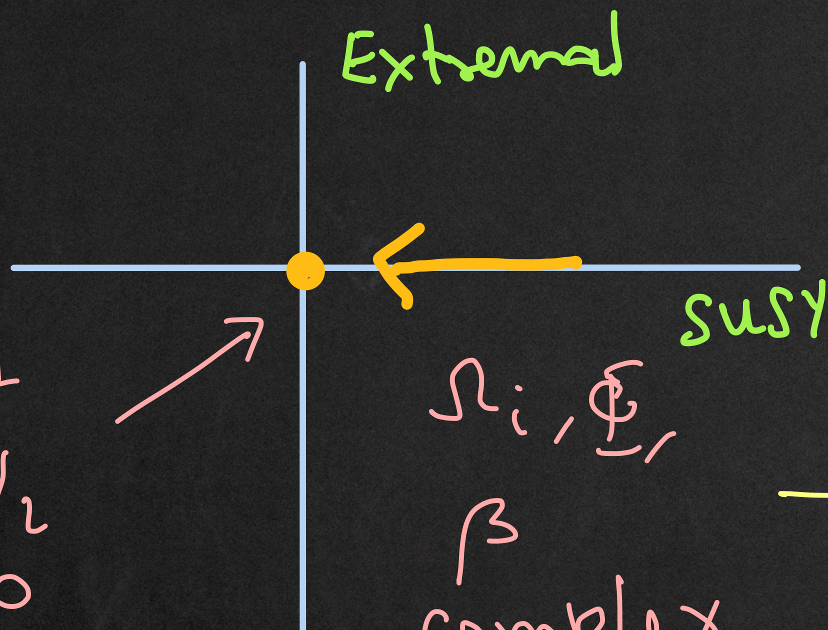
<u>Universal</u>

AdS_5 SUSY BH as a limit

[CCMM '18]
(cf Silva '06).

- Extremal $\Rightarrow$ $r_+ = r_0$

- SUSY $\Rightarrow$ $q = \dfrac{m}{1+a+b}$



$\Omega_i^* = 1$
$\Phi^* = 3/2$
$\beta = \infty$
(BH)

$\Omega_i, \Phi,$
β
complex

$$\longrightarrow \boxed{\begin{aligned} &\beta\left(1+\Omega_1+\Omega_2 - 2\Phi\right) \\ &= 2\pi i \end{aligned}}$$

Meaning of constraint : global regularity

<u>KSE</u>: $\left(\nabla_\mu - i (\Gamma \cdot F)_\mu - \frac{1}{\ell} \Gamma_\mu - i A_\mu^R \right) \varepsilon = 0$

$$V \cdot \text{KSE} \Rightarrow \frac{1}{2}\left(1 + \Omega_1 + \Omega_2 - 2\Phi \right) = \frac{\pi i}{\beta}$$

$$V = \partial_t + \Omega_1 \partial_\phi + \Omega_2 \partial_\psi$$

Euclidean BH

Smoothness $\Rightarrow \varepsilon$ anti-periodic

$\Rightarrow$ Need $A_\mu^R \neq 0$ at ∞!

Supersymmetric "Thermodynamics"

$$\omega_i := \beta(\Omega_i - \Omega_{i*})$$
$$\varphi := \beta(\Phi - \Phi_*)$$

$\beta \to \infty$, $\Omega_i \to \Omega_{i*}$ $\Phi \to \Phi_*$

ω_i, φ finite.

$$\Rightarrow \boxed{\omega_1 + \omega_2 - 2\varphi = 2\pi i}$$

- Action: $\boxed{I = \frac{2\pi}{27} \frac{\varphi^3}{\omega_1 \omega_2}}$ (Note: $\frac{\partial I}{\partial \beta} = 0$!)

- QSR:

$$I = \beta E - S_{BH} - \sum_i \beta \Omega_i J_i - \beta \Phi R$$
$$= \beta(\underbrace{E - \Omega_{i*} J_i - \Phi_* Q}_{\{Q, Q^\dagger\} = 0\,!}) - S_{BH} - \omega_i J_i - \varphi R$$

SUSY BH Thermodynamics

[CCMM '18]

BH entropy follows from on-shell action I as constrained Legendre transform

$$I(\omega_i,\varphi) = -S_{BH} - \omega_i J_i - \varphi R - \Lambda(\omega_1+\omega_2-2\varphi-2\pi i) \quad (*)$$

- $S_{BH} = A/4 = \pi\sqrt{3R^2 - N^2 J}$ (Exercise)

(*) Inspiring observation of Hosseini - Hristov - Zaffaroni 2017

What have we achieved?

- Non-linear constraint $(J_i; R)$

$\rightsquigarrow$ linear constraint $(\omega_i; \varphi)$

- Entropy $\rightsquigarrow$ Canonical ensemble

$$S_{BH} = \text{extr}\left(-I(\varphi, \omega_i) - \omega_i J_i - \varphi R - \Lambda\left(\omega_1 + \omega_2 - 2\varphi - 2\pi i\right)\right)$$

(reg) on-shell action of AdS_5 BH

$$Z_{AdS}(w_i,\varphi) = e^{-I_{grav}(w_i,\varphi)}$$

$$\Vert$$

$$\boxed{Z_{SCFT}(w_i,\varphi)}$$