GWs : the (Q)FT approach

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– Problem

GWs from binary compact objects in GR

$$\lambda_{GW} \underset{(v)}{\gg} r \underset{(v^2)}{\gg} R \sim G_N m$$

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

$(\hbar = c = 1)$

LISA/ET

LIGO/VIRGO

Inspiral Regime.

$$\left.\begin{array}{l} \dfrac{GM}{r} \sim v^2 \ll 1 \\ \lambda_{GW} \sim r/v \end{array}\right\}$$

$$\frac{R}{r} = \frac{R}{GM}\,\frac{GM}{r} \simeq * \, v^2 \ll 1$$

"UV" scale

"New Physics"

[EoS of NS; BHs; ALPs]

∘ Perturbation theory in ratio of scales

$$(R/r)^{1/2} \sim \frac{r}{\lambda} \sim v$$

$\Rightarrow$ EFT ?? (Effective Field theory)

— Task: Solve for $\Omega(t)$. High-accuracy in v.
so-called Post-Newtonian expansion. (nPN $\sim v^{2n}$)

$$\omega_{GW} \simeq \underbrace{2\Omega(t)}_{\text{quadrupolar}} \implies \phi_{GW}(t) = \int_{t_i}^{t} \omega_{GW}(\tilde{t})\, d\tilde{t}$$

this is what LIGO/VIRGO measures!

Adiabatic Expansion

$$\dot{\Omega}/\Omega^2 \ll 1$$

"Conservative" $E(\Omega) \leftarrow$ w/out flux. $[\mathcal{F}=0]$

"Dissipative" $\mathcal{F}(\Omega) \overset{*}{=} \frac{d}{dt}E = \langle \vec{F}_{RR} \cdot \vec{v} \rangle_a$ average

Flux in GWs: $\frac{dE}{dt} = \underbrace{\frac{dE}{d\Omega}}_{\dot{\Omega}=0} \frac{d\Omega}{dt} = \mathcal{F}[x(\Omega)]$ "$\ddot{x} = -\nabla V$"

$$\implies \phi_{GW} \simeq \int_{w_i}^{w_c} \frac{\Omega}{\mathcal{F}} \frac{dE}{d\Omega} d\Omega$$

$\mathcal{F}$: dissipative; $\frac{dE}{d\Omega}$: conservative

$$2\int \omega\, dt = \int \frac{\Omega}{\dot{\Omega}} d\Omega$$

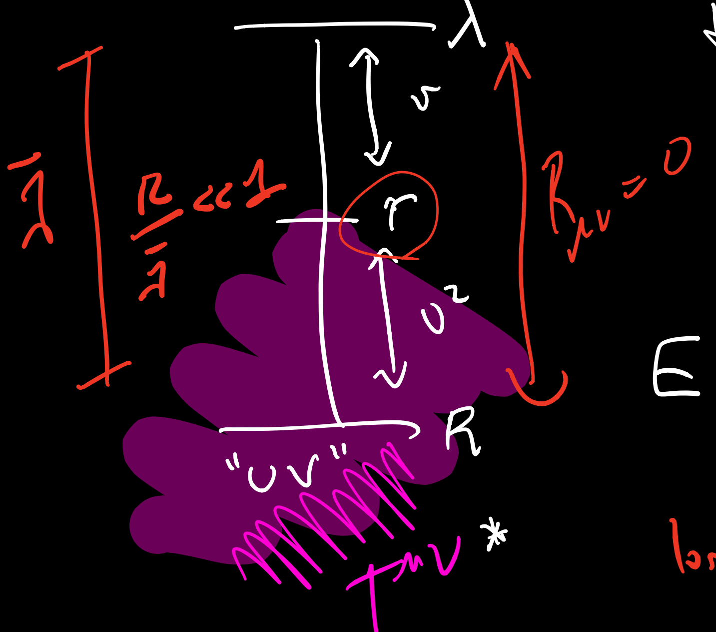
* We can also compute $\vec{F}_{RR}$ directly!
$\langle in | out \rangle^J$ vs $\langle in | in \rangle^J$

Even in the PN-regime it is **VERY HARD!**

Many scales in a non-linear theory: "mode-coupling"

Goal: Separate scales one at a time

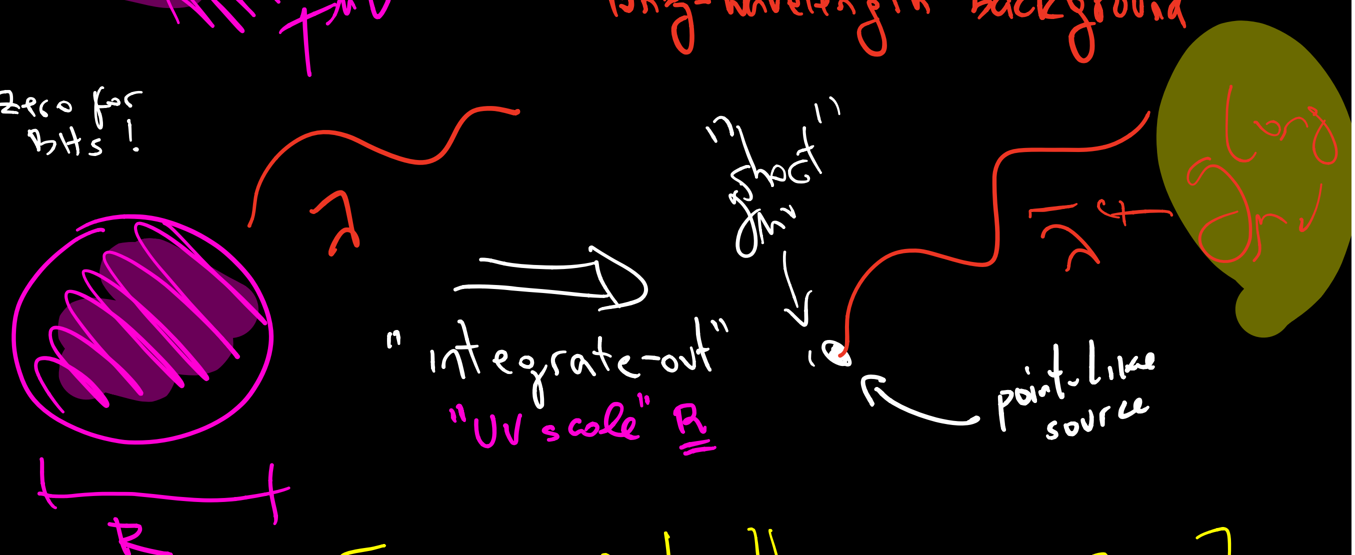
$\Longrightarrow$ **EFT!!**



the scales $(r, \lambda) \equiv \bar{\lambda}$ exist in the 2-body problem

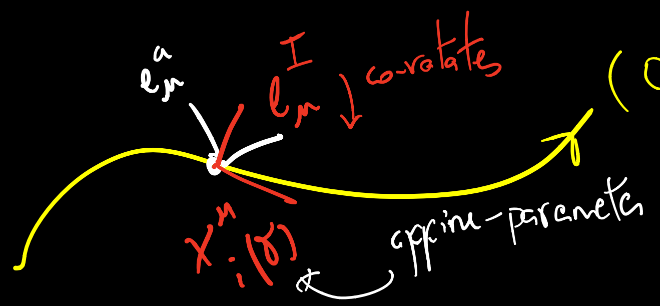
EFT for compact objects in gravitational long-wavelength background

* zero for BHs!



[We will do the same for the scale $\underline{\underline{r}}$ $\Leftarrow$ binary]

Worldline (WL) EFT



(0+1 field theory)

Locally-flat:

$$e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu}$$
$$e_\mu^a e_\nu^b g_{\mu\nu} = \eta^{ab}$$

"fields":

* $x_i^\mu(\sigma) \quad (i = 1,2)$
* $e_\mu^I(\sigma) = \Lambda^I{}_a(\sigma) e_\mu^a$;

Lorentz transformation

S_{EH} "Wilson coeffs" R-scale

"Long"

$$\Rightarrow \underline{\text{Construct } S_{eff}} = \int d\sigma\, \mathcal{L}_{eff}\Big(g^{L}_{\alpha\beta},\ \underbrace{x, v, e, \dot{e}}_{\text{light-fields}},\ C_i\Big)$$

$$\boxed{\text{GBs of } P_\mu, J_{\alpha\beta}}$$

Symmetries:
- Diff invariance
- Local Lorentz invariance
- RPI $(\sigma \to \sigma')$

derivative expansion

$$S_{eff}^{(n)} = -\sum_i \int d\sigma_i \underbrace{\sqrt{v^\mu v_\mu}\left[m_i + \dots\right]}_{\substack{n=0 \\ \text{(monopole)}}} \leftarrow \mathcal{O}(R/\lambda)^n$$

"mass" ?

$\leftarrow q^n A_\mu$

$\Rightarrow$ "$C_i\, \partial^n g_{Long}[x]\, e\, v$"

What about higher multipoles?

- Spin

"$p\dot{q}$"

$$\mathcal{L}_{spin} \supset \frac{1}{2} S^{ab} \Omega_{ab}$$

angular velocity $(e\dot{e})$ $De=\Omega e$

conjugated momentum $\left(\frac{\partial \mathcal{L}}{\partial \Omega} = S\right)$

coupling to gravity

$$\frac{1}{2} \omega_\mu^{ab} S_{ab} v^\mu$$

Ricci rotation coefs.

II ⟹

- Quadrupole (what about mass dipole?)

$$S^{(2)}_{eff} = \int d\tau\, Q^{(E)}_{ab}(\tau)\, e^a_\mu e^b_\nu E^{\mu\nu}_{Long} + (E \leftrightarrow B)$$

$\rightarrow O(h)$

May be promoted to a WL field to describe QNM

[resembles the $\vec{D}\cdot\vec{E}$ in EM]

$$E_{\mu\nu} = W_{\mu\alpha\nu\beta} v^\alpha v^\beta \quad (\text{"Electric"})$$

$E^\alpha_\alpha = 0$

$E_{\alpha\beta} v^\beta = 0$

* Why Weyl tensor?
* What do these Q_{ab} describe? (no absorption for now)