What about higher multipoles?

- Spin

"p q̇" → angular velocity ($e\dot{e}$) $De \sim \Omega e$

$$\mathcal{L}_{spin} \supset \frac{1}{2} S^{ab} \Omega_{ab}$$

conjugated momentum $\left(\frac{\partial \mathcal{L}}{\partial \Omega} = S\right)$

coupling to gravity

$$\frac{1}{2} \omega_\mu^{ab} S_{ab} v^\mu$$

Ricci rotation coefs.

- Quadrupole (what about mass dipole?)

$$S^{(2)}_{eff} = \int d\tau\, Q^{(E)}_{ab}(\tau)\, e^a_\mu e^b_\nu E^{\mu\nu}_{Long} + (E \leftrightarrow B)$$

→ $O(h)$

May be promoted to a WL field to describe QNM

[resembles the $\vec{D} \cdot \vec{E}$ in EM]

$$E_{\mu\nu} = W_{\mu\alpha\nu\beta} v^\alpha v^\beta \quad (\text{"Electric"})$$

$E^\alpha_\alpha = 0$

$E_{\alpha\beta} v^\beta = 0$

* Why Weyl tensor?

* What do these Q_{ab} describe? (no absorption for now)

traces: $R^{\alpha}{}_{\alpha}$, $R_{\mu\nu} v^{\mu} v^{\nu}$

$\Rightarrow$ $c_R \int R \, d\tau$? $\quad R_{\mu\nu} \propto T_{\mu\nu} \propto \delta^{(4)}$

(bulk) 0 $\quad$ ∞ (boundary)

$$S_{EH} = -2 M_{Pl}^2 \int \sqrt{-g_L} R_L d^4 x \quad \left(M_{Pl}^2 \equiv \frac{1}{32\pi G_N} \right)$$

field redefinition

$$\delta g^{\mu\nu}(x) \propto c_R \int d\tau \frac{\delta^{(4)}(x^\alpha - x^\alpha(\tau))}{\sqrt{-g}} g(x) \quad *$$

$$\rightarrow \delta S_{EH} \propto \int d^4x \, G_{\mu\nu} \delta g^{\mu\nu} \sqrt{-g}$$

$$\rightarrow -c_R \int R \, d\tau$$

kills the WL operator!

the same with $c_V \int R_{\alpha\beta} v^\alpha v^\beta$

Terms which vanish "on-shell" can be removed via field-redef. from observables!

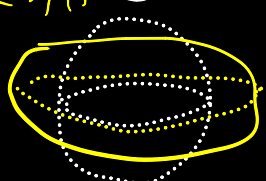
$*$ what about δg on WL term? $\Rightarrow$ Darwin! : $\delta(x_1 - x_2)$

Background and Response

$$Q_E^{ab}(\tau) \begin{cases} \nearrow \langle Q_E^{ab}(\tau)\rangle_S & \text{"short"} \leftarrow \text{modes with } |K| > \bar{R}^{-1} \\ \searrow (Q_E^{ab})_R & \text{"long"} \leftarrow |K|^{-1} \sim \bar{\lambda} > R \end{cases}$$

$\langle Q_E^{ab}(\tau)\rangle_S \leftarrow$ permanent Quadrupole (Isolated)

$Q \sim S^2/M$



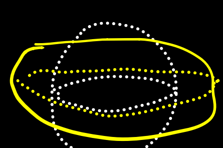
spin-induced

e.g. $\dfrac{C_{ES^2}}{2m} S^{ac} S^{b}{}_{c}$ $\quad \boxed{\begin{matrix} C_{ES^2} = 1 \\ \text{for Kerr} \end{matrix}}$

If spherically sym. in isolation $\Rightarrow \langle Q^{ab}\rangle_S \propto \delta^{ab}$ such that the coupling to (STF) E_{ab} vanishes.

(Birkoff's theorem)

$(Q_E^{ab})_R \longrightarrow$ response to a long-wavelength perturbation ($\bar{\lambda} > R$)



external field

H.S. Love 1700's ?

LRT: $(Q \cdot E)$ $\quad C_E\, E^{\mu\nu} E(\tau)\, e^a_\mu\, e^b_\nu + \ldots$

Tidal "Love" number!

this is the same as in EM (dipole)

$(\vec{P})_{\ell} = \chi_{(\omega)} \vec{E}_{ext}$ "$\mathrm{Re}\,\chi(\omega \to 0) \sim C_E$"

static susceptibility

$\boxed{\begin{array}{l} * \ \mathrm{Im}\,\chi \neq 0 \to \mathrm{Re}\,\chi \neq 0 \ \text{(KK relations)} \\ \text{(absorption)} \\ \text{puzzle: BHs absorve but do not deform!} \end{array}}$

$\Rightarrow C_E \int E_{\mu\nu} E^{\mu\nu} d\tau$ + ...

in the WL theory

"higher-dim" operator

First term beyond minimal coupling for non-rotating BHs

two (wilson) coefs $\{M, C_E\}$ that know about "UV-scales"

New Physics?!

Matching!!

$E_{\mu\nu} \propto \partial^2 g$

$E^2 \propto \partial^4 g$

many derivatives!

(eq. principle)

No more R scale $\to$ EFT "=" full theory

$\hookrightarrow$ (wilson coef.)

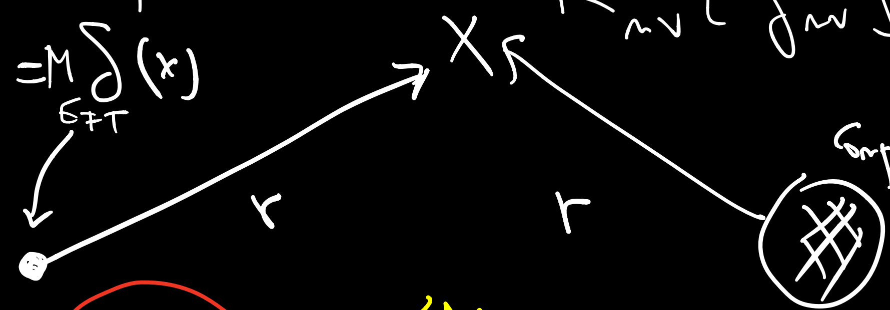
$(R/\bar{\lambda}) \ll 1$ expansion

||

Mass

$\Box h_{\mu\nu} = M_{EFT}\,\delta^4(x)$
$+ \dots$

$R_{\mu\nu}[g_{\mu\nu}] \propto T_{\mu\nu}$



Compact objects of mass M

$$h_{00} = 2G_N \frac{M_{EFT}}{r} \overset{r \gg GM}{\iff} g_{00} \to 1 + 2G_N \frac{M}{r}$$

Matching.

(includes binding)

$$M_{EFT} = M \neq \sum_i m_i \,!$$

What about BHs with $T^{full}_{\mu\nu} = 0$!!

Notice in the EFT $T^{EFT}_{\mu\nu} \neq 0$

$$\underbrace{G_{\mu\nu} = 0}_{\text{full theory}} \Rightarrow G_{\mu\nu}[g^{Long}_{\mu\nu}] = -G_{\mu\nu}[g^{short}_{\mu\nu}]$$

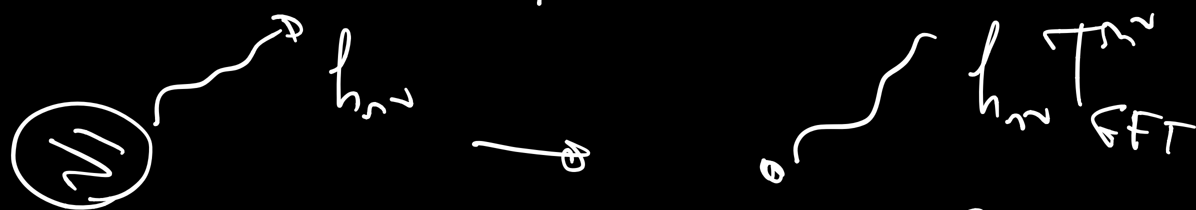
Worldline EFT

M_{EFT} matches also into short-distance geometry!

Quadrupole

1) For the background part we can do the same $\Longrightarrow$ $\Phi \propto \frac{\langle Q^{ij}\rangle x^i x^j}{r^5}$

For all the multipole it is more convenient to match the $T_{\mu\nu}$'s.

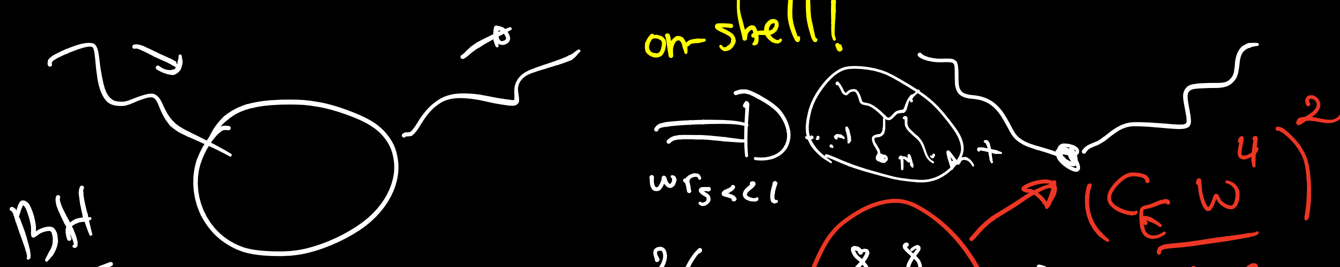


and compute e.g. $\langle Q^{ij}\rangle_S = \int d^3x \, T^{00} x^i x^j$ + ...

(see below)

this will be useful with the binary as a WL theory.

2) For the response — Love numbers —



$$\sigma_{\text{scattering}}(\omega) = r_s^2 \left(1 + \dots \omega r_s + \dots\right)$$

$$\left(\frac{C_E \, \omega^4}{m_{Pl}}\right)^2$$

$\Rightarrow \; c_E = \cancel{\#}\, r_s^5 / G_N$ (other objects: $\sim M R^4$) ← Neutron stars.

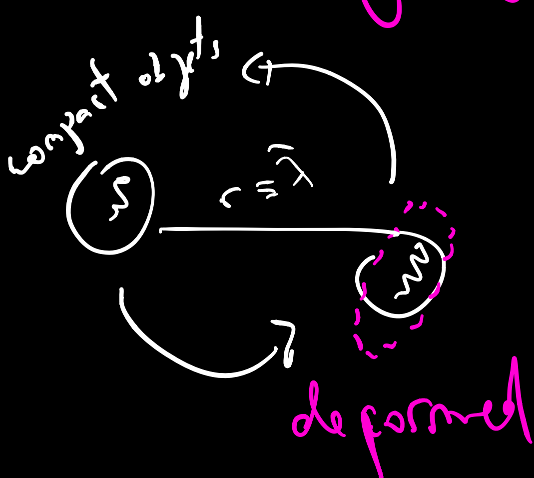
Naturalness $O(1)$?

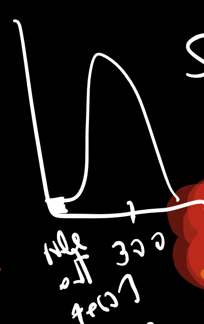
$$\boxed{\text{turns out (GR)} \quad c_{E,B}^{BH} = 0 \;!!}$$

$d=4$

For NS there's an enhancement of $\left(\frac{R}{GM}\right)^4 \sim 10^3$

How are we going to tell?





The effect in the waveforms scales like $\left(\frac{r_s}{r}\right)^5 \sim v^{10}$

5PN order!

Say $m \gg 3 M_\odot$ ← Not a NS!

and $c_E \neq 0$ ← Not a BH in GR!

this can be DM! Alps condensate and deform.

We need high-accuracy!!

What is it?!

(gravitational collider physics)