$\hookrightarrow$ <u>IBP</u> $\quad v_1^i v_2^j \int \frac{p^i p^j}{\vec{p}^4} e^{i\vec{p}\cdot\vec{r}} dt \qquad \vec{r} \equiv \vec{x}_1 - \vec{x}_2$

$$\frac{p^i p^j}{\vec{p}^4} \to A\underbrace{\left(\frac{r^i r^j}{r^2} - \frac{\delta^{ij}}{3}\right)}_{TF} + B\underbrace{\delta^{ij}}_{\text{trace}}$$

$$\left(\frac{(\vec{v}_1\cdot \vec{r})(\vec{v}_2\cdot\vec{r})}{r} - \frac{\vec{v}_1\cdot\vec{v}_2}{r}\right)$$

We need $\{P_{0i0j}\, ; P_{ij00}\}$ couplings

(also the v^2 from $\sqrt{\eta v v}$)

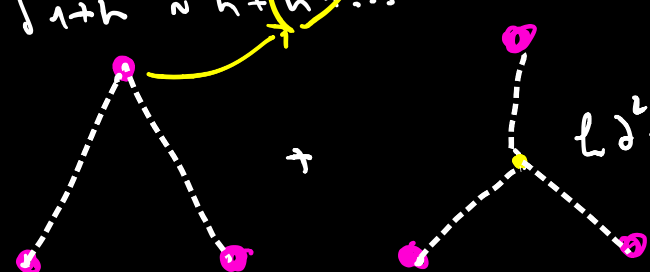
this completes the 1PN at $\mathcal{O}(G)$

What are we missing?!

Recall $\frac{GM}{r} \sim v^2 \Rightarrow \mathcal{O}(G^2 v^0) \sim \mathcal{O}(G v^2) \Leftarrow$ mixing of G's!

We need non-linear couplings!

$\sqrt{1+h} \sim h + h^2 + \ldots$



$h\partial^2 h + h^2\partial^2 h + \ldots$

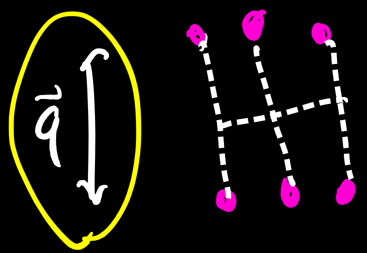
3-graviton

$\frac{G^2 m_1 m_2^2}{r^2} + (1 \leftrightarrow 2)$

"one-loop" integrals.

Exercise: Compute the Einstein-Infeld-Hoffman potential

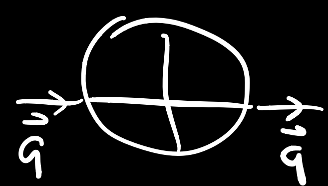
State of the art.



$$\left[\int_{\vec k_n} \frac{A(k_1 \ldots k_n, q)}{k_1^2\, k_2^2\, (k_1+k_2+q)^2 \cdots}\right] e^{i \vec q \cdot \vec r} \quad FT$$

$\Uparrow$

n-loop massless "self-energy" integral in $D=3$



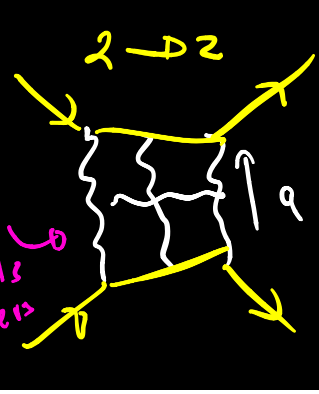
Master integrals

We now know up to 4-loops! $\longrightarrow$ 5PN potentials and G^4 at 6PN!

Caveat: this is not the whole story! there are "Lamb-shift"-type corrections too, from conservative Radiation-Reaction.

New developments (amplitudes)

Classical Limit!



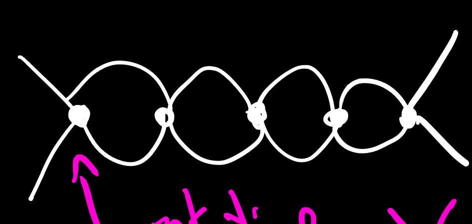
Keep only potentials to all orders

$\hbar/J \ll 1$

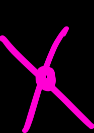
(large M, b)

$\Longrightarrow$

w/out Feynman!



potential interaction



$V(q)$

Matching to EFT with $V(q)$ interaction

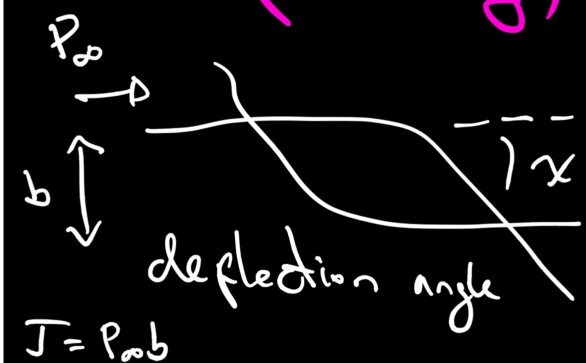
$$\rightarrow V(r) = \int V(q)\, e^{i q \cdot r} \leftarrow \text{(gauge choice)}$$

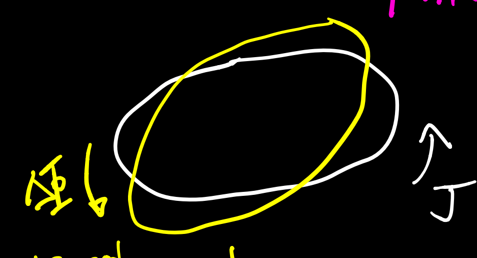
But we can also do better and relate

gauge invariant scattering $\longleftrightarrow$ gauge-invariant Bound State

(Boundary) B2B map (Bound)

1910.03008
1911.09310





$$\frac{\Delta\Phi(\mathcal{E},J)}{2\pi} = \frac{\chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})}{2\pi}$$

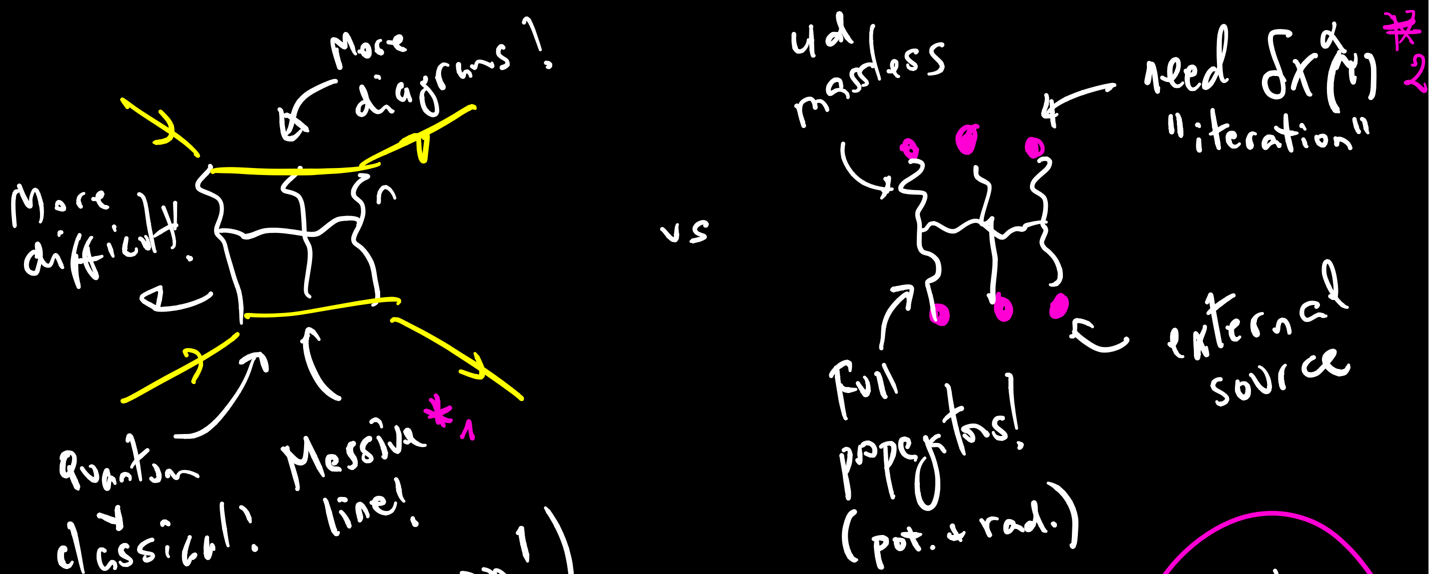
$\mathcal{E}<0$ analytic continuation

$\rightarrow$ All observables from χ !!

BONUS:
Naturally a (relativistic) expansion in G to all orders in v/c !

We can then construct an EFT to compute $\mathcal{X}$ in a "Post-Minkowskian" (PM) expansion! — Next time ☺

A comment on: INTEGRALS!



(but double copy!)

differential equations for loop integrals!

potential

radiation ("soft")

boundary conditions

*1 In the classical limit mass prop. collapses. (HQET)

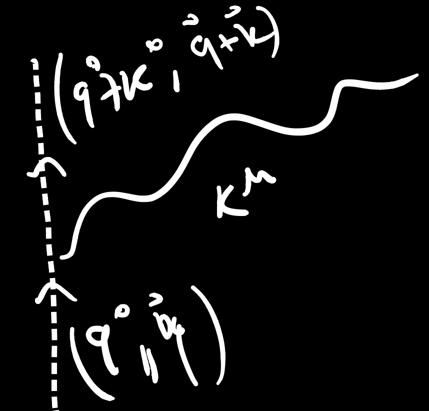
*2 Automatic in iteration

$$\frac{1}{k\cdot u \pm i\epsilon}$$

Radiation Problem: Q_{ij} ?!

We now match the one-point functions.

Power-counting:



the $\vec{q}+\vec{k}$ doesn't scale homogenously!

$$\frac{1}{(\vec{q}+\vec{k})^2} \simeq \frac{1}{\vec{q}^2}\left(1-\frac{\vec{k}^2}{\vec{q}^2}+\cdots\right)$$

Such that the $\vec{k}$'s go "upstairs" as in the multipole expansion.

Multipoles:

at LO in ∂_i-exp. $\sim \frac{r}{\lambda} \sim \mathcal{O}(v)$

$$Q^{ij}(t) = \int T^{00}_{(t,\vec{x})} x^i x^j d^3\vec{x}$$

$$\tilde{T}^{00}(t,\vec{k}) = \int d^3\vec{x}\, T^{00}(t,x)\, e^{i\vec{k}\cdot\vec{x}}$$

expand. $\mathcal{O}(k^2)$

$$\frac{1}{2}\partial_{ij}\tilde{T}^{00}(t,\vec{0})\, k^i k^j = \int \frac{(i\vec{k}\cdot\vec{x})^2}{2} T^{00}(t,\vec{x})$$

$$Q^{i_L}(t) = i^l \partial_L \tilde{T}^{00}(t,\vec{0}) \qquad i_L = i_1 \dots i_l$$

At $\mathcal{O}(G^0)$ the matching is trivial.

$$T^{\mu\nu}(t,x) \underset{\text{flat}}{=} \sum_a \int m_a\, d\tau_a\, \delta^4(x - x_a(\tau_a))\, v^\mu v^\nu$$

* PN corrections.

$(dS = -m\sqrt{v^2}\, d\tau)$

* At each $\mathcal{O}(\partial^n)$ we have many PN contributions.

$\Rightarrow$ What do we need for 1PN?

* 1) $\mathcal{O}(G^0 v^2)$ in $\mathcal{O}(\partial^2)$ quadrupole ✓

2) $\mathcal{O}(G^0 v^0)$ in $\mathcal{O}(\partial^3)$ octupole ✓ $[Q^{ijk}\partial_k E_{ij}]$

3) $\mathcal{O}(G^1)$ in $\mathcal{O}(\partial^2)$ quadrupole ⇐

4) $\underbrace{\mathcal{O}(G^0 v^2)}_{\text{LO!}}$ in $\mathcal{O}(\partial^2)$ current-quadrupole ✓

* there is a caveat. the Q_{ij} has corrections from decomposition of higher orders in the derivative expansion:

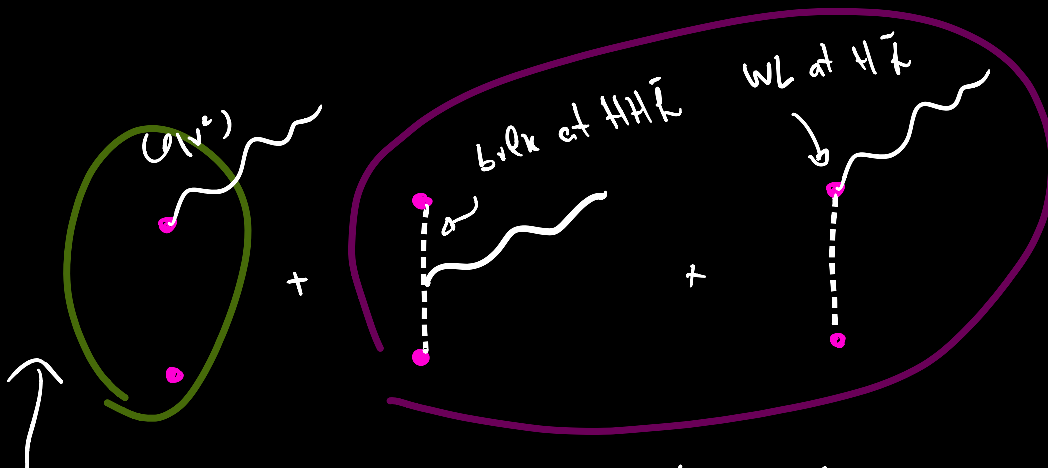
$$\text{e.g.} \int \underbrace{T^{ij} x^k x^l}_{\substack{2 \otimes 2 \\ \text{contains a "2"}}} \longrightarrow Q^{ij} \supset \left\{ \begin{array}{l} T^{ll} x^i x^j \\ \ddot{T}^{00} x^2 x^i x^j \\ \dot{T}^{0k} x^k x^i x^j \end{array} \right.$$

must be also PN expanded!

$$\text{PN} \left\{ \begin{array}{ll} T^{00} & \text{starts at } G^0 v^0 \\ T^{ll} & \text{start at} \\ \ddot{T}^{00} x^2 & \mathcal{O}(G^0 v^2) \\ \dot{T}^{0k} x^k & \text{w/out spin} \end{array} \right.$$

this is the price to pay for correlated scales!

At the end we have:

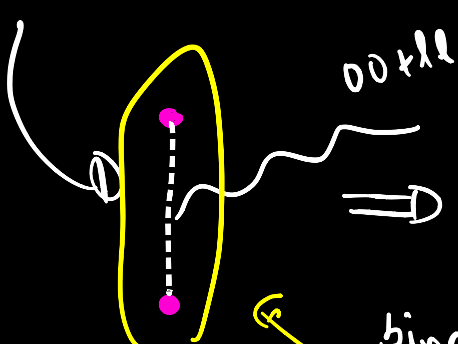


the LO radiation is relativistic; but $Q^{(0)}_{ij}$ is "Newtonian" $\mathcal{O}(v^0)$ (mass)

the $\left|\frac{d^3}{dt^3}Q^{ij}\right|^2$ can be computed with $\vec{a} \sim \frac{G\hat{r}}{r^2}$

At $\mathcal{O}(G^1)$ the binding radiates!

Moreover, at $\mathcal{O}(G^1)$ both T^{00} and T^{ll} enter at the same order!



$$\Rightarrow \sum_{a,b} \frac{m_a m_b}{m_{Pl}^2} \int \frac{1}{\vec{q}^2} \frac{(\vec{q}\cdot\vec{k}+\vec{q}^2)}{(\vec{q}+\vec{k})^2} e^{i\vec{q}\cdot\vec{r}} \, e^{i\vec{k}\cdot x_b}$$

localized

binding

"complete squares" $\underbrace{\vec{k}\cdot\vec{q}}_{2} = (\vec{k}+\vec{q})^2 - \vec{k}^2 - \vec{q}^2$

Expand in $|\vec{k}|^n$ and extract Q^{ij} at $\mathcal{O}(|k|^2)$

$$\Rightarrow \sum_b \frac{G m_1 m_2}{|\vec{r}|} e^{i\vec{k}\cdot\vec{x}_b}$$

e.g. $\underline{\underline{00}}$ $\quad \bar{m}_1 = m_1\left(1 + \frac{1}{2}\vec{v}_1^{\,2} - \frac{G m_2}{|\vec{r}|}\right)$

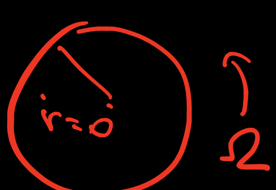
$$Q^{ij}_{(1)} = \sum_a \bar{m}_a \left[x_a^i x_a^j\right]_{TF} + \dots$$

recall $M = \int T^{00} d^3x$!

$\left[T^{ll} + * \dot{T}^{0k} x^k + * \ddot{T} x^2\right] x^i x^j$

$\propto \delta(\omega - 2\Omega)$

circular $\Rightarrow$ $Q^{ij}(t) = \eta\left(1 - \left(\frac{1}{42} + \frac{39}{42}\nu\right) x_\omega\right)\left[r^i r^j\right]_{TF}$



$$+ \frac{11}{21}\eta\, x_\omega^2 (1 - 3\nu)\left[v_i v_j\right]_{TF}$$

$x_\omega = (GM\Omega)^{2/3}$

in the CoM

$\vec{r} = \vec{x}_1 - \vec{x}_2$; $\eta = \frac{m_1 m_2}{M}$; $\nu = \mu/M$; $M = m_1 + m_2$

Now we are almost done! $\Rightarrow$

Use EOM from 1PN EIH to "take the dots" on $\dot{E} \propto |\dddot{Q}^{ij}|^2$

(we need also current-Q and octupole!)

$$\hookrightarrow \frac{\dot{E}_{1PN}}{\dot{E}_{LO}} = 1 - \left(\frac{1247}{336} + \frac{35}{12}\nu\right) x_\omega$$

$\dot{E}_{LO}$ ← chirp mass

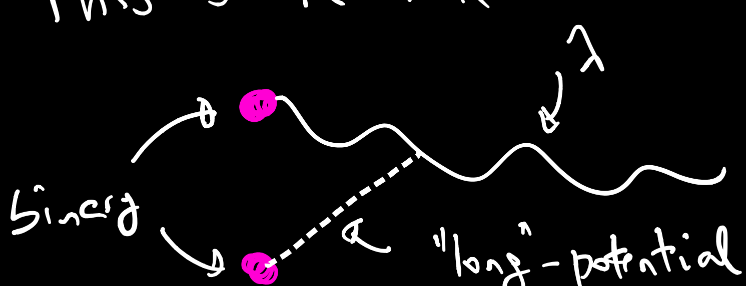
ν ← mass-ratio!

$$+ \dots \; x_\omega^5$$

x_ω^5 ← Love number!

there is one more scale I didn't tell you about: $\frac{GM}{\lambda} \sim \frac{GM}{r}\frac{r}{\lambda} \sim v^3$

this is the "tail"



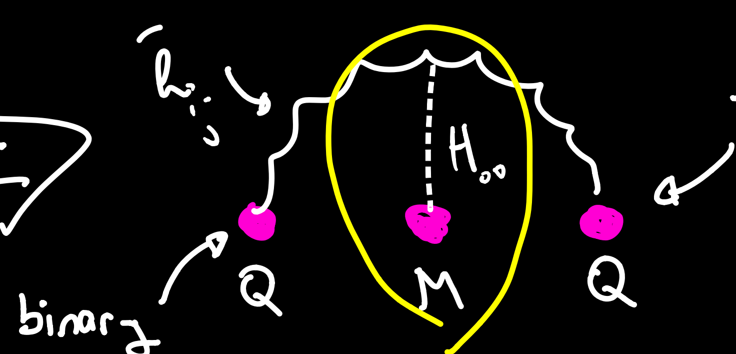
⇐ logarithm! ("Lamb-shift")

Renormalization/RG flow

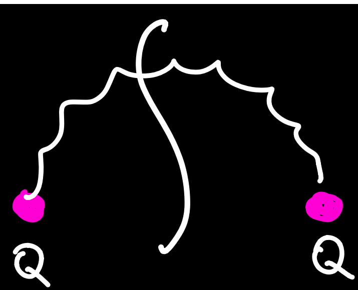
Let's compute the radiation-reaction (RR) contribution to the S_{eff}.

This is subtle! We need to "track" the field. We have been using Feynman $i\varepsilon$-prescription in an $\langle in|out\rangle^J$ computation.
To include RR we need the "in-in" formalism.
(otherwise RR $\to 0$! if we force $|0,out\rangle$!)

However! for CONSERVATIVE RR effects the in-out formalism captures the result (up to a factor of 2 – opt thm.)

$\Rightarrow$ 

$-M\,dt + \frac{1}{2} Q^{ij} E_{ij} + \ldots$

Recall 

$\Longrightarrow$ Flux. starts at v^5

Imaginary with $i\varepsilon$-prescription

and the "tail" part adds a factor of v^3

$\Longrightarrow$ Conservative RR terms at 4PN!

"LAMB SHIFT"

$$\mu \frac{d}{d\mu} M = -2G^2 M \langle \dddot{Q}\dddot{Q} \rangle$$

$\mu \sim r^{-1}$

Flux!

$$\Delta E \propto G^2 M (\dddot{Q})^2 \log v$$

circular

$v = \Omega r$

"IR log"