Particle Physics teaching

at Vilnius University

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Vilnius University

CERN Baltic Conference, 06/2021

Teaching: Courses for Bachelor, Master, and Ph.D. level

Research in theory: investigating the Grimus-Neufeld model

Standard Model (SM) + one fermionic singlet + two Higgs doublets

- [G-N] W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

The Particle Physics group of Vilnius University

- staff:
  - Aurelijus Rinkevičius: “CERN department”, CMS
  - Andrius Juodagalvis: CMS data analysis
  - Darius Jurčiukonis: multi Higgs doublet models, modelling
  - Thomas Gajdosik: theory, Grimus-Neufeld model

- postdoc:
  - Vytautas Dudenas: QFT, renormalization, Grimus-Neufeld model

- doctoral students:
  - Simonas Draukšas: QFT, renormalization, Grimus-Neufeld model
  - Marijus Ambrozas: CMS data analysis,
    DAQ software development for the CMS tracker

Particle Physics related courses at Vilnius University

- bachelor studies:
  - Unix systems by Andrius Juodagalvis
    - bash, root, etc. . . .
  - Elementary Particle Physics 1
    - first part of the book *Introduction to Elementary Particles* by David Griffiths
  - Introduction to High Energy Physics Analysis by Aurelijus Rinkevičius
    - basics of: particle physics, statistics, Monte Carlo methods, data analysis
  - Elementary Particle Physics 2
    - second part of the book *Introduction to Elementary Particles* by David Griffiths
- general university studies (i.e.: courses for everybody except physicists)
  - World of Particles by A. Juodagalvis, A. Rinkevičius, Christoph Schäfer, Albinas Plėšnys, TG
    - outreach ! aims to help critical thinking, countering fake news

Particle Physics related courses at Vilnius University

- master studies
  - Cosmology
    - SR, GR, ΛCDM; concepts only; no star evolution
  - elementary particle physics (before: modern theoretical physics; then: quantum field theory)
    - introduction to QFT; concepts only; hand-waving Standard Model
  - planned : QFT 1
    - based on the lectures of David Tong
  - planned : QFT 2 and/or Standard Model
- doctoral studies:
  - Quantum Field Theory
    - based on the book of Matthew D. Schwartz : *Quantum Field Theory and the Standard Model*

Particle Physics theory research at Vilnius University

- we use the Grimus-Neufeld model (GNM)
  [G-N] W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.
  - it extends the Standard Model (SM)
  - with a single fermionic singlet
    * the Majorana mass term for this gauge singlet allows the seesaw mechanism
  - and a second Higgs doublet
    * its different coupling to the leptons allows for radiative neutrino masses

- the GNM can accomodate
  - the measured neutrino mass differences
  - the measured neutrino mixing angles (PMNS matrix)

⇒ the GNM can be seen as a neutrino extension to a generic 2HDM

Why choosing the Grimus-Neufeld model (GNM) as the research area?

- as an extension of the SM:
  we have the opportunity to teach the SM, while still doing something new
  - this requirement of being something new comes regularly
    in bachelor and master thesis defenses . . .
    . . . as if a bachelor student is able to fully understand the SM . . .
- there is very little published about the GNM
  - we can slowly work on our better understanding of the model
    * and still publish something that is genuinely new
- the different parts of the model highlight interesting theory mechanisms:
  - the seesaw mechanism
  - mixing of states
  - spontaneous symmetry breaking
  - interplay between tree-level and loop-level

Why choosing the Grimus-Neufeld model (GNM) as the research area?

- the GNM is complicated and simple at the same time

complicated:

- in order to reproduce the neutrino sector couplings have to fulfill tight relations
  - technically: neutrino masses have to be analytically obtained
    for example, using the Grimus-Lavoura approximation
  - mass functions and neutrino mixing matrix have to be inverted

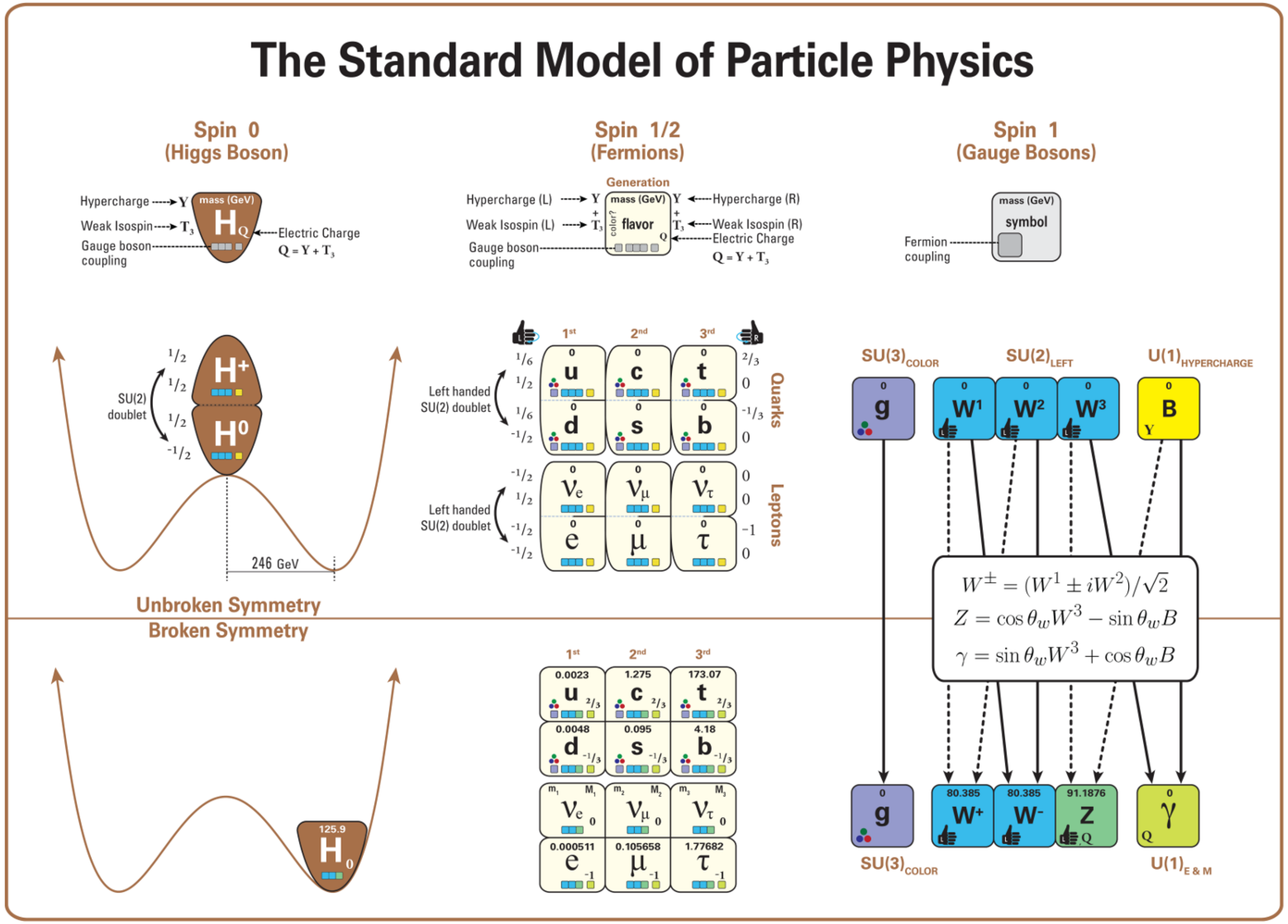
  $\Rightarrow$ we get a reduced parameter set for the model

simple:

- once these relations are implemented in the GNM
  - students can investigate simple processes and still do something new
    * Higgs decays and branching ratios
    * neutrino production at colliders
    * investigating RGE running, . . .

The GNM as an extension of the Standard Model of Particle Physics (SM)

- the particle content of the SM
  - one Higgs boson
  - quarks and leptons
  - gauge bosons
    - gluon
    - $W^{\pm}$-bosons
    - Z-boson
    - photon



The GNM as an extension of the SM ... in terms of particles

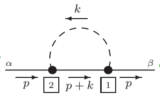
- the particle content of the SM
  - Higgs boson
  - quarks and leptons
    - ∗ originally there are only 3 left-handed massless neutrinos
  - gauge bosons
- in the Two Higgs Doublet Model part the GNM adds to the SM
  - one charged scalar boson (upper part of the scalar $SU(2)_L$-doublet)
  - two neutral scalar bosons (lower part of the scalar $SU(2)_L$-doublet)
    - ∗ usually written as scalar and pseudo-scalar
- in the neutrino sector the GNM adds only a single Majorana fermion
  - it is a singlet under all the SM gauge groups
    - $\Rightarrow$ no gauge boson couplings
  - it has a Majorana mass term

The GNM as an extension of the SM . . . in terms of parameters

- the parameters of the SM
  - gauge-Higgs sector:
    - theory: gauge couplings $g_{SU(3)} = g_s$, $g_{SU(2)}$, $g_{U(1)}$ and Higgs potential μ^2, λ
    - experiment: couplings g_s, α_{em}, G_{F}, angle $\cos\theta_w$, and Higgs mass m_H
  - fermion sector:
    - theory: Yukawa couplings Y_U, Y_D, Y_E
    - experiment: masses m_t, m_c, m_u, m_b, m_s, m_d, m_τ, m_μ, m_e, and mixing matrix V_{CKM}
- in the Two Higgs Doublet Model (THDM) part the GNM adds to the SM
  - the Higgs potential $V(\phi_1, \phi_2)$ with parameters: (m_1^2), m_2^2, m_3^2, (λ_1), λ_2, λ_3, λ_4, λ_5, λ_6, λ_7
  - the additional Yukawa couplings $Y_U^{(2)}$, $Y_D^{(2)}$, $Y_E^{(2)}$
- in the neutrino sector the GNM adds
  - the Majorana mass M_N
  - the additional Yukawa couplings $Y_N^{(1)}$, and $Y_N^{(2)}$

Using the Grimus-Lavoura approximation for calculating light neutrino masses

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

- taking the interaction eigenstates of the neutral leptons ν_j and N_R
  - they couple to the first Higgs doublet and the vev by the Yukawa coupling $(Y_N^{(1)})_j$
- calculating the $(3+1) \times (3+1)$ symmetric mass matrix M_ν
  - at tree-level M_ν has rank 2 $\Rightarrow$ only two masses are non-zero , one of them the "heavy" N_R
- considering only the "light" neutrinos ν_j leads to an effective 3×3 mass matrix $\mathcal{M}_\nu$
- approximating in the corrections to $\mathcal{M}_\nu$: only the loop with ν_α  ν_β matters
- parametrizing $(Y_N^{(1)})_k = \frac{\sqrt{2} m_D}{v} u_{3k}$ and $(Y_N^{(2)})_k := d\, u_{2k} + d' u_{3k}$
- calculating the effective $\mathcal{M}_\nu^{\text{1-loop}}$ and diagonalizing it by the Takagi decomposition:

$\Rightarrow$ we get two light neutrinos, $\hat{m}_r$ and $\hat{m}_s$, as analytic functions

  - $*$ which we invert to detemine d and $|d'|$ from the measured neutrino mass differences

$\Rightarrow$ we determine the Yukawa couplings $(Y_N^{(1)})_k$ and $(Y_N^{(2)})_k$ (12 real parameters)

- in terms of $\Delta m^2_{\text{atm/sol}}$ and V_{PMNS} and two free parameters: m_D^2 and $\phi' = \arg[d']$

Conclusions

- Bachelor students can work on processes involving the fixed $(Y_N^{(i)})_k$
  - up to now we managed only one simple bachelor thesis looking at an overly simple assumption
- The full renormalization of the model is still missing $\Rightarrow$ advanced topics
  - see later talks . . .

Thank you

for listening

and for discussion and comments

and of course for the conference! ☺

Backup slides

explaining the theoretical steps of building the model

Features of the Grimus-Neufeld model (GNM)

- an extension of the SM to describe neutrino masses
  - no change to the stong sector
  - only lepton- and Higgs-sector are modified
    * by adding only one fermionic singlet
    * and a second Higgs doublet

The particle spectrum contains additional to the SM

- four Majorana neutrinos
  - one heavy — the added fermionic singlet
  - and three light Majorana neutrinos
    * at tree-level two of them are massless
    * at loop-level one of them gains a radiative mass
- the second Higgs doublet gives
  - a charged scalar H^+ and a two neutral scalars H_k^0

The rôle of seesaw mechanism and of THDM in the GNM

- "normal" seesaw gives a small mass for each heavy mass

  ⇒ one heavy fermionic singlet gives only one light neutrino

  – the other two SM-like light neutrinos stay massless

- in the SM fermion masses come from Yukawa couplings

  – massless light neutrinos have vanishing Yukawa couplings

    ⇒ massless light neutrinos stay massless

- in the GNM the THDM allows different Yukawa couplings

  – the Yukawa couplings to the second Higgs doublet
    can generate radiative masses for the light neutrinos

  ⇒ the THDM has to be general (i.e. not a type-I or type-II or . . .)

The Grimus-Lavoura approximation for calculating light neutrino masses

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

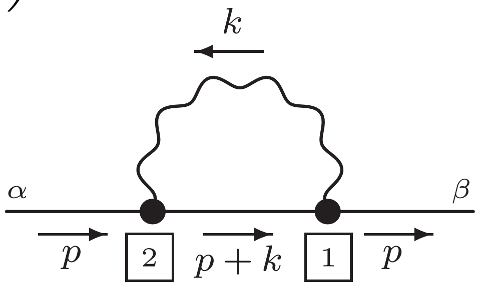
- taking the interaction eigenstates of the neutral leptons ν_j and N_R
  - coupled to first Higgs doublet and vev by the Yukawa coupling $(Y_N^{(1)})_j$
- calculating the $(3+1) \times (3+1)$ symmetric mass matrix

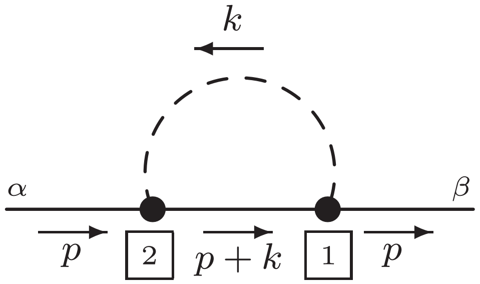
$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} \quad \text{with} \quad \begin{aligned} M_L &= 0_{3\times 3} \quad \ldots \text{ at tree level} \\ M_D &= (m_{De}, m_{D\mu}, m_{D\tau}) = \frac{v}{\sqrt{2}} Y_N^{(1)\top} \end{aligned} \qquad (1)$$

  - at tree-level M_ν has rank 2 $\Rightarrow$ only two masses are non-zero
- considering the loop corrections to M_ν:

$$\delta M_\nu = \begin{pmatrix} \delta M_L & \delta M_D^\top \\ \delta M_D & \delta M_R \end{pmatrix} \quad \text{with} \quad \delta^{\text{ct}} M_L = 0_{3\times 3}\ , \quad \text{since} \quad M_L = 0_{3\times 3} \qquad (2)$$

  - but $\delta M_L \neq 0$,

    coming from $\Sigma_V^{[2]}(p^2) =$ 

$\Sigma_S^{[2]}(p^2) =$ 

- reducing the problem to the "light" neutrinos
- leads to an effective 3×3 mass matrix $\mathcal{M}_\nu$
  - at tree level $\mathcal{M}_\nu^{\text{tree}} = -M_D^\top M_R^{-1} M_D$ has rank 1 ,
  - at one-loop level $\mathcal{M}_\nu^{\text{1-loop}} = \mathcal{M}_\nu^{\text{tree}} + \delta\mathcal{M}_\nu$ can have rank > 1 ,

    with $\delta\mathcal{M}_\nu = \delta M_L - \delta M_D^\top M_R^{-1} M_D - M_D^\top M_R^{-1} \delta M_D + M_D^\top M_R^{-1} \delta M_R M_R^{-1} M_D$
- the approximation assumes
  - δM_R is irrelevant (as M_R is a free unmeasured parameter, set $\delta M_R = 0$)
  - corrections with δM_D are subdominant
    - suppressed by $Y^2 m_{\ell^\pm}/m_D$ or $g^2 m_{\ell^\pm}/m_D$ compared to $\mathcal{M}_\nu^{\text{tree}}$
  - the correction δM_L is of the same order as $\mathcal{M}_\nu^{\text{tree}}$

$\Rightarrow$ at 1-loop only neutral bosons contribute to δM_L
  - calculated from $\Sigma^{[2]}_{V=Z^0}(p^2)$ and $\Sigma^{[2]}_{S=G^0,h,H,A}(p^2)$
    - Z^0 and G^0 sum up to a gauge invariant contribution

Using the Grimus-Lavoura approximation

- parametrize the Yukawa couplings as

$$(Y_N^{(1)})_k = \frac{\sqrt{2}m_D}{v} u_{3k} \qquad (Y_N^{(2)})_k := d\, u_{2k} + d' u_{3k} \tag{3}$$

  – with three orthonormal vectors $\vec{u}_\alpha = u_{\alpha k}$
    * that mix the three neutrinos $\nu_k = u_{\alpha k} P_L \zeta_\alpha^M$ at tree-level

- calculate the effective 1-loop 3×3 mass matrix

$$(\mathcal{M}_\nu^{\text{1-loop}})_{jk} = u_{2j}u_{2k}A + (u_{2j}u_{3k} + u_{3j}u_{2k})B + u_{3j}u_{3k}C \tag{4}$$

  * which is obviously only rank 2 :

$$u^*_{\alpha j}(\mathcal{M}_\nu^{\text{1-loop}})_{jk}u^*_{\beta k} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & B \\ 0 & B & C \end{pmatrix}_{\alpha\beta} , \tag{5}$$

  – with $A = d^2 f_1$, $B = d' d f_1 + i d \frac{m_D}{v} f_2$, $C = d'^2 f_1 + 2 i d' \frac{m_D}{v} f_2 + \frac{m_D^2}{v^2} f_3$,
  and the f_i depending on the parameters of the Higgs sector (and the SM)

- diagonalize $\mathcal{M}_\nu^{\text{1-loop}}$ by the Takagi decomposition :

$$V_{\text{PMNS}}^\top \mathcal{M}_\nu^{\text{1-loop}} V_{\text{PMNS}} = \text{diag}(\hat{m}_o = 0, \hat{m}_r, \hat{m}_s) \tag{6}$$

Using the Grimus-Lavoura approximation

- since we get $\widehat{m}_r$ and $\widehat{m}_s$ as analytic functions
  - we can invert these functions to determine parameters
- we choose to detemine d and $|d'|$
- since $\widehat{m}_o = 0$, the measured neutrino mass differences
  - determine $\tilde{m}_2 = \sqrt{\Delta m^2_{\text{sol}}}$ and $\tilde{m}_3 = \sqrt{\Delta m^2_{\text{atm}} + \Delta m^2_{\text{sol}}}$
    - * attention: there are several possibilities of ordering the neutrinos
- the Takagi decomposition also determines the mixing matrix V_{PMNS}
  - which determines the vectors $u_{\alpha k}$ that define the Yukawa couplings

$\Rightarrow$ we determine the Yukawa couplings $(Y_N^{(1)})_k$ and $(Y_N^{(2)})_k$ (12 real parameters)

- in terms of SM parameters
- the parameters of the neutrino sector: $\Delta m^2_{\text{atm/sol}}$ and V_{PMNS}
- parameters of the Higgs sector: m_h^2, m_H^2, m_A^2, mixing angle $\beta - \alpha$
- and two free parameters: m_D^2 and $\phi' = \arg[d']$

What we achieved is

- a one-loop improved parametrization for the GNM
  - this parametrization reproduces neutrino data exactly
    - the masses at one loop
    - the mixing matrix within the approximation
  - but not a full renormalization
    - . . . this is still a goal for the (far) future
- We can avoid doing the renormalization ourselves :
  - by using a generic tool: a spectrum calculator
  - a spectrum calculator does the renormalization numerically
  - the problem in our case: implementing the seesaw numerically
- with limiting the seesaw scale to $\sim 10^4$ GeV
  - FlexibleSUSY (FS) provides qualitative correct neutrino masses
    - studies in FS suggest a promising seesaw scale of $\sim 10^{-6}$ GeV