Connection between tadpole renormalization and gauge dependences

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Motivation: tadpoles are confusing

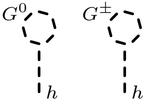
- Standard QFT textbook (like in Peskin&Schroeder) tadpole renormalization:
  - SSB and tree level minimum:

$$
\begin{aligned}
V(\phi) &= \mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2, \mu^2 < 0 \\
&\Rightarrow \ \phi \to v + \phi', \ \frac{\partial}{\partial \phi'} V\left(\phi'\right) = 0 \leftarrow \ \textbf{minimum condition} \\
&\Rightarrow v^2 = -\frac{\mu^2}{\lambda}.
\end{aligned}
$$

  - the **minimum condition** imposed at every order is called a ***tadpole condition***.
  - then no tadpole diagrams appear anywhere, anymore.

Motivation: tadpoles are confusing

- SM in R_ξ
  - Tadpoles **are** gauge dependent:



  - then v and mass parameters are too..
    - even more of a problem in extended models, where OS is not possible.
  - **not enough** renormalization constants generated from $p \to (1 + \delta_p)\, p$ and $\phi \to \sqrt{Z_\phi}\phi$ to absorb divergences.
- Common interpretation on the usage of tadpoles is still missing.
  - even though, there a hints in works from 70's that all these features were already known (see e.g. T. Appelquist et.al. in PRD 8 (1973) 1747)
- We looked at different approaches to tadpoles and tried to relate them in a common language.

Divergences in R_ξ – explicit symmetry breaking

- The Higgs doublet:

$$\phi = \begin{pmatrix} G_W^+ \\ \frac{1}{\sqrt{2}}(v + h + iG_Z) \end{pmatrix}.$$

- The R_ξ gauge fixing Lagrangian (for W and Z only):

$$\mathscr{L}_{GF} = -\frac{1}{2\xi} F_Z F_Z - \frac{1}{\xi} F_W^+ F_W^-, \quad F_Z = \partial^\mu Z_\mu - \xi m_Z G_Z, F_W^\pm = \partial^\mu W_\mu^\pm \mp i\xi m_W G_W^\pm.$$

- G's are components of $SU(2)$ doublet $\Rightarrow$ R_ξ breaks the global symmetry explicitly (unless $\xi \to 0$).
- One expect any $SU(2)-$breaking divergences.
  - Symmetry preserving $p \to Z_p p$ and $\phi \to \sqrt{Z_\phi}\phi$ are not enough.
  - Which symmetry breaking counterterms to include?

Divergences in R_ξ with background fields

- Split fields into quantum + background:

$$\phi \to \phi + \hat{\phi}$$

- background$-R_\xi-$gauge is constructed invariant under the original global symmetry (quantum+background)
  - it reduces to usual R_ξ, when set $\hat{\phi} \to 0$
  - All the renormalized Green functions are made finite with redefinitions:

$$p \to (1+\delta_p)\,p\,,\ \phi \to \sqrt{Z_\phi}\phi\,,\ \hat{\phi} \to \sqrt{Z_{\hat{\phi}}}\hat{\phi}$$

- set $\hat{\phi} \to 0$ to recover "quantum" Lagrangian and look at counterterm structure that is left.
  - these counterterms are then enough to absorb all the divergences.

Divergences in R_ξ

- For the SM, the single renormalization constant (RC), $Z_{\hat{\phi}}$, *leaks* into "quantum" Lagrangian
  - to be interpreted as an independent VEV RC or Higgs background field RC.
- difference between vev and Higgs field RCs collect all spurious gauge dependent divergences from tadpoles, i.e.

$$\hat{\delta}_\phi \equiv 1 - \sqrt{Z_{\hat{\phi}}/Z_\phi} \sim \xi$$

- $\hat{\delta}_\phi$– only independent constant with gauge dependent divergences.
  - but $\hat{\delta}_\phi$ usually enters into definitions of mass counterterms.

Gauge dependence in parameter RCs

- The VEV renormalization can be written

$$v \to v + \delta_v v \,,\, \delta_v = \bar{\delta}_\phi + \hat{\delta}_\phi$$

where $\bar{\delta}_\phi$ has no gauge dependent divs, and $\hat{\delta}_\phi \sim \xi$.

- Take a fermion mass renormalization in the SM:

$$\begin{aligned} m_f &= \frac{y_f v}{\sqrt{2}} \\ &\Rightarrow \delta m_f = \frac{y_f v}{\sqrt{2}} \delta_{y_f} + \frac{y_f v}{\sqrt{2}} \delta_v \\ &= m_f \left(\delta_{y_f} + \bar{\delta}_\phi + \hat{\delta}_\phi \right) \\ &\Rightarrow \frac{\partial}{\partial \xi} \delta m_f \neq 0 \end{aligned}$$

- The same is true for any mass parameter as they include v.
- How to get rid of ξ dependence?

Getting rid of ξ: FJ-scheme

- The FJ-scheme [Fleischer and Jegerlehner, PRD 23 (2001)] in short:
  - One has to shift the VEV to the "true one-loop minimum" by full tadpole contribution to get gauge independent mass counterterms
  - It is equivalent to including tadpoles to all diagrams
- How is this related to $\hat{\delta}_\phi$ and how it gets rid of gauge dependences?
- One can relate PRTS δ_v to FJ $\delta_v|_{\mathrm{FJ}}$ by a trivial redefinition:

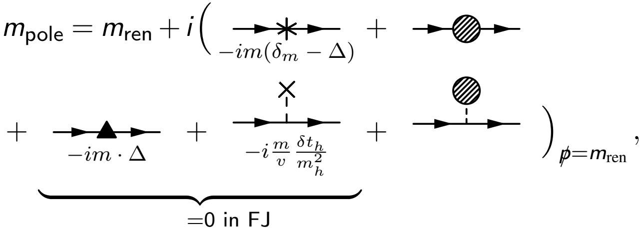
$$\delta_v = \delta_v - \Delta + \Delta\,, \quad \delta_v|_{\mathrm{FJ}} \equiv \delta_v - \Delta$$

Δ is an FJ-term, defined:

$$\Delta = \frac{T_h}{v m_h^2}\,,\ T_h - \text{ all tadpole diagrams}\,.$$

FJ-scheme

- Δ in turn appears everywhere in the Lagrangian, where v is.
- A fermion pole mass expression:



- The sum of last two terms is zero by tadpole condition.
- The "standard" (PRTS) scheme is recovered by $\Delta \to 0$.
- How this saves gauge independence?

FJ-scheme, UV terms

- Δ in terms of "original" renormalization constants, δ_ps, $\bar{\delta}_\phi$ and $\hat{\delta}_\phi$:

$$\Delta = \frac{-\delta t_h}{v m_h^2} = \frac{1}{2}\left(\delta_\lambda - \delta_{\mu^2} + \bar{\delta}_\phi + \hat{\delta}_\phi\right)$$

  where δt_h stands for a tadpole counterterm.

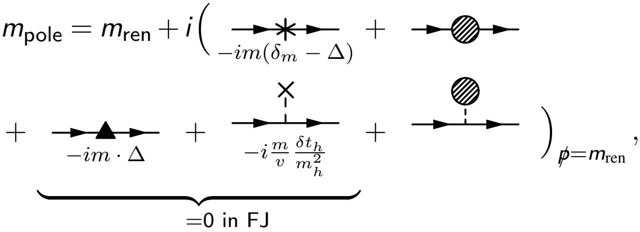
- The FJ-vev renormalization does not have $\hat{\delta}_\phi$ in definition:

$$\delta_v|_{\mathrm{FJ}} = \delta_v - \Delta = \left(\bar{\delta}_\phi + \hat{\delta}_\phi\right) - \Delta$$
$$= \frac{1}{2}\left(\delta_{\mu^2} - \delta_\lambda\right)$$

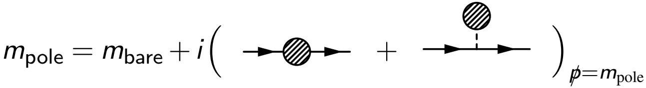
  has no $\xi-$dependent UV divergence.

- In turn, mass counterterms are defined gauge independenty with $\delta_v|_{\mathrm{FJ}}$.

FJ-scheme finite terms



- Bare perturbation theory, m_{pole} and m_{bare} are gauge independent:



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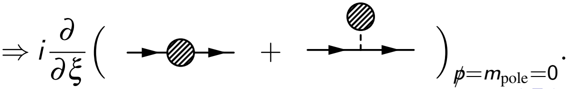
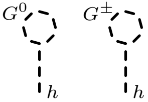


(2)

FJ vs PRTS, which to use

- The parameters of an FJ scheme can be related to the ones of the PRTS with the help of Δ,
  - we can really look at the different "tadpole schemes" more like parameter definitions, then truly schemes (tadpole condition is the same anyways).
- Δ is gauge dependent by



  - One can account for gauge dependencies in definitions by Δ.
- Numerically, FJ might behave worse then PRTS, as it might introduce large cancellations:
  - taking the SM example, the 1-loop finite VEV-shifts in numbers:

$$\delta v|_{\mathrm{PRTS}} = 10.155 \text{ GeV}, \quad \delta v|_{\mathrm{FJ}} = -138.456 \text{ GeV}, \quad \Delta v = 148.612 \,\text{GeV}.$$

Conclusions

- The gauge dependence in parameter definitions come from the explicit breaking of the global symmetry in the R_ξ gauge.
- FJ-scheme shifts away this gauge dependencies from definitions.
- We can still use whatever tadpole scheme we like, since:
  - **if** we are consistent, no physical process can depend on a gauge.
  - With the help of Δ-term, we can go from parameters of one "scheme" to the other.
- **But** we still must be careful with gauge dependent definitions
  - They can easily slip into physical interpretations.
  - The accounting of tadpoles by an FJ procedure helps being transparent in that.

Do **not** forget about tadpoles

Thank you :)