On the On-Shell Renormalization of Fermion Masses, Fields and Mixing Matrices at 1-loop

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- Usually fermion masses and fields are renormalized as

$$m_i^0 \to m_i + \delta m_i$$
$$\psi_j^0 \to \tilde{Z}_{ji}\psi_i$$

- then *no-mixing conditions on external legs* are imposed [1]

$$\frac{1}{\not{p}-m_j}\Sigma_{ji}(p^2)u_i = 0, \qquad \bar{u}_j\Sigma_{ji}(p^2)\frac{1}{\not{p}-m_i} = 0$$

$\Longrightarrow \delta\tilde{Z}_{L,R}$ *and* $\delta\tilde{Z}^\dagger_{L,R}$ in terms of self-energy scalar functions (no †)!

Introduction II

- ***Overspecification*** of field renormalization constants [2]

$$\Sigma \neq \gamma^0 \Sigma^\dagger \gamma^0 \qquad \Longrightarrow \qquad \left(\tilde{Z}_{L,R}\right)^\dagger \neq \tilde{Z}^\dagger_{L,R}$$

  - (pseudo-)Hermiticity violated by **absorptive parts**
  - Relax the no-mixing condition or add a set of constants $\bar{Z}$
    - $\mathcal{L} \neq \mathcal{L}^\dagger$, but OK for external legs
- In [3], the CKM matrix counterterm is gauge-dependent [4]

$$\delta V \sim -\delta \tilde{Z}_L^{A,u} V + V \delta \tilde{Z}_L^{A,d} \qquad \Longrightarrow \qquad \partial_\xi \delta V \neq 0$$

- In [5, 6], one no-mixing condition is dropped and $\partial_\xi \delta V = 0$, but
  - No explicit field renormalization definition
  - On-Shell scheme, but with „choices“
  - Self-energy functions only for the SM

Introduction III

- Want to keep the Lagrangian hermitian $\Longrightarrow$ keep no-mixing condition only for incoming particles

$$\boxed{\frac{1}{\not{p} - m_j}\Sigma_{ji}(p^2)u_i = 0}$$

  - Outgoing particles still mix, but only due to absorptive parts!
- Want to separate gauge-dependence into field and mass contributions $\Longrightarrow$ introduce off-diagonal mass counterterms

$$\delta m_i \to \delta m_{ji}$$

- Want to have *universal* mass, field, and mixing matrix counterterm definitions in terms of scalar self-energy functions $\Longrightarrow$ ...?

Setup I

- The mass and fields are renormalized as

$$\bar{\psi}\left(m + \delta m^L P_L + \delta m^R P_R\right)\psi \xrightarrow{Hermiticity} \left(\delta m^L\right)^\dagger = \delta m^R$$

$$\psi_0 \to Z^{1/2}\psi, \qquad \bar{\psi}_0 \to \bar{\psi}\gamma^0 Z^{1/2\dagger}\gamma^0$$

$$Z = Z_L P_L + Z_R P_R, \quad Z_{L,R} = 1 + \delta Z_{L,R}$$

- Self-energy is decomposed as

$$\begin{aligned}\Sigma_{ji}(p^2) = & \Sigma^L_{ji}(p^2)\not{p}P_L + \Sigma^R_{ji}(p^2)\not{p}P_R + \Sigma^{sL}_{ji}(p^2)P_L + \Sigma^{sR}_{ji}(p^2)P_R \\ & + \frac{1}{2}\left(\delta Z^\dagger_{Lji} + \delta Z_{Lji}\right)\not{p}P_L + \frac{1}{2}\left(\delta Z^\dagger_{Rji} + \delta Z_{Rji}\right)\not{p}P_R \\ & - \left(\boxed{\delta m^L_{ji}} + \frac{1}{2}\delta Z^\dagger_{Rji}m_i + \frac{1}{2}m_j\delta Z_{Lji}\right)P_L \\ & - \left(\boxed{\delta m^R_{ji}} + \frac{1}{2}\delta Z^\dagger_{Lji}m_i + \frac{1}{2}m_j\delta Z_{Rji}\right)P_R\end{aligned}$$

Setup II

(Hereinafter $i \neq j$!)

No-mixing on incoming particles $\Longrightarrow$ *relation* between field and mass renormalization

$$(m_i^2 - m_j^2)\,\delta Z_{Lji} - 2m_j \delta m_{ji}^L - 2m_i \delta m_{ji}^R = \underbrace{-2\left(m_i^2 \Sigma_{ji}^L(m_i^2) + m_i m_j \Sigma_{ji}^R(m_i^2) + m_j \Sigma_{ji}^{sL}(m_i^2) + m_i \Sigma_{ji}^{sR}(m_i^2)\right)}_{\text{standard piece}}$$

$$(m_i^2 - m_j^2)\,\delta Z_{Rji} - 2m_j \delta m_{ji}^R - 2m_i \delta m_{ji}^L = \underbrace{-2\left(m_i^2 \Sigma_{ji}^R(m_i^2) + m_i m_j \Sigma_{ji}^L(m_i^2) + m_j \Sigma_{ji}^{sR}(m_i^2) + m_i \Sigma_{ji}^{sL}(m_i^2)\right)}_{\text{standard piece}}$$

- *Off-diagonal* field renormalization, but “shifted” by mass counterterms
- 2 equations and 4 unknowns $\Longrightarrow 4 > 2$ — end of the road?

Setup III

Not the end yet!

- $\left(\delta m^L\right)^\dagger = \delta m^R \implies$ relation to the anti-hermitian part of field renormalization

$$\boxed{\left(m_i^2 - m_j^2\right)}\delta Z^A_{Lji} - 2m_j\delta m^L_{ji} - 2m_i\delta m^R_{ji} = -\left(m_i^2\Sigma^L_{ji}(m_i^2) + m_i m_j\Sigma^R_{ji}(m_i^2)\right.$$
$$\left. + m_j\Sigma^{sL}_{ji}(m_i^2) + m_i\Sigma^{sR}_{ji}(m_i^2)\right) + \boxed{H.C.}$$

- Note the $H.C.$ — self-energy functions are hermitian conjugated!
- Very *distinct mass structure* in front of field renormalization!

Let us explore the properties of $\boxed{m_i^2 - m_j^2}$ terms

Exploration: Gauge-dependence

In general, from Nielsen Identities we have [7, 8, 9]

$$\partial_\xi \Sigma_{ji}(p^2) = \Lambda_{jj'}\Sigma_{j'i}(p^2) + \Sigma_{ji'}(p^2)\bar{\Lambda}_{i'i}$$

$$\xrightarrow{1-loop} \partial_\xi \Sigma_{ji}(p^2) = \Lambda_{ji}\left(\not{p} - m_i\right) + \left(\not{p} - m_j\right)\bar{\Lambda}_{ji}$$

- $\Lambda's$ also have Dirac structure and contain all the gauge-dependence at 1-loop

$$\boxed{\left(m_i^2 - m_j^2\right)}\partial_\xi \delta Z^A_{Lji} - 2m_j \partial_\xi \delta m^L_{ji} - 2m_i \partial_\xi \delta m^R_{ji}$$

$$= -\boxed{\left(m_i^2 - m_j^2\right)}\left(m_i \bar{\Lambda}^R_{ji}(m_i^2) + \bar{\Lambda}^{sL}_{ji}(m_i^2)\right) + H.C.$$

$\Longrightarrow \boxed{m_i^2 - m_j^2}$ mass structure carried by gauge-dependent terms!

Exploration: UV divergences I

- At 1-loop contributions of various particles *species* to fermion self-energies are well known in terms of Passarino-Veltman functions [10]
- For scalars and vectors (SV) we have

$$\Sigma^L(p^2) = f_L B_1(p^2, m^2_{\psi\text{loop}}, m^2_{\text{SV}})$$
$$\Sigma^R(p^2) = f_R B_1(p^2, m^2_{\psi\text{loop}}, m^2_{\text{SV}})$$
$$\Sigma^{sL}(p^2) = m_{\psi\text{loop}} f_s B_0(p^2, m^2_{\psi\text{loop}}, m^2_{\text{SV}})$$
$$\Sigma^{sR}(p^2) = m_{\psi\text{loop}} f_s^\dagger B_0(p^2, m^2_{\psi\text{loop}}, m^2_{\text{SV}})$$

- For fermion tadpoles we have

$$\Sigma^{sL}(p^2) = f_T m_{\psi\text{loop}} A_0(m^2_{\psi\text{loop}})$$
$$\Sigma^{sR}(p^2) = f_T^\dagger m_{\psi\text{loop}} A_0(m^2_{\psi\text{loop}})$$

- f's are appropriate couplings and $f^\dagger_{L,R} = f_{L,R}$

Exploration: UV divergences II

UV divergences of PV functions are also well-known!

$$[B_1(p^2, m_1^2, m_2^2)]_{\text{div.}} = \frac{1}{D-4}$$
$$[B_0(p^2, m_1^2, m_2^2)]_{\text{div.}} = -\frac{2}{D-4}$$
$$[A_0(m^2)]_{\text{div.}} = -\frac{2m^2}{D-4}$$

- D is the spacetime dimension ($4-\epsilon$ or $4-2\epsilon$)

$\Longrightarrow$ Can easily look at UV divergences!

- We have

$$\left[\left(m_i^2 - m_j^2\right)\delta Z^A_{Lji} - 2m_j\delta m^L_{ji} - 2m_i\delta m^R_{ji}\right]^{\mathrm{SV}}_{\mathrm{div.}} = \frac{1}{D-4}\Big(-f_L\boxed{(m_i^2+m_j^2)} - f_R\boxed{2m_im_j} + 4m_{\psi\mathrm{loop}}f_s\boxed{m_j} + 4m_{\psi\mathrm{loop}}f_s^\dagger\boxed{m_i}\Big)$$

$$\left[\left(m_i^2 - m_j^2\right)\delta Z^A_{Lji} - 2m_j\delta m^L_{ji} - 2m_i\delta m^R_{ji}\right]^{\mathrm{tad.}}_{\mathrm{div.}} = \frac{1}{D-4}\Big(4m^3_{\psi\mathrm{loop}}f_T\boxed{m_j} + 4m^3_{\psi\mathrm{loop}}f_T^\dagger\boxed{m_i}\Big)$$

- Only $(m_i^2 + m_j^2)$, $2m_im_j$, m_i and m_j on the r.h.s and no $m_i^2 - m_j^2$ mass structure!
  - Dimensionless mass counterterms would give the additional structures on l.h.s.

$\Longrightarrow$ No UV divergent terms with $\boxed{m_i^2 - m_j^2}$ factors!

- Gauge dependence always comes with $\boxed{m_i^2 - m_j^2}$ factors
  - Gauge-*independent* terms may also have this structure
  - All the other mass structures are gauge-independent
- No UV divergences with $\boxed{m_i^2 - m_j^2}$ factors
- UV divergences accompanied by $m_i^2 + m_j^2$, $2m_i m_j$, m_j, and m_i mass structures

Definitions: Field renormalization

Define the anti-hermitian part of field renormalization as the coefficient of $\boxed{m_i^2 - m_j^2}$ $(i \neq j)$

$$(m_i^2 - m_j^2)\delta Z^A_{Lji} = -\left[m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2) + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2) + H.C.\right]_{(m_i^2 - m_j^2)}$$

$$(m_i^2 - m_j^2)\delta Z^A_{Rji} = -\left[m_i^2 \Sigma^R_{ji}(m_i^2) + m_i m_j \Sigma^L_{ji}(m_i^2) + m_j \Sigma^{sR}_{ji}(m_i^2) + m_i \Sigma^{sL}_{ji}(m_i^2) + H.C.\right]_{(m_i^2 - m_j^2)}$$

- Only *finite* terms consisting of Log's (and Disc functions)!
- Contains all possible gauge dependence!
- Universal definition in terms of self-energies and restrictions
- The hermitian part is unchanged w.r.t. the usual approach

Definitions: Mass renormalization

Having defined field renormalization we SOLVE for $\delta m^{L,R}$ $(i \neq j)$!

$$\delta m^L_{ji} = \frac{1}{2}\left(m_i \Sigma^R_{ji}(m_i^2) + \Sigma^{sL}_{ji}(m_i^2) + m_j \Sigma^{L\dagger}_{ji}(m_j^2) + \Sigma^{sR\dagger}_{ji}(m_j^2)\right) + \frac{1}{2}\left(m_i \delta Z^A_{Rji} - m_j \delta Z^A_{Lji}\right)$$
$$\delta m^R_{ji} = \frac{1}{2}\left(m_i \Sigma^L_{ji}(m_i^2) + \Sigma^{sR}_{ji}(m_i^2) + m_j \Sigma^{R\dagger}_{ji}(m_j^2) + \Sigma^{sL\dagger}_{ji}(m_j^2)\right) + \frac{1}{2}\left(m_i \delta Z^A_{Lji} - m_j \delta Z^A_{Rji}\right)$$

- Gauge dependence canceled by field renormalization!
- Hermiticity relation $\left(\delta m^L\right)^\dagger = \delta m^R$ holds by construction!
- Universal expression in terms of self-energies
- No $\widetilde{\mathrm{Re}}$ or Re
  - Exceptions for Majorana particles
- Can ***extend the real part to the diagonal***, then $\mathrm{Re}\left(\delta m^L_{ii}\right) = \mathrm{Re}\left(\delta m^R_{ii}\right)$ and also matches the results in [2]

The *Need* to Renormalize CKM?

- In [3] $\delta\tilde{Z}_L^A$ (no δm's) is used to define the SM CKM counterterm

$$\delta V \sim -\delta\tilde{Z}_L^{A,u} V + V\delta\tilde{Z}_L^{A,d}$$

- Argued on the basis of the UV divergent $W\bar{u}d$ vertex

But now $\delta Z_{L(R)}^A$ is ***finite!***

$\Longrightarrow$ There is *no need* for a CKM counterterm on the basis of finiteness!

- *Need* to define *off-diagonal* δm — otherwise *double counting of UV divergences*

Conclusions

We defined a new fermion renormalization scheme that

- ✓ Is *universal*
- ✓ Relies on (incoming) no-mixing conditions and *mass structures*
- ✓ Does *not rely on dropping the absorptive parts*
- ✓ Has *gauge–independent* mass counterterms
- ✓ Has *finite* anti-hermitian part of field counterterms
- ✓ Avoids *double counting* of divergences and keeps the Lagrangian Hermitian

Also

- ✓ There is no need to define the CKM counterterm on the basis of UV divergences!

Not in the presentation...

- Masless particles and radiative masses, explicit computations in the Grimus-Neufeld model...

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BACKUP

Lightning Fast Intro to the Grimus-Neufeld Model

The Grimus-Neufeld model [11, 12, 13, 14] is the SM extended with

- Heavy Singlet Majorana neutrino $\Longrightarrow$ seesaw mechanism

$$\begin{pmatrix} 0_{3\times 3} & M_D \\ M_D^T & M_R \end{pmatrix} \Longrightarrow \operatorname{diag}(0, 0, m_3, m_4)$$

- Second Higgs doublet $\Longrightarrow$ radiative mass
  - General CP conserving THDM potential + Higgs basis
  - Additional physical Higgs particles: H, A, $H^\pm$
  - Yukawa couplings to the second doublet $G_{u,d,l}$ and G_ν
    - For neutrinos $(Y_\nu)_i \bar{N}.(L_i \tilde{H}_1) + (G_\nu)_i \bar{N}.(L_i \tilde{H}_2)$

Main features:

- 2 massless neutrinos at tree-level
- 1 heavy and 1 light neutrino at tree-level
- Radiative mass for 1 massless tree-level neutrino at 1-loop

Majorana Particles

- The *bare* Majorana condition $\nu = \nu^c$ at 1-loop implies

$$\nu = \nu_L + \nu_L^c \quad \Longrightarrow \quad Z_L \nu_L + Z_R \nu_L^c \quad \Longrightarrow \quad \delta Z_L = \delta Z_R^\star$$

- *But this does not hold due to absorptive parts*

$$\boxed{\delta Z_L \neq \delta Z_R^\star}$$

  - The same problem of overspecification
- The *Majorana condition does not hold* if no-mixing is imposed on incoming(outgoing) particles *at 1-loop*
- Treat as Dirac particles beyond tree-level

$\Longrightarrow$ ***bi**-unitary 1-loop rotation*

Massless Particles and Radiative Masses I

How to use the mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} ? & ? \\ \hline ? & \begin{array}{c} h, \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark \text{ out of the box } \checkmark \end{array} \end{array}\right)$$

How to use the mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \text{diag}\left(0, 0, m_3, m_4\right)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{matrix}\delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}\left(0\right)\end{matrix} & h, \delta Z, \delta m, m_i \neq 0 \\ \hline h, \delta Z, \delta m, m_j \neq 0 & \begin{matrix} h, \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark \text{ out of the box } \checkmark \end{matrix} \end{array}\right)$$

$$\delta m^L_{ji} = \frac{1}{2}\left(m_i \Sigma^R_{ji}(m_i^2) + \Sigma^{sL}_{ji}(m_i^2) + m_j \Sigma^{L\dagger}_{ji}(m_j^2) + \Sigma^{sR\dagger}_{ji}(m_j^2)\right) + \frac{1}{2}\boxed{m_i \delta Z^A_{Rji} - m_j \delta Z^A_{Lji}}$$

Massless Particles and Radiative Masses III

How to use mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \begin{array}{c} \delta m, m_{j,i} = 0 \\ \Sigma^{sL,sR}(0) \Leftrightarrow \not{m}\ \text{limit} \end{array} & \begin{array}{c} \Leftarrow \\ h, \delta Z, \delta m, m_i \neq 0 \\ \not{m_j}\ \text{limit} \end{array} \\ \hline \begin{array}{c} \Uparrow \\ h, \delta Z, \delta m, m_j \neq 0 \\ \not{m_i}\ \text{limit} \end{array} & \begin{array}{c} \Leftarrow\Uparrow \\ h, \delta Z, \delta m, m_{j,i} \neq 0 \\ \checkmark\ \text{out of the box}\ \checkmark \end{array} \end{array}\right)$$

$$\left(m_i^2 - m_j^2\right) \delta Z^A_{Lji} = -\left[m_i^2 \Sigma^L_{ji}(m_i^2) + m_i m_j \Sigma^R_{ji}(m_i^2) + m_j \Sigma^{sL}_{ji}(m_i^2) + m_i \Sigma^{sR}_{ji}(m_i^2) + H.C.\right]_{(m_i^2 - m_j^2)}$$

How to use mass structures if particles are massless?

- Say we have 4 particles, two of which are massless at tree level, i.e. $m = \mathrm{diag}\,(0, 0, m_3, m_4)$
- Then at 1-loop

$$m_{ji} + \delta m_{ji} \sim \left(\begin{array}{c|c} \delta m, m_{j,i} = 0 & \\ \Sigma^{sL,sR}(0) & \cdots \\ \hline & \\ \cdots & \cdots \end{array}\right)$$

- No 1-loop $\implies$ $\Sigma^{sL,sR}(0)$ diagonalized with leftover freedom from the tree-level

$\implies$ *Radiative mass!* [11]

Tips & Tricks I

- Might find gauge-dependent terms with incorrect mass structures in some steps, e.g. for up-quarks

$$\left((m_i^u)^2 - (m_j^u)^2\right) \delta Z_{Lji}^{A,u} \subset \frac{V_{jk} V_{ik}^\star}{2^D \pi^{D-2} v^2} \left((m_i^u)^2 + (m_j^u)^2\right) A_0(\xi_W m_W^2)$$

- However, this term vanishes for $i \neq j$ due to unitarity of the CKM matrix V
- Nielsen identities guarantee that terms like these vanish

Tips & Tricks II

- Might find gauge-independent terms with the mass structure $\left(m_i^2 - m_j^2\right)$— this is *not* forbidden by Nielsen identities
- In the Grimus-Neufeld model neutrino interaction with the SM Higgs is not diagonal
  - SM Higgs is from the same doublet as are the Goldstone bosons
  - Gauge dependence comes with the first Higgs doublet
- $\Longrightarrow$ SM Higgs terms can carry $\left(m_i^2 - m_j^2\right)$

Tips & Tricks III

- Gauge-independent terms with $\left(m_i^2 - m_j^2\right)$ mass structure can sometimes intertwine with $\left(m_i^2 + m_j^2\right)$
- It is simple to write the following

$$am_i^2 + bm_j^2 = \frac{a+b}{2}\left(m_i^2 + m_j^2\right) + \frac{a-b}{2}\left(m_i^2 - m_j^2\right)$$

- Then it is possible to define δZ^A
  - This is the case for neutrinos