Role of meson spectroscopy in unfolding the character of EHM in the SM

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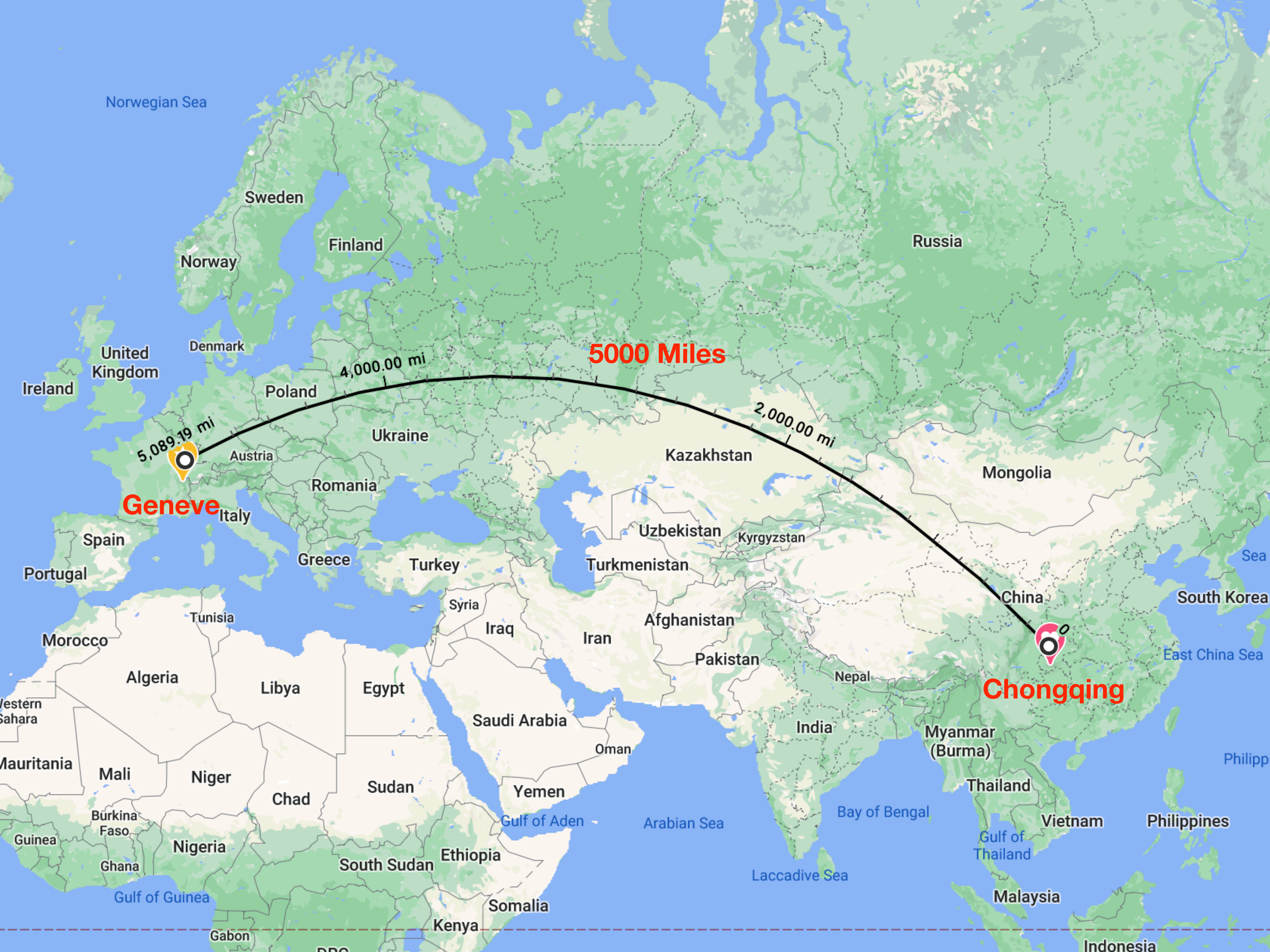

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Norwegian Sea
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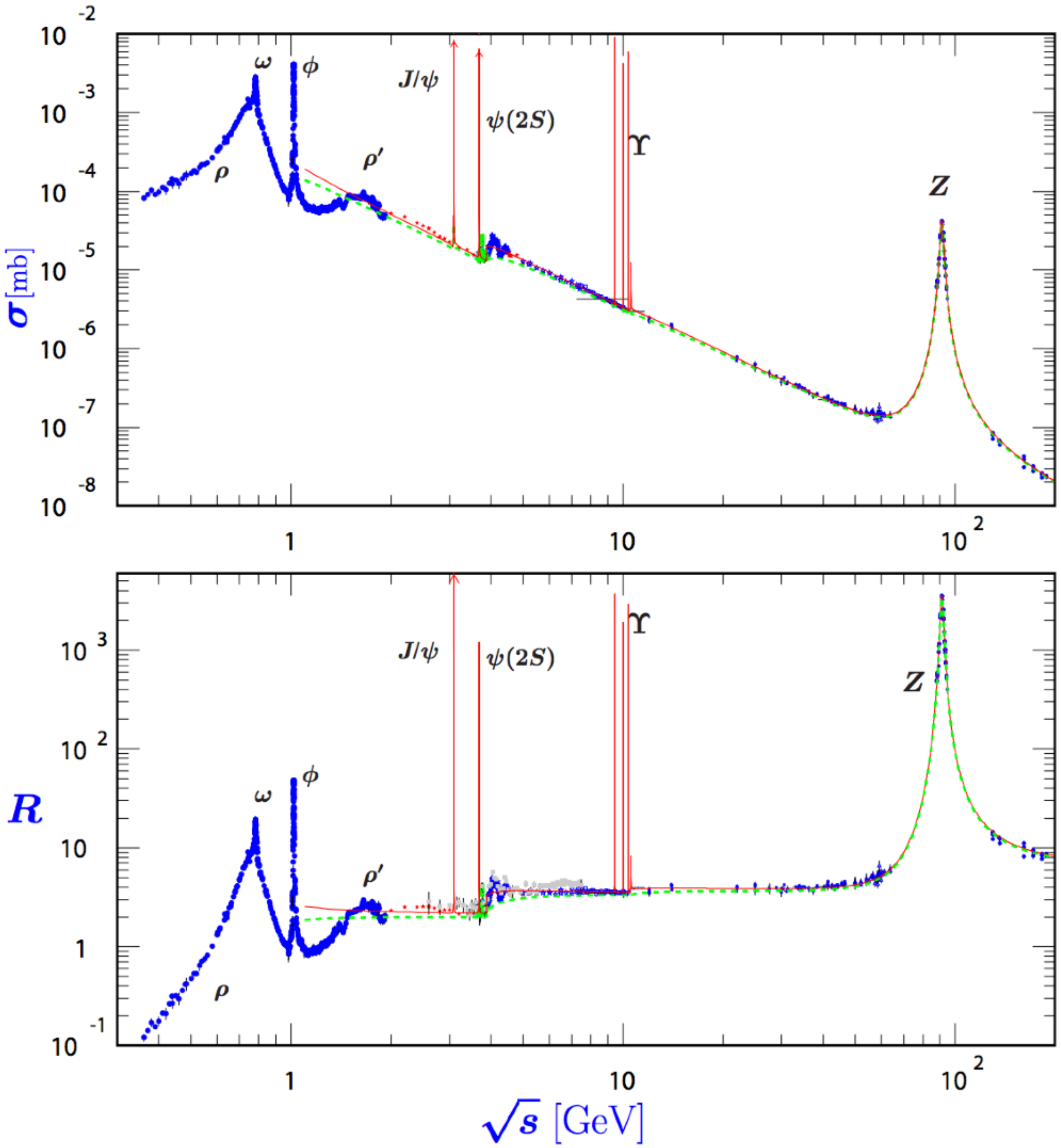
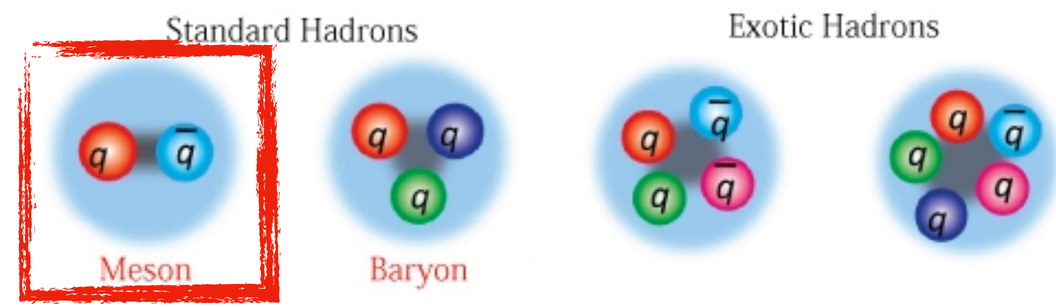
Italy
Spain
Portugal
Greece
Turkey
Uzbekistan
Kyrgyzstan
Turkmenistan
China
South Korea
Sea
Syria
Iraq
Iran
Afghanistan
Pakistan
Nepal
East China Sea
Tunisia
Morocco
Algeria
Libya
Egypt
Saudi Arabia
Oman
India
Myanmar (Burma)
Philipp
Mali
Niger
Chad
Sudan
Yemen
Thailand
Burkina Faso
Gulf of Aden
Arabian Sea
Bay of Bengal
Vietnam
Philippines
Guinea
Nigeria
Ethiopia
Gulf of Thailand
Ghana
South Sudan
Laccadive Sea
Gulf of Guinea
Somalia
Malaysia
Kenya
Gabon
Indonesia

Background: Hadrons as QCD bound-states



Cross section of $e^+ e^-$ hadronic annihilation



Quantum Field Theory

Feynman amplitudes

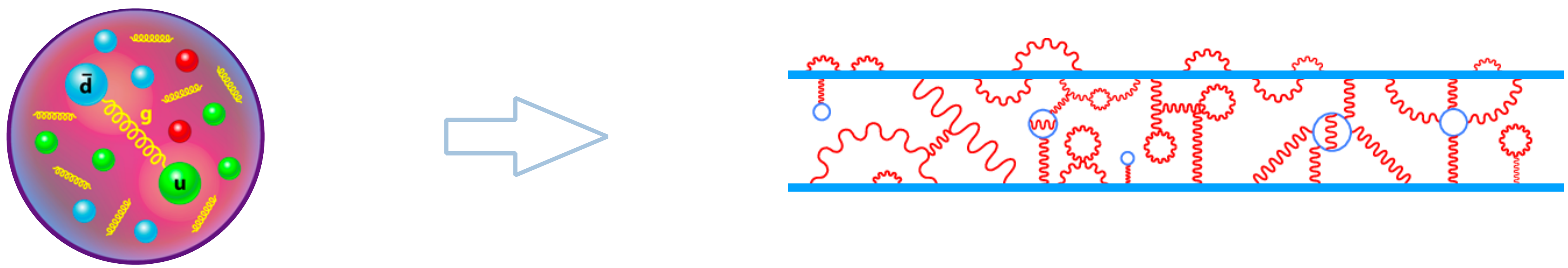
↓

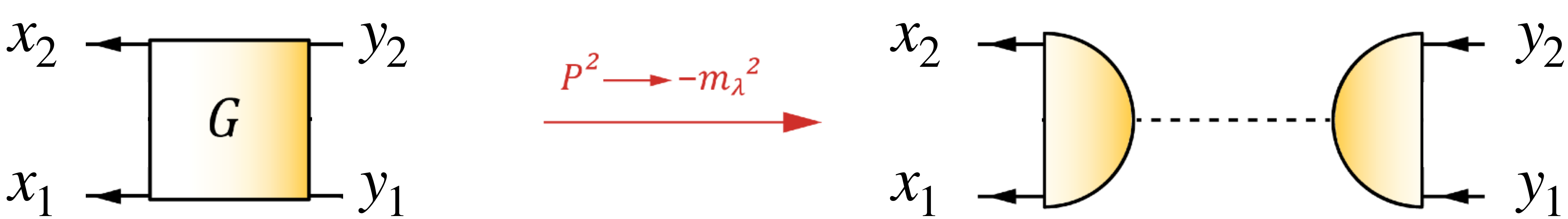
Green functions

↓

Bound-state equations

Background: Mesons as two-body bound-states

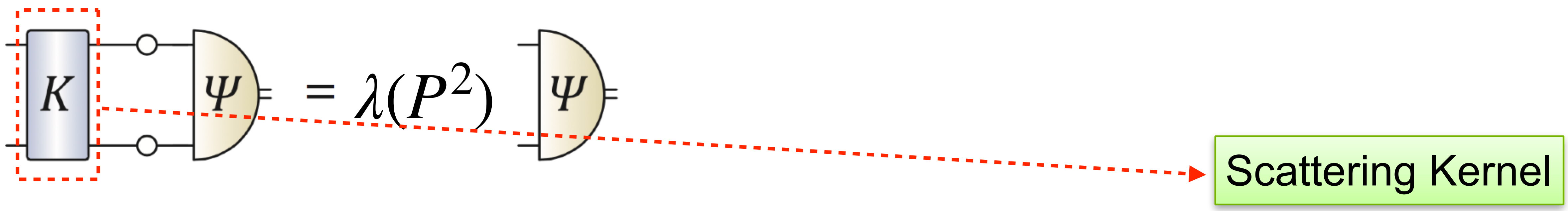




$$G^{(4)}(x_1, x_2; y_1, y_2) = \langle \Omega \,|\, \bar{q}(x_1) q(x_2) \bar{q}(y_1) q(y_2) \,|\, \Omega \rangle$$

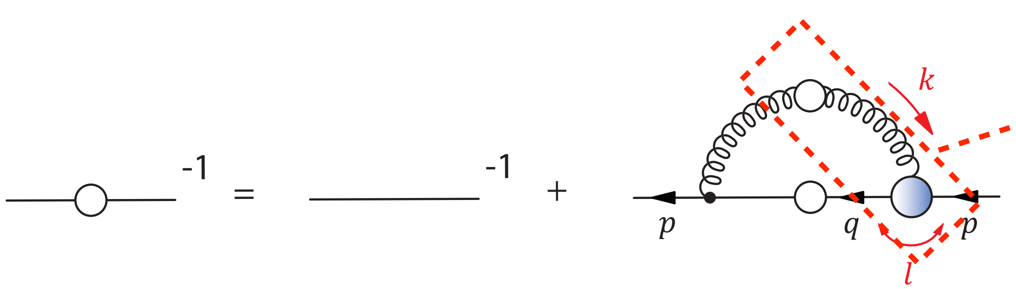
Background: Bethe-Salpeter equation for mesons

- **Two-body Bethe-Salpeter equation**

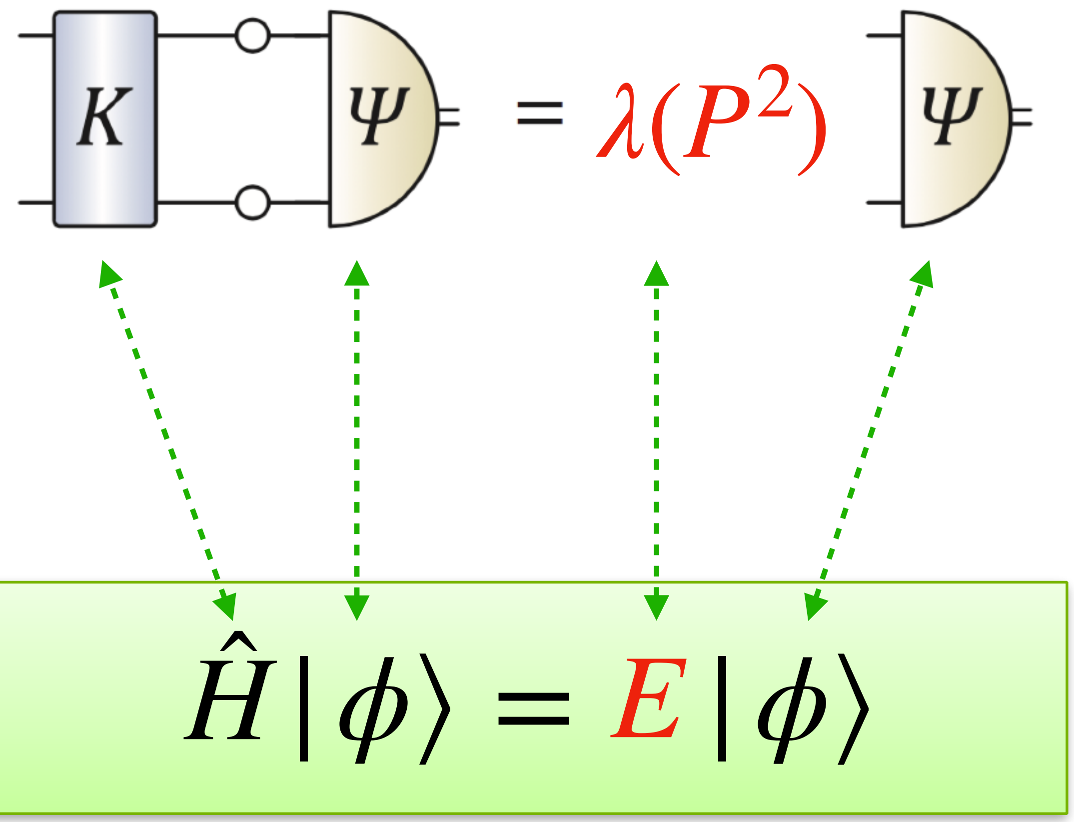


Interaction Kernel

- **One-body gap equation**



Background: Bethe-Salpeter equation for mesons



- ✦ The kernel (or the Hamiltonian) must respects all QCD's symmetries.
- ✦ Quarks are relativistic; and infinitely many virtual quarks are created.

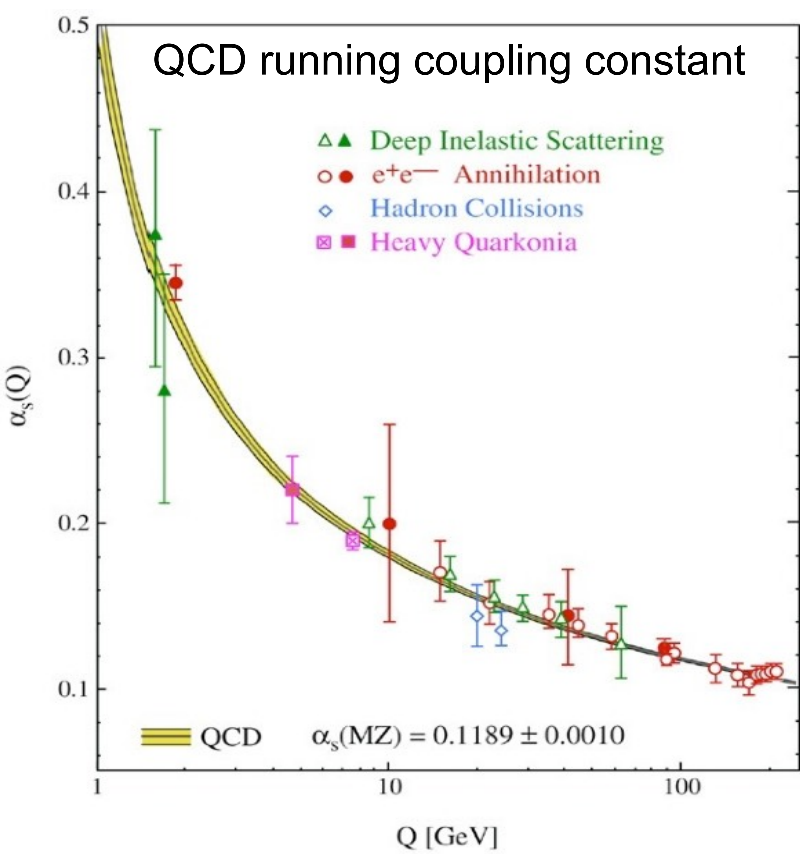
Background: Bethe-Salpeter equation for mesons

- **Relativistic bound states**

"These problems are those involving bound states […] such problems necessarily involve a breakdown of ordinary perturbation theory. […] The pole therefore can only arise from a divergence of the sum of all diagrams […]"

The QFT book vol1 p564 Weinberg

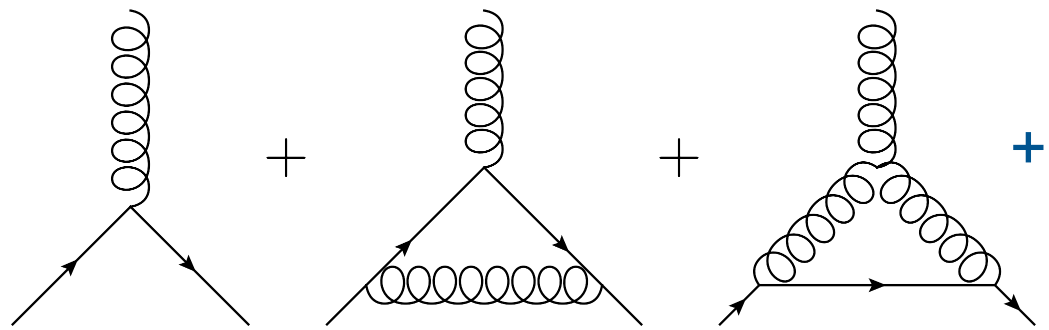
- **Strongly coupled systems**



- **Asymptotic Freedom**: Bonds between particles become asymptotically weaker as energy increases and distance decreases (Solved, Nobel Prize).
- **Color Confinement**: No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon (Millennium Problems).
- **Dynamical Chiral Symmetry Breaking**: Mystery of bound state masses, e.g., current quark mass (Higgs) is small, and no degeneracy between *parity partners.*

Development (i): DCSB in the interaction kernel

Quark-gluon vertex:



+ … = ?

"Symmetry dictates interaction."

Spacetime

- ❑ **Poincaré symmetry**

Fields

- ❑ **Gauge symmetry**
- ❑ **Chiral symmetry**

Development (i): DCSB in the interaction kernel

- **Gauge symmetry:** Vector WGTI

$$i q_\mu \Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)$$

$\nabla \cdot \Phi$

- **Chiral symmetry:** Axial-vector WGTI

$$q_\mu \Gamma^A_\mu(k,q) = S^{-1}(k) i\gamma_5 + i\gamma_5 S^{-1}(p) - 2im\Gamma_5(k,p)$$

- **Lorentz symmetry + :** Transverse WGTIs

$\nabla \times \Phi$

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \epsilon_{\lambda\mu\nu\rho} \Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p)\,, \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = {} & S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ & + t_\lambda \epsilon_{\lambda\mu\nu\rho} \Gamma_\rho(k,p) \\ & + V^A_{\mu\nu}(k,p)\,, \qquad \sigma^5_{\mu\nu} = \sigma_{\mu\nu}\gamma_5 \end{aligned}$$

✦ The WGTIs express the curls and divergences of the vertices.

✦ The WGTIs of the vertices in different channels couple together.

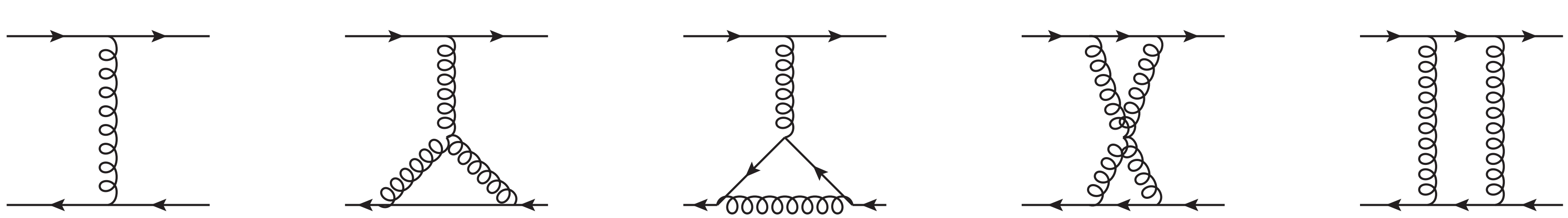
✦ The WGTIs involve contributions from high-order Green functions.

$$\Gamma_\mu(k,q) \sim \Delta_B(k^2, q^2)$$

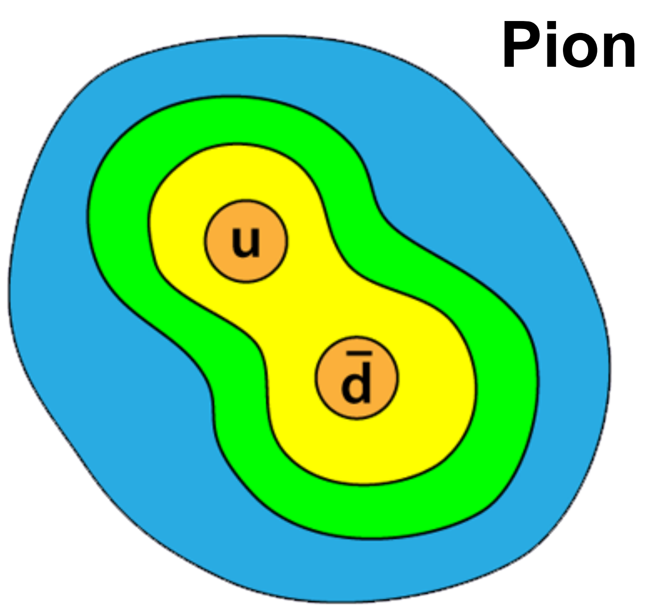
Interaction → **DCSB**

See, e.g., PLB722, 384 (2013)

Development (ii): Symmetries of the scattering kernel



+ … = ?



Bound state of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

Goldstone's theorem: If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

Development (ii): Symmetries of the scattering kernel

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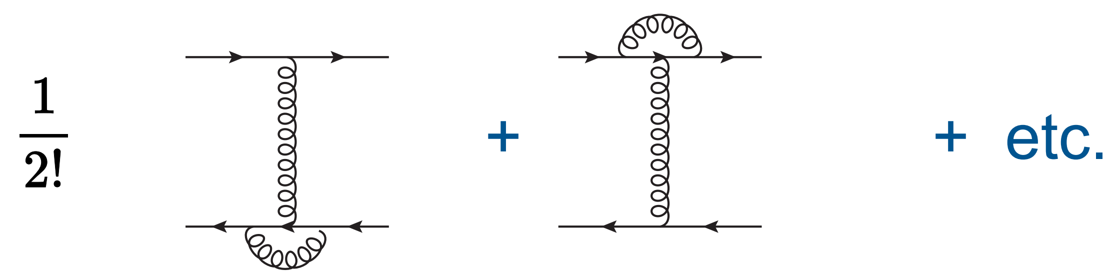
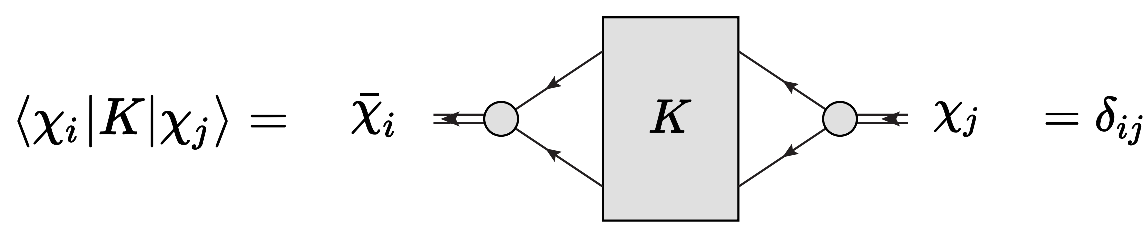
✦ **Permutation:** $\mathscr{P}\,\mathcal{K}(q_\pm, k_\pm) = \mathcal{K}^*(q_\pm, k_\pm) = C\,K_R^\mu(-q_\mp, -k_\mp)\,C^{-1} \otimes C\,K_L^\mu(-q_\mp, -k_\mp)\,C^{-1}$



✦ **Charge-conjugation:** $\mathsf{C}\,\mathcal{K}(q_\pm, k_\pm) = \overline{\mathcal{K}}(q_\pm, k_\pm) = C\,K_L^\mu(-k_\pm, -q_\pm)^T\,C^{-1} \otimes C\,K_R^\mu(-k_\pm, -q_\pm)^T\,C^{-1}$



✦ **P and T symmetries:** $\mathsf{P}\,\mathcal{K}(q_\pm, k_\pm) = \widehat{\mathcal{K}}(q_\pm, k_\pm) = P\,K_L^\mu(q_\pm, k_\pm)\,P^{-1} \otimes P\,K_R^\mu(q_\pm, k_\pm)\,P^{-1}$

$$K = \mathbf{1}\otimes\mathbf{1} + \gamma_5\otimes\gamma_5 + \mathbf{1}\otimes\gamma_5 + \gamma_5\otimes\mathbf{1}$$

Lorentz covariance guarantees CPT-symmetry; T-symmetry is obtained for free.

Development (ii): Symmetries of the scattering kernel

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The Bethe-Salpeter equation and the quark gap equation are written as

$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'},$$

$$S^{-1}(k) = S_0^{-1}(k) + \int_q D_{\mu\nu}(k-q)\gamma_\mu S(q)\Gamma_\nu(q,k),$$

The color-singlet axial-vector and vector WGTIs are written as

$$P_\mu\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-),$$

$$iP_\mu\Gamma_\mu(k,P) = S^{-1}(k_+) - S^{-1}(k_-).$$

The derived WGTIs between the scattering kernel and the interaction kernel:

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)],$$

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)].$$

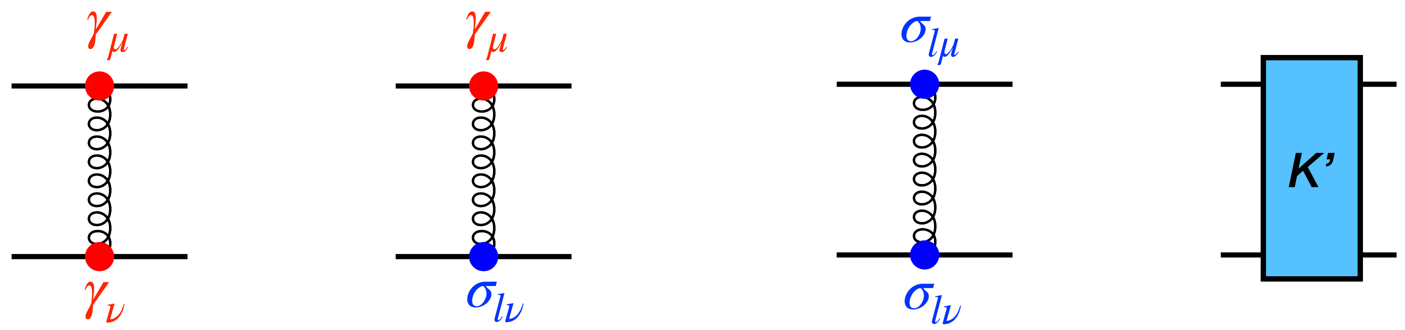
Development (ii): Symmetries of the scattering kernel

✦ A deep connection between one-body and two-body problem:

$$f_\pi E_\pi(k^2) = B(k^2)$$

- **Pion** exists if, and only if, the **quark mass** is dynamically generated.
- **Two-body problem** solved, almost completely, once solution of **one-body** problem is known.

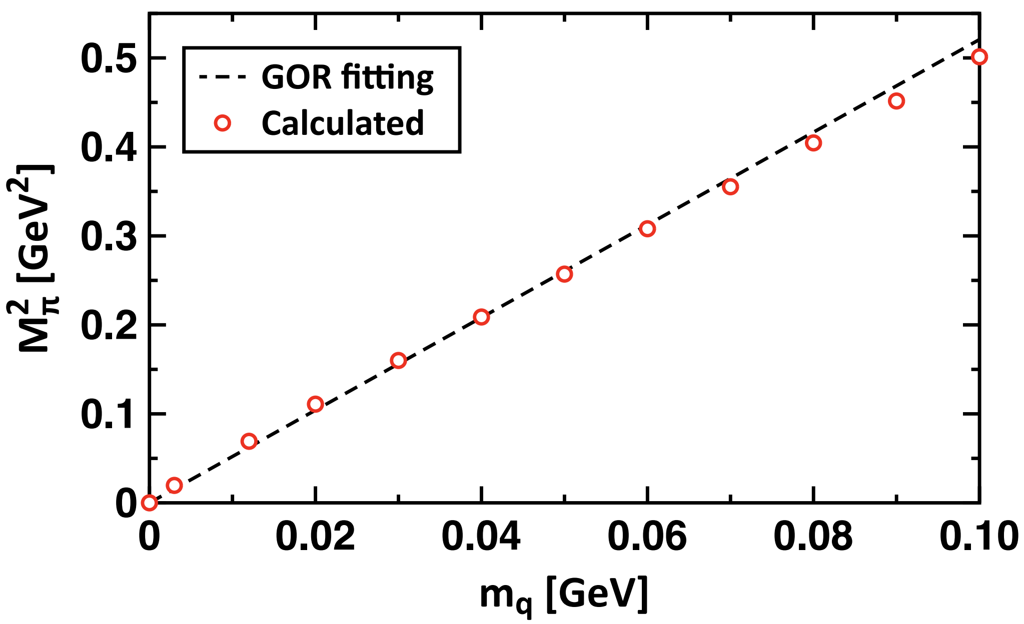
✦ A minimal kernel involves the Dirac terms and the Pauli terms:



See, e.g., arXiv:2009.13637 (2020)

Results: Meson spectroscopy

✦ Gell-Mann-Oakes-Renner relation:



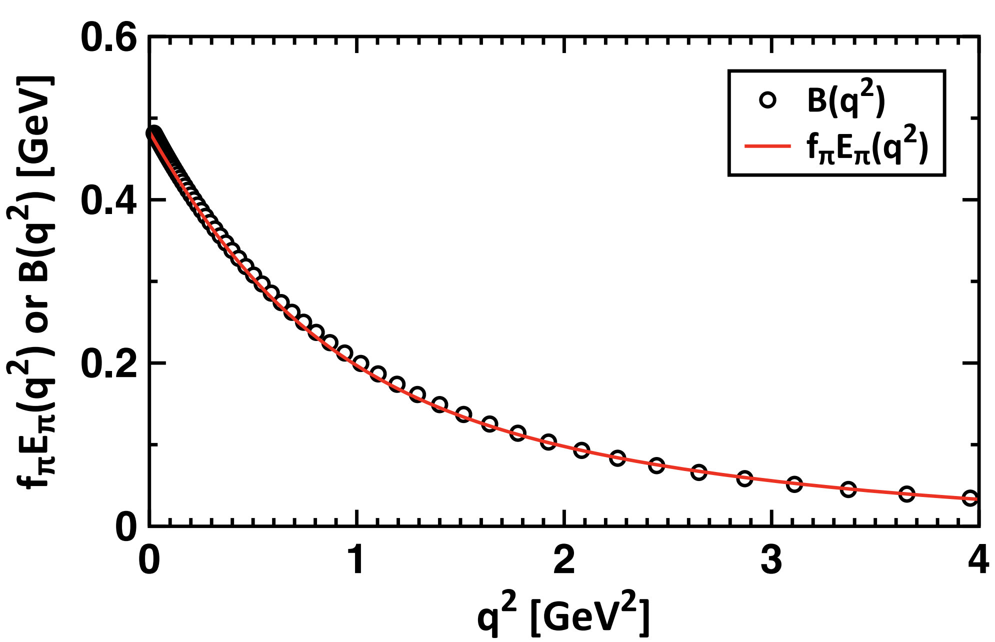
✦ The square of pion mass is proportional to the quark current mass:

$$\text{quadratic}: \quad m_\pi^2 = m \times 5.40(1 - 0.077\, m/m_{\rm m}) \,,$$
$$\text{linear}: \quad m_\pi^2 = m \times 5.07 \,,$$

where the extracted chiral condensate:

$$\text{quadratic}: \quad -\langle \bar{q}q \rangle = (0.286\,\text{GeV})^3,$$
$$\text{linear}: \quad -\langle \bar{q}q \rangle = (0.280\,\text{GeV})^3.$$

✦ Goldberger-Treiman relation:



✦ In the chiral limit, the mass function is proportional to the BSA:

$$f_\pi^0 E_\pi^0(k^2; P^2 = 0) = B_0(k^2)$$

where the normalized BSA:

$$\Gamma_\pi(k;P) = \gamma_5 \left[iE_\pi(k;P) + \gamma \cdot P F_\pi(k;P) \right.$$
$$\left. + \gamma \cdot k G_\pi(k;P) + \sigma_{kP} H_\pi(k;P) \right]$$

See, e.g., arXiv:2009.13637 (2020)

Results: Meson spectroscopy

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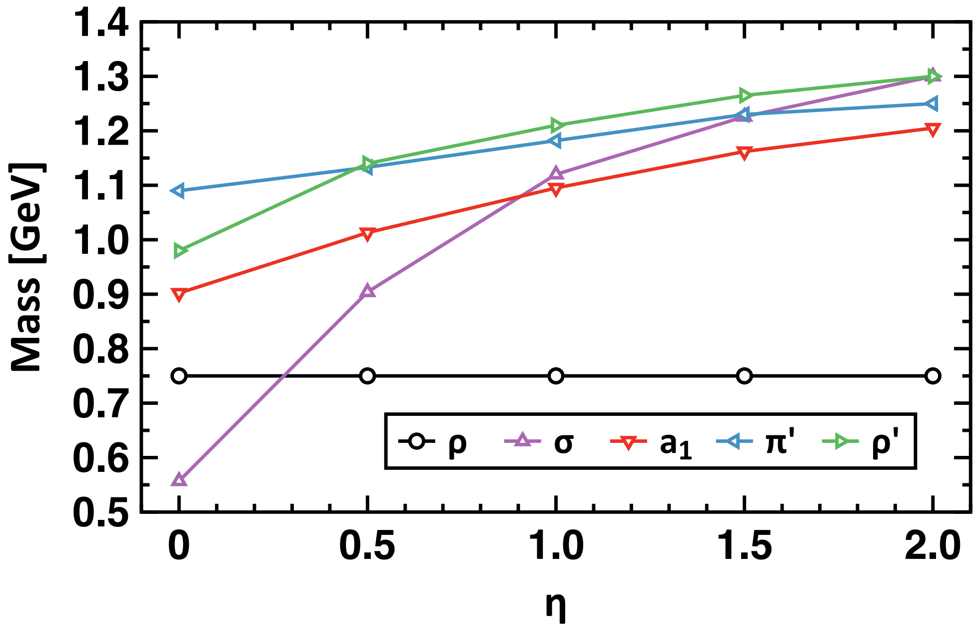
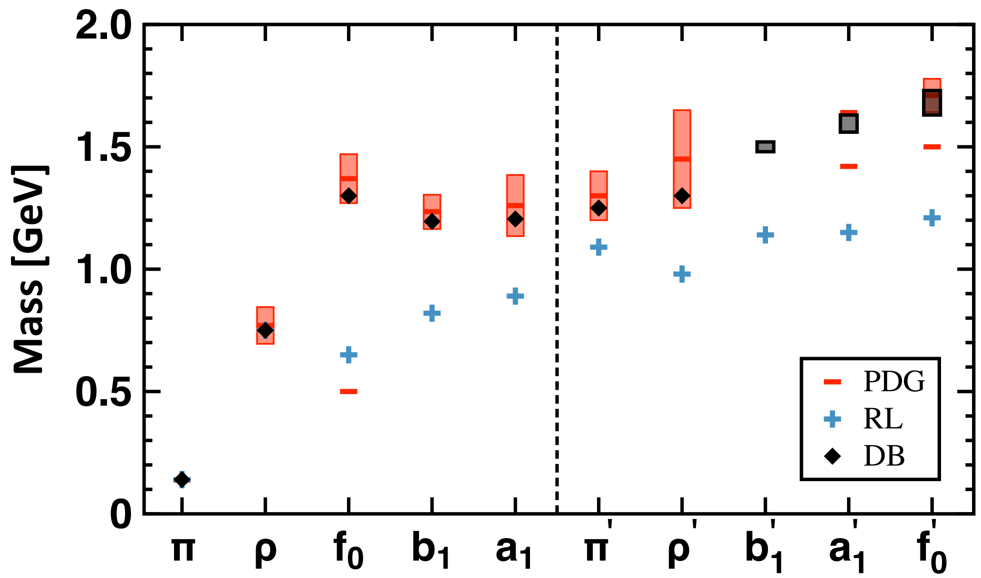
✦ Impact of the Pauli term (AM):



✦ Light-flavor meson spectrum:



✦ With increasing the AM strength, the a_1−ρ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced vertex increases spin-orbit repulsion.

✦ The spin-orbit boosted quark-core mass of the f_0 is greater than the empirical value, and matches an estimate the result obtained using chiral perturbation theory.

✦ The magnitude and ordering of radial excitation states are fixed with the DCSB-enhanced vertex.

See, e.g., arXiv:2009.13637 (2020)

Summary

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- **Quark-gluon vertex**: Solve the WGTIs resulting from the fundamental symmetries (gauge, chiral, and Lorentz symmetries). The vertex is significantly modified by DCSB feedback.

- **Scattering kernel**: Analyze discrete and continuous symmetries, i.e., color-singlet WGTIs. The kernel realizes pion's twofold role and produces full array of ground and excited mesons.

Outlook

- With the **sophisticated** approach, we can push it to a much wider range of applications in **two-body** (meson) and **three-body** (baryon) problems of QCD.

- Hopefully, based on more and more **successful applications**, we may provide a **faithful path** to understand QCD, and the **ultimate questions** may be addressed.

Backup

Development (ii): Symmetries of the scattering kernel

✦ In the chiral limit, the color-singlet axial-vector WGTI (**chiral symmetry**) is written as

$$\partial_\mu J^\mu = 0 \qquad P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

✦ Assuming **DCSB**, i.e., the mass function is nonzero, we have the following equation

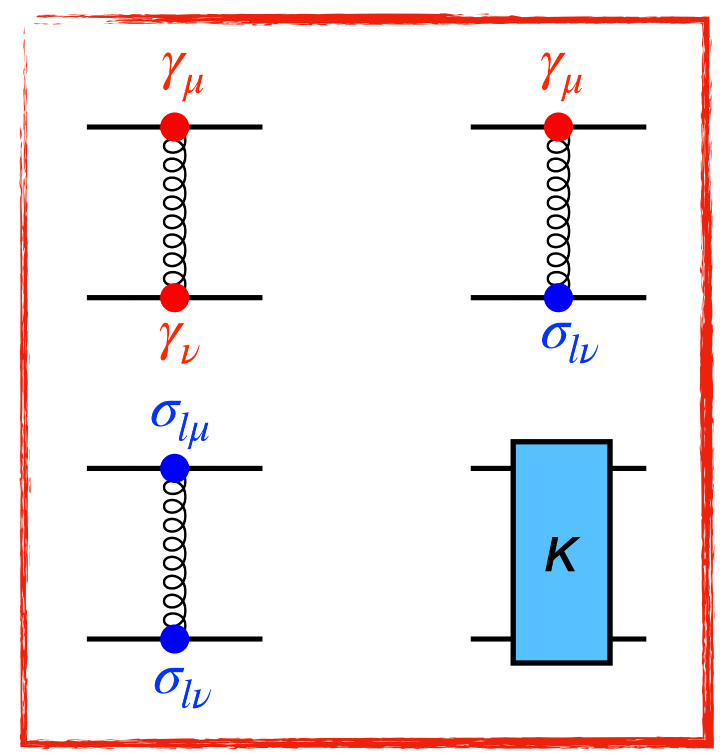
$$\lim_{P\to 0} P_\mu \Gamma_{5\mu}(k,P) = 2i\gamma_5 B(k^2) \neq 0$$

✦ The axial-vector vertex must involve a pseudo scalar pole (Goldstone's theorem)

$$\Gamma_{5\mu}(k,P) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2} \qquad f_\pi E_\pi(k^2) = B(k^2)$$

Pion exists if, and only if, **mass** is dynamically generated.

Two-body problem solved, almost completely, once solution of **one-body** problem is known.



See, e.g., PLB733, 202 (2014)