NANJING UNIVERSITY

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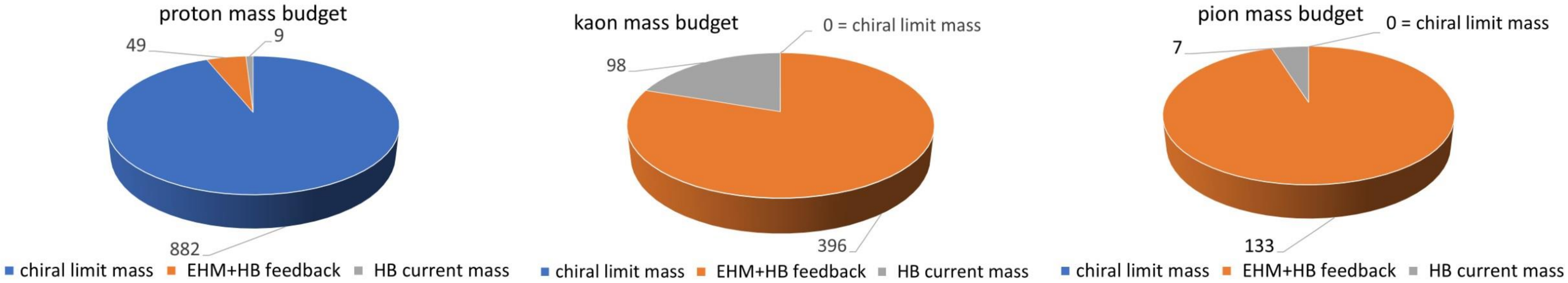
# On Mass and Matter

# Trace Anomaly

$$\Theta := T^{\rm QCD}_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu} + [1+\gamma(\alpha(\zeta))] \sum_{i=u,d,\ldots} m_i^\zeta \, \bar{q}_i q_i$$

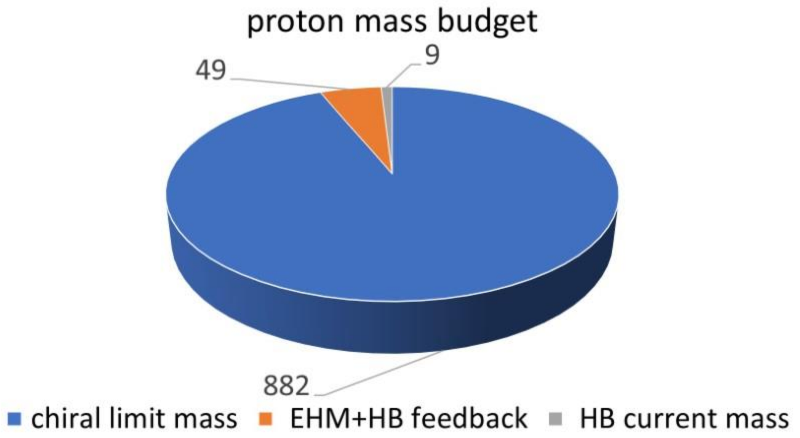
- It's not a miracle
- Straightforward consequence of quantum definition of interacting field theory
  - Electroweak theory has trace anomalies
  - Never talk about them because their consequences are expressed perturbatively
- The coupling runs if and only if there is a trace anomaly
- The masses and operators run if and only if the coupling runs
- Trace anomaly enables mass scales to emerge in the absence of a Higgs mechanism.
  - By itself, it doesn't guarantee such an outcome
- Magnitude of QCD trace anomaly in Nature is an empirical observable
  - Theory cannot predict it
  - But theory can elucidate its manifold connections, correlations, and manifestations

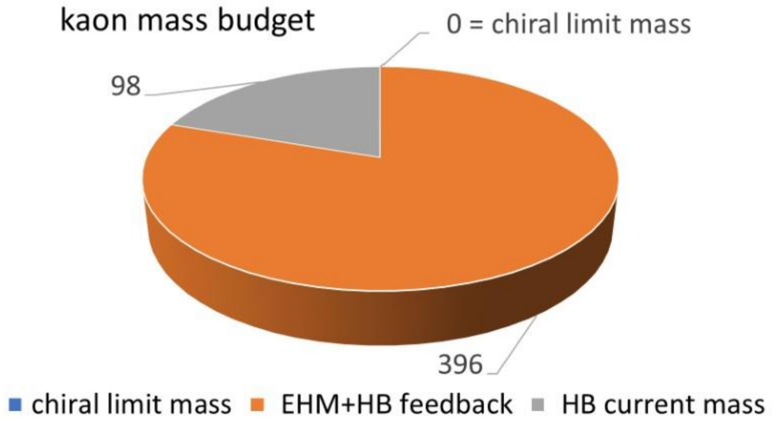
# Mass Budgets

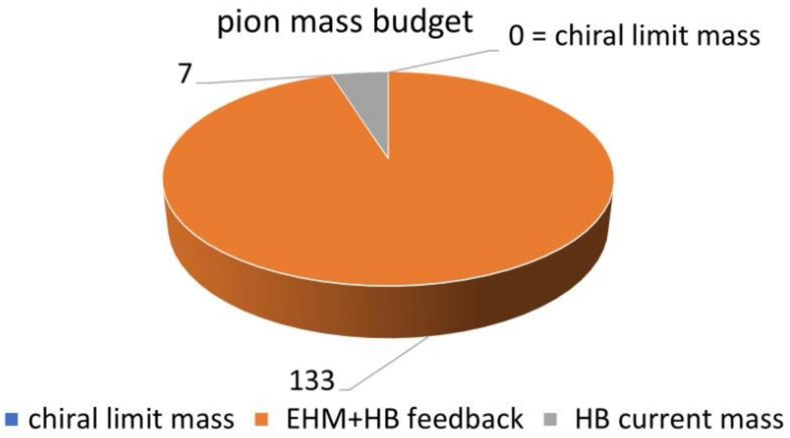



- Poincaré invariant and scale independent decompositions of masses of selected light-quark hadrons
- Proton
  - Higgs contribution
    - Renormalisation group invariant current-quark masses $\hat{m}_u \approx$ 4.5 MeV & $\hat{m}_d \approx 2\,\hat{m}_u$
    - Sum = 2% of proton mass
  - EHM+HB interference = constructive = 4% of proton mass
  - EHM alone = 94% of proton mass

# Mass Budgets

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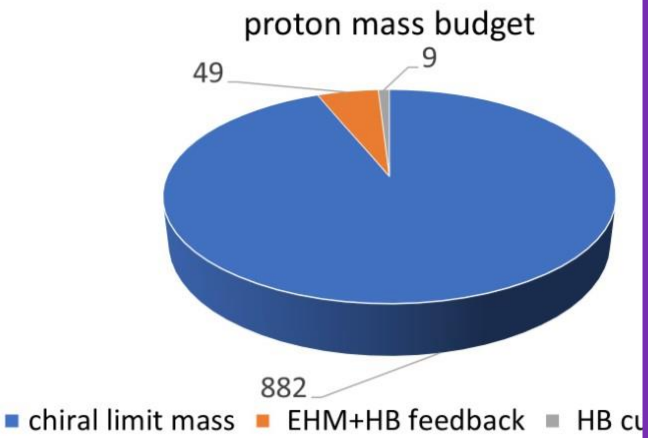
    Assessment and estimate of size from low-energy ChPT analyses
  - EHM+HB interference = constructive = 4% of proton mass

    Assessment and estimate from analysis of low-energy π N scattering ~ nucleon σ-term

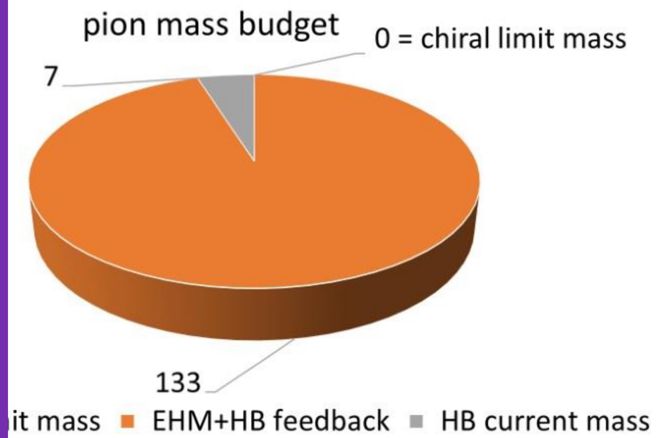
# Mass Budgets




Assuming validity of all assumptions needed to justify formula for charmonium production, then analysis of GlueX data

$\Rightarrow \varsigma_h = 0.07(17) =$ small

Confirms that bulk of nucleon mass is emergent = EHM



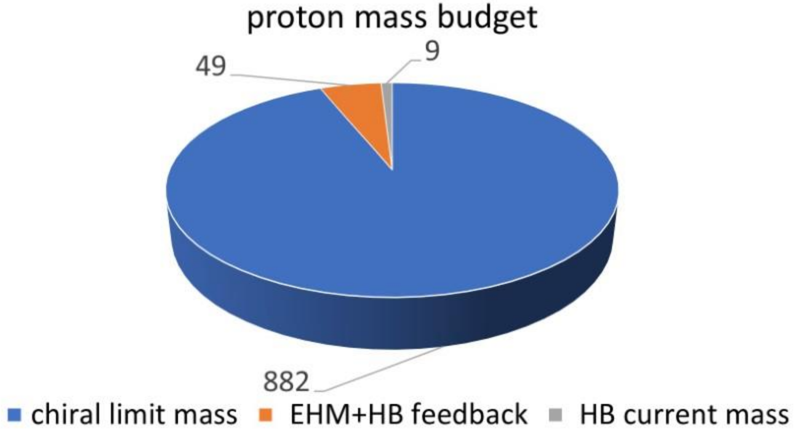

- Proton
  - EHM alone = 94% of proton mass
  - Heavy-meson production at threshold – operator product expansion and low-energy theorems,

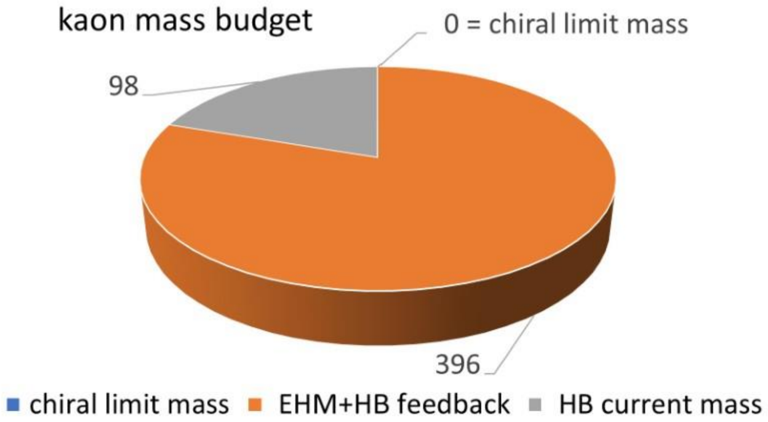
$$\mathcal{M}_{\Upsilon \mathrm{N}} \simeq 2 m_\Upsilon r_0^3 d_2 \frac{2\pi^2}{27} \left( 2m_N^2 - \left\langle N \middle| \sum_{i=u,d,s} m_i \bar{q}_i q_i \middle| N \right\rangle \right) \simeq 2 m_\Upsilon r_0^3 d_2 \frac{2\pi^2}{27} 2m_N^2 (1 - \varsigma_h)$$

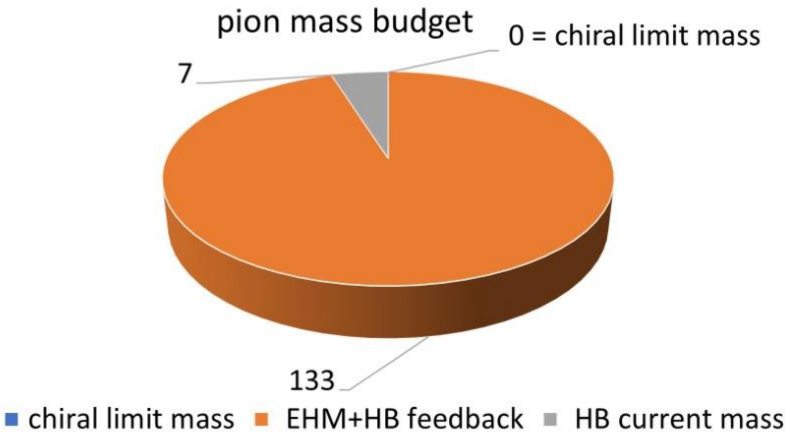
Such measurements do not provide news on the trace anomaly – instead, they may provide indirect information via the light-quark and s-quark in-nucleon σ-terms

# Mass Budgets

*How are these manifest differences expressed in parton distributions that characterise these hadrons?*




- Kaon – highlights Higgs modulation
  - Higgs contribution
    - Sum of current-quark mass = 20% of pion mass
  - EHM+HB interference = constructive = 80% of kaon mass
  - No Blue Zone because kaon, too, is NG mode ⇒ NO mass in chiral limit

- Pion
  - Higgs contribution
    - Sum of current-quark mass = 5% of pion mass
  - EHM+HB interference = constructive = 95% of pion mass
  - No Blue Zone because pion = NG mode ⇒ there is NO mass in chiral limit

# Trace Anomaly

$$\Theta := T_{\mu\mu}^{\rm QCD} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu} + [1+\gamma(\alpha(\zeta))] \sum_{i=u,d,\ldots} m_i^\zeta \, \bar{q}_i q_i$$

- Expressed in terms of the QCD Lagrangian degrees-of-freedom
- Poincaré invariant, but everything depends on the probe scale, ζ
- The trace anomaly is not homogeneous in the running coupling, α(ζ); so, renormalisation-group-invariance does not imply form invariance of the right-hand-side.
- Material point because
  - Many discussions implicitly assume that all operators and associated identities are expressed with reference to a partonic basis, *i.e*. using elementary field operators that can be renormalised perturbatively
  - In this basis, the state-vector for any hadron is an extremely complicated wave function = intractable problem in any approach based on parton-basis Fock space expansion

# Trace Anomaly

$$\Theta := T_{\mu\mu}^{\rm QCD} = \frac{1}{4}\beta(\alpha(\zeta))G_{\mu\nu}^{a}G_{\mu\nu}^{a} + [1+\gamma(\alpha(\zeta))] \sum_{i=u,d,\ldots} m_i^{\zeta}\, \bar{q}_i q_i$$

- Different perspective is required at probe scales $\zeta < m_p$
- Metamorphosis from parton-basis to quasiparticle-basis occurs. Under such reductions in $\zeta$,
  - Light partons evolve into heavy dressed objects, corresponding to complex and highly nonlinear superpositions of partonic operators
  - Using the associated quasiparticle operators, the wave functions can be expressed in a relatively simple form.

# Dressed-partons are quasiparticles … they are real

- Lane:1974he & Politzer:1976tv
- QCD has a renormalization group invariant running quark mass

$$M(-Q^2) \underset{Q^2\to\infty}{=} \frac{c}{Q^2}\left[\ln(Q^2/\Lambda_{QCD}^2)\right]^{d_M-1} + m\left[\frac{\ln(\mu^2/\Lambda_{QCD}^2)}{\ln(Q^2/\Lambda_{QCD}^2)}\right]^{d_M}$$

$$\frac{1}{Q^2} = \frac{1}{\Lambda_{QCD}^2} e^{-\pi d_M/\alpha(Q^2)}$$

- ✓ Essentially nonperturbative
- ✓ Nonzero in chiral limit
- ✓ "c" is the pseudoscalar projection of the pion wave function onto the origin – direct measure of EHM = chiral condensate
- ✓ Power-law × standard QFT scaling violation
- ✓ Inaccessible in perturbation theory

Essential singularity in the running coupling

- ✓ Now known to be origin of constituent-quark mass scale, introduced by Gell-Mann and Zweig

Higgs-induced
Runs as logarithm
Perturbative contribution can be computed loop-by-loop
BUT loop-by-loop always gives contribution proportional to "m"; so vanishes in chiral limit

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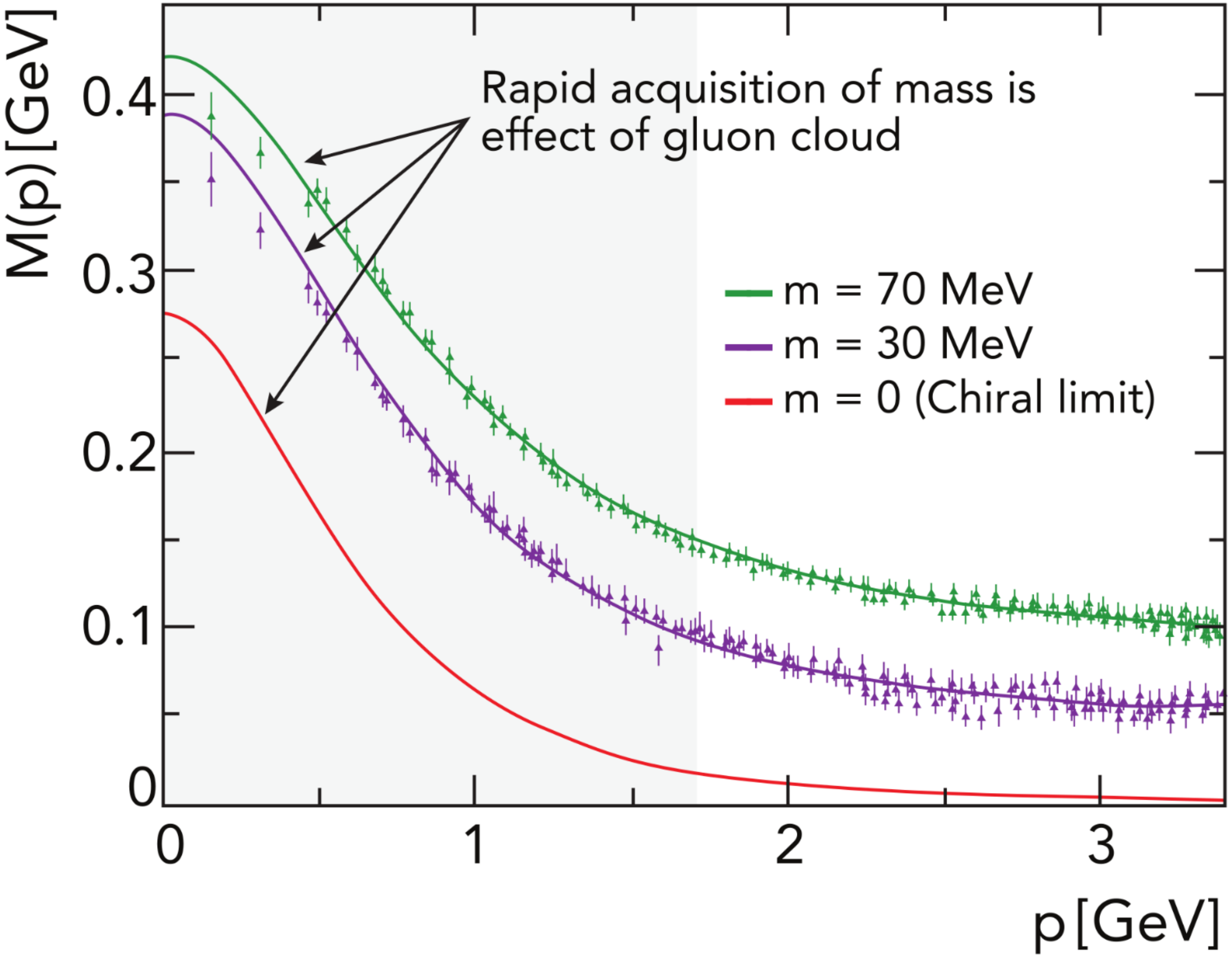


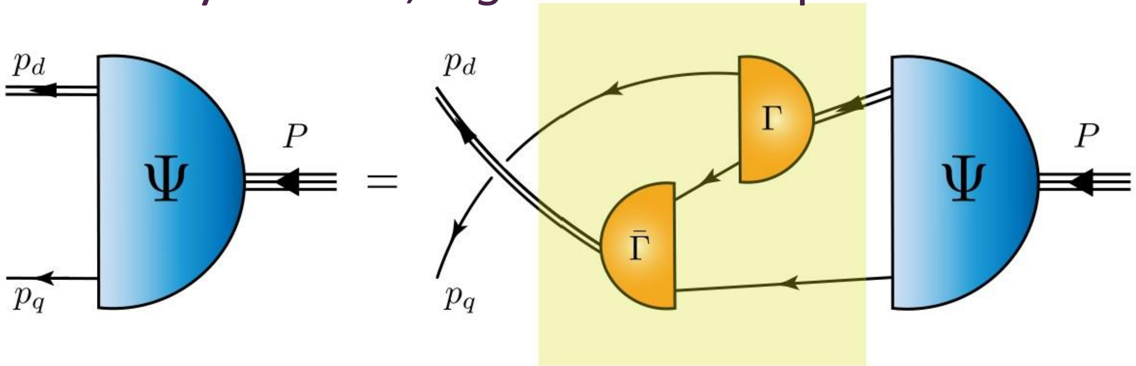
# Metamorphosis expressed in Schwinger function field equations

Parton basis $\langle p(P)|\Theta|p(P)\rangle \stackrel{\zeta\gg m_p}{=} \langle p(P)| \left[ \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu} + [1+\gamma(\alpha(\zeta))] \sum_{i=u,u,d,} m_i^\zeta\, \bar{q}_i q_i \right] |p(P)\rangle$

Quasiparticle basis $\stackrel{\zeta\lesssim m_p}{=} \langle \tilde{p}(P)| \left[\mathcal{D}_3 + I_3\right] |\tilde{p}(P)\rangle$

$$\mathcal{D}_3 = \sum_{i=u,u,d} M_f(\zeta)\bar{Q}_f(\zeta)Q_f(\zeta)\,, \quad I_3 = \frac{1}{4}\left[\beta(\alpha(\zeta))\mathcal{G}^a_{\mu\nu}(\zeta)\mathcal{G}^a_{\mu\nu}(\zeta)\right]_{2PI}$$

- Achieved mathematically by resumming infinite towers of diagrams in one-body, two-body, three-body sectors, *e.g*. Faddeev equation for the proton mass and wave function




*Merit of EHM resummed basis = the wave function can be calculated*

*Enables verifiable predictions to be made and connections to be drawn with other applicable computational schemes*

- Parton basis ⇒ *almost* all proton mass carried (somehow) by partonic/free-field gluons
- EHM-resummed basis ⇒ bulk of proton mass stored in dressed-quark mass-scale with residual inter-particle interactions providing the refinements

# Avoid a Mass Crisis

- Steer away from decompositions whose interpretations are observer (frame) specific and/or whose interpretations change “weirdly” with scale
  - *e.g.*, exists at least one decomposition and scheme in which gluons contribute more than 100% of the proton’s mass and quarks a negative amount
- Focus on the Poincaré invariant form factors of the stress-energy tensor in hadron states
  - Beware … like spin and orbital angular momentum, any attempt to decompose the form factors into contributions from different parton species is frame and scale dependent
  - OK, if one is using a mathematical tool that can draw reliable connections between the frames and scales
- Think before you leap

$$\langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

can only mean that pion is empty of glue in a theory that doesn’t produce a pion

- All bound states possess an interaction energy
- Rigorous mathematical statements in QCD reveal how Dyson and Bethe-Salpeter equations conspire to make a massless bound state from emergent massive constituent quasiparticles, themselves possessing a traceable, nonperturbative connection to partons in QCD’s Lagrangian