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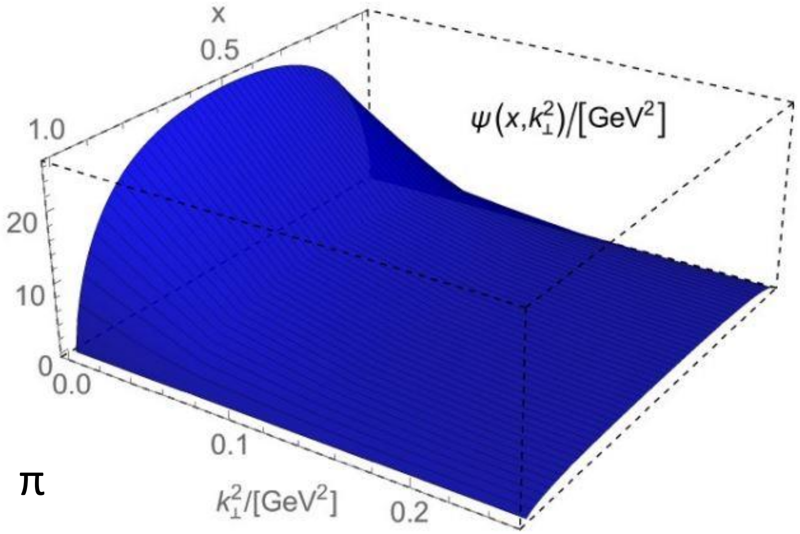
Access to DAs with AMBER?

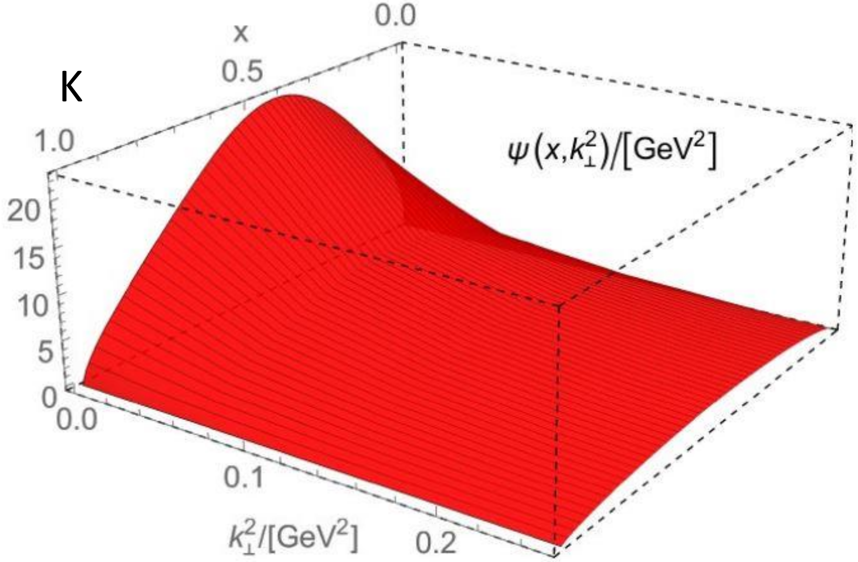
Light Front Wave Function

- Quantum mechanics ⇒ Wave Functions are the key
- Relativistic quantum field theory, many appealing features of Schrödinger wave functions are preserved if one works with the light-front projections of their covariant analogues
- Light Front Wave Function ⇒ overlaps preserve probability interpretation because particle number conserved within each Fock-space sector
- LFWF correlates all observables
- EHM is expressed in every hadron LFWF
- The “tricks” are
  - to find a way to compute the LFWF
  - then to find a way to “measure” it



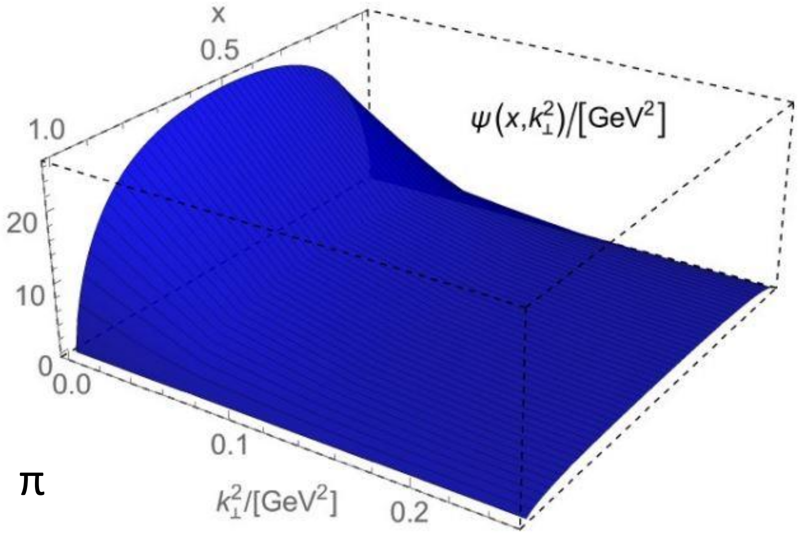
π

K

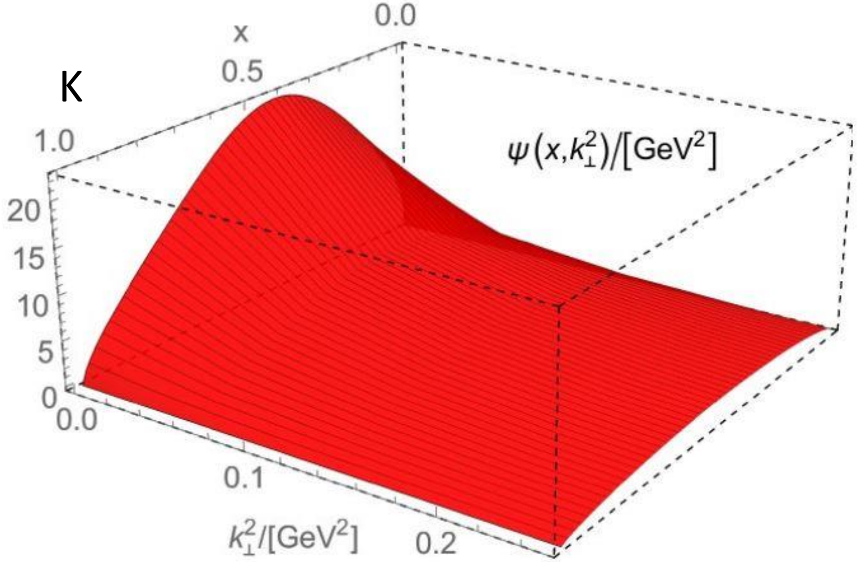


Light Front Wave Function

- Experiments sensitive to differences in LFWFs are sensitive to EHM
- Excellent examples are π & K DAs and DFs
  - Two sides of the same coin
  - Accessible via different processes
  - Independent measurements of the same thing
  - Great check on consistency



π



K

PDAs & PDFs

- Relationship between leading-twist PDAs and valence-quark PDFs, expressed via a meson's light-front wave function (LFWF):

$$\varphi(x) \sim \int d^2k_\perp \psi(x, k_\perp^2)\,,$$
$$q(x) \sim \int d^2k_\perp |\psi(x, k_\perp^2)|^2$$

- Given that factorization of LFWF is a good approximation for integrated quantities, when the wave function has fairly uniform support, then at the hadronic scale, ζ_H:

$$q_{\pi,K}(x;\zeta_H) \propto \varphi^{q}_{\pi,K}(x;\zeta_H)^2$$

Proportionality constant is fixed by baryon number conservation

- Owing to parton splitting effects, this identity is not valid on $\zeta > \zeta_H$.

(Think about DGLAP and ERBL regions for a GPD.)

- Nevertheless, evolution equations are known; so connection not lost, it just metamorphoses.

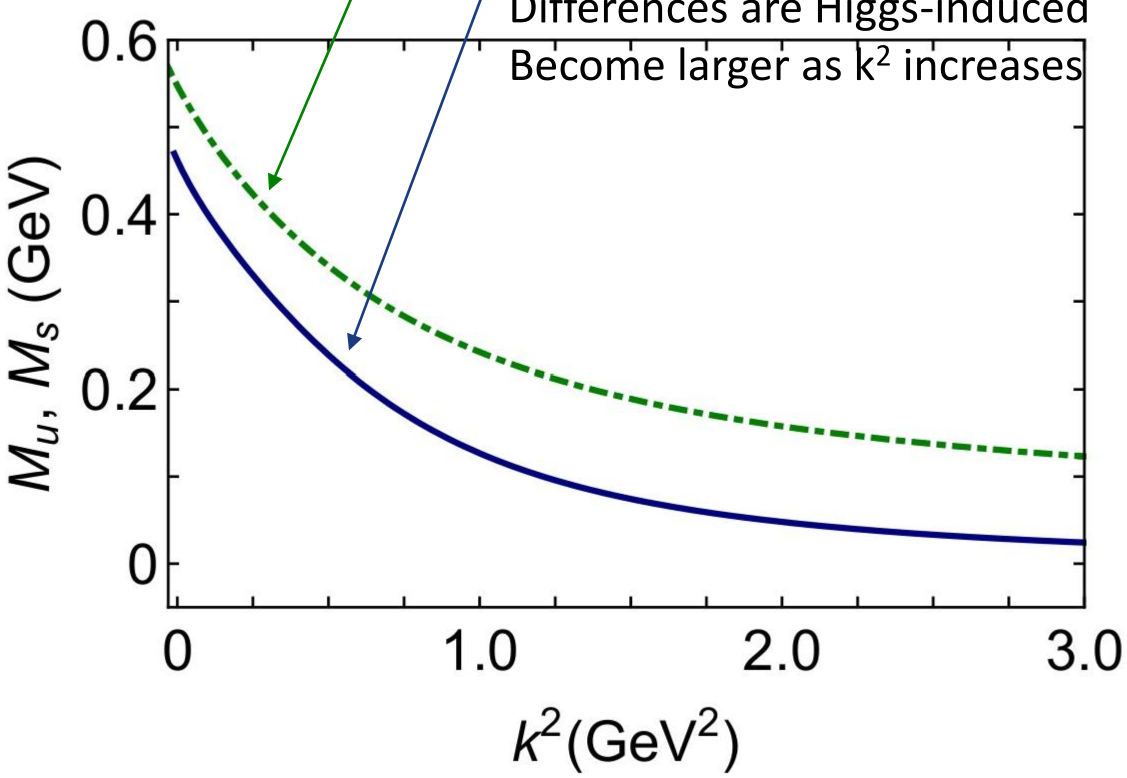
Meson valence-quark DFs

- Owing to these relations

$$\varphi(x) \sim \int d^2k_\perp \psi(x, k_\perp^2)\,,$$

$$q(x) \sim \int d^2k_\perp |\psi(x, k_\perp^2)|^2$$

- Broadening of DAs feeds into broadening of DFs
- Necessary consequence of EHM
- Moreover, any Higgs-boson related modulations of EHM in the DA will also be expressed in the DF
- Pion – Kaon comparisons great place to study interference between the Standard Model's two mass-generating mechanisms



Dressed quark mass functions

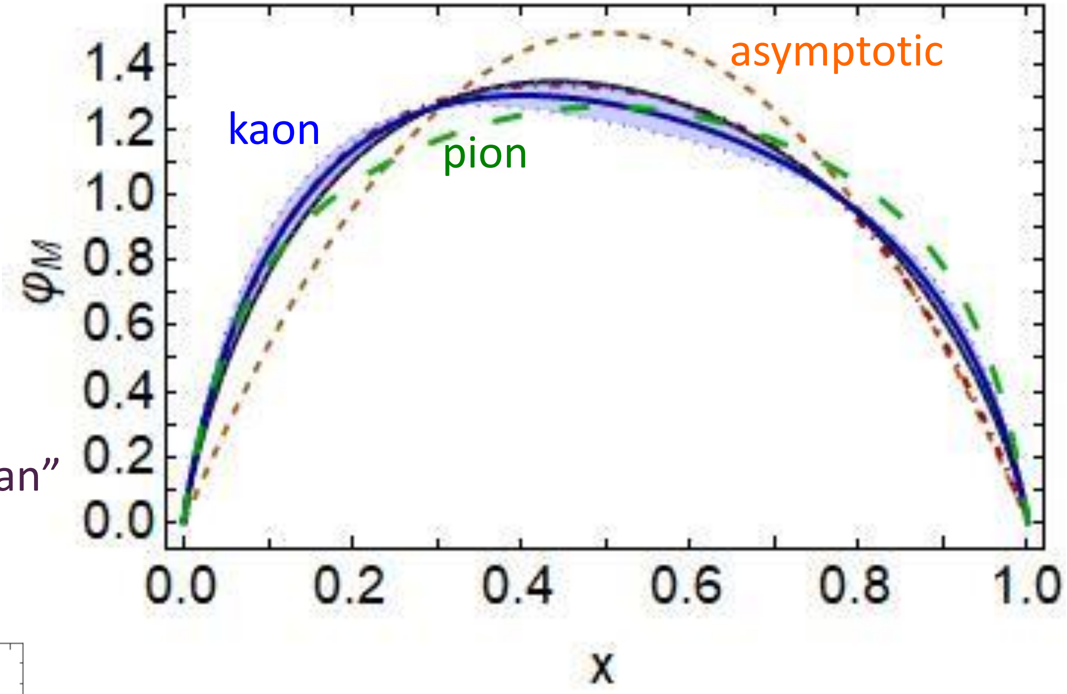
strange & up

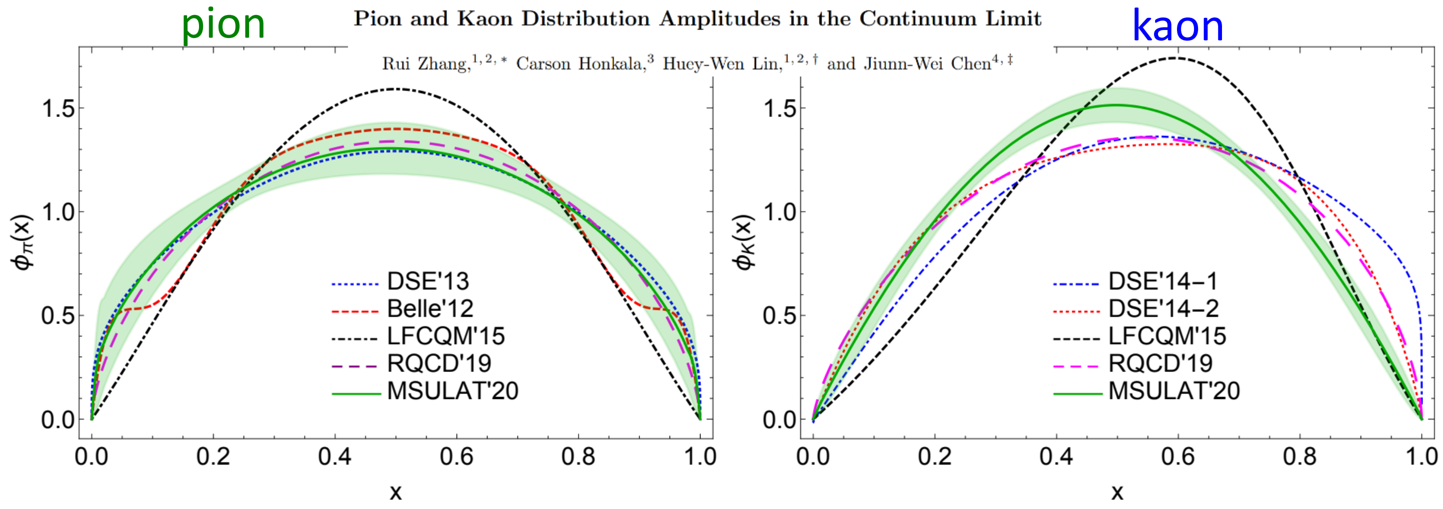
Differences are Higgs-induced

Become larger as k^2 increases

Meson leading-twist DAs

- Continuum results exist & lQCD results arriving
- Common feature = broadening
- Origin = EHM
- NO differences between π & K if EHM is all there is
  - Differences arise from Higgs-modulation of EHM mechanism
  - “Contrasting π & K properties reveals Higgs wave on EHM ocean”




Pion and Kaon Distribution Amplitudes in the Continuum Limit

Rui Zhang,[1,2,*] Carson Honkala,[3] Huey-Wen Lin,[1,2,†] and Jiunn-Wei Chen[4,‡]

FIG. 10. Fit of the $P_z = 4\frac{2\pi}{L}$ pion (left) and kaon (right) data to the analytical form in Bjorken-x space, compared with previous calculations (with only central values shown). Although we do not impose the symmetric condition $m = n$, both results for the pion and kaon are symmetric around $x = 1/2$ within error.

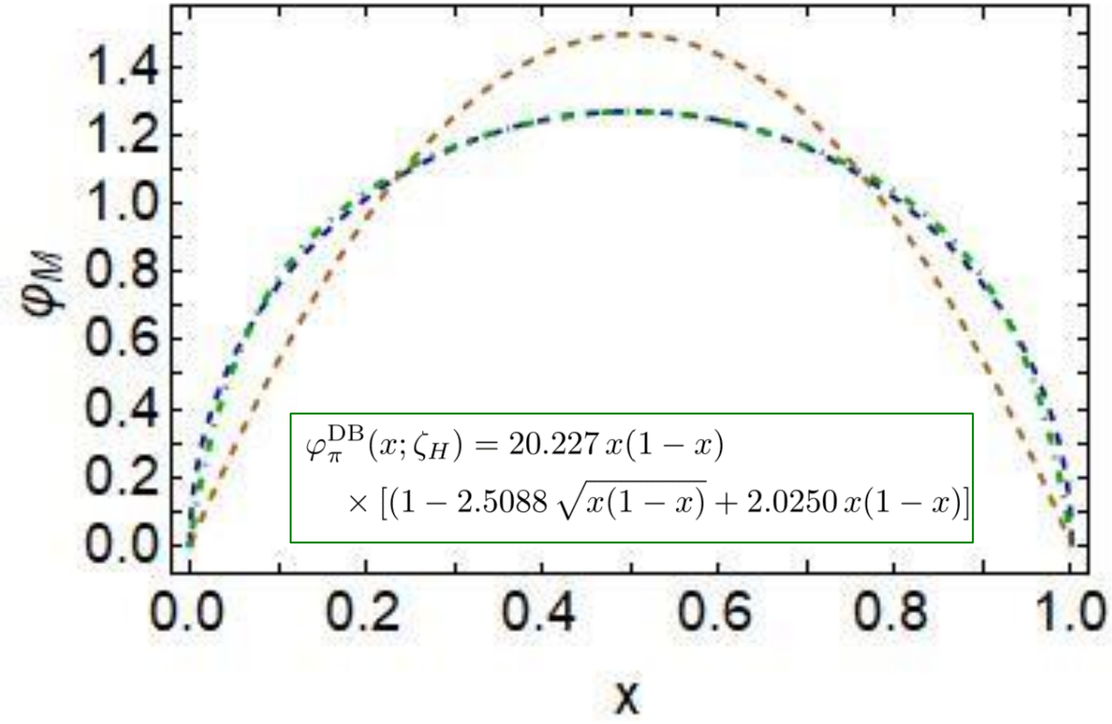
- Kaon DA vs pion DA
  - almost as broad
  - peak shifted to x=0.4(5)
  - ⟨ξ²⟩ = 0.24(1), ⟨ξ⟩ = 0.035(5)
- ERBL evolution logarithmic
- Broadening & skewing persist to *very* large resolving scales – beyond LHC

Meson leading-twist DAs and valence-quark DFs

- Broadening need not and should not disturb the DA's endpoint behaviour
- QCD: $\varphi(x) = x\,(1-x) f(x), f(x \simeq 0) = constant_1, f(x \simeq 1) = constant_2$
- Many models that express EHM-induced broadening violate this constraint
- Typically not a problem, unless endpoint behaviour is taken too seriously
- Example AdS/QCD: $\varphi(x) = \frac{8}{\pi}\sqrt{x(1-x)}$
- Practically identical to the continuum prediction that preserves QCD constraint:

  blue dashed *vs* green dot-dashed

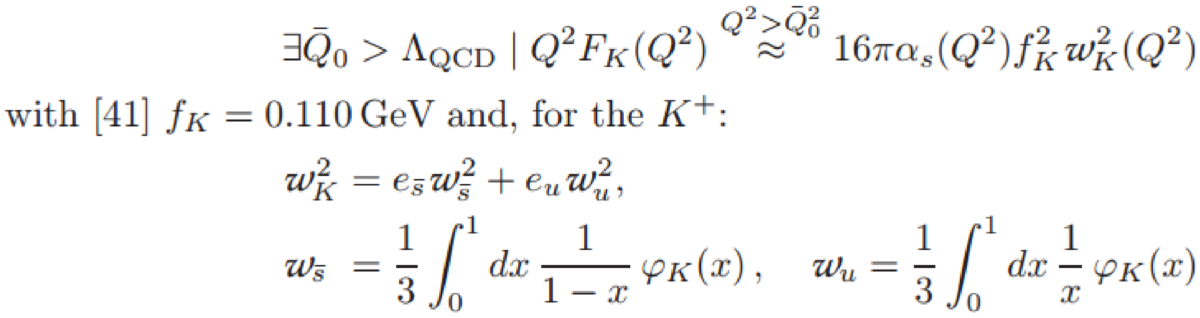
  - However, AdS/QCD practitioners use DA to argue for $x \simeq 1 \Rightarrow q^{\pi}(x;\zeta_{\rm H}) \propto (1-x)^1$
  - Endpoint behaviour taken "too seriously"



$$\varphi_\pi^{\rm DB}(x;\zeta_H) = 20.227\,x(1-x) \times [(1 - 2.5088\,\sqrt{x(1-x)} + 2.0250\,x(1-x)]$$

Meson Observables Sensitive to DAs

- Electromagnetic elastic and transition form factors of π & K



$$\exists \bar{Q}_0 > \Lambda_{\rm QCD} \mid Q^2 F_K(Q^2) \overset{Q^2 > \bar{Q}_0^2}{\approx} 16\pi \alpha_s(Q^2) f_K^2 w_K^2(Q^2)$$

with [41] $f_K = 0.110\,{\rm GeV}$ and, for the K^+:

$$w_K^2 = e_{\bar{s}} w_{\bar{s}}^2 + e_u w_u^2 ,$$

$$w_{\bar{s}} = \frac{1}{3}\int_0^1 dx\, \frac{1}{1-x}\, \varphi_K(x)\,, \quad w_u = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_K(x)$$

- Sensitive to the $\langle \frac{1}{x} \rangle$ moment of the DA ⇒ very sensitive to endpoint behaviour
- Prediction that EHM ⇒ DA dilation means form factors very sensitive to EHM

Pion DA & form factor

- QCD is not found in scaling … it is found in scaling violations
- Continuum predictions
  - Match existing data
  - Suggest that Jlab 12 could potentially be first to reveal scaling violations in a hard-scattering process = see QCD in a hard-scattering process
- Simulations indicate that EIC is certainly capable of doing so.
- Normalisation of the form-factor curve is a measure of the level of DA broadening; hence, size of EHM

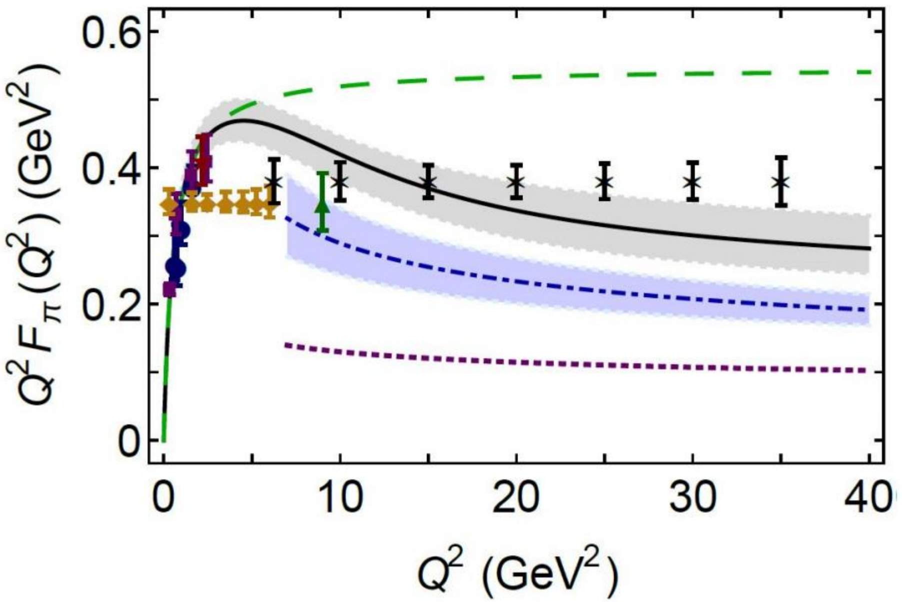


FIG. 9: Projected EIC pion form factor data as extracted from a combination of electron-proton and electron-deuteron scattering, each with an integrated luminosity of 20 fb^{-1} – black stars with error bars. Also shown are projected JLab 12-GeV data from a Rosenbluth-separation technique – orange diamonds and green triangle. The long-dashed green curve is a monopole form factor whose scale is determined by the pion radius. The black solid curve is the QCD-theory prediction bridging large and short distance scales, with estimated uncertainty [41]. The dot-dashed blue and dotted purple curves represent the short-distance views [79–81], comparing the result obtained using a modern DCSB-hardened PDA and the asymptotic profile, respectively.

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Exposing strangeness: projections for kaon electromagnetic form factors, Fei Gao et al., arXiv:1703.04875 [nucl-th], Phys. Rev. D ***96*** *(2017) 034024*

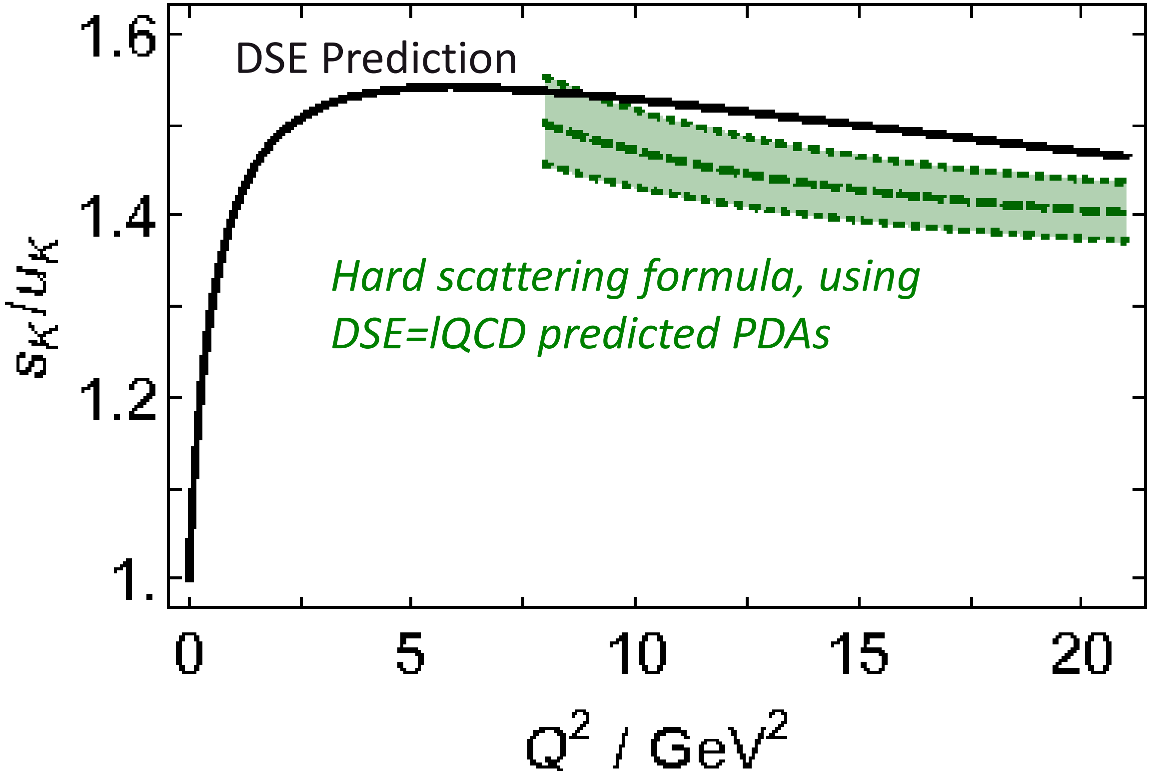
Kaon form factor
- flavour separation

- Current conservation: $F_{uss}(0) = F_{uus}(0)$
- Under evolution:

  $\varphi_K \rightarrow 6\,x\,(1\text{-}x) \Rightarrow \omega_{\bar{s}} \rightarrow \omega_u \Rightarrow \text{Ratio} \rightarrow 1$
- Agreement between direct calculation and hard-scattering formula, using consistent PDA
- Ratio never exceeds 1.5 and

  Logarithmic approach to unity
- Typical signal of EHM-dominance in flavour-symmetry breaking, taming the large Higgs-produced current-quark mass difference:
  - *ms ~ 30 mu* $\Rightarrow M_s(0) \sim 1.25\, M_u(0)$
  - scale difference does finally become irrelevant under evolution, but only at very large scales

$[\, \bar{s}\,\gamma\, s\, u_{\text{spectator}} \,/\, \bar{u}\,\gamma\, u\, s_{\text{spectator}}\,]^2 \leq 1.5$



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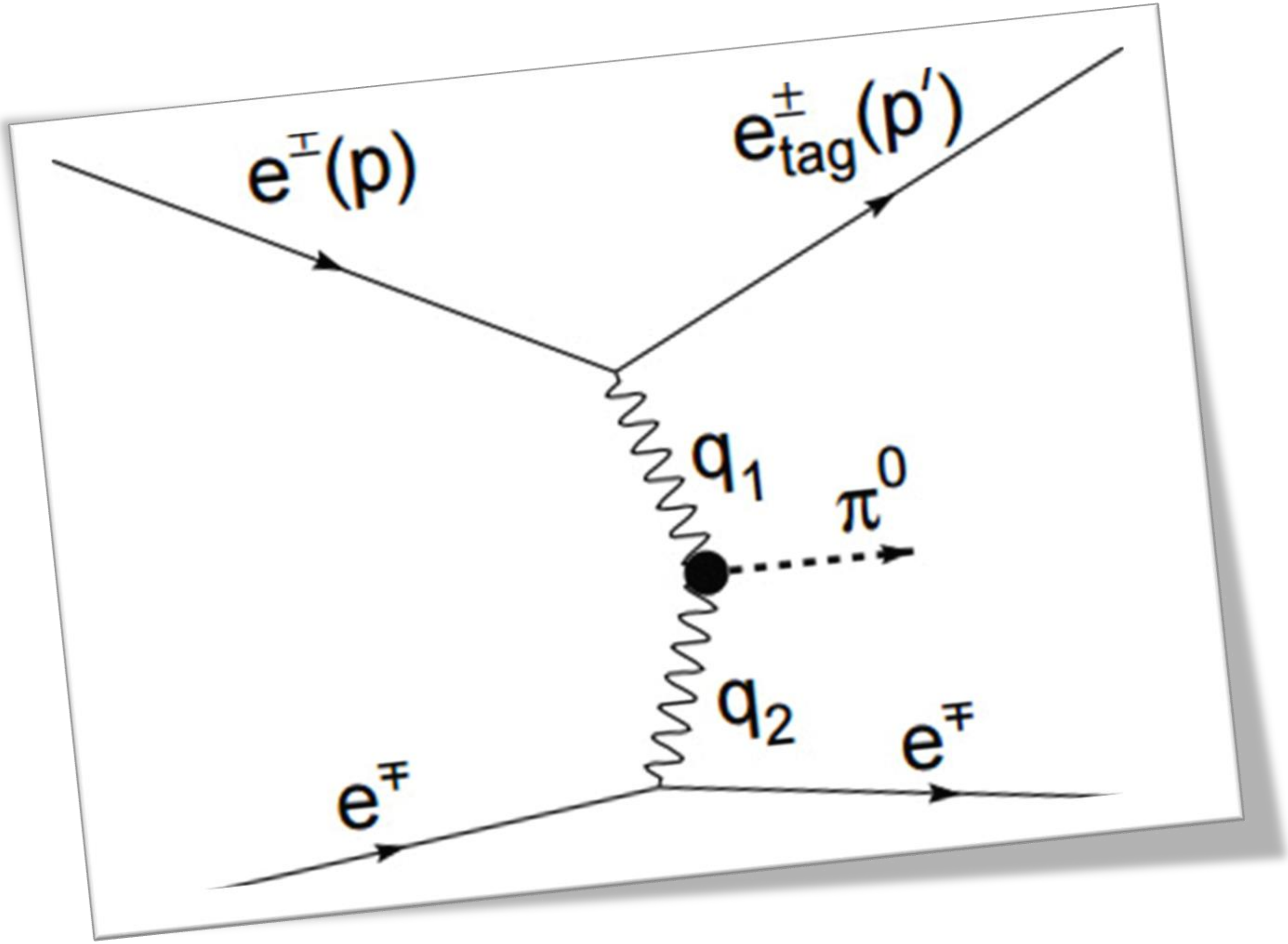


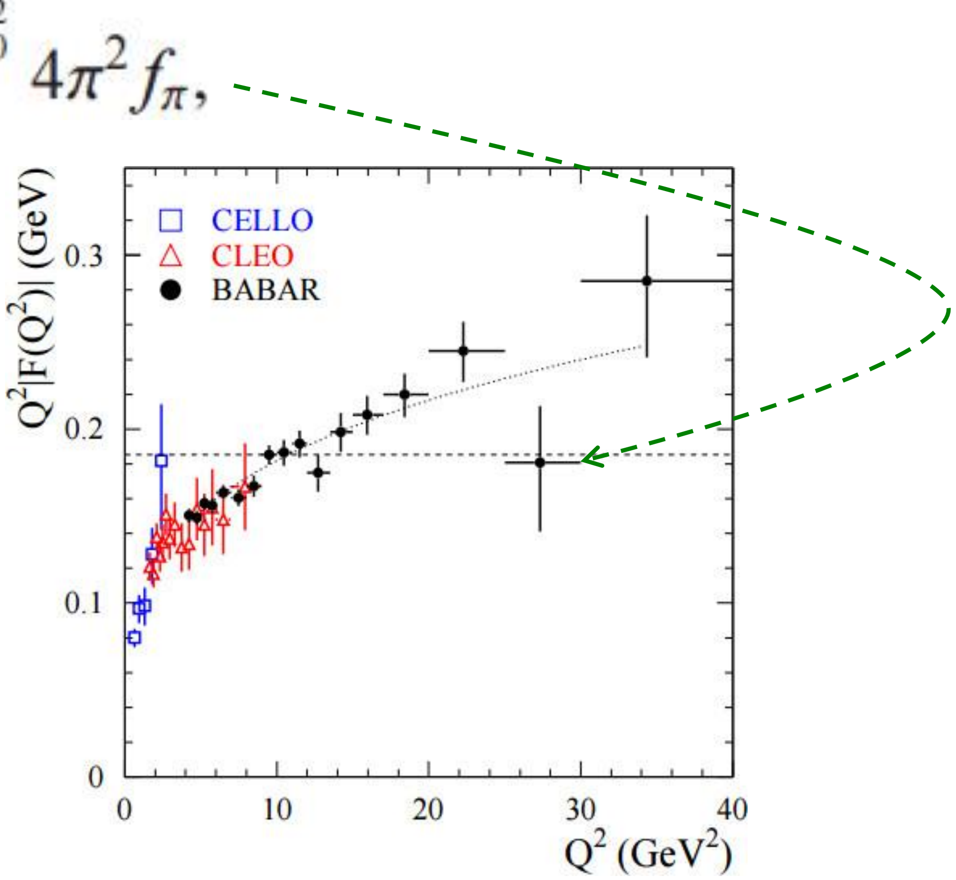
γ^* π γ transition

Neutral Pion Transition Form Factor

- Factorisation in QCD hard scattering processes leads to an *inviolable* prediction:

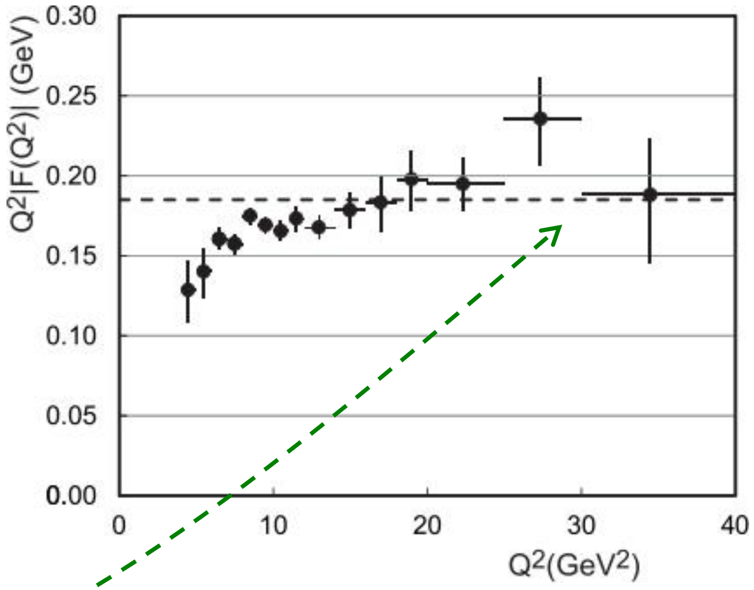
$$\exists Q_0 > \Lambda_{\rm QCD} \mid Q^2 G(Q^2) \overset{Q^2>Q_0^2}{\approx} 4\pi^2 f_\pi ,$$

- Problem?! BaBar (2009)
  - Whilst the new data agree with earlier experiments on their common domain of momentum-transfer, they are unexpectedly far *above* the QCD prediction on Q^2>10GeV^2.



Neutral Pion Transition Form Factor

- Numerous authors have attempted to reconcile the BaBar measurements with the QCD prediction … typically, produce a transition form factor whose magnitude on Q^2>10GeV^{2}exceeds the ultraviolet limit, without explaining how that limit might finally be recovered or how their results might be reconciled with modern measurements of $F_\pi(Q^2)$
- However, others, including "us" argue that the BaBar data is not an accurate measure of the transition form factor
- Significantly, data subsequently published by the Belle Collaboration appear to be in general agreement with QCD prediction



Structure of the neutral pion and its electromagnetic transition form factor, K. Raya *et al*., arXiv:1510.02799 [nucl-th] , Phys. Rev. D**93** (2016) 074017/1-9
$\gamma_*\gamma\to\eta,\eta'$ transition form factors, Minghui Ding, Khépani Raya, *et al*., arXiv:1810.12313 [nucl-th], Phys. Rev. D **99** (2019) 014014/1-17

$\gamma^* \pi^0 \gamma$ Transition

- Computation of $\gamma^* \pi^0 \gamma$ transition –unifies explanation of this form factor with
  - charged pion electromagnetic form factor,
  - Pion valence-quark distribution amplitude,
  - and numerous other quantities,
- Novel analysis techniques enable computation of $G(Q^2)$ on entire domain of spacelike momenta for 1st time in framework with direct connection to QCD.
- Enabled demonstration that a fully self-contained and consistent treatment can readily connect a broad, concave pion PDA at hadronic scale with perturbative QCD prediction for the transition in hard photon limit.

Structure of the neutral pion and its electromagnetic transition form factor, K. Raya *et al.*, arXiv:1510.02799 [nucl-th] , Phys. Rev. D**93** (2016) 074017/1-9
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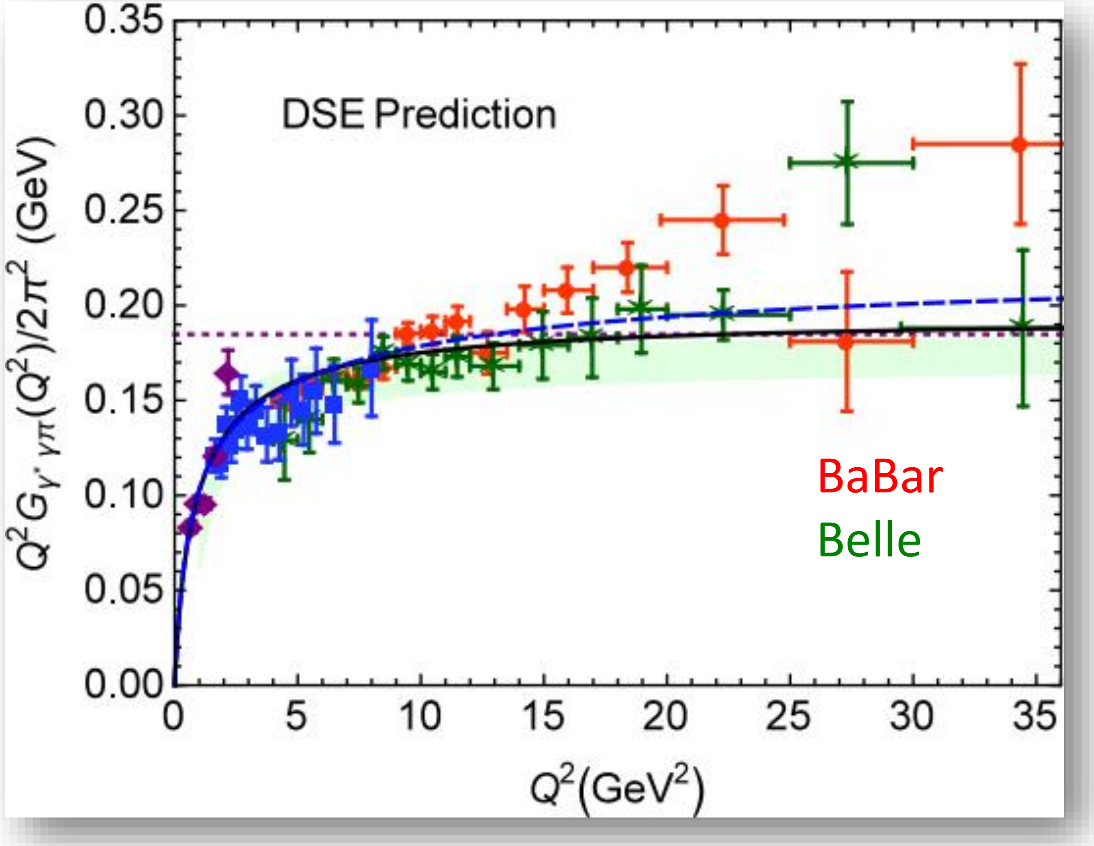
γ^* π^0 γ Transition



- QCD sum rules analysis, exhibiting the variation resulting from a wide range of possible sources of error … *green band*
- DSE Prediction without evolution … *dashed-blue*
- Best DSE prediction, including ERBL evolution of the Bethe-Salpeter wave function … *solid-black*

- *NB*. Normalisation of γ^* π^0 γ transition form factor's hard scattering limit is set by the f_π, whose magnitude is fixed by scale of EHM.
- Thus, in order to claim understanding of Standard Model, it's crucial to obtain new, accurate and precise transition form factor data on Q^2>10GeV^2 so that QCD predictions can reliably be tested.

$\gamma^*\ (\eta,\ \eta')\ \gamma$ Transition

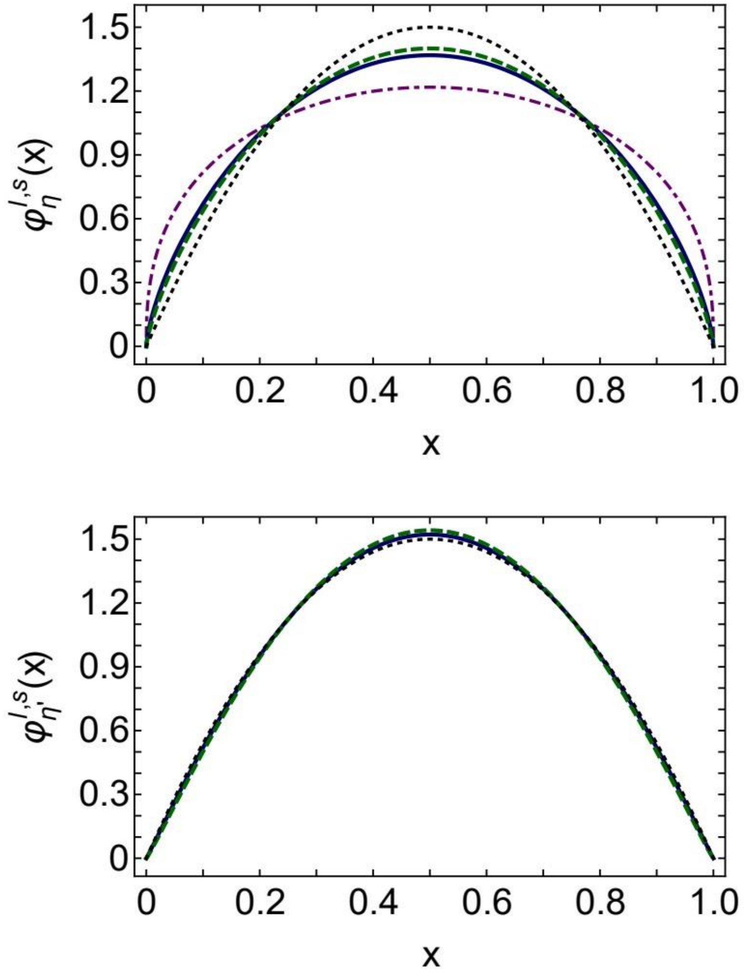
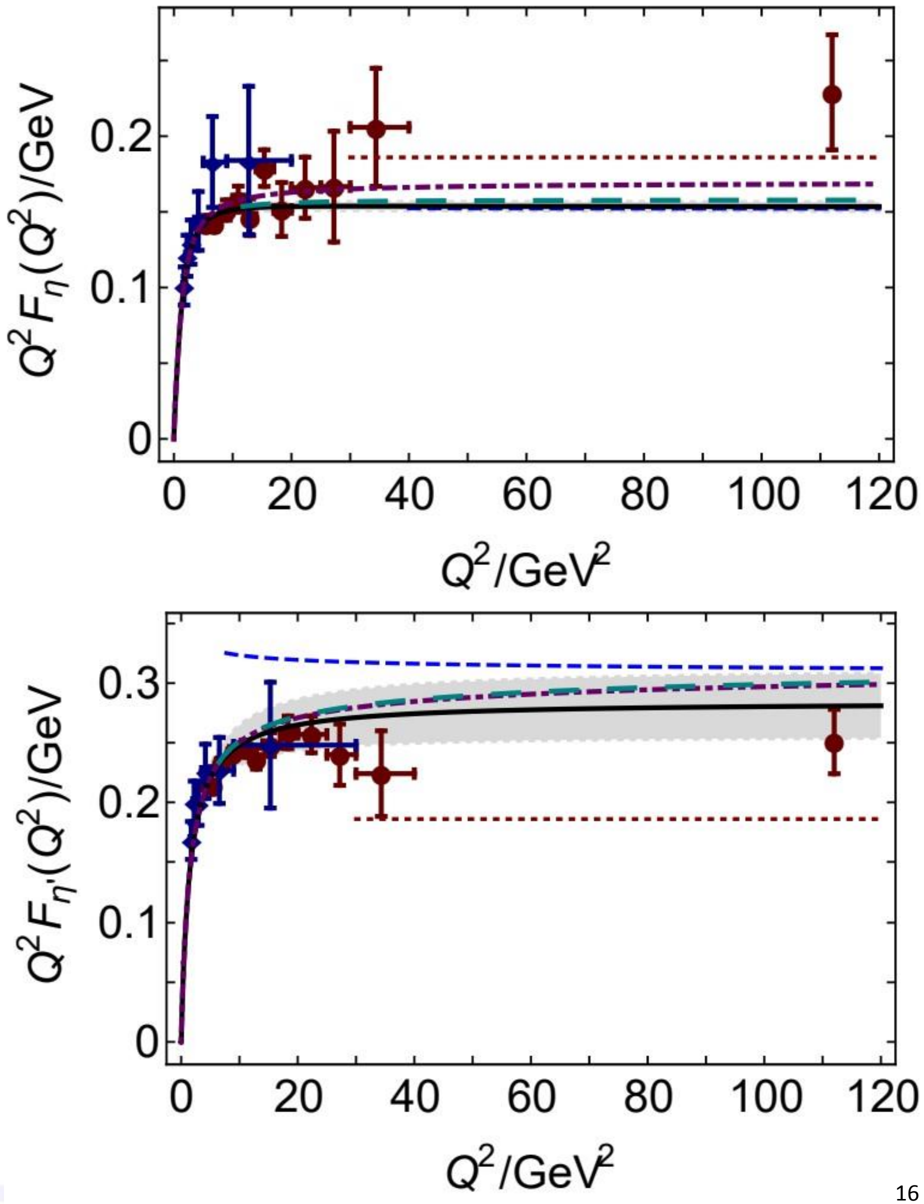


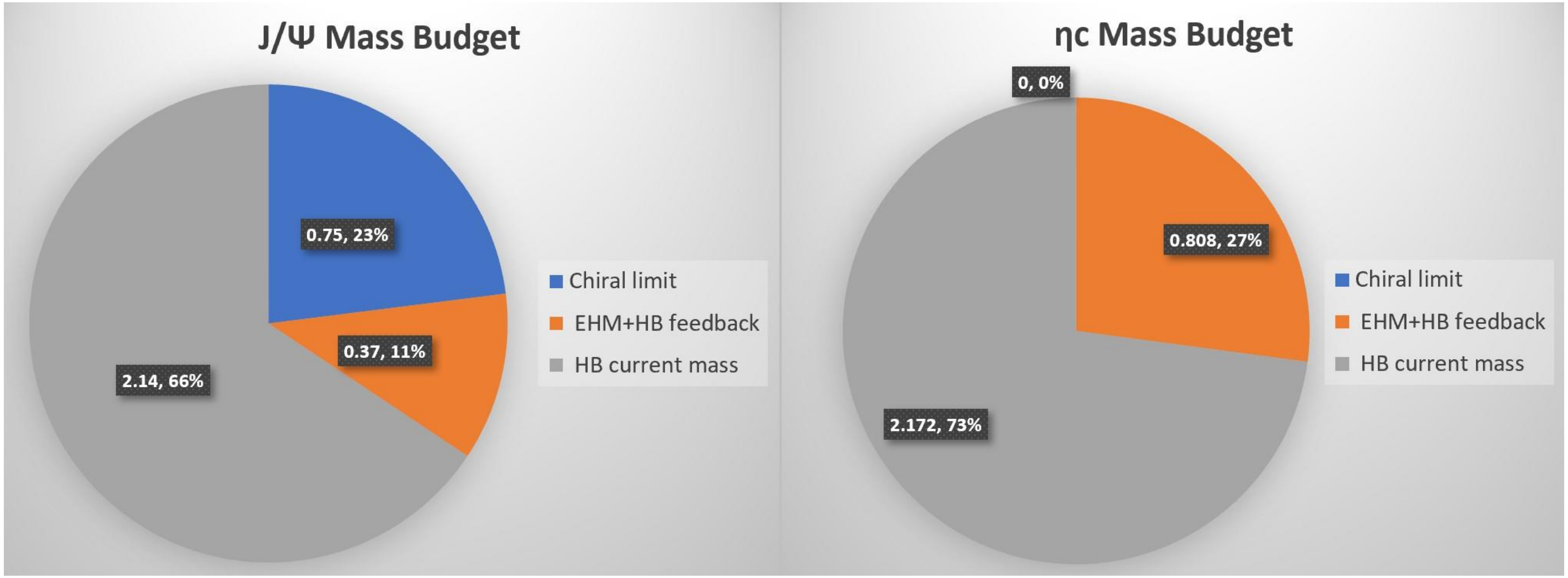
FIG. 3. Computed light-quark (solid blue curve) and s-quark (dashed green curve) component DAs of the η-meson (upper panel) and η'-meson (lower panel), determined at $\zeta = 2\,\text{GeV} =: \zeta_2$, listed in Eqs. (37). For comparison: upper panel, dot-dashed (purple) curve – pion's dressed-valence-quark distribution amplitude [87, 88]; and both panels, dotted black curve – asymptotic profile, Eq. (4).

- Difference in mass between η & η′
  - introduced by non-Abelian anomaly
  - strength set by EHM

  translate into differences between the DAs in these states
- Difference in large Q^2 limit of transition form factors measures strength of EHM as expressed in the meson leptonic decay constants



EHM modulation of Higgs mass generation



- c-quark … solve quark Dyson/gap equation
  - $\hat{m}_c = 1.7\, GeV \Rightarrow m_c := M_c(\zeta_2) = 1.25\, GeV$ and $m_c^D := M_c(0) = 1.6\, GeV$
  - EHM contribution to all quark masses in IR is roughly uniform $M_u(0) = 0.375\, GeV$

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EHM modulation of Higgs mass

- c-quark … solve quark Dyson/gap equation
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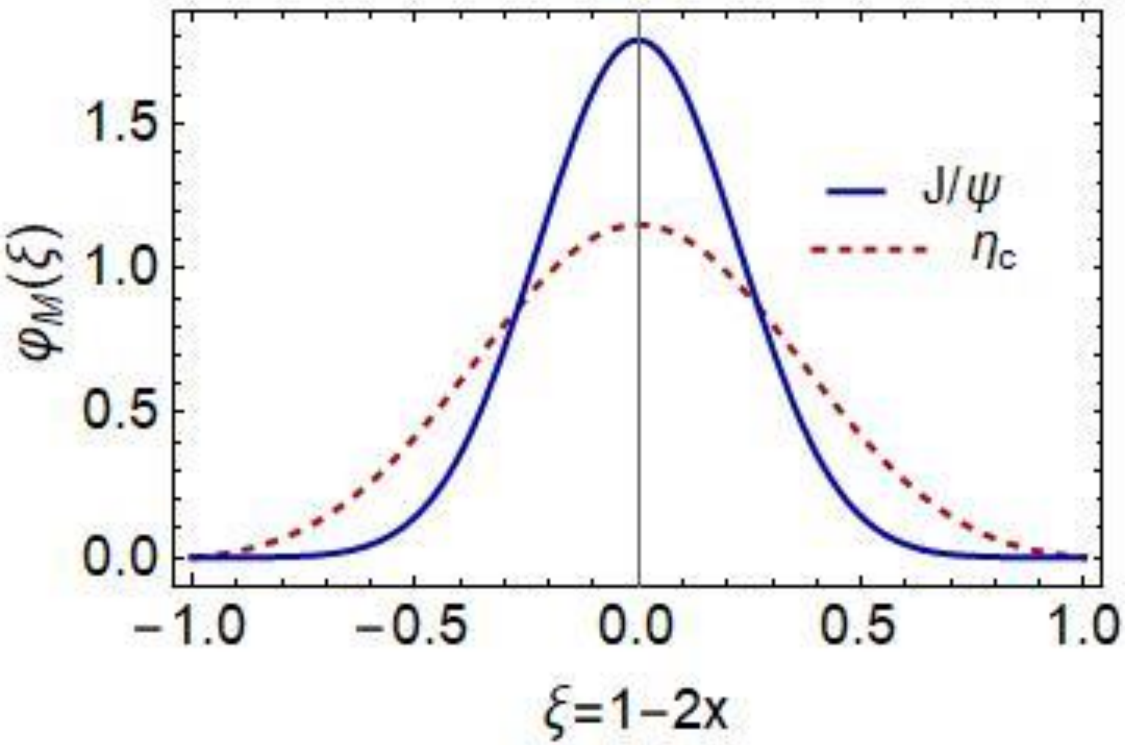
  - EHM contribution to all quark masses in IR is roughly uniform $M_u(0) = 0.375\ GeV$
- DAs are different – Why?
- NG modes
  - Complete cancellation between 2*1-body and 1*2-body dressing in chiral limit
  - Memory fades slowly with increasing HB contribution
- Vector meson
  - Always incomplete cancellation between 2*1-body and 1*2-body dressing in chiral limit



- DA → DF means far greater concentration of support around $x = \frac{1}{2}$ in valence c-quark distribution of vector meson *cf*. pseudoscalar meson
- Impact on sea and glue?

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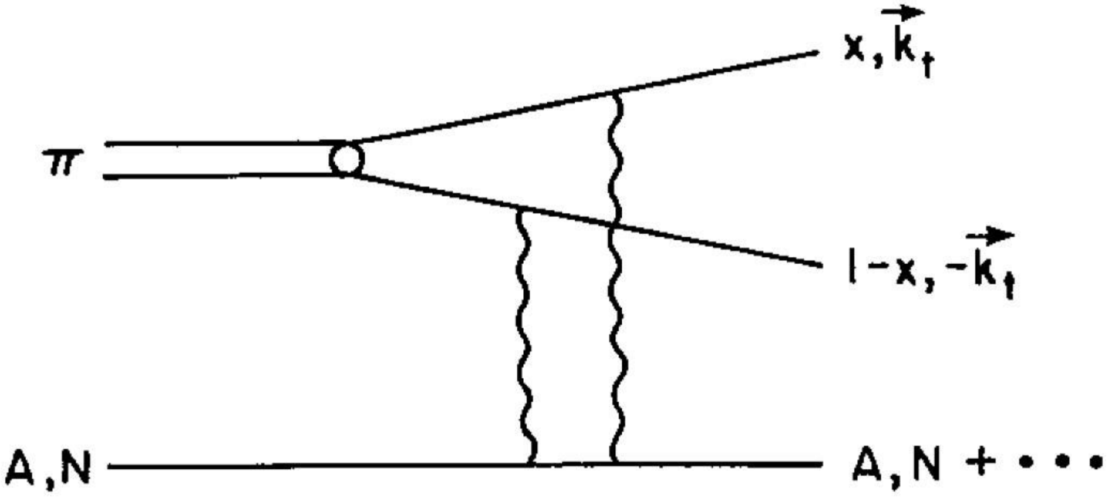
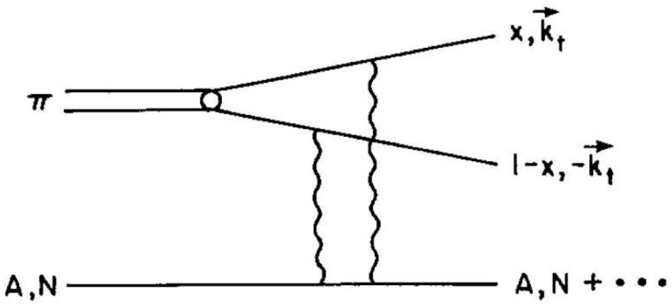
Meson Observables Sensitive to DAs
Diffractive Dissociation of Meson into di-jets

- E791 Collaboration, E. Aitala *et al*., Phys. Rev. Lett. 86, 4768 (2001)
- "Direct measurement of the pion valence quark momentum distribution, the pion light-cone wave function squared"
- "The measurements were carried out using data on diffractive dissociation of 500 GeV/c π^- into di-jets from a platinum target at Fermilab experiment E791."
- As drawn, measurement sensitive to gluon distribution in the target and wave function (DA) of the projectile



L.L. Frankfurt, G.A. Miller, and M. Strikman, Phys. Lett. B304, 1 (1993).

Diffractive Dissociation of Meson into di-jets



- Assume transverse q̅ q separation = b = constant
- Focus on di-jets with total momentum ≈ P_M
- Invariant amplitude

$$\mathcal{M}(N) = \int d^2 b\, \psi_\pi(x,\boldsymbol{b})\, \frac{f(b^2)}{2}\, e^{i\boldsymbol{\kappa}_t\cdot\boldsymbol{b}}$$

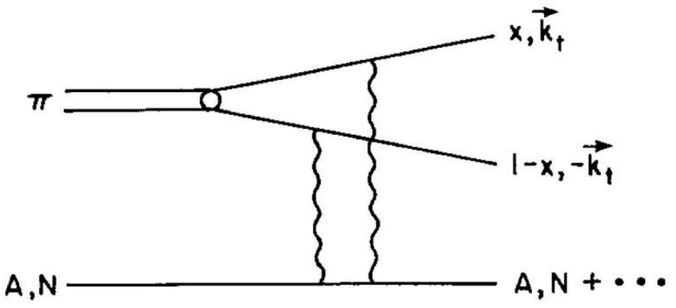
  - ψ_M = meson wave function
  - $f(b^2)$ = is the forward q̅ q scattering amplitude normalized according to optical theorem

    Im $f(b^2) = s\,\sigma(b^2)$
  - $e^{ik_t\cdot b}$ final-state wave function of two "jets" with high relative momentum, k_t
  - Allows cross-section to be written in terms of $\tilde{\psi}$ = Fourier transform of ψ

$$\mathcal{M}(N) = i\,\tfrac{1}{2}\,s\sigma(-\nabla^2_{\kappa_t})\,\tilde{\psi}_\pi(x,\boldsymbol{\kappa}_t)$$

  - Evaluate derivative after assuming factorisation at large k_t^2, viz. $\tilde{\psi} \sim \frac{\varphi(x)}{k_t^2}$
  - Then, $M(N) \propto \varphi_M(x)$ … one can be more sophisticated in choice of k_t^2-dependence

Diffractive Dissociation of Meson into di-jets

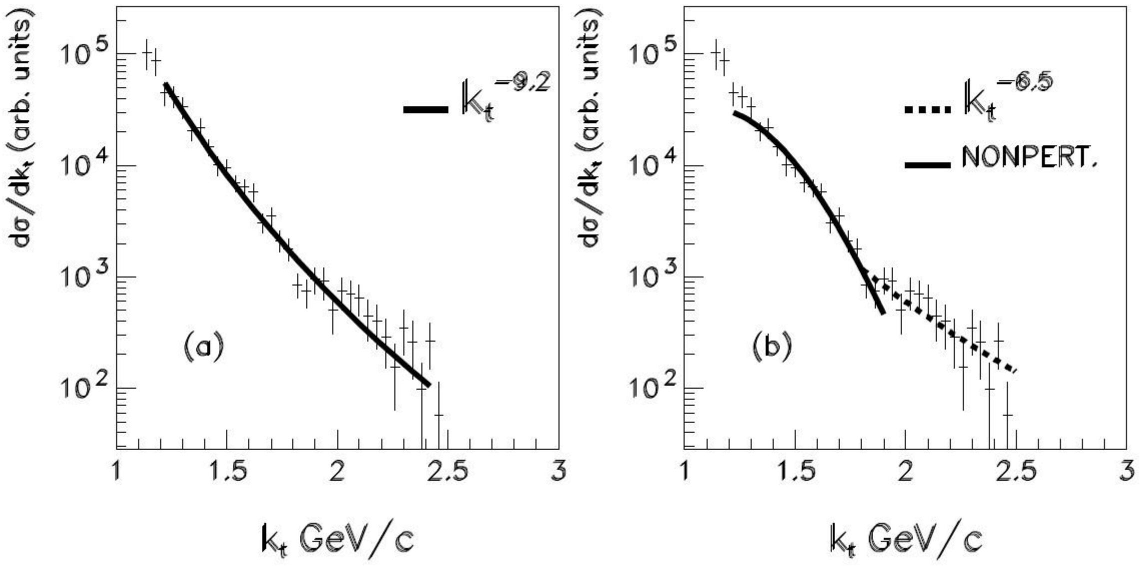


$$\frac{d\sigma}{dk_t^2} \propto \left|\alpha_s(k_t^2) x_{Bj} G(x_{Bj}, k_t^2)\right|^2 \left|\frac{\partial^2}{\partial k_t^2}\psi(x, k_t^2)\right|^2$$

- With factorisation assumption

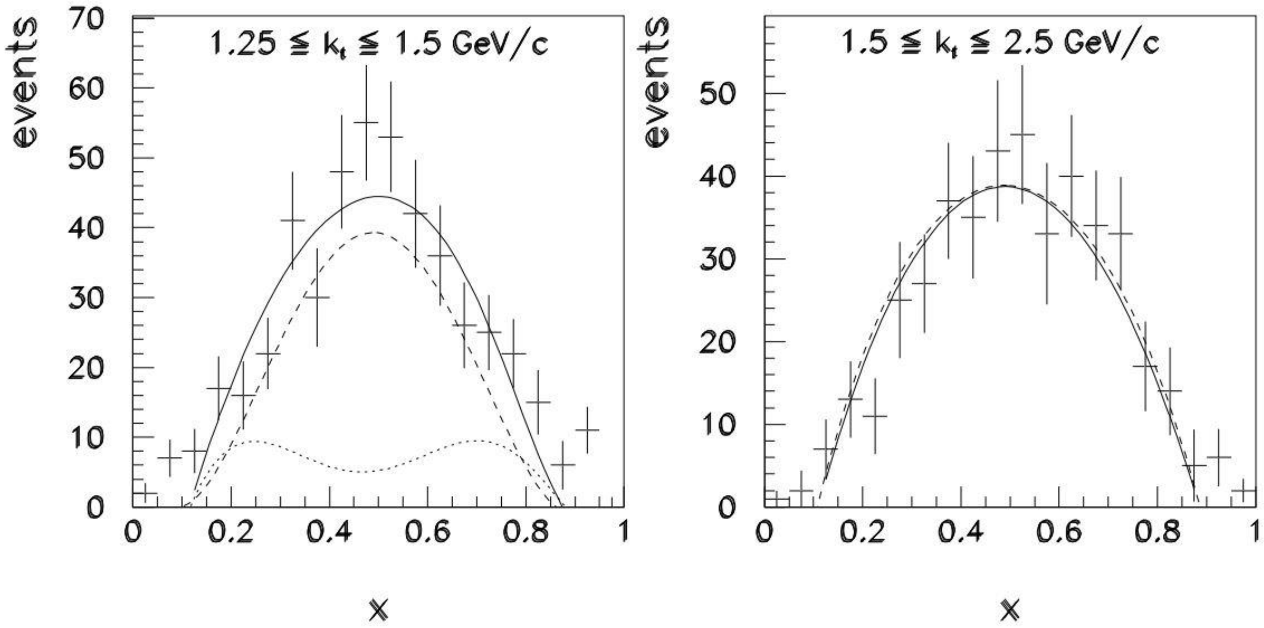
  $$\psi(x, k_t^2) = \varphi_M(\mathrm{x})\, \Psi(k_t^2),$$

  can study data and infer best-fit form for $\Psi(k_t^2)$
- This can be done in conjunction with "intelligent" theory
  - E791 found $\left[\frac{1}{k_t^2}\right]^{9/2}$, which is a theoretically unlikely outcome
  - But found $\left[\frac{1}{k_t^2}\right]^{6/2}$ on $|k_t| > 1.5$ GeV, if a two-step function is assumed
- $k_t^2 > 2GeV^2$ is "reasonable" definition of large-k_t^2



Diffractive Dissociation of Meson into di-jets

- E791 used that “separation” to define “soft” and “hard” domains
- Claim:
  - Description of data on soft domain requires non-asymptotic DA
  - Hard domain is described by asymptotic DA
- Credible that soft domain requires non-asymptotic DA
- Incredible that soft-DA becomes asymptotic DA under logarithmic evolution from $\zeta = 2 \to 3$ GeV



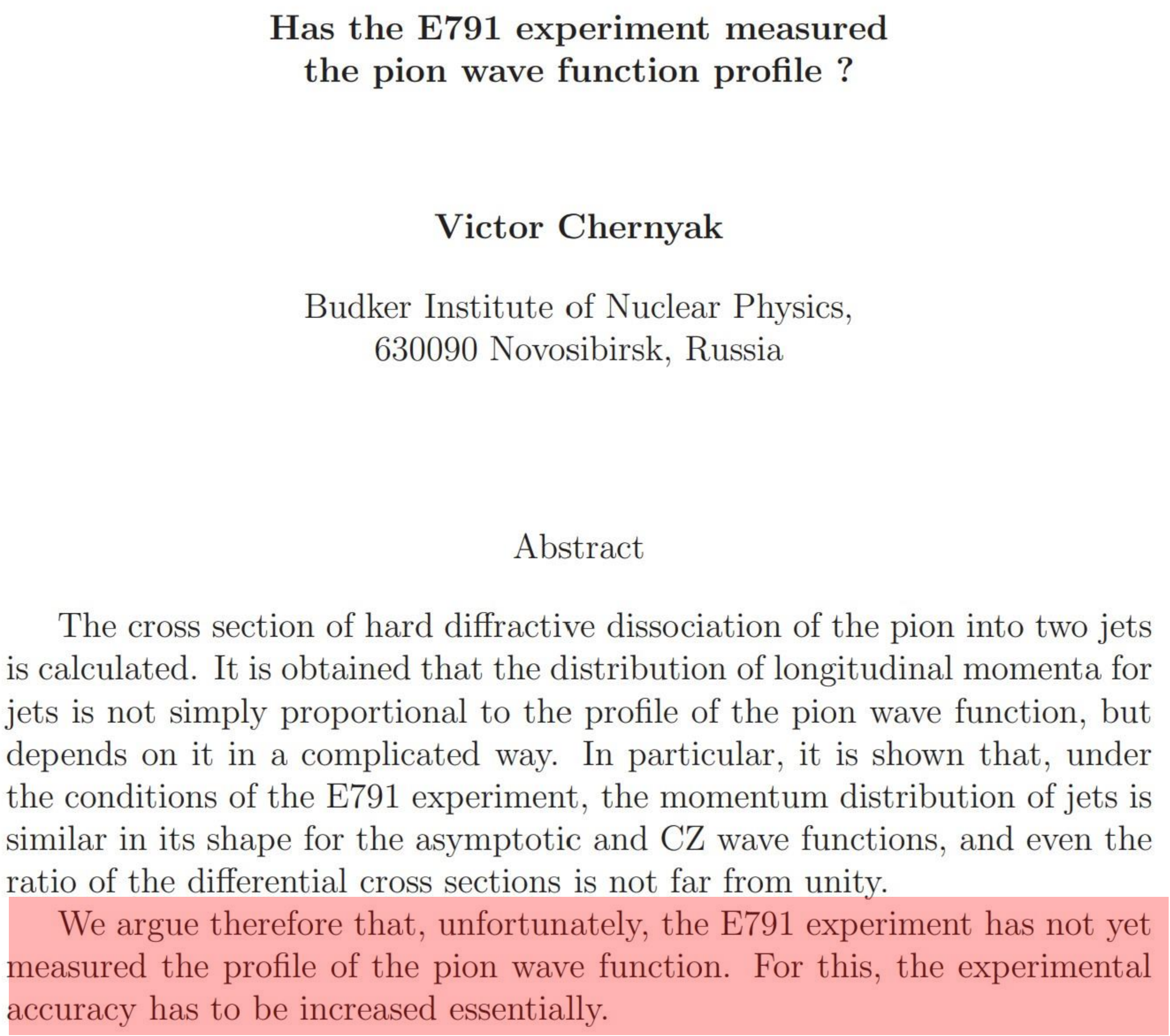
$Q^2 = 4$ GeV^2

$Q^2 = 10$ GeV^2

$$Q^2 \sim M_J^2 = \frac{k_t^2}{x(1-x)}$$

Diffractive Dissociation of Meson into di-jets

- Phys. Lett. **B** 516 (2001) 116-122
  - e-Print: hep-ph/0103295 [hep-ph]



Has the E791 experiment measured the pion wave function profile ?

Victor Chernyak

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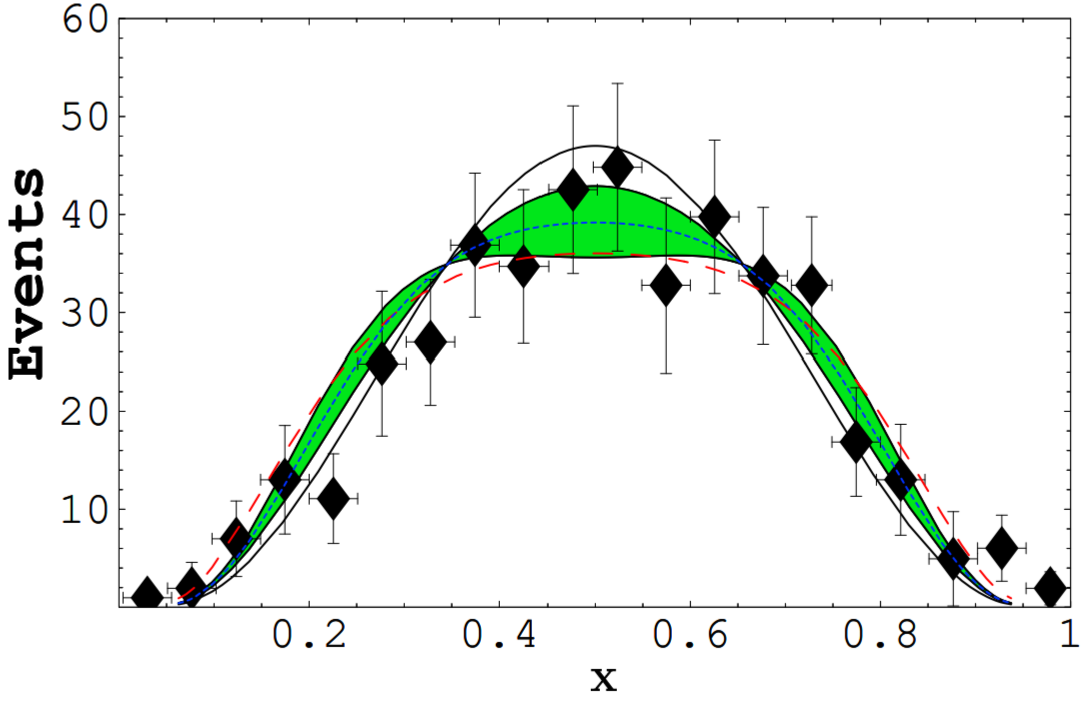
Abstract

The cross section of hard diffractive dissociation of the pion into two jets is calculated. It is obtained that the distribution of longitudinal momenta for jets is not simply proportional to the profile of the pion wave function, but depends on it in a complicated way. In particular, it is shown that, under the conditions of the E791 experiment, the momentum distribution of jets is similar in its shape for the asymptotic and CZ wave functions, and even the ratio of the differential cross sections is not far from unity.

We argue therefore that, unfortunately, the E791 experiment has not yet measured the profile of the pion wave function. For this, the experimental accuracy has to be increased essentially.

Diffractive Dissociation of Meson into di-jets

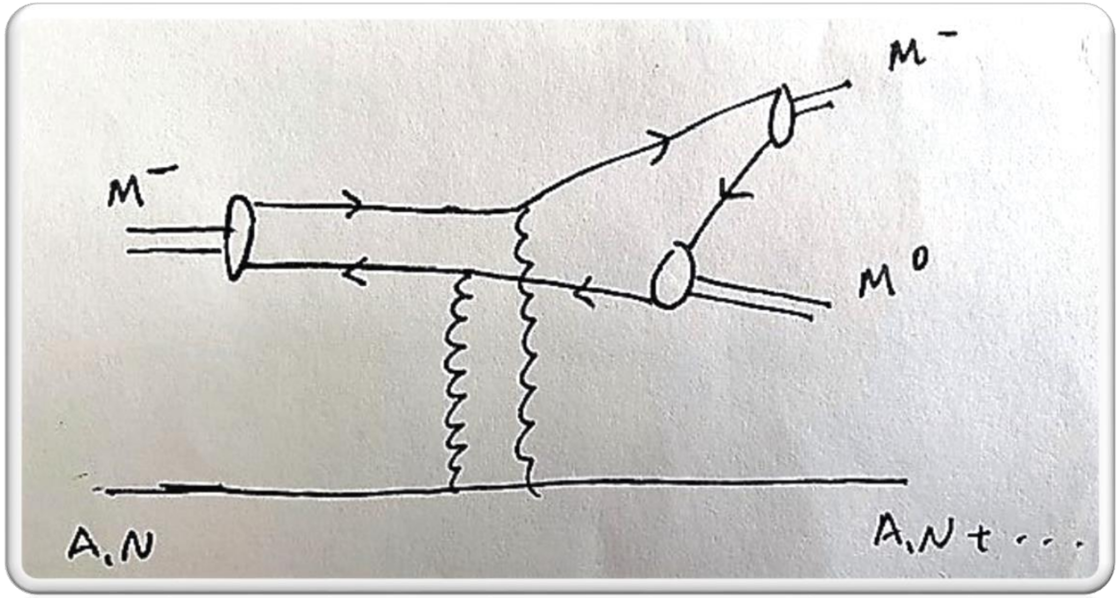
- Another perspective: *CLEO and E791 data: A Smoking gun for the pion distribution amplitude?* A. Bakulev, S. Mikhailov, N. Stefanis, Phys. Lett. B **578** (2004) 91-98
- One might be sceptical of the simple arguments used to relate diffractive dissociation into di-jets – at least, one can look deeper
- Notwithstanding that, the E791 data and analysis can and should be improved



E791 data are consistent with a large variety of DAs, including the asymptotic DA

Diffractive Dissociation of Meson into di-meson final states

- Can one obtain information on meson DAs via di-meson final states
- 1st guess answer = No
- If the diagram at right is the sort of thing one would look for, then following problems are encountered:
  - Two additional LFWFs ⇒ additional $\frac{1}{k_t^8}$ suppression introduced to cross-section
  - Integration over the loop means pointwise information on x-dependence is lost



Access to pion and kaon DAs

- E791 used 500 GeV beam ⇔ COMPASS++/AMBER has ~ 200 GeV beam
  - Is di-jet region accessible?
- COMPASS++/AMBER typically looks for single/few hadrons in the final state
  - Detectors unable to identify jets?
- Jets inaccessible? Then seems COMPASS++/AMBER can only access moments of π & K DAs
- Can anything be done using the Primakoff effect?
  - π & K elastic form factors at large Q^2?
  - Strong interaction background too large?
- Could muon beam (100 GeV) be used to produce π^0, η, η′ at large one-photon virtuality?
  - Access to pion DA through Abelian anomaly
- Sullivan process using muon beam (100 GeV) on proton target to measure π & K elastic form factors at large Q^2?

Thankyou

Controversy over pion valence DF

- Parton model prediction for the valence-quark DF of a spin-zero meson:

$$x \simeq 1 \Rightarrow q^{\pi}(x;\zeta_{\rm H}) \propto (1-x)^2$$

- The hadronic scale is not empirically accessible in Drell-Yan or DIS processes.
  (Matter of conditions necessary for data to be interpreted in terms of distribution functions.)

- For such processes, QCD-improvement of parton model leads to the following statement:
  At any scale for which experiment can be interpreted in terms of parton distributions, then

$$x \simeq 1 \Rightarrow q^{\pi}(x;\zeta) \propto (1-x)^{\beta=2+\gamma}, \gamma > 0$$

- Simple restatement of the following:
  - The parton model gives us scaling and scaling laws.
  - QCD's gluon corrections give us scaling violations
  - Scaling violations do NOT alter the integer-number that characterises scaling powers [L&B-1980 Lepage:1980fj]
  - Certainly don’t reduce 2 → 1 (or 3 → 2 for nucleon valence) – scaling violations increase power logarithmically

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- Consequence
  - Any analysis of DY or DIS (or similar) experiment which returns a value of $\beta < 2$ conflicts with QCD.
- Observation
  - All existing internally-consistent calculations preserve connection between large-k^2 behaviour of interaction and large-x behaviour of DF.
    - $J=0 \ldots (1/k^2)^n \Leftrightarrow (1-x)^{2n}$
- No existing calculation with n=1 produces anything other than $(1-x)^2$
- Internally-consistent calculation that preserve RG properties of QCD,

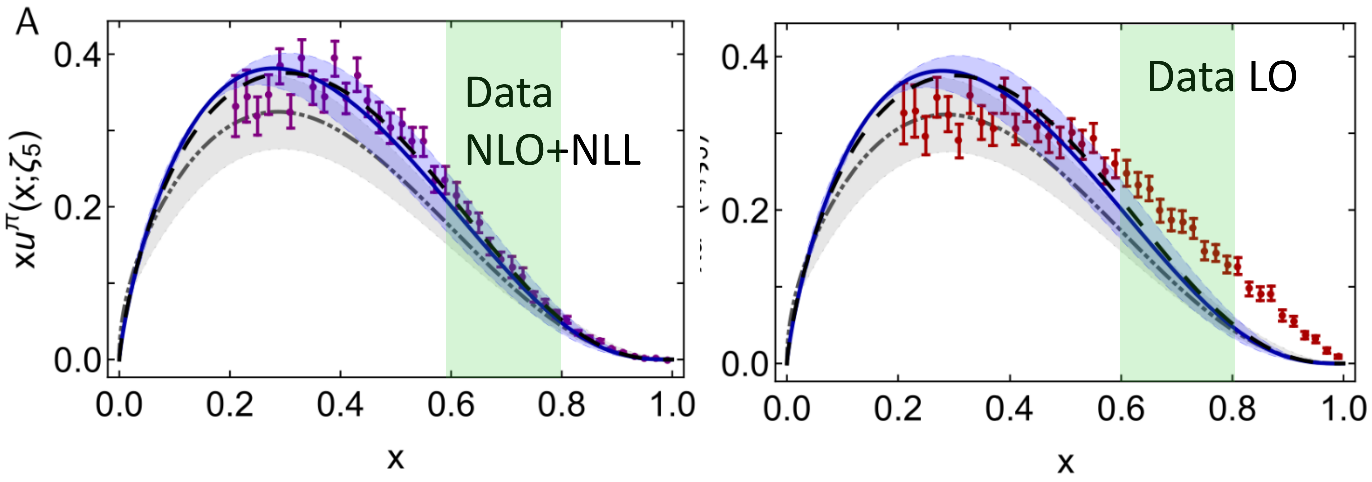
  then $2 \rightarrow 2+\gamma$, $\gamma>0$, at any factorisation-valid scale
- Controversy:
  - Ignore threshold resummation, then data analysis yields $(1-x)^{1+\gamma}$
  - Include threshold resummation, then data analysis yields $(1-x)^{2+\gamma}$

Where is “2” to be seen?

- Use DSE DF … prediction … NOT fit to data
  - Within uncertainty, brackets DF points obtained in NLO+NLL analysis
    - Central curve: χ^2/dof = 1.66
  - By same measure, inconsistent with LO E615
    - Central curve: χ^2/dof = 19.4 – order of magnitude larger
- Valence domain begins after peak, at which point 2 x V(x) > x (S(x)+G(x))
- Power discriminating function – local (x-dependent) exponent:

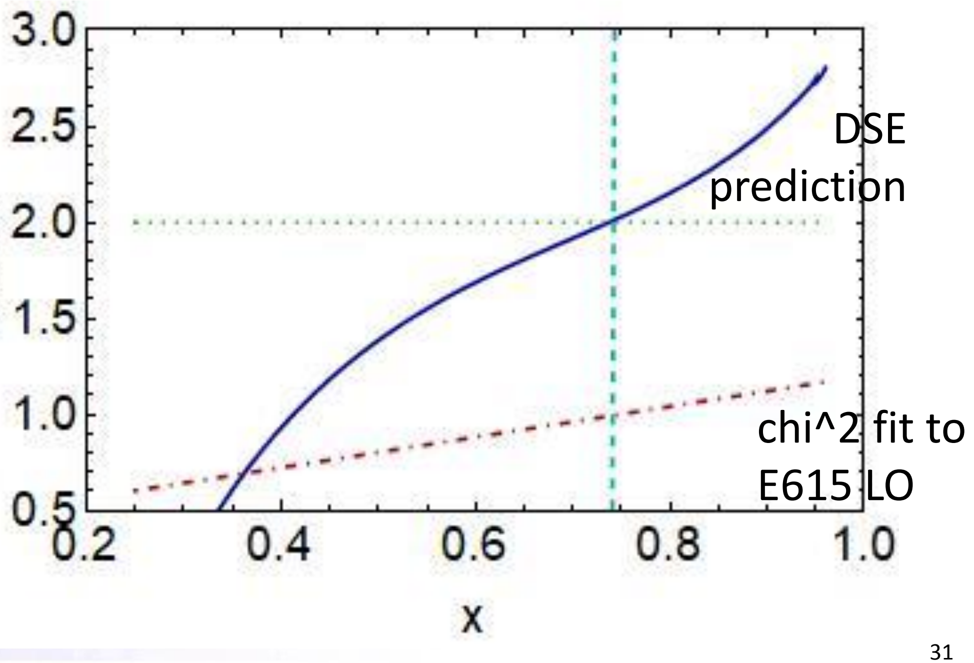
$$\beta(x) = -\frac{1-x}{q_V^{\pi}(x)}\frac{dq_V^{\pi}(x)}{dx}$$

  - “Active” power greater > 2 on x > 0.75



Precise data & sound extraction on 0.6 < x < 0.8 sufficient to test QCD prediction: 2 ≠ 1

Effective $\beta(x)$



This is not the end, this is not even the beginning of the end, this is just perhaps the end of the beginning.