Meson Distribution Amplitude in the Continuum Limit from Lattice QCD

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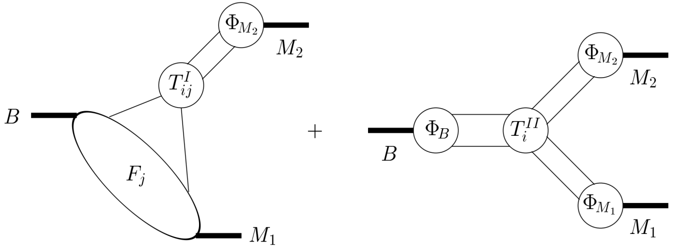
Based on PhysRevD.102.094519

This work is supported by US National Science Foundation under grant PHY 1653405 "CAREER: Constraining PartonDistribution Functions for New-Physics Searches".

Meson DA in exclusive processes

Factorization of exclusive processes [Beneke et al., NPB 2001]:



$$\langle M_1 M_2|Q_i|B\rangle = F_0^{B\to M_1} \otimes T^I_{M_2,i} \otimes \Phi_{M_2} + T^{II}_i \otimes \Phi_B \otimes \Phi_K \otimes \Phi_\pi$$

Meson DA: probability of finding the meson in $q_1 - \bar{q}_2$ fock state.

$$\tilde{\phi}_M(x,\mu) = \frac{i}{f_M}\int \frac{d\xi}{2\pi} e^{i(x-1)P\cdot\xi n}\langle M(P)|\bar{\psi}(0)n\cdot\gamma\gamma_5\Gamma(0,\xi n)\psi(\xi n)|0\rangle$$

Properties of meson DA

Similarities to parton distribution functions:

- Universal quantity
- Non-perturbative nature
- Light-like correlation

On the other hand:

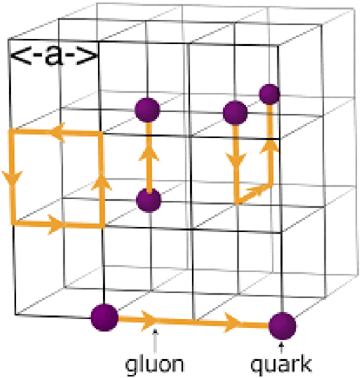
- Less contrained by experiments
- Various model-dependent calculations
- No global-fitting result to compare with

A direct calculation from first principle may put more constraints on this quantity.

Lattice QCD - Numerical Simulation of QCD

- Discretization of the 4-d spacetime
- Wick rotation to Euclidean lattice
- Quark fields on lattice sites
- Gauge links connecting lattice sites
- Use of a heavier-than-physical M_π
- Random generation of the gauge configurations with probability $\frac{e^{-S_E(U_i,\psi_i)}}{\int \mathcal{D}\psi \mathcal{D}U e^{-S_E(U_i,\psi_i)}}$
- Evaluation of non-perturbative observables

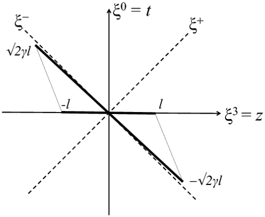
$$\langle \tilde{\mathcal{O}} \rangle = \lim_{N\to\infty} \frac{1}{N} \sum_i^N \tilde{\mathcal{O}}_i$$



Credit: *M. Savage@NNPSS2015*

lightcone-DA to quasi-DA

The lightlike correlation cannot be calculated on an Euclidean lattice.



Alternative approach:
spatial separation
↓ *boost*
light-like separation

Credit: *Ji,PRL.2013*

- quasi-DA [Ji,PRL.2013]:

$$\tilde{\phi}_M(x, \mu_R, P_z) = \frac{i}{f_M}\int \frac{dz}{2\pi} e^{-i(x-1)P_z z}\langle M(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z)|0\rangle$$

- Factorization relating quasi-DA to light-cone DA

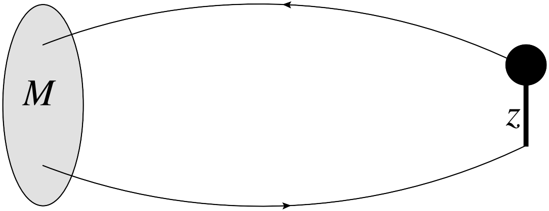
$$\tilde{\phi}_M(x, \mu_R, P_z) = \int_0^1 dy\, C_\phi(x, y, \mu, \mu_R, P_z)\phi_M(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{m_M^2}{P_z^2}\right),$$

quasi-DA from lattice

- matrix element in coordinate space

$$\tilde{h}_M(z, P_z) = \langle M(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z)|0\rangle$$

- Meson boosted with large momentum in $\hat{z}$ direction.
- Disconnected contribution for flavor singlet η_s is ignored.



Lattice Set Up

We do the measurements for three mesons π, K, η_s on three different lattices,

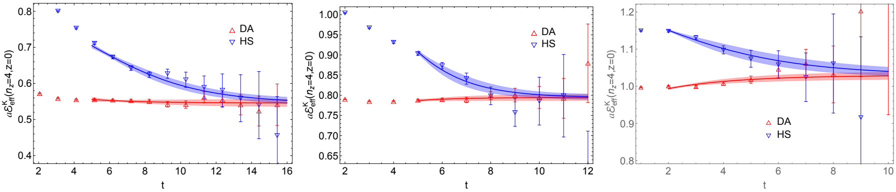
- $2+1+1$ dynamic Highly Improved Staggered Quark (HISQ) in the sea
- Clover action for valence quarks
- Hypercubic smeared gauge configuration
- $M_\pi \approx 310$ MeV
- $M_{\eta_s} \approx$ 690MeV

a(fm)	V	P_z (GeV)	$\#_{meas}$
0.06	$48^3 \times 144$	$0.87/1.31/1.74$	2496
0.09	$32^3 \times 96$	$0.87/1.31/1.74$	9216
0.12	$24^3 \times 64$	$0.87/1.31/1.74$	3072

Fit of Quasi-PDF Matrix Elements

The correlators can be expanded in energy eigenstates:

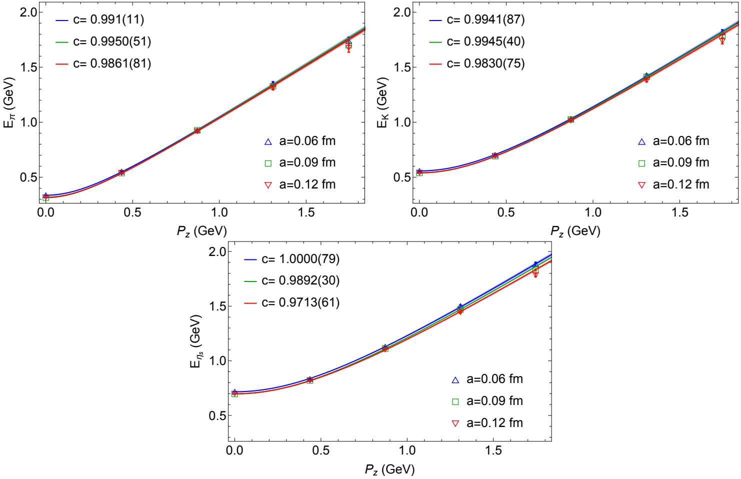
$$C^{\text{HS}}(t,z,P_z) = \mathcal{A}_0^{\text{HS}}(z,P_z)e^{-E_0 t} + \mathcal{A}_1^{\text{HS}}(z,P_z)e^{-E_1 t} + \dots$$
$$C^{\text{DA}}(t,z,P_z) = \mathcal{A}_0^{\text{DA}}(z,P_z)e^{-E_0 t} + \mathcal{A}_1^{\text{DA}}(z,P_z)e^{-E_1 t} + \dots$$



The matrix elements are obtained by taking the ratio of the ground state coefficients $h(z,P_z) = \mathcal{A}_0^{\text{DA}}(z,P_z)/\mathcal{A}_0^{\text{DA}}(0,P_z)$.

Dispersion Relations

The energies as functions are fitted to form $E(P_z)^2 = P_z^2 c^2 + E(P_z = 0)^2$.

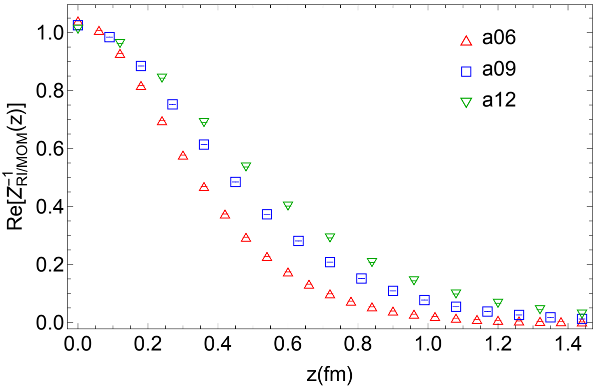


Light speed is consistent with 1 on finest lattice.

Non-Perturbative Renormalization

Non-perturbative renormalization condition in RI/MOM scheme:

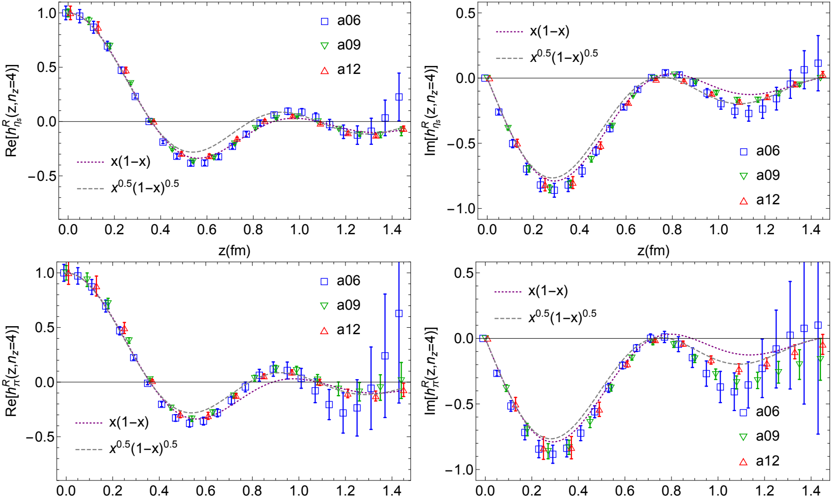
$$Z_{\gamma_z\gamma_5}(z)\langle p|\bar{\psi}(\vec{x})\gamma^z\gamma_5\Gamma(\vec{x},\vec{x}+z)\psi(\vec{x}+z)|p\rangle_{p^2=-\mu_R^2} = \langle p|\bar{\psi}(\vec{x})\gamma^z\gamma_5\Gamma(\vec{x},\vec{x}+z)\psi(\vec{x}+z)|p\rangle_{\text{tree}}$$



RI/MOM scale is chosen to be $\mu_R = 3.8$ GeV, $p_R^z = 0$.

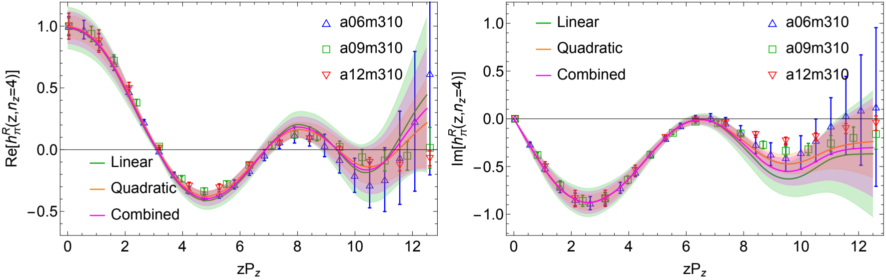
Renormalized Matrix Elements

η_s (upper) is closer to $\phi(x) = 6x(1-x)$ (asymptotic form), pion (lower) is closer to $\phi(x) = \frac{8}{\pi}\sqrt{x(1-x)}$ (a form with closer first moment),



Extrapolation to continuum

- Three data sets: $a = 0.06, 0.09, 0.12$ fm.
- Two simplest forms with two free parameters:
  - Linear: $h^R_M(z, a) = h^R_1(z) + c_{M,1}a$
  - Quadratic: $h^R_M(z, a) = h^R_2(z) + c_{M,2}a^2$
- Interpolating $h(z)$ with bootstrap before continuum extrapolation.
- Weight-averaged (AIC) fit results: $h^R_M(z) = \frac{h^R_{M,1}(z)e^{-\chi^2_1/2} + h^R_{M,2}(z)e^{-\chi^2_2/2}}{e^{-\chi^2_1/2} + e^{-\chi^2_2/2}}$

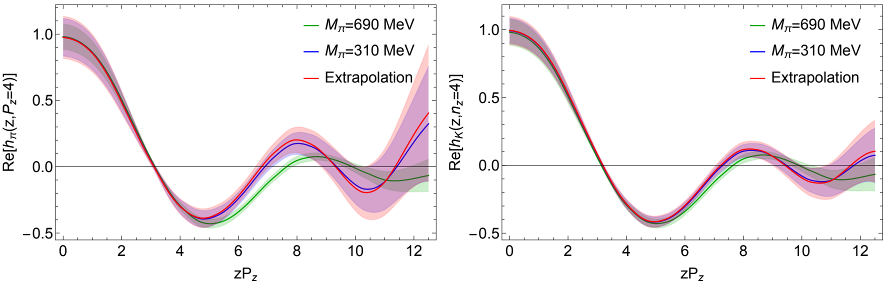


Combined results are closer to linear extrapolations.

Chiral Extrapolation

$\eta_s \to M_\pi = 690$ MeV. A naive linear extrapolation with two M_π:

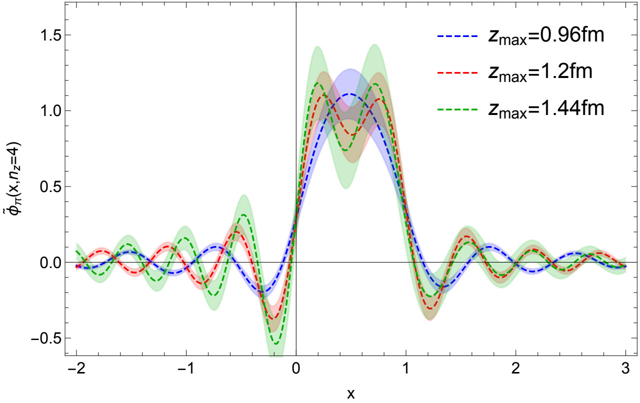
$$h_M^R(M_\pi, a = 0) = s_M M_\pi^2 + h(0),$$



The result is very close to the light pion result.

Extracting x-dependenc of light-cone DA

A truncated FT and inverse matching may not be a good approach:

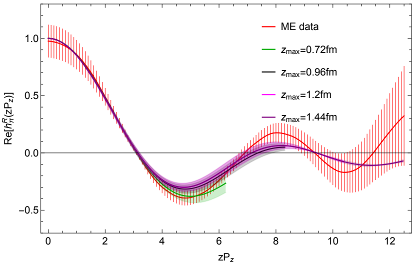


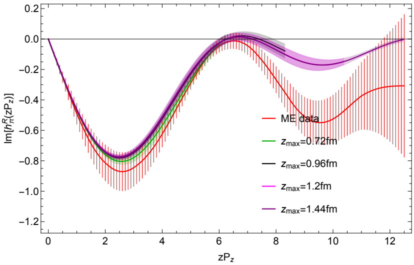
- Shape of peak is sensitive to z_{max}.
- High-frequency oscillations with large z_{max}.
- Derivative method does not work well on DA.

An attempt in fitting to analytical form

Fit $h(z)$ to ME obtained from $f_{m,n} = x^m(1-x)^n / B(m+1, n+1)$:

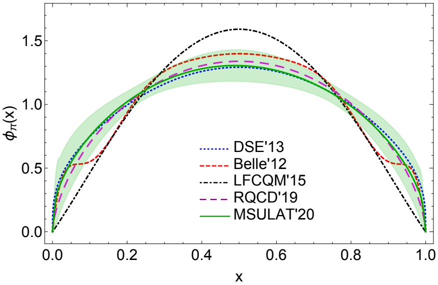
$$h_{m,n}(z, \mu_R, P_z) = \int_{-\infty}^{\infty} e^{i(1-x)zP_z} dx \int_0^1 dy\, C_\phi(x, y, \mu, \mu_R, P_z) f_{m,n}(y),$$

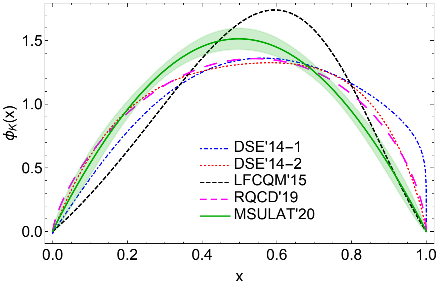


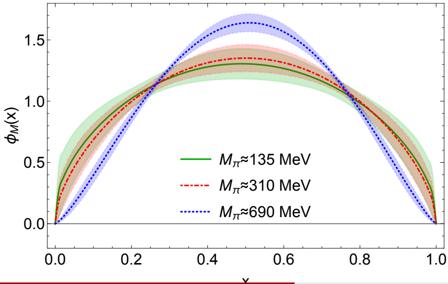


The model fail to recover the large amplitude of the secondary peak.
No improvement with an additional term $f_m = x^m(1-x)^m(1 + tx(1-x))$.

Fit results in x-space







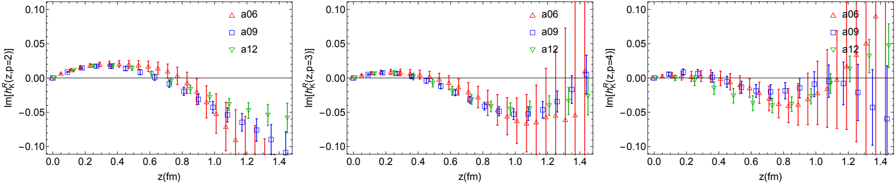
- Heavier pion mass favors larger $\{m, n\}$, a narrower distribution.
- No obvious kaon asymmetry.

Kaon asymmetry revisit

The kaon asymmetry can be checked directly in coordinate space:

$$\tilde{H}^R(zP_z, p_z^R, \mu^R) = e^{-izP_z/2} h^R(zP_z, p_z^R, \mu^R) \quad (1)$$

Asymmetry $\Leftrightarrow Im[\tilde{H}^R] \neq 0$

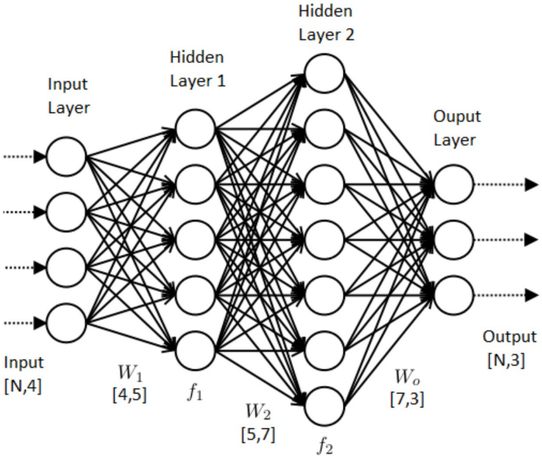


- The asymmetry vanishes when extrapolated to continuum at $n_z = 4$.
- Not necessarily goes to zero at larger momentum.

Machine Learning Approach?

Machine learning models are effective in extracting complicated dependence of the output data on input data.

$\mathrm{Re}[h(z)]$, $\mathrm{Im}[h(z)]$



$\phi(x)$

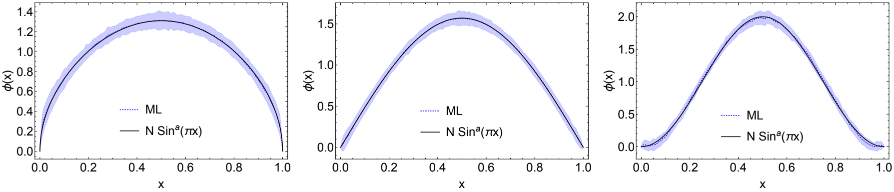
Credit: *VIASAT*

The machine learning approach

Train a MLP regressor with pseudo-data generated from random $f_{m,n}$.

- 3 hidden layers
- 100 perceptrons on each layer
- activation function $g(x) = max(x, 0)$
- minimizing mean suqare loss function

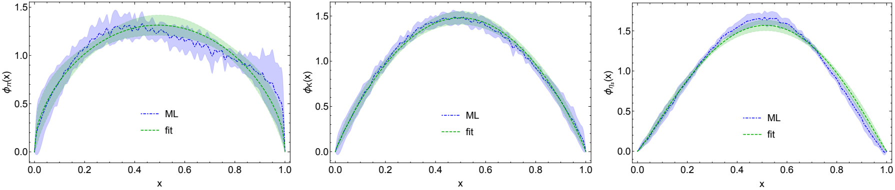
Test on another function form $\sin^a(\pi x)$:



Thus the prediction on unknown fucntion form is promising.

Comparison of the two approach

The predictions from machine learning model are very close to our fit results because of the limited function form we used to train the model.



- Need more generalized training data

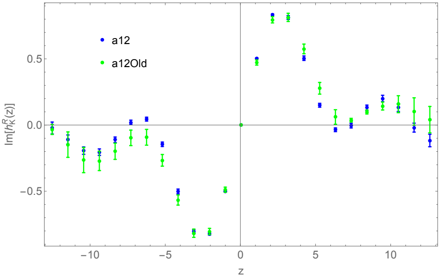
Summary

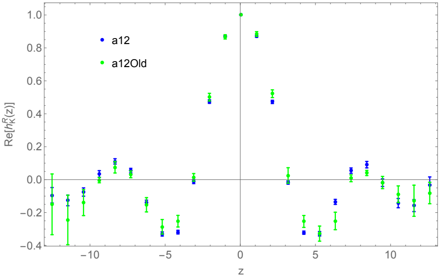
- Calculated quasi-DA of pion, kaon and η_s on three lattice spacings;
- Renormalized matrix elements in RI/MOM scheme;
- Extrapolated to continuum;
- Naively chiral extrapolated to physical pion mass.
- Fit the matrix element to a function form in x-space.
- Attemptted to obtain the x-dependence DA from machine learning results.

Comparison with old result from LP3

Statistics:

- Old results: 3868
- New results: 2496





They're consistent up to 2σ.