Lattice parton distributions and global QCD analysis

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Confronting lattice parton distributions with global QCD analysis

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We present the first Monte Carlo based global QCD analysis of spin-averaged and spin-dependent parton distribution functions (PDFs) that includes nucleon isovector matrix elements in coordinate space from lattice QCD. We investigate the degree of universality of the extracted PDFs when the lattice and experimental data are treated under the same conditions within the Bayesian likelihood analysis. For the unpolarized sector, we find rather weak constraints from the current lattice data on the phenomenological PDFs, and difficulties in describing the lattice matrix elements at large spatial distances. In contrast, for the polarized PDFs we find good agreement between experiment and lattice data, with the latter providing significant constraints on the spin-dependent isovector quark and antiquark distributions.

U. Maryland

JLab Theory Center

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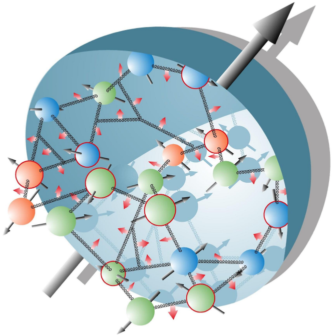
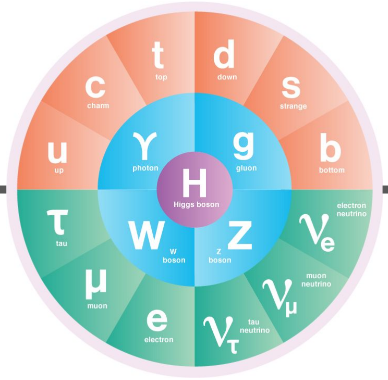
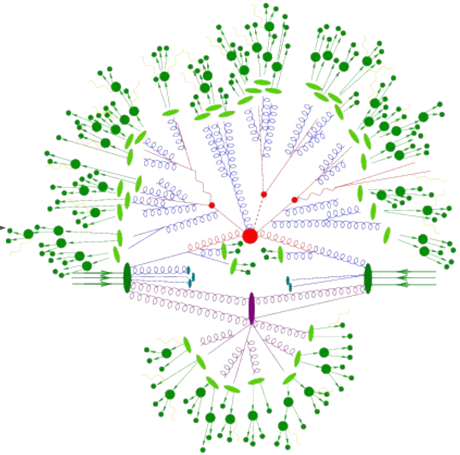
Temple U.

Motivations



Nucleon Structure





Hadronization

Hadrons are **emergent phenomena** of QCD

What do we mean by "hadron structure" ? (1D)

$$\xi = \frac{k^+}{P^+}$$

Parton momentum fraction relative to parent hadron

$$f_i(\xi) = \int \frac{\mathrm{d}w^-}{4\pi} e^{-i\xi p^+ w^-} \langle N|\bar{\psi}_i(0, w^-, \mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle$$

parton distribution function (PDF)

Interpretation in non-interacting QCD

$$\psi_i(x) = \sum_{k,\alpha} b_{k,\alpha}(x^+) u_{k,\alpha} e^{-ik^+x^- + ik_\mathrm{T}\cdot x_\mathrm{T}} + d^\dagger_{k,\alpha}(x^+) u_{k,-\alpha} e^{ik^+x^- - ik_\mathrm{T}\cdot x_\mathrm{T}}$$

$$f_i(\xi) \sim \sum_\alpha \int \mathrm{d}^2k_\mathrm{T} \langle N| \underbrace{b^\dagger_{k,\alpha} b_{k,\alpha}(\xi p^+, k_\mathrm{T}, \alpha)}_{\text{number operator}} |N\rangle$$

What do we mean by "hadronization" ? (1D)

$$\zeta = \frac{p_h^+}{k^+}$$

hadron momentum fraction relative to parent parton

$$d_{h/j}(\zeta) = \frac{\mathrm{Tr}_{\text{color,Dirac}}}{4N_{c,j}} \sum_X \zeta \int \frac{dw^+}{2\pi} e^{i(p_h^-/\zeta)w^+}$$
$$\times \gamma^- \langle 0 | \bar{\psi}_j(0, w^+, \mathbf{0}_\mathrm{T}) \; | p_h, X \rangle \langle p_h, X | \; \psi_j(0) | 0 \rangle$$

Fragmentation functions (FFs)

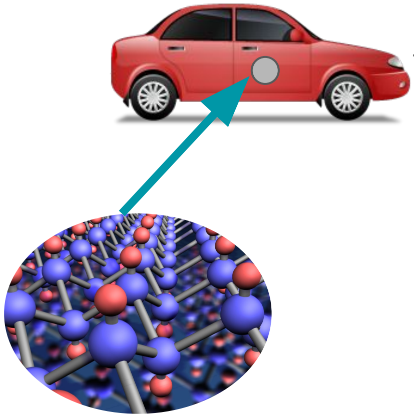
X = all states except detected hadron h

So how do we get hadron structure from experimental data?

What part of this is the
"**internal structure**"?

Factorization

Want to see
internal structure



But we only see **debris**

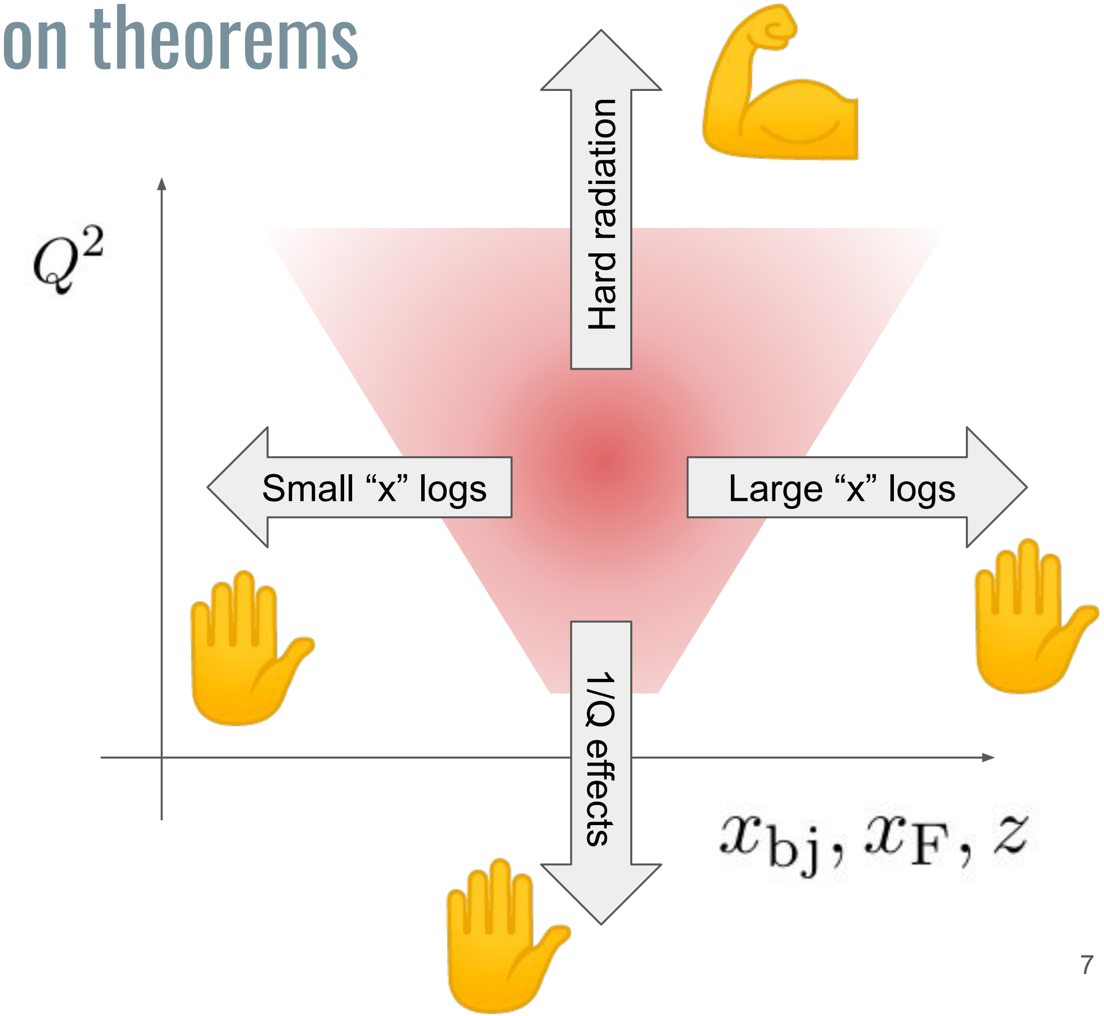
e.g. Collinear Factorization theorems

$$d\sigma_{\mathrm{DIS}} = \sum_i H_i^{\mathrm{DIS}} \otimes f_i$$

$$d\sigma_{\mathrm{DY}} = \sum_{ij} H_{ij}^{\mathrm{DY}} \otimes f_i \otimes f_j$$

$$d\sigma_{\mathrm{SIA}} = \sum_i H_i^{\mathrm{SIA}} \otimes d_i$$

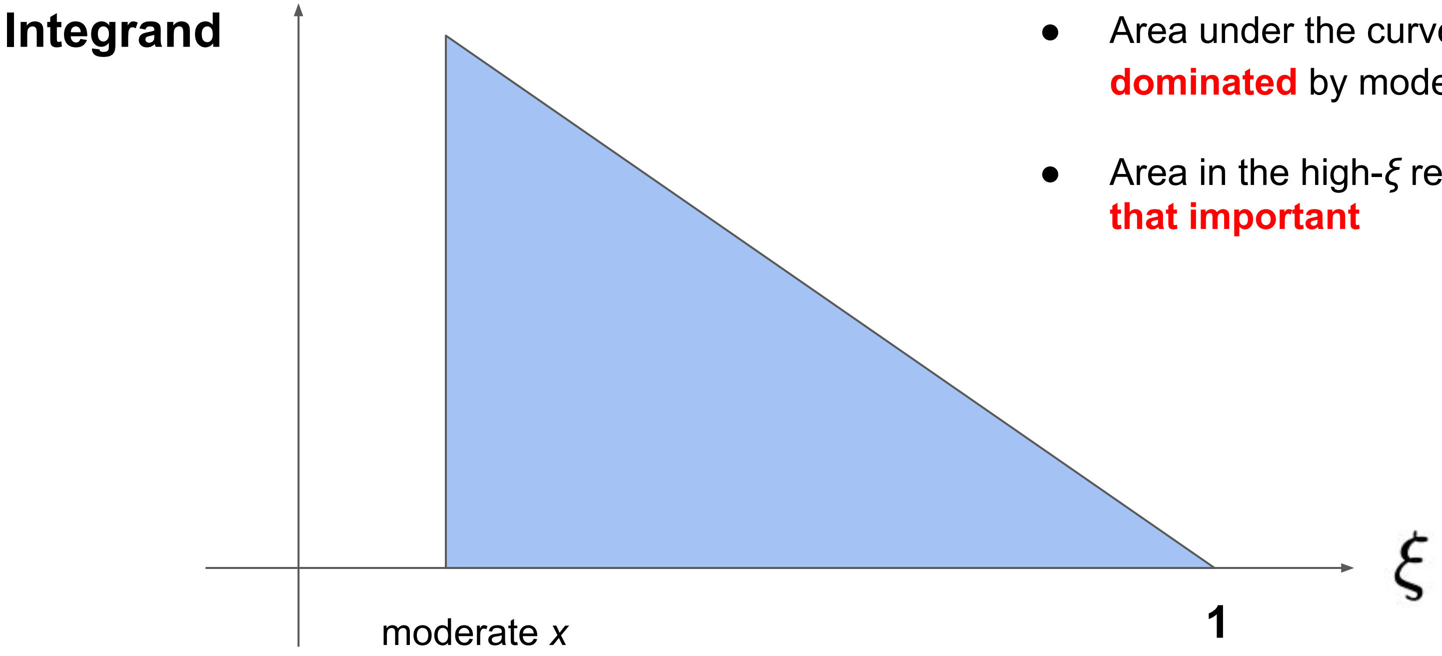
$$d\sigma_{\mathrm{SIDIS}} = \sum_{ij} H_{ij}^{\mathrm{SIDIS}} \otimes f_i \otimes d_j$$



Moderate "x"

@ moderate Q

$$\frac{\mathrm{d}\sigma^{\mathrm{DIS}}}{\mathrm{d}x_{\mathrm{bj}}\mathrm{d}Q^2} = \sum_i \int_{x_{\mathrm{bj}}}^{1} \frac{\mathrm{d}\xi}{\xi} H_i\left(\frac{x}{\xi}, Q, \frac{\mu}{Q}\right) f_i(\xi, \mu)$$

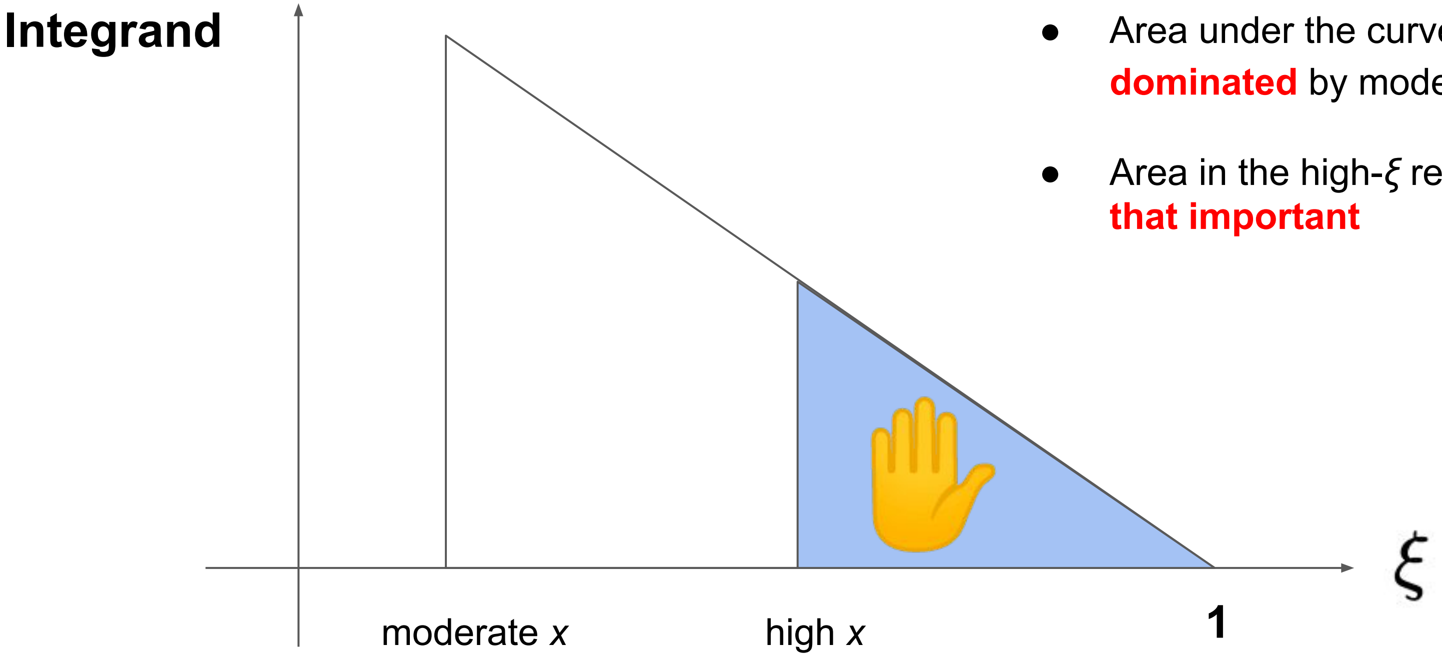


- Area under the curve is **dominated** by moderate-x region
- Area in the high-ξ region is **not that important**

High "x"

@ moderate *Q*

$$\frac{d\sigma^{\mathrm{DIS}}}{dx_{\mathrm{bj}}dQ^2} = \sum_i \int_{x_{\mathrm{bj}}}^{1} \frac{d\xi}{\xi} H_i\left(\frac{x}{\xi}, Q, \frac{\mu}{Q}\right) f_i(\xi, \mu)$$

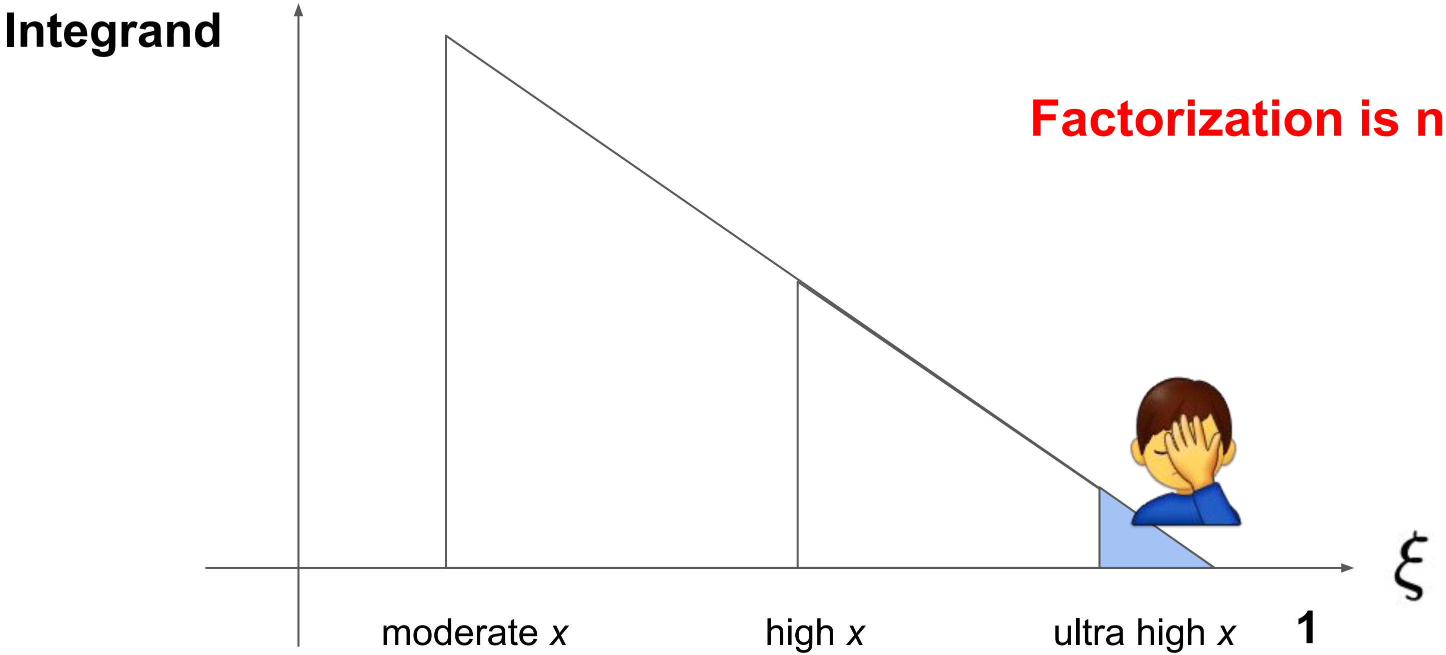


- Area under the curve is **dominated** by moderate-x region
- Area in the high-ξ region is **not that important**

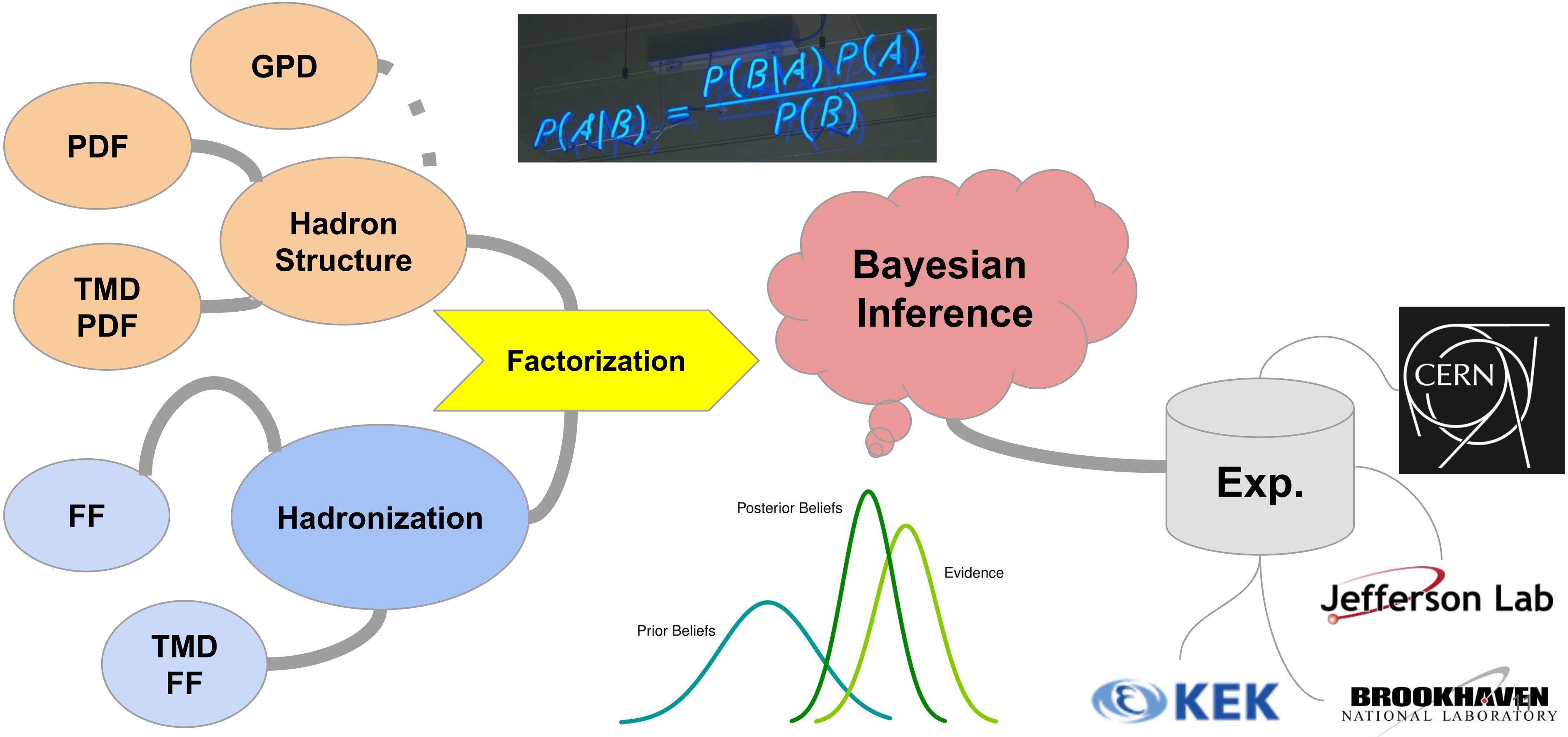
Ultra high “x”
@ moderate Q

$$\frac{d\sigma^{\text{DIS}}}{dx_{\text{bj}}dQ^2} = \sum_i \int_{x_{\text{bj}}}^1 \frac{d\xi}{\xi} H_i\left(\frac{x}{\xi}, Q, \frac{\mu}{Q}\right) f_i(\xi, \mu)$$

Factorization is not valid



JAM global QCD analysis paradigm



Bayesian inference

Experiments = theory + errors

$$d\sigma_{\mathrm{DIS}} = \sum_i H_i^{\mathrm{DIS}} \otimes f_i$$

$$d\sigma_{\mathrm{DY}} = \sum_{ij} H_{ij}^{\mathrm{DY}} \otimes f_i \otimes f_j$$

$$d\sigma_{\mathrm{SIA}} = \sum_i H_i^{\mathrm{SIA}} \otimes d_i$$

$$d\sigma_{\mathrm{SIDIS}} = \sum_{ij} H_{ij}^{\mathrm{SIDIS}} \otimes f_i \otimes d_j$$

Hadron Structure

Hadronization

RGE boundary conditions

$$f_i(\xi, \mu_0^2) = N_i \xi^{a_i} (1-\xi)^{b_i} (1 + \ldots)$$

$$d_i(\zeta, \mu_0^2) = N_i \zeta^{a_i} (1-\zeta)^{b_i} (1 + \ldots)$$

$$\mathbf{a} = (N_i, a_i, b_i, \ldots)$$

Posterior distribution

Prior distribution

$$\rho(\mathbf{a}|\mathrm{data}) \sim \mathcal{L}(\mathbf{a}, \mathrm{data}) \pi(\mathbf{a})$$

Likelihood

$$\mathcal{L}(\boldsymbol{a}, \mathrm{data}) = \exp\left[-\frac{1}{2}\chi^2(\boldsymbol{a}, \mathrm{data})\right]$$

$$\mathrm{E}[f_i(\xi, \mu^2)] = \int d^n\mathbf{a}\, \rho(\mathbf{a}|\mathrm{data}) f_i(\xi, \mu^2; \mathbf{a})$$

$$\mathrm{V}[f_i(\xi, \mu^2)] = \int d^n\mathbf{a}\, \rho(\mathbf{a}|\mathrm{data}) \left[f_i(\xi, \mu^2; \mathbf{a}) - \mathrm{E}[f_i(\xi, \mu^2)]\right]^2$$

So how do we get hadron structure from lattice?

$$\mathcal{M}^q_{[\Gamma]}(z,\mu) = Z_\Gamma(z,\mu)\,\langle N(P_3)|\,\overline{\psi}_q(0,z)\,\Gamma\, W_3(z)\,\psi_q(0,0)\,|N(P_3)\rangle$$

Perturbative matching (analogous to factorization)

$$\widetilde{f}_q(x,\mu,P_3) = P_3 \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP_3 z}\, \mathcal{M}_q(z,\mu)$$

$$\Delta\widetilde{f}_q(x,\mu,P_3) = P_3 \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP_3 z}\, \mathcal{M}_{\Delta q}(z,\mu)$$

$$\mathcal{M}_q(z,\mu) = \int_{-\infty}^{\infty} dx\, e^{-ixP_3 z} \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C_q\!\left(\frac{x}{\xi}, \frac{\mu}{\xi P_3}\right) f_q(\xi,\mu)$$

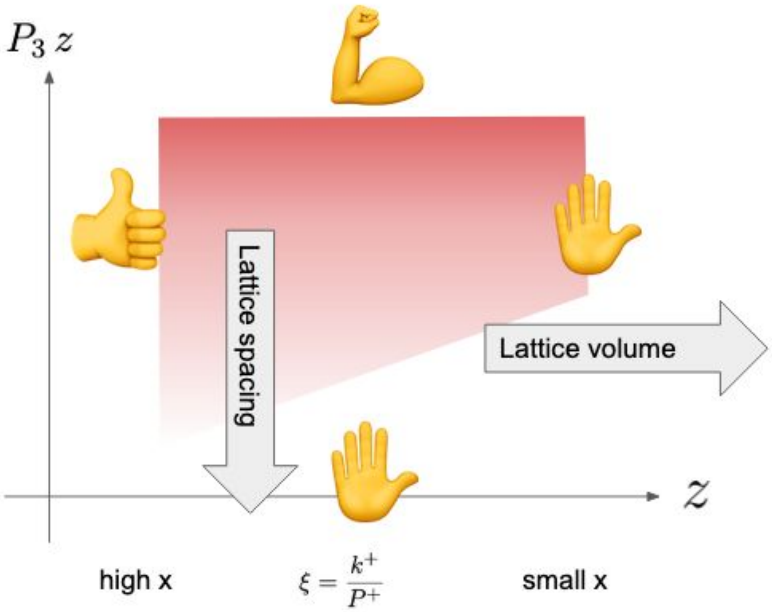
$$\mathcal{M}_{\Delta q}(z,\mu) = \int_{-\infty}^{\infty} dx\, e^{-ixP_3 z} \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C_{\Delta q}\!\left(\frac{x}{\xi}, \frac{\mu}{\xi P_3}\right) \Delta f_q(\xi,\mu)$$

PDF "reconstruction" approach
pro: no need to involve additional data
con: relies on unmeasured |z| --> infinity regions

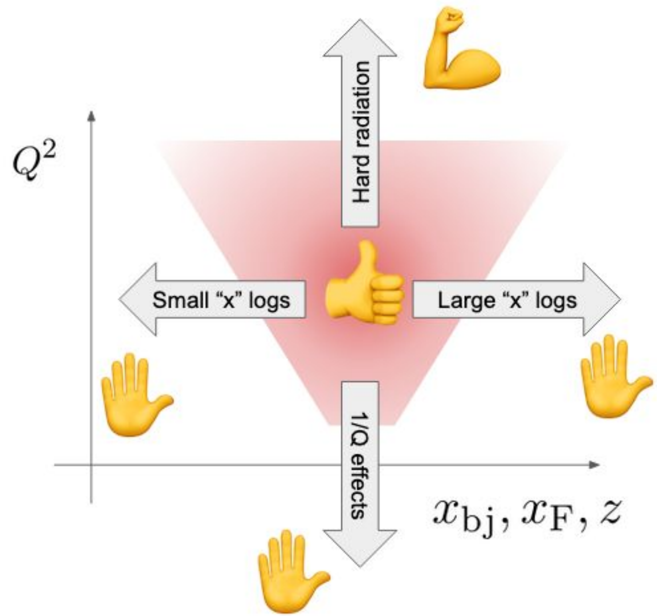
PDF fitting approach
pro: can be combined with experimental data
con: PDF modeling is needed

Applicability of lattice "factorization"

Lattice

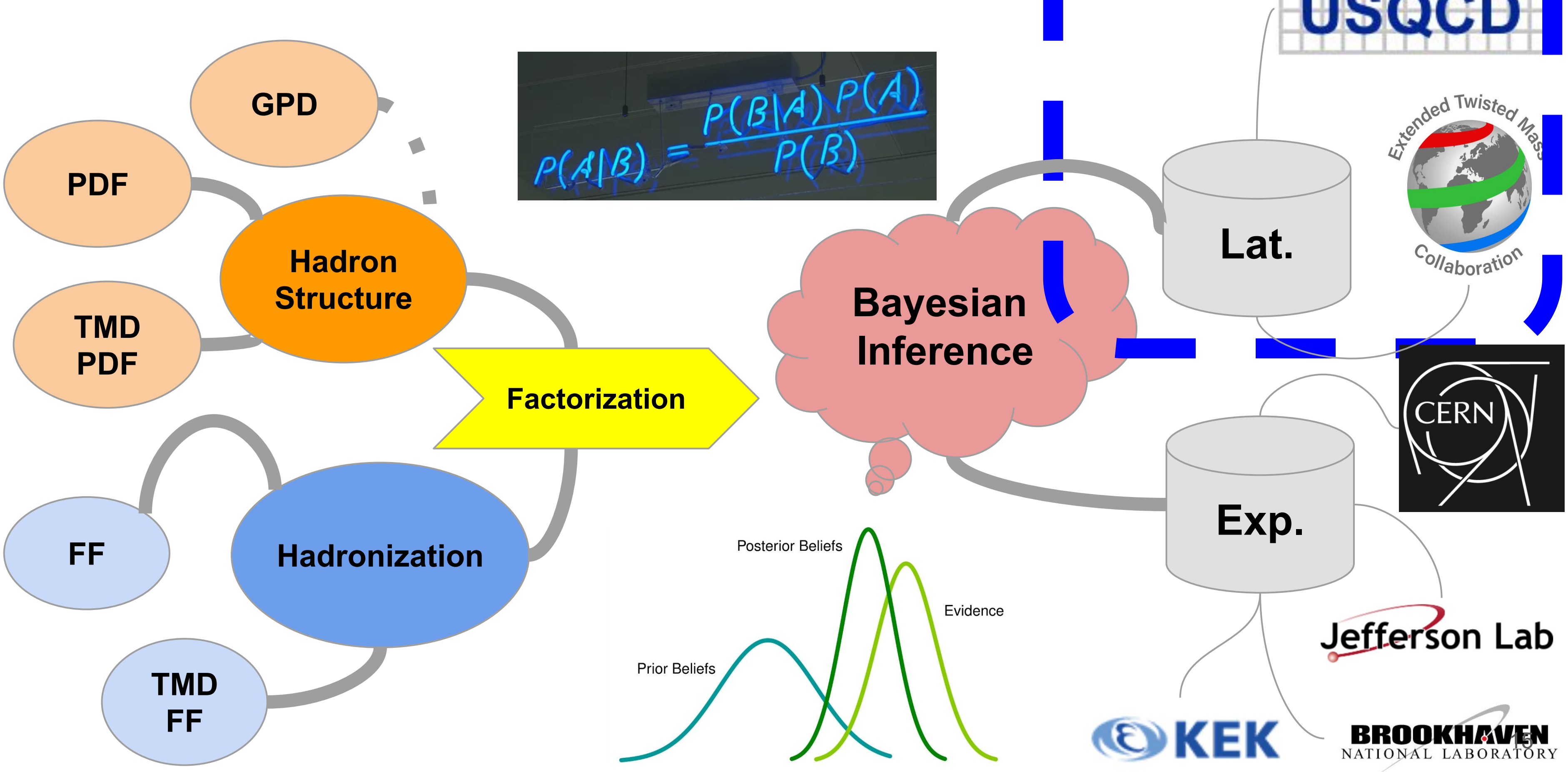


Experiments



$$\mathcal{M}^q_{[\Gamma]}(z,\mu) = Z_\Gamma(z,\mu)\,\langle N(P_3)|\,\overline{\psi}_q(0,z)\,\Gamma\,W_3(z)\,\psi_q(0,0)\,|N(P_3)\rangle$$

JAM+lattice QCD global analysis paradigm

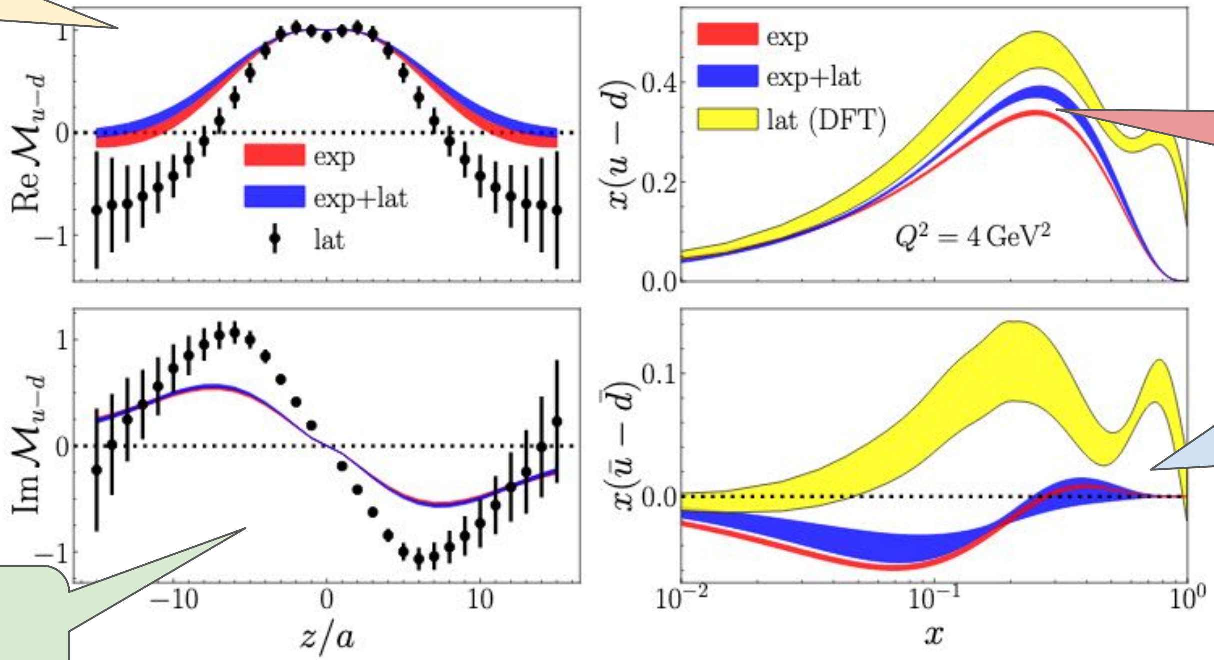


Unpolarized PDF analysis

$$\mathrm{Re}\,\mathcal{M}_q(z,\mu) = -\int_0^1 dy\,\cos(yP_3z)\,[q(y)-\bar{q}(y)] + \mathcal{O}(\alpha_s^2),$$

$$\mathrm{Im}\,\mathcal{M}_q(z,\mu) = \int_0^1 dy\,\sin(yP_3z)\,[q(y)+\bar{q}(y)] + \mathcal{O}(\alpha_s^2),$$

Sensitive only to valence



No significant impact

Opposite dbar-ubar asymmetry!

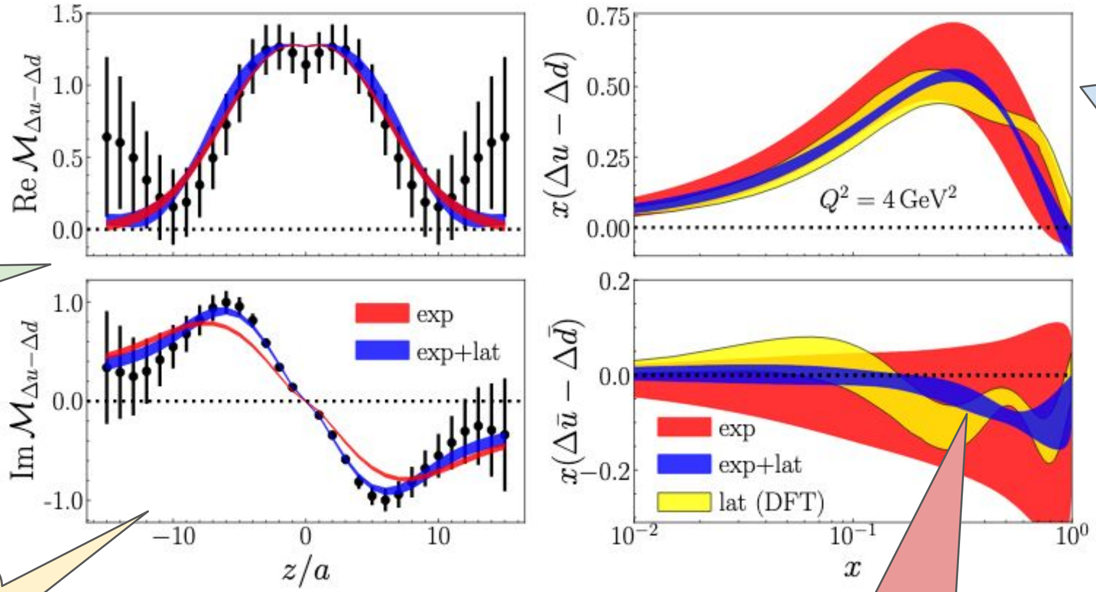
Sensitive to sea-quark PDFs

Polarized PDF analysis

$$\mathrm{Re}\,\mathcal{M}_{\Delta q}(z,\mu) = -\int_0^1 dy\,\cos(yP_3z)\,[\Delta q(y)+\Delta\bar{q}(y)] + \mathcal{O}(\alpha_s^2),$$

$$\mathrm{Im}\,\mathcal{M}_{\Delta q}(z,\mu) = \int_0^1 dy\,\sin(yP_3z)\,[\Delta q(y)-\Delta\bar{q}(y)] + \mathcal{O}(\alpha_s^2),$$

Sensitive to sea-quark PDFs

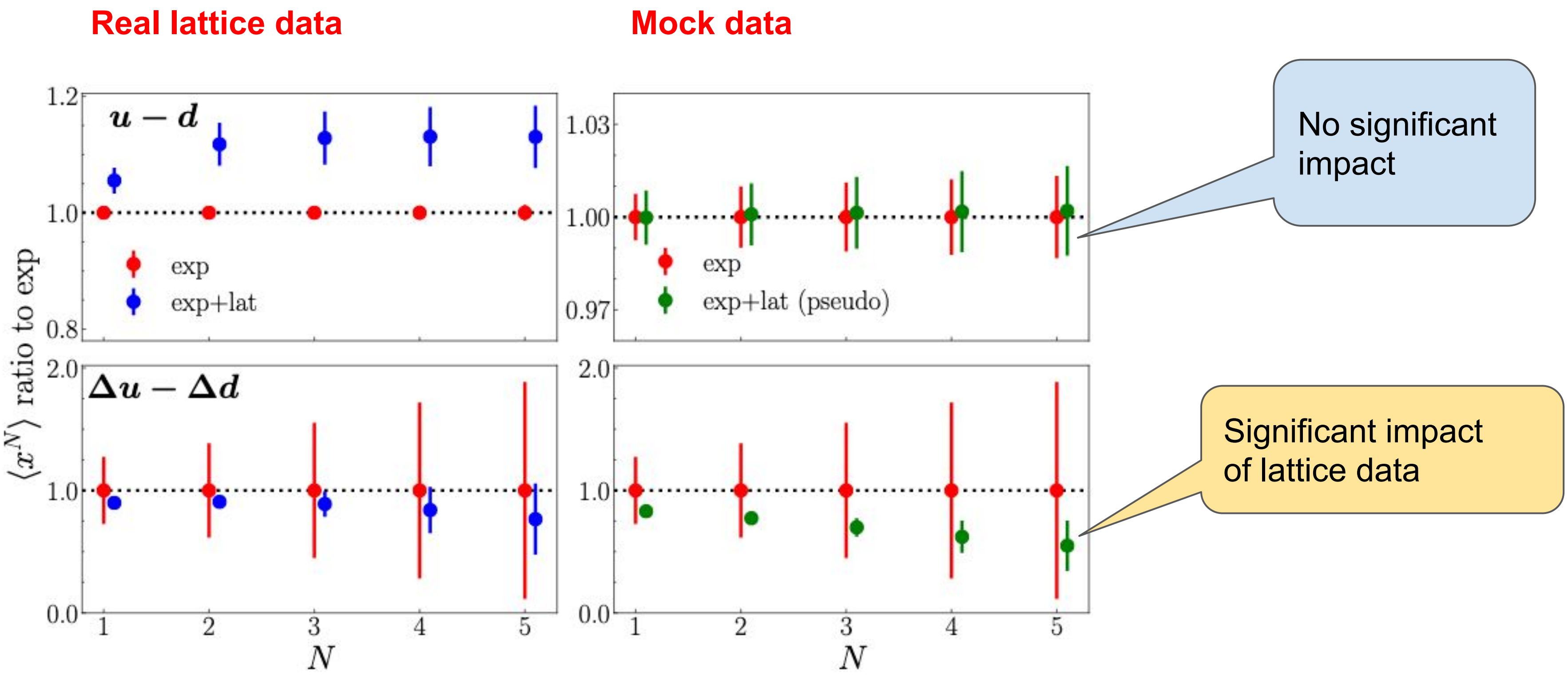


Compatibility with exp. data

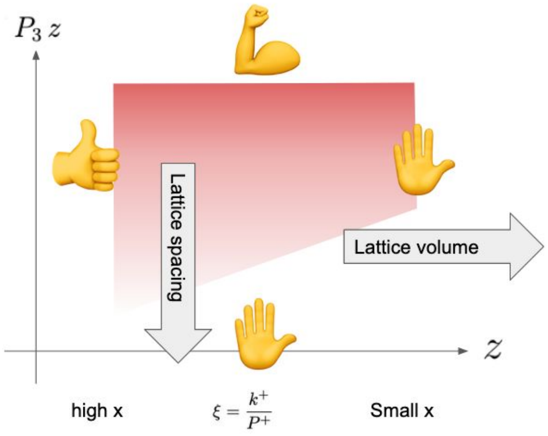
Sensitive only to valence

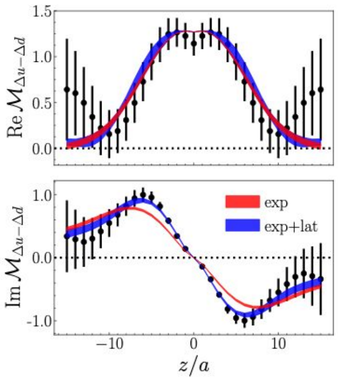
Significant impact of lattice data

Impact of lattice data

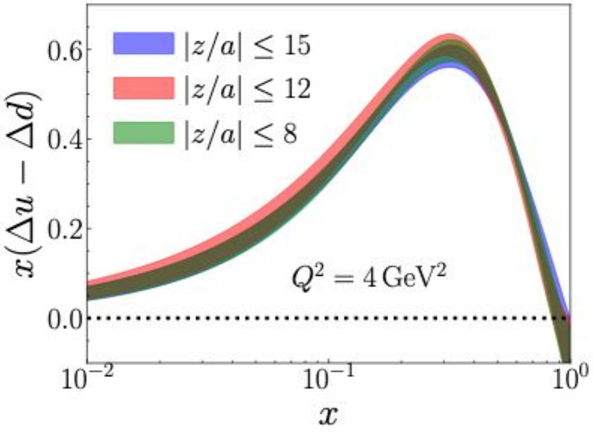


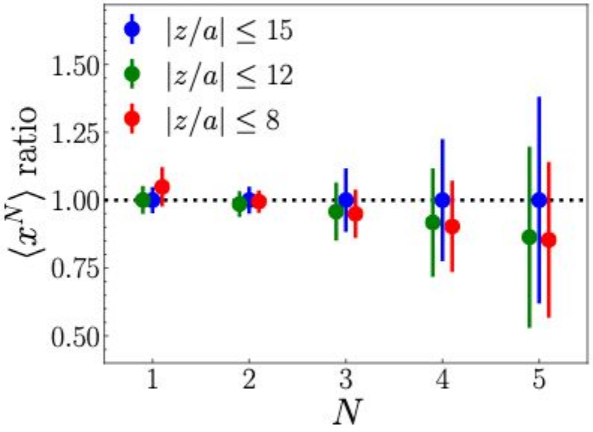
Importance of high-|z| lattice data





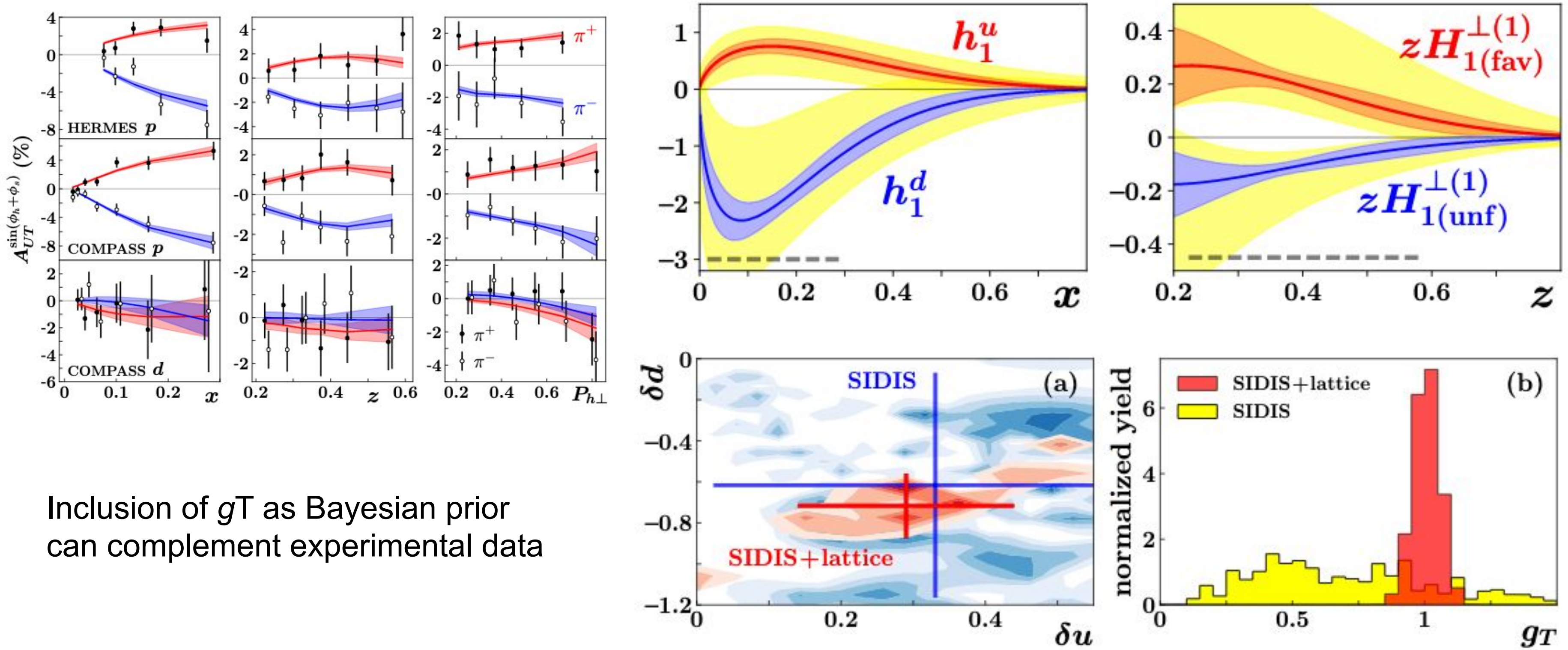
PDFs are not sensitive to large *z*





JAM’18 (3D experiment + lattice QCD: gT moment)

Lin, Melnitchouk, Prokudin, NS, Shows

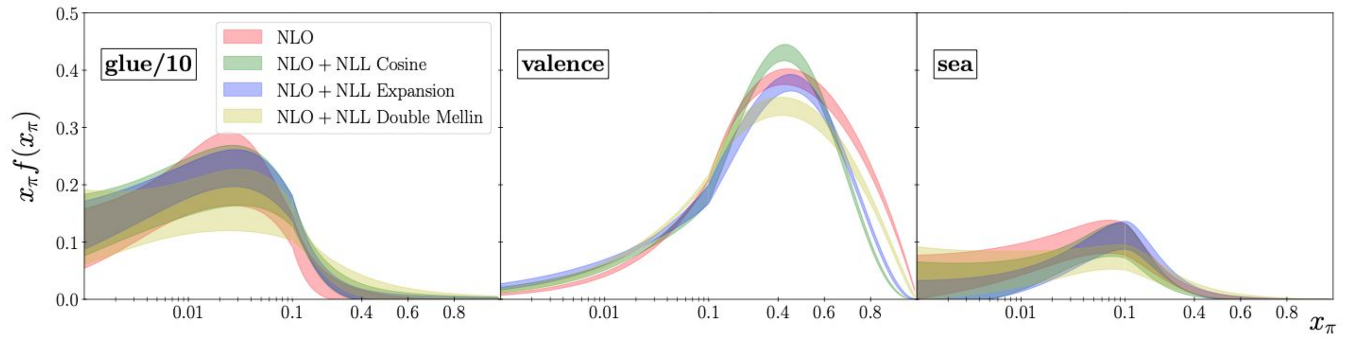


Inclusion of g_T as Bayesian prior can complement experimental data

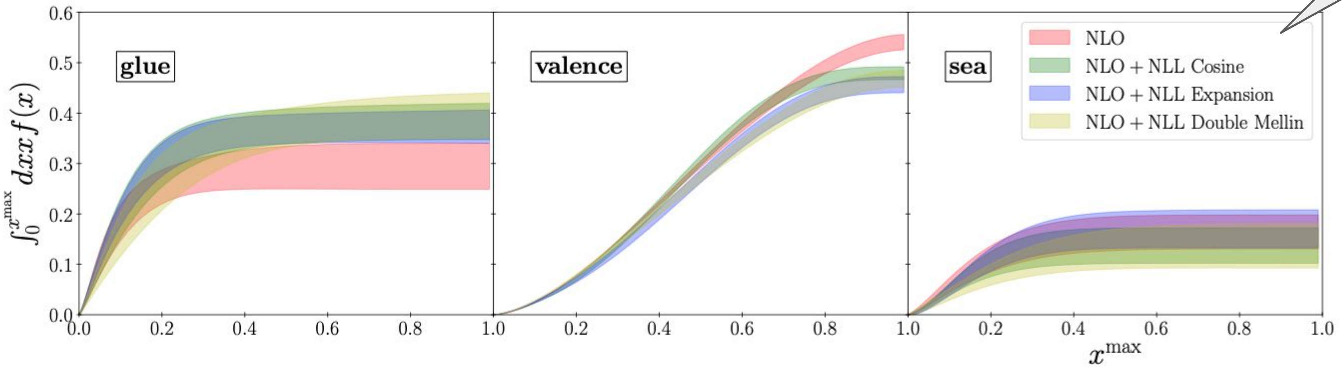
JAM PIONS with Threshold Resummation

Barry, NS, Ji, Melnitchouk
(in preparation)

Pion PDFs at input scale



Truncated momentum fractions

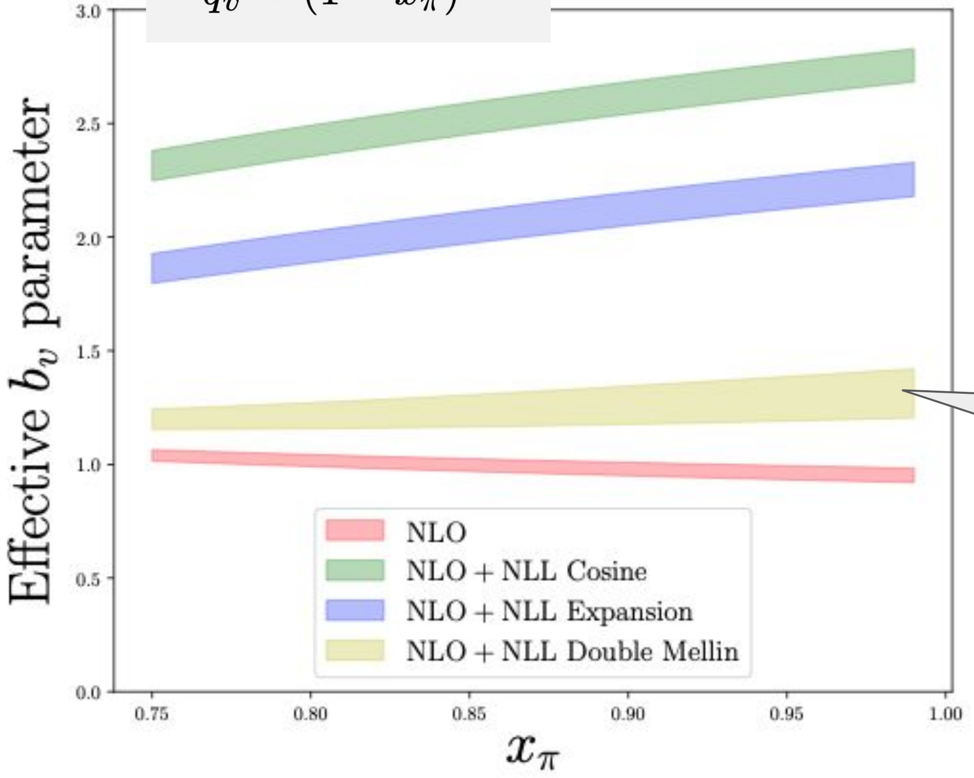


Threshold resummation **is scheme dependent**

JAM PIONS with Threshold Resummation

Barry, NS, Ji, Melnitchouk
(in preparation)

$$q_v \sim (1 - x_\pi)^{b_v}$$



Exponents close to 1 even with threshold resummation

Summary and Outlook

A new paradigm

- Inclusion of lattice data with experimental data
- MC methods for reliable uncertainty quantification

Near future

- Global analysis of lattice polarized twist 3 (gT)
- Meson exp data + lattice data

JAM Collaboration

Staff / Faculty

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Students / Postdocs

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