

Diquark correlations and where to find them?

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Perceiving the Emergence of Hadron Mass through AMBER@CERN

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Emergence

Low-level rules producing high-level phenomena with enormous apparent complexity

Start from the QCD Lagrangian:

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}(i\not{D}-m)\psi - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \frac{1}{2\xi} (\partial^\mu A_\mu^a)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc} (\partial^\mu \bar{c}^a) A_\mu^b c^c.$$



Lattice-regularized QCD, Continuum Schwinger-function methods, ...

And obtain:

- ☞ Dynamical generation of fundamental mass scale in pure Yang-Mills (gluon mass).
- ☞ Quark constituent masses and dynamical chiral symmetry breaking.
- ☞ Bound state formation: mesons, baryons, glueballs, hybrids, multiquark systems...
- ☞ Signals of confinement.

These (emergent) phenomena is not apparent in the QCD Lagrangian; however, they characterized the nonperturbative regime of QCD where hadrons live

Non-perturbative QCD: Dynamical generation of gluon mass

Emergent phenomena could be associated with dramatic, dynamically driven changes in the analytic structure of QCD's Green functions, which are solutions of the DSEs

Dressed-gluon propagator in Landau gauge:

$$i\Delta_{\mu\nu} = -iP_{\mu\nu}\Delta(q^2), \quad P_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2$$

- An inflexion point at $q^2 > 0$.
- Breaks the axiom of reflexion positivity.
- Gluon mass generation \leftrightarrow Schwinger mechanism.

A.C. Aguilar *et al.*, Phys. Rev. D78 (2008) 025010;

I.L. Bogolubsky *et al.*, Phys. Lett. B676 (2009) 69.

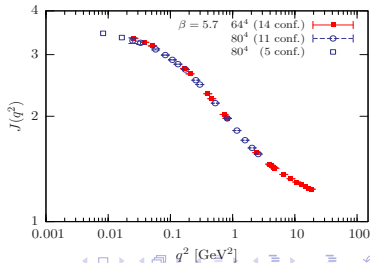
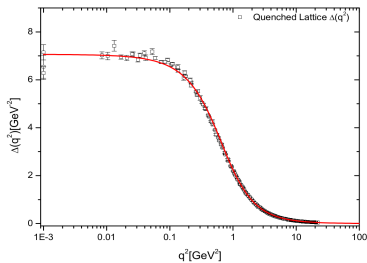
Dressed-ghost propagator in Landau gauge:

$$G^{ab}(q^2) = \delta^{ab} \frac{J(q^2)}{q^2}$$

- No power-like singular behavior at $q^2 \rightarrow 0$.
- Good indication that $J(q^2)$ reaches a plateau.
- Saturation of ghost's dressing function.

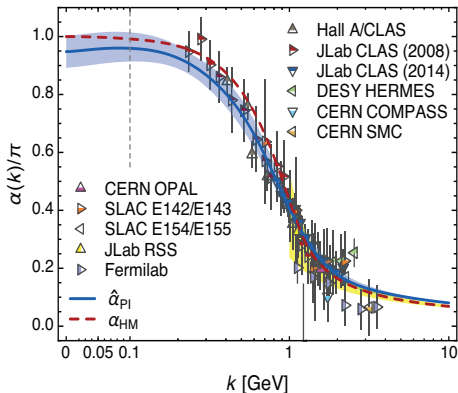
Ph. Boucaud *et al.*, JHEP 0806 (2008) 099;

C. Fischer *et al.*, Annals Phys. 324 (2009) 2408.



Non-perturbative QCD: Saturation at IR of process-independent effective-charge

D. Binosi *et al.*, Phys. Rev. D96 (2017) 054026;
A. Deur *et al.*, Prog. Part. Nucl. Phys. 90 (2016) 1-74.



☞ Perturbative regime:

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\text{MS}}}(k^2) \left[1 + 1.14\alpha_{\overline{\text{MS}}}(k^2) + \dots \right]$$

$$\hat{\alpha}_{\text{PI}}(k^2) = \alpha_{\overline{\text{MS}}}(k^2) \left[1 + 1.09\alpha_{\overline{\text{MS}}}(k^2) + \dots \right]$$

☞ Data = running coupling defined from the Bjorken sum-rule.

$$\int_0^1 dx \left[g_1^p(x, k^2) - g_1^n(x, k^2) \right] = \frac{g_A}{6} \left[1 - \frac{1}{\pi} \alpha_{g_1}(k^2) \right]$$

☞ Curve determined from combined continuum and lattice analysis of QCD's gauge sector (massless ghost and massive gluon).

☞ The curve is a running coupling that does NOT depend on the choice of observable.

- No parameters.
- No matching condition.
- No extrapolation.

☞ It predicts and unifies an enormous body of empirical data via the matter-sector bound-state equations.

Non-perturbative QCD: Dynamical generation of quark mass

☞ Dressed-quark propagator in Landau gauge:

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m) + \Sigma(p) = \left(\frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \right)^{-1}$$

- Mass generated from the interaction of quarks with the gluon-medium.
- Light quarks acquire a **HUGE** constituent mass.
- Responsible of the 98% of proton's mass, the large splitting between parity partners, ...

☞ Goldberger-Treiman relation at the quark level:

$$\text{Quark propagator: } S^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2),$$

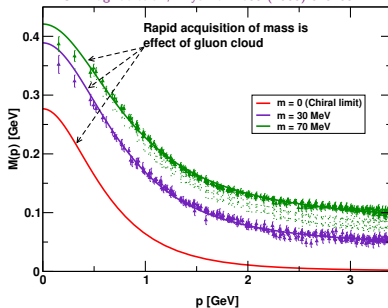
$$\text{Pion's BS-amplitude: } \Gamma_\pi(p, P) \propto \gamma^5 E_\pi(p; P).$$

$$\mathbf{f}_\pi \mathbf{E}_\pi(\mathbf{p}; 0) = \mathbf{B}(p^2)$$

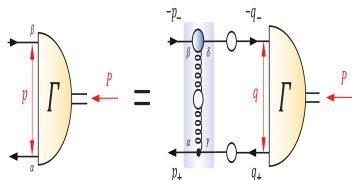
Properties of the massless pion are a direct measure of the dressed-quark mass function

Cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe

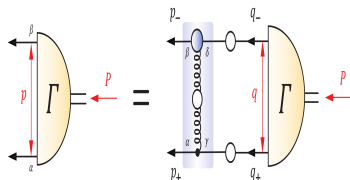
M.S. Bhagwat et al., Phys.Rev. C68 (2003) 015203.



Any interaction able to create Goldstone modes as bound-states of light dressed-quark and -antiquark will generate strong $\bar{3}_c$ correlations between any two dressed quarks.



Meson BSE



Diquark BSE

☞ Owing to properties of charge-conjugation, a diquark with spin-parity J^P may be viewed as a partner to the analogous J^{-P} meson:

$$\Gamma_{q\bar{q}}(p; P) = - \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q+P) \Gamma_{q\bar{q}}(q; P) S(q) \frac{\lambda^a}{2} \gamma_\nu$$

$$\Gamma_{qq}(p; P) C^\dagger = - \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q+P) \Gamma_{qq}(q; P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu$$

☞ Whilst no pole-mass exists, the following mass-scales express the strength and range of the correlation:

$$m_{[ud]_{0+}} = 0.7 - 0.8 \text{ GeV}, \quad m_{\{uu\}_{1+}} = 0.9 - 1.1 \text{ GeV}, \quad m_{\{dd\}_{1+}} = m_{\{ud\}_{1+}} = m_{\{uu\}_{1+}}$$

☞ Diquark correlations are soft, they possess an electromagnetic size:

$$r_{[ud]_{0+}} \gtrsim r_\pi, \quad r_{\{uu\}_{1+}} \gtrsim r_\rho, \quad r_{\{uu\}_{1+}} \gg r_{[ud]_{0+}}$$

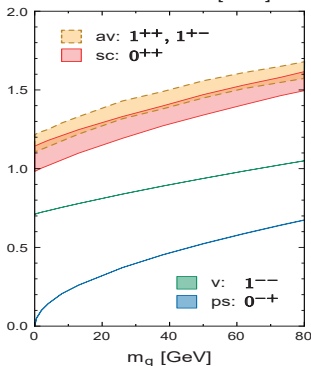
Octet and decuplet baryons

	[nn]	{nn}	[ns]	{ns}	{ss}
N	●	●			
Δ		●			
Λ	●		●	●	
Σ		●	●	●	
Ξ			●	●	●
Ω					●

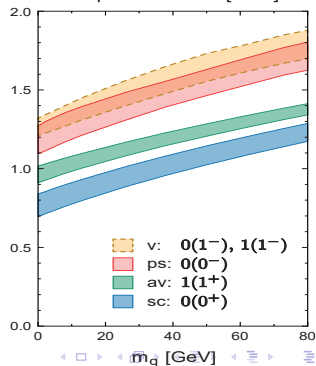
Other baryons as parity partners

- ☞ $[I = 0, J^P = 0^+]$: Isoscalar-scalar.
- ☞ $[I = 1, J^P = 1^+]$: Isovector-pseudovector.
- ☞ $[I = 0, J^P = 0^-]$: Isoscalar-pseudoscalar.
- ☞ $[I = 0, J^P = 1^-]$: Isoscalar-vector.
- ☞ $[I = 1, J^P = 1^-]$: Isovector-vector.

Meson masses [GeV]



Diquark masses [GeV]

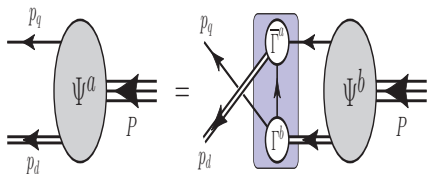


M. Yu. Barabanov *et al.*,
arXiv:hep-ph/2008.07630

The quark+diquark structure of a baryon

A baryon can be viewed as a **Borromean bound-state**, the binding within which has two contributions:

- Formation of tight diquark correlations.
- Quark exchange depicted in the shaded area.



The exchange ensures that diquark correlations within the baryon are **fully dynamical**: no quark holds a special place.

The rearrangement of the quarks guarantees that the baryon's wave function complies with **Pauli statistics**.

The number of states in the **spectrum of baryons obtained is similar** to that found in the three-constituent quark model, just as it is in today's LQCD calculations.

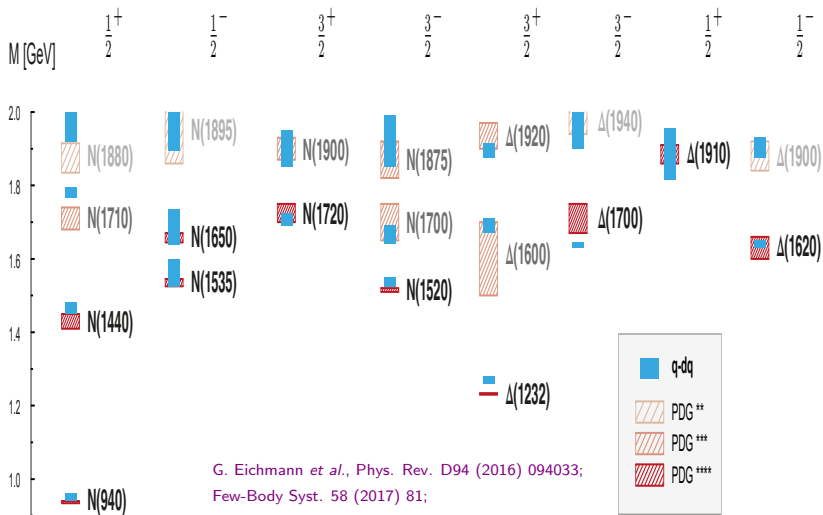
Modern diquarks are **different from the old static, point-like diquarks** which featured in early attempts to explain the so-called missing resonance problem.

Modern diquarks enforce certain **distinct interaction patterns** for the singly- and doubly-represented valence-quarks within the baryon.

S.-S. Xu *et al.*, Phys. Rev. D92 (2015) 114034; Y. Lu *et al.*, Phys. Rev. C96 (2017) 015208;
C. Chen *et al.*, Phys. Rev. D100 (2019) 054009; P.-L. Yin *et al.*, Phys. Rev. D100 (2019) 034008.

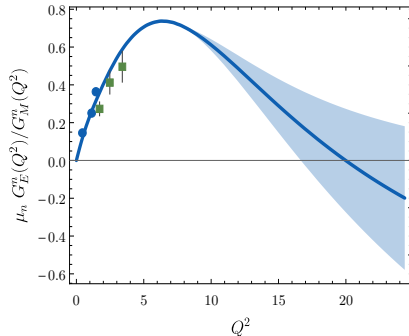
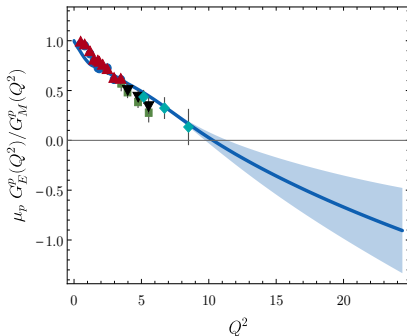
Baryon spectrum within the quark+diquark picture

- ☞ Spectrum in one to one agreement with experiment.
- ☞ Correct level ordering (without coupled-channels effects).
- ☞ Three-body agrees with quark-diquark where applicable.



G. Eichmann *et al.*, Phys. Rev. D94 (2016) 094033;
 Few-Body Syst. 58 (2017) 81;
 Prog. Part. Nucl. Phys. 91 (2016) 1-100.

Structure functions: Nucleons elastic form factors (I)

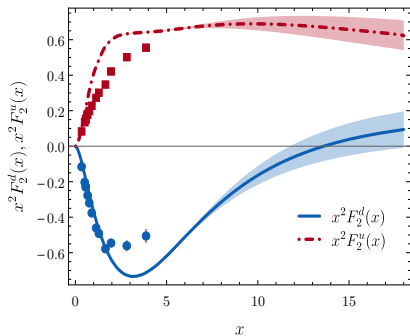
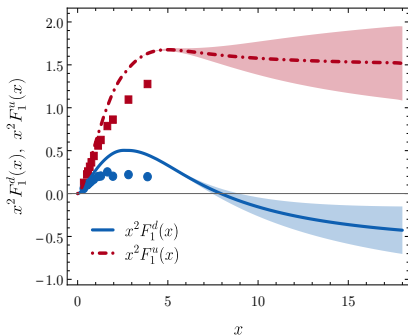


Observations:

- There is no evidence for scaling in Dirac and Pauli form factors, and thus in the electromagnetic Sachs form factors.
- Our analysis predicts a zero for the proton's electromagnetic ratio at $Q^2 = 10.3^{+1.1}_{-0.7} \text{ GeV}^2$.
- The neutron's electromagnetic ratio has a peak at $Q^2 \approx 6 \text{ GeV}^2$ and then crossed zero for $Q^2 = 20.1^{+10.6}_{-3.5} \text{ GeV}^2$.
- All these features can be related with both quark-quark and angular momentum correlations within the nucleon.

Z.-F. Cui *et al.*, *Phys. Rev. D*102 (2020) 014043.

Structure functions: Nucleons elastic form factors (II)

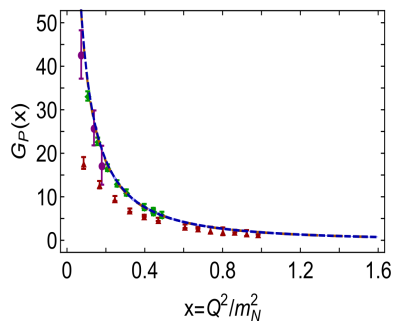
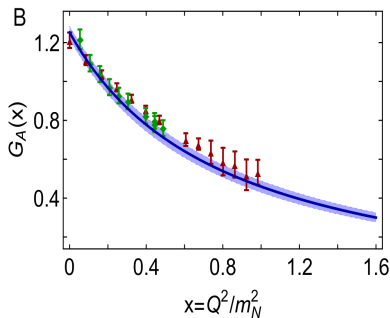


Observations:

- F_1^d is smaller than F_1^u , even allowing for the difference in normalisation, and decreases more quickly as x increases.
- The location of the zero in F_1^d is a measure of the relative probability of finding pseudovector and scalar diquarks in the proton.
- The u - and d -quark Pauli form factors are roughly equal in magnitude on $x \lesssim 5$; *i.e.* F_2^d is suppressed with respect F_2^u but only at large momentum transfer.
- There are contributions playing an important role in F_2 , like the anomalous magnetic moment of dressed-quarks or meson-baryon final-state interactions.

Z.-F. Cui *et al.*, Phys. Rev. D102 (2020) 014043.

Structure functions: Form factors of the nucleon axial current



Observations:

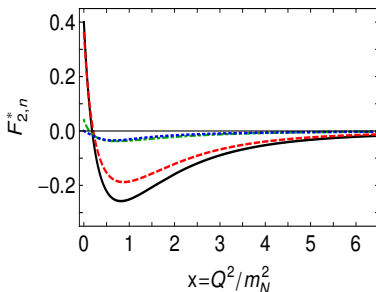
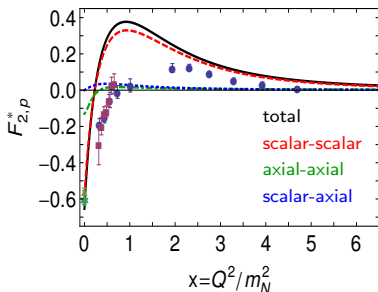
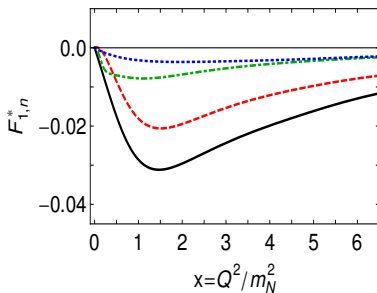
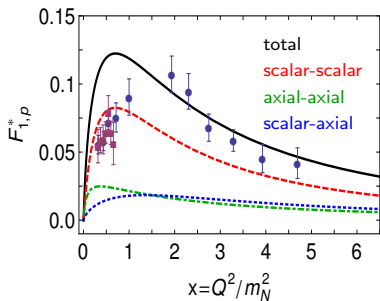
C. Chen *et al.*, arXiv:hep-ph/2011.14026.

- The result for G_A can reliably be represented by a dipole form factor characterised by an axial charge $g_A = G_A(0) = 1.25(3)$ and a mass-scale $M_A = 1.23(3)m_N$.
- The pion pole dominance *ansatz* is found to provide a reliable estimate of the directly computed result of the induced pseudoscalar form factor, G_P .
- The ratio $g_A^d/g_u^A = -0.16(2)$ expresses a marked suppression of the d -quark component and owes to the presence of strong diquark correlations.

G_A should provide a sound foundation for analyses of the (anti-)neutrino–nucleus cross-sections that are relevant to modern accelerator neutrino experiments

Structure functions: electro-production of nucleon resonances (I)

Transition form factor of $\gamma^* N \rightarrow N(1440)_{\frac{1}{2}}^+$



C. Chen *et al.*, Phys. Rev. D99 (2019) 034013.

Structure functions: electro-production of nucleon resonances (II)

Consequence of solving Poincaré-covariant bound-state equations

non-relativistic

Mesons: $P = (-1)^{L+1}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}
1	1	0^{++}

relativistic

~~$P = (-1)^{L+1}$~~

Bethe, Salpeter, Llewelyn-Smith 1950ies

$$\Gamma_{\pi}(P, p) = \gamma_5 [F_1(P, p) \quad \text{s-wave} \\ + F_2(P, p) i \not{P} \\ + F_3(P, p) p P i \not{p} \quad \text{p-wave} \\ + F_4(P, p) [\not{p}, \not{P}]]$$

Baryons: $P = (-1)^L$

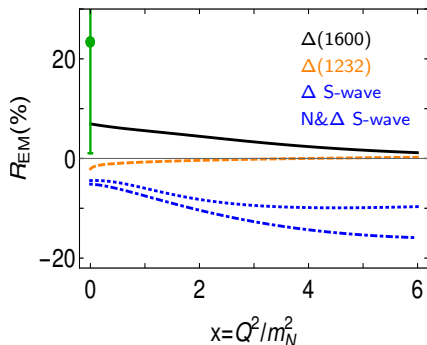
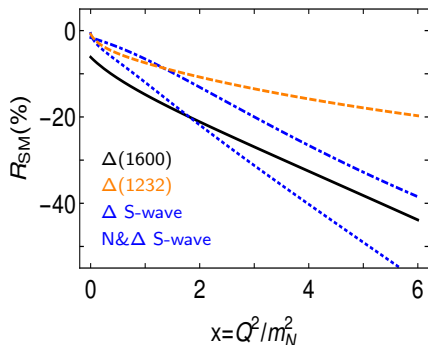
S	L	J^P
1/2	0	$1/2^{+}$
3/2	2	

~~$P = (-1)^L$~~

J^P	total	s-wave	p-wave	d-wave	f-wave
$1/2^{+}$	64	8	36	20	
$3/2^{+}$	128	4	36	60	28

Structure functions: electro-production of nucleon resonances (III)

Transition form factors of $\gamma^* N \rightarrow \Delta(1232)_{\frac{3}{2}^+}$, $\Delta(1600)_{\frac{3}{2}^+}$



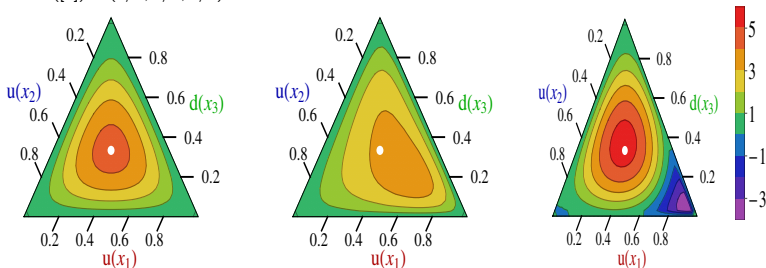
Observations:

- R_{SM} and R_{EM} are different from zero indicating that $\Delta(1232)$ and $\Delta(1600)$ are not simple S-wave ground and radial-excitation states of the Δ -baryon.
- R_{SM} and R_{EM} for the $\Delta(1600)$ transition are far larger in magnitude than the analogous results for the $\Delta(1232)$.
- Points above are an observable manifestation of important higher orbital angular momentum components in both states.
- In particular, there is an enhanced D-wave strength in the $\Delta(1600)$ relative to that in the $\Delta(1232)$.

Ya Lu et al., Phys. Rev. D100 (2019) 034001.

Structure functions: Nucleon and Roper PDAs (I)

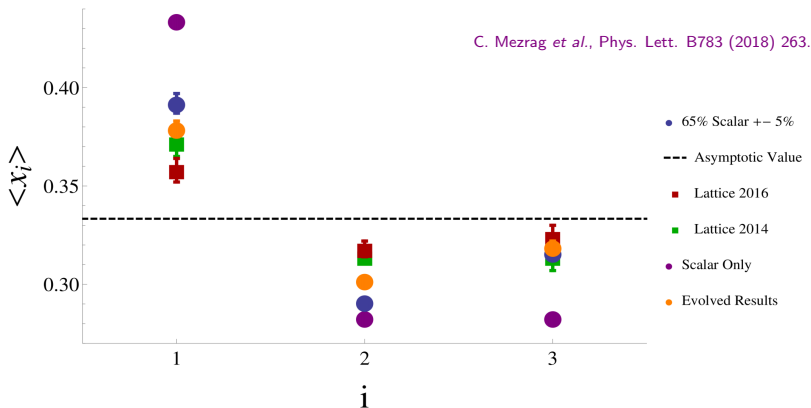
Barycentric plots: *left panel* – conformal limit PDA, $\varphi_N^{\text{cl}}([x]) = 120x_1x_2x_3$; *middle panel* – computed proton PDA evolved to $\zeta = 2$ GeV, which peaks at $([x]) = (0.55, 0.23, 0.22)$; and *right panel* – Roper resonance PDA at $\zeta = 2$ GeV. The white circle in each panel serves only to mark the centre of mass for the conformal PDA, whose peak lies at $([x]) = (1/3, 1/3, 1/3)$.



Observations:

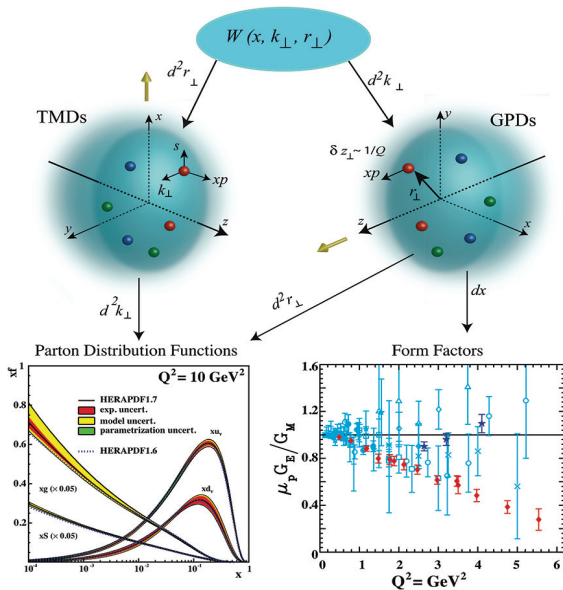
- Proton: The leading-twist PDA of the ground-state nucleon is both broader than $\varphi_N^{\text{cl}}([x])$ and decreases monotonically away from its maximum in all directions.
- Proton: The peak of the φ -distribution is shifted toward the region where the single quark carries most of the nucleon light-cone momentum fraction.
- Roper: The excitation's PDA is not positive definite which echoes features of the wave function for the first radial excitation of a quantum mechanical system.

C. Mezrag *et al.*, Phys. Lett. B783 (2018) 263.



	Scalar	S+AV	Evolved	Braun:2014wpa	Bali:2015yqx
$\langle x_1 \rangle_\varphi$	0.434	0.392(5)	0.379(4)	0.372(7)	0.358(6)
$\langle x_2 \rangle_\varphi$	0.283	0.291(2)	0.302(1)	0.314(3)	0.319(4)
$\langle x_3 \rangle_\varphi$	0.283	0.316(4)	0.319(3)	0.314(7)	0.323(6)
$10^3 f_N$ (GeV ²)	2.97	4.05	3.78(14)	2.84(33)	3.60(6)

Hard (semi-)inclusive and (semi-)exclusive reactions are coming to the forefront of nuclear and particle physics with data expected from COMPASS, JLab12, and EIC



The GPDs are a new class of hadron observables which combine the physics of electromagnetic form factors and Feynman parton distributions, and are related to quantum phase-space distributions of the partons through Fourier transformation.

☞ Apart from the renormalization scale, GPDs depend on:

- The momentum transfer t , as in a form factor;
- Light-cone momentum x , as in a parton distribution;
- The projection of the momentum transfer along the light-cone direction ξ , also known as the skewness parameter.

☞ In the hard scattering regime, $Q^2 \rightarrow \infty$, the nucleon's Dirac form factor can be written as:

$$Q^4 F_1(Q^2) = \frac{8\pi^2}{27} \int [dx] [dy] \left[f_N(\zeta_y^2) \varphi_N^*(y_j; \zeta_y^2) \right] T_F(x_i, y_j; Q^2; \zeta_x^2, \zeta_y^2) \left[f_N(\zeta_x^2) \varphi(x_i; \zeta_x^2) \right],$$
$$[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3),$$

where φ is the nucleon parton distribution amplitude evaluated at scale ζ , f_N is the nucleon's normalization, and T_F the hard scattering kernel that one can compute using perturbation theory.

☞ In the regime, $\Lambda_{\text{QCD}} \ll -t \ll Q^2$, the nucleon's GPD H_q can be written as:

$$H_q(x, \xi, t) = \int [dx] [dy] \left[f_N(\zeta_y^2) \varphi_N^*(y_j; \zeta_y^2) \right] T_{H_q}(x_i, y_j; x, \xi, t; \zeta_x^2, \zeta_y^2) \left[f_N(\zeta_x^2) \varphi(x_i; \zeta_x^2) \right],$$

$$[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3),$$

where φ is the nucleon parton distribution amplitude evaluated at scale ζ , f_N is the nucleon's normalization, and T_{H_q} the hard scattering kernel that one can compute using perturbation theory.

P. Hoodbhoy *et al.*, Phys. Rev. Lett. 92 (2003) 012003

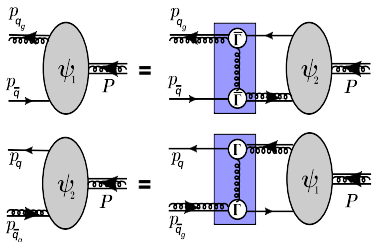
Nobody has tried to compute it in such a limit for two main reasons:

- There was no realistic models of nucleon PDA \Rightarrow DSEs prediction of nucleon's PDA exists now.
- The kinematical range $\Lambda_{\text{QCD}} \ll -t \ll Q^2$ was out of reach experimentally \rightarrow New experimental facilities JLab12, COMPASS, EIC.

J. Morgado-Chavez *et al.*, work in progress

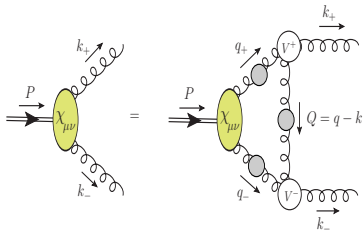
Other kind of two-body correlations?

Hybrid mesons: Existence of strong two-body correlations in the gluon-quark, $q_g = [gq]$, and gluon-antiquark, $\bar{q}_g = [g\bar{q}]$ channels.

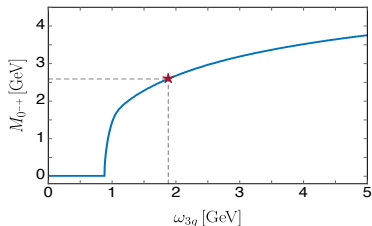
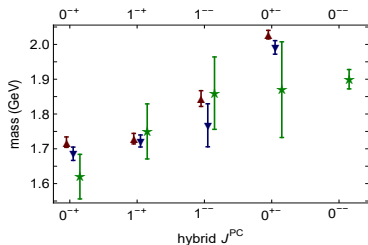


S.-S. Xu et al., Eur. Phys. J. A Lett. 55 (2019) 113.

Glueballs: The infrared suppression of the three-gluon vertex is essential to achieve agreement with lattice-QCD predictions for the 0^{-+} glueball mass.



E.V. Souza et al., Eur. Phys. J. A56 (2020) 25.



Modern facilities will probe hadronic interiors as never before.

- JLab12 will push form factor measurements to unprecedented values of momentum transfer and use different charge states, enabling flavour separations.
- COMPASS, EIC and EicC would measure valence-quark distribution functions with previously unattainable precision.
- Collaborations like BaBar, Belle, BESIII, LHCb, are discovering new hadrons whose structure does not fit once viable paradigms.

The wealth of new and anticipated information demands that the issue of correlations within hadrons be settled.

- The features of baryons, and their unification with the properties of mesons, depend on a veracious expression of DCSB in the hadron's bound-state and scattering problems.
- The existence of non-pointlike, fully dynamical quark-quark correlations, as well as quark-gluon and gluon-gluon ones, is an important consequence of DCSB.
- There is evidence for such clusters in simulations of IQCD; and their presence within baryons is predicted to have numerous observable consequences, some of which already have strong experimental support.

M. Yu. Barabanov *et al.*, "Di-quark correlations in hadron physics: Origin, impact and evidence"
arXiv:hep-ph/2008.07630.

Modern diquarks are confined, with mass-scales that express the strength and range of the correlation inside the hadron; they are fully dynamical, with no quark holding a special place because each one participates in all correlations to the fullest extent allowed by its quantum numbers; they have electromagnetic sizes, which enforce certain distinct interaction patterns; and there are different species, amongst which isoscalar-scalar and isovector-pseudovector correlations are the strongest but others play a key role in nucleon excited states.

M. Yu. Barabanov et al., "Diquark correlations in hadron physics: Origin, impact and evidence"
arXiv:hep-ph/2008.07630.

