Perceiving the Emergence of Hadron Mass through AMBER@CERN - IV

2020-12-03

Distribution amplitudes (DAs) and distribution functions (DFs) of diquark correlations

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FONDAZIONE BRUNO KESSLER

FUTURE BUILT ON KNOWLEDGE

EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

Motivation I: Diquark in experiment and phenomenology

- The neutron and proton structure functions in Feynman parton model

$$\frac{1}{x}F_2^n(x) = \frac{1}{9}\left[u(x) + \bar{u}(x)\right] + \frac{4}{9}\left[d(x) + \bar{d}(x)\right]$$
$$\frac{1}{x}F_2^p(x) = \frac{4}{9}\left[u(x) + \bar{u}(x)\right] + \frac{1}{9}\left[d(x) + \bar{d}(x)\right]$$

- Neutron-to-proton structure function ratio

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{\left[u(x) + \bar{u}(x)\right] + 4\left[d(x) + \bar{d}(x)\right]}{4\left[u(x) + \bar{u}(x)\right] + \left[d(x) + \bar{d}(x)\right]}$$

- Zero strangeness quarks for simplicity

- Consider $x \to 1$, expect the sea contribution to be small.

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{u(x) + 4\,d(x)}{4\,u(x) + d(x)}$$

- Naively $u(x) = 2d(x)$, spin-flavour SU(6) symmetry

$$\frac{F_2^n(x)}{F_2^p(x)} \xrightarrow{x \to 1} \frac{2}{3}$$

Motivation I: Diquark in experiment and phenomenology

- Experimentally $F_2^n(x)/F_2^p(x)$ is smaller than $2/3$ as $x \to 1$.

- Special case: $F_2^n(x)/F_2^p(x) \xrightarrow{x\to1} 1/4$.

  * $d(x)$ falls faster than $u(x)$.

  * Realistic DF, $d(x)/u(x)$ in $x \to 1$ is with large uncertainty.

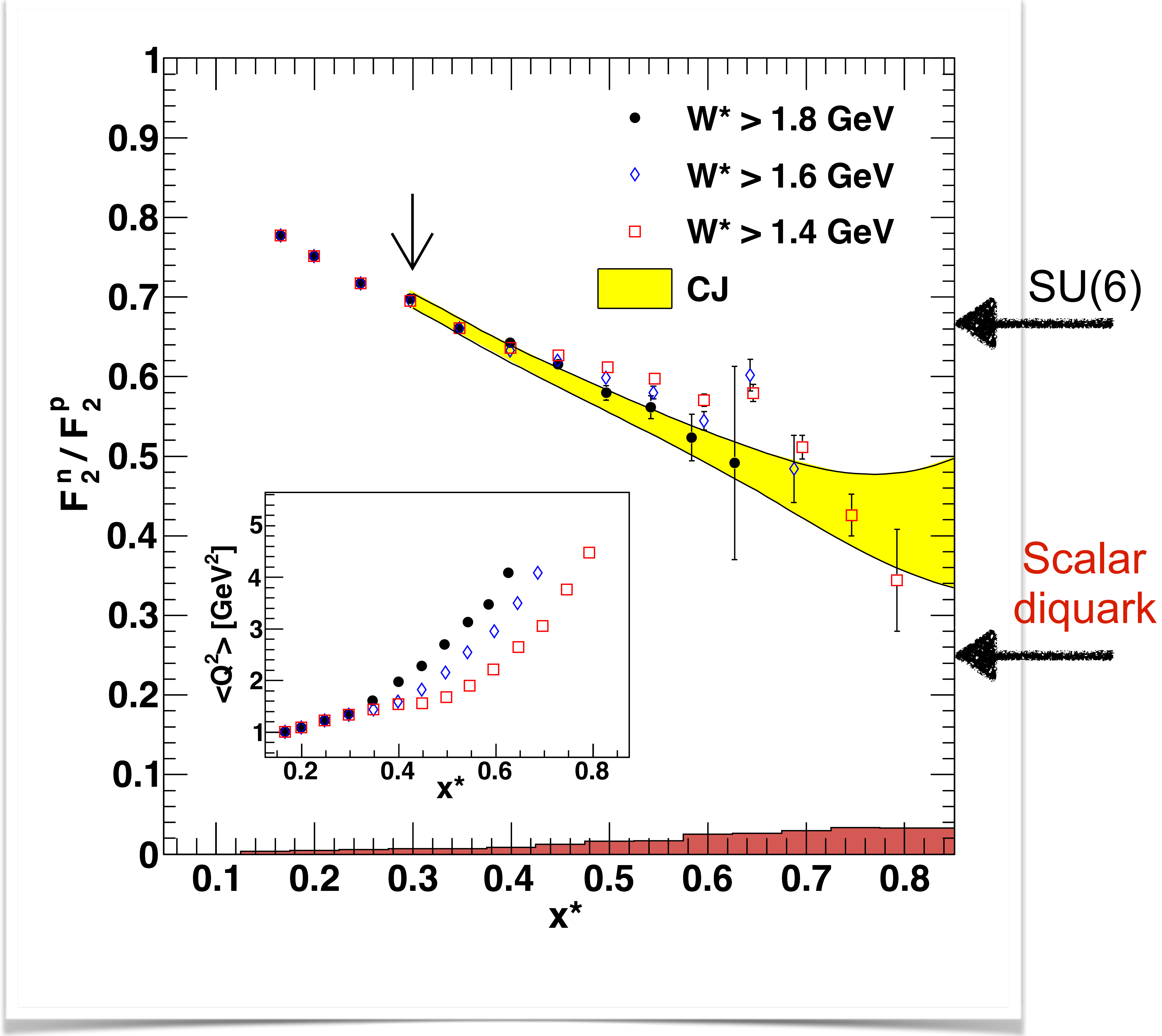
  * Leading term is scalar diquark

$$I(J^P) = 0(0^+),\ [ud] = \frac{1}{\sqrt{2}}(ud - du)$$

  * The fast parton is only up quark.

  * $$\frac{F_2^n(x)}{F_2^p(x)} = \frac{u(x)}{4u(x)} = \frac{1}{4}.$$

R. P. Feynman, Photon-hadron interactions (1972).



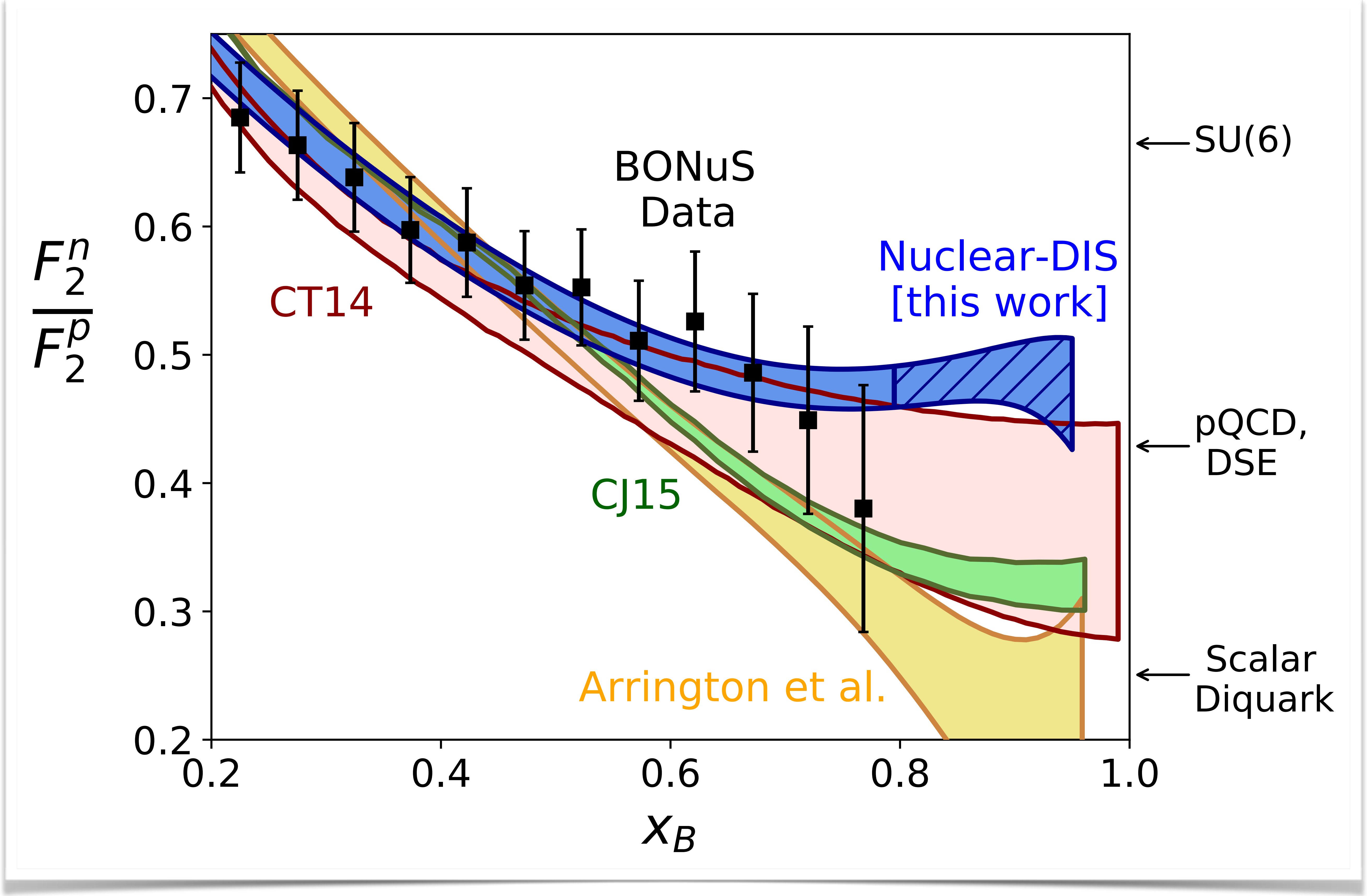
N. Baillie et al, Phys. Rev. Lett. 108, 199902 (2012).

Motivation I: Diquark in experiment and phenomenology

- Modern $F_2^n(x)/F_2^p(x)$ becomes constant for $x \geq 0.6$, equaling 0.47 ± 0.04 as $x \to 1$.

- In agreement with theoretical predictions of perturbative QCD and the Dyson-Schwinger equation (DSE).

- DSE: scalar diquark + 25-35% axial-vector diquark contributions to proton.



E. P. Segarra et al, Phys. Rev. Lett. 124, 092002 (2020).

Motivation II: Diquark in Emergence of Hadron Mass (EHM)

- Proton can be considered as a Borromean bound-state.

  - A system constituted from three bodies, no two of which can combine to produce an independent, asymptotic two-body bound-state. J. Segovia, C D. Roberts, S. M. Schmidt, Phys. Lett. B 750 (2015) 100-106.

    - Diquark clusters
    - Dynamically breakup and reformation.

- Diquark clustering is an emergent phenomenon, driven by emergence of hadronic mass (EHM).

- Diquarks are colored and confined.

  - This is not true if leading-order (rainbow-ladder) truncation, used in the associated scattering problem, corrections to rainbow-ladder truncation are critical in diquarks.

- Modern diquarks are soft.

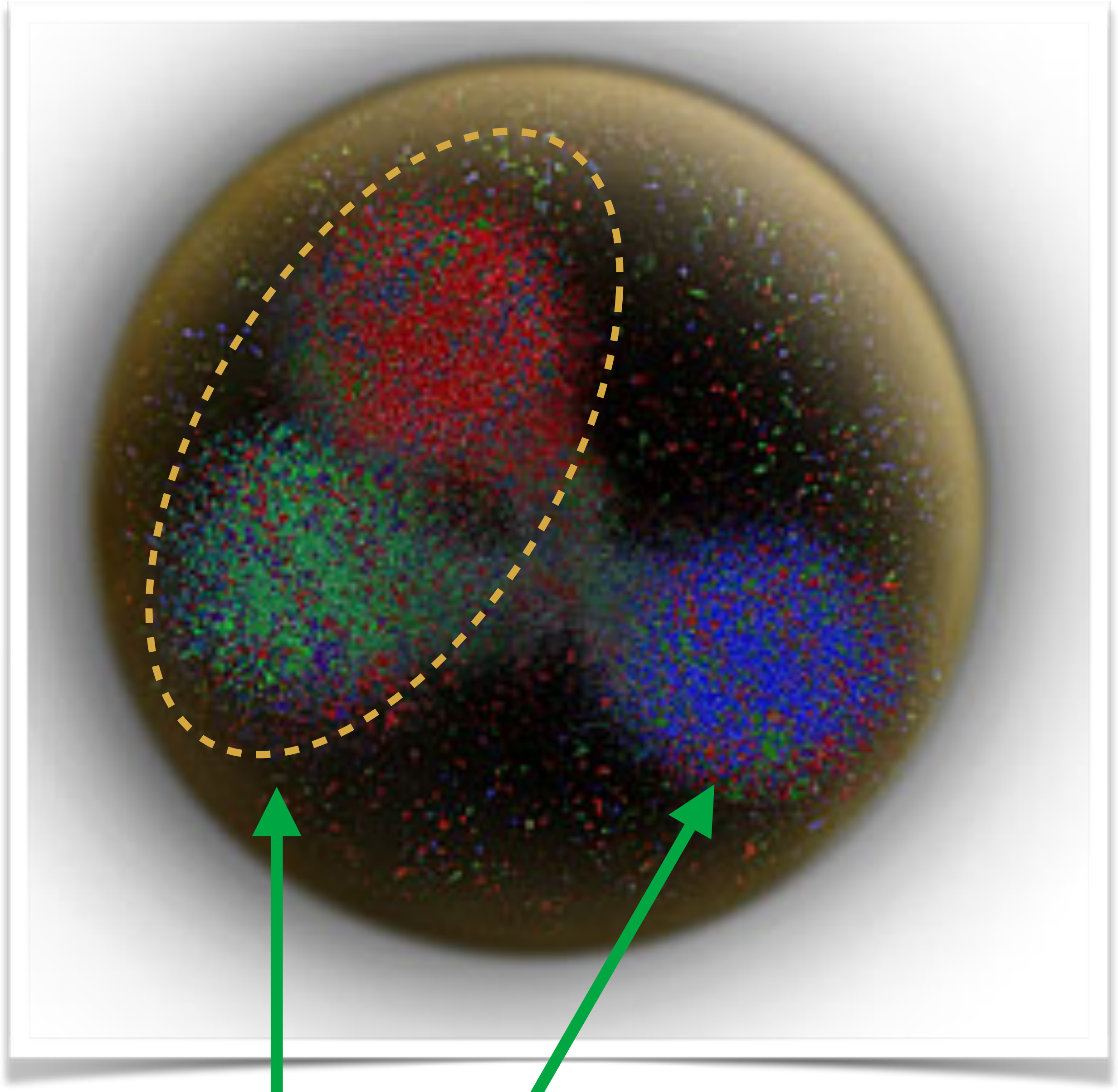
  - They possess an electromagnetic size that is bounded below by that of the analogous mesonic system.

    - Charge radius $r_{[ud]_{0+}} \gtrsim r_\pi$, $r_{\{uu\}_{1+}} \gtrsim r_\rho$ and $r_{\{uu\}_{1+}} > r_{[ud]_{0+}}$.

see talk by J. Segovia

- Quark distribution in proton

$$f_{q/P}(x) = \sum_{Q} \int_0^1 dy \int_0^1 dz\, \delta(x - yz) f_{Q/P}(y) f_{q/Q}(z)$$



- $f_{Q/P}(y)$: parent distribution in proton.
- $f_{q/Q}(z)$: quark distribution in parent (quark/diquark).
- Q : parent, quark or diquark.

H. Mineo, W. Bentz, and K. Yazaki, Phys. Rev. C 60 (1999) 065201.

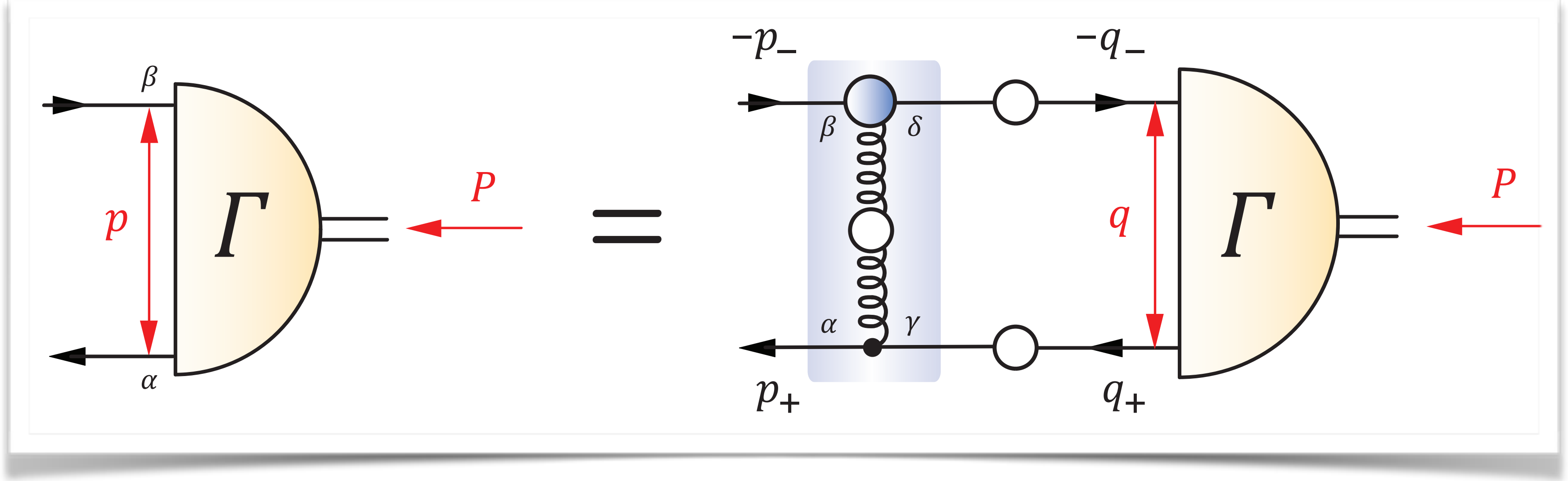
- Diquark wave function must be anti-symmetric:

$$\psi = \phi_{flavor}\chi_{spin}\xi_{color}\eta_{space}$$

- Color wave function ξ_{color}, anti-triplet, anti-symmetric.
- Spatial wave function η_{space}, ground-state no internal orbital angular momentum, $P = (-1)^{L=0} = +$, symmetric.
- $\Rightarrow \phi_{flavor}\chi_{spin}$ symmetric.
- Spin $S = 0$, isospin $I = 0$, $I(J^P) = 0(0^+)$, scalar diquark, dominate configuration.
- Spin $S = 1$, isospin $I = 1$, $I(J^P) = 1(1^+)$, axial-vector diquark.

- Meson BSE in Ladder approximation

$$\Gamma_M(k;P) = -Z_2^2 \int_{dq} g^2 D_{\mu\nu}(k-q) \frac{\lambda^a}{2}\gamma_\mu S(q_+)\,\Gamma_M(q;P) S(q_-)\,\frac{\lambda^a}{2}\gamma_\nu$$

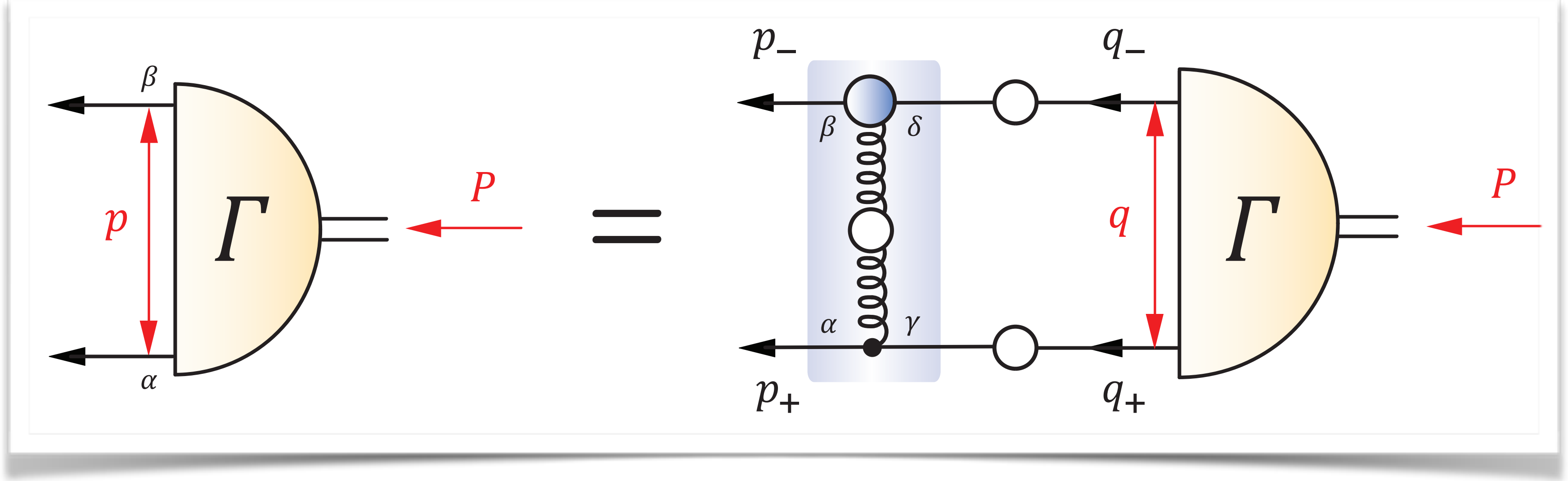


✱ Color structure of the meson (color-singlet) BSE:

$$\Gamma_{il} = \Gamma_{jk}\lambda_{ji}^{*a}\lambda_{kl}^{a}/4 = \delta_{jk}\lambda_{ji}^{*a}\lambda_{kl}^{a}/4 = \frac{4}{3}\Gamma_{il}$$

- Diquark BSE in Ladder approximation

$$\Gamma_D(k;P) = -Z_2^2 \int_{dq} g^2 D_{\mu\nu}(k-q) \frac{\lambda^a}{2}\gamma_\mu S(q_+)\,\Gamma_D(q;P) S^T(-q_-)\,\frac{\lambda^{aT}}{2}\gamma_\nu^T$$



✱ Color structure of the diquark (color-antitriplet) BSE:

$$\Gamma_{il} = \Gamma_{jk}\lambda_{ji}^{a}\lambda_{kl}^{a}/4 = 2(\Gamma_{li} - \frac{1}{3}\Gamma_{il})/4 = -\frac{2}{3}\Gamma_{il}$$

R.T. Cahill, C. D. Roberts, and J. Praschifka, Phys. Rev. D 36 (1987) 2804.

Diquark in Dyson-Schwinger Equations (DSEs)

- The Meson and Diquark Bethe-Salpeter equations in Ladder approximation are:

$$\Gamma_M(k;P) = -\frac{4}{3} Z_2^2 \int_{dq} g^2 D_{\mu\nu}(k-q)\gamma_\mu S\left(q_+\right) \Gamma_M(q;P) S\left(q_-\right)\gamma_\nu\,,$$

$$\Gamma_D^C(p;P) = -\frac{2}{3} Z_2^2 \int_{dq} g^2 D_{\mu\nu}(k-q)\gamma_\mu S\left(q_+\right) \Gamma_D^C(q;P) S\left(q_-\right)\gamma_\nu\,.$$

* Defining $\Gamma_D^C(p;P) = \Gamma_D(p;P)C$, and using $C^{-1}\gamma_\mu^T C = -\gamma_\mu$.

* Diquark BSE kernel is smaller than meson kernel by a factor of two. see talk by J. Segovia

* Effective interaction in diquark is reduced by a factor of two compared to interaction in meson.

* Solve meson BSE with eigenvalue $\lambda_M = 2$, the corresponding diquark state will have $\lambda_D = 1$.

P. Maris, Few Body Syst. 32 (2002) 41-52.

Diquark Bethe-Salpeter amplitude (BSA)

- Scalar diquark $\Gamma^C_{[ud]_{0^+}}(k;P)$ has exactly the same Dirac bases as a pseudoscalar meson BSA.

$$\Gamma^C_{[ud]_{0^+}}(k;P) = \sum_{i=1}^{4} \tau_\pi^i(k;P) f^i_{[ud]_{0^+}}(k;P)\,,$$

- Axial-vector diquark $\Gamma^C_{\{uu\}_{1^+}}(k;P)$ has exactly the same Dirac bases as a vector meson BSA.

$$\Gamma^C_{\{uu\}_{1^+}}(k;P) = \sum_{i=1}^{8} \tau_\rho^i(k;P) f^i_{\{uu\}_{1^+}}(k;P)\,,$$

* $\{uu\}_{1^+} = \{dd\}_{1^+} = \{ud\}_{1^+}$ in the isospin symmetric limit.

$$
\begin{aligned}
\tau_\pi^1 &= i\gamma_5\,,\\
\tau_\pi^2 &= \gamma_5 \gamma\cdot P\,,\\
\tau_\pi^3 &= \gamma_5 k\cdot P(P^2\gamma\cdot k - k\cdot P\gamma\cdot P)\,,\\
\tau_\pi^4 &= \gamma_5 \sigma_{\mu\nu} P_\mu k_\nu\,,\\
\tau_\rho^1 &= i\gamma_\mu^T,\\
\tau_\rho^2 &= i[3k_\mu^T \gamma\cdot k^T - \gamma_\mu^T k^T\cdot k^T],\\
\tau_\rho^3 &= ik_\mu^T\, k\cdot P\,\gamma\cdot P,\\
\tau_\rho^4 &= i[\gamma_\mu^T \gamma\cdot P\,\gamma\cdot k^T + k_\mu^T \gamma\cdot P],\\
\tau_\rho^5 &= k_\mu^T,\\
\tau_\rho^6 &= k\cdot P[\gamma_\mu^T \gamma^T\cdot k - \gamma\cdot k^T \gamma_\mu^T],\\
\tau_\rho^7 &= (k^T)^2(\gamma_\mu^T \gamma\cdot P - \gamma\cdot P\gamma_\mu^T) - 2k_\mu^T \gamma\cdot k^T \gamma\cdot P,\\
\tau_\rho^8 &= k_\mu^T \gamma\cdot k^T \gamma\cdot P,
\end{aligned}
$$

Si-xue Qin et al, Phys. Rev. C 85 (2012) 035202.

Diquark static properties and structure functions

- Static properties:
  - Mass
    R.T. Cahill, C. D. Roberts, and J. Praschifka, Phys. Rev. D 36 (1987) 2804.
    P. Maris, Few Body Syst. 32 (2002) 41-52.
    H.L.L. Roberts, L. Chang, I.C. Cloet, C.D. Roberts, Few Body Syst. 51 (2011) 1.
    G. Eichmann, C.S. Fischer, H. Sanchis-Alepuz, Phys. Rev. D 94 (2016) 094033.
    P. Yin, C. Chen G. Krein, C. D. Roberts, J. Segovia, Shu-Sheng Xu, Phys. Rev. D 100 (2019) 3, 034008. eta al.
  - Decay constant — P. Maris, Few Body Syst. 35 (2004) 117-127.
- Structure functions: Diquarks are not pointlike objects.
  - Electromagnetic form factor — P. Maris, Few Body Syst. 35 (2004) 117-127.
  - Light front quantities
    - Distribution amplitude (DA)
    - Distribution function (DF)

Diquark decay constant

- Scalar diquark

$$if_{[ud]_{0^+}} P_\mu = \langle 0 | \bar{\psi} \gamma_\mu \gamma_5 \psi | [ud]_{0^+}(P) \rangle$$

$$f_{[ud]_{0^+}} P_\mu = Z_2 \,\mathrm{tr} \int_{dq} i\gamma_5 \gamma_\mu \chi^C_{[ud]_{0^+}}(q;P)$$

- Axial-vector diquark: longitudinal

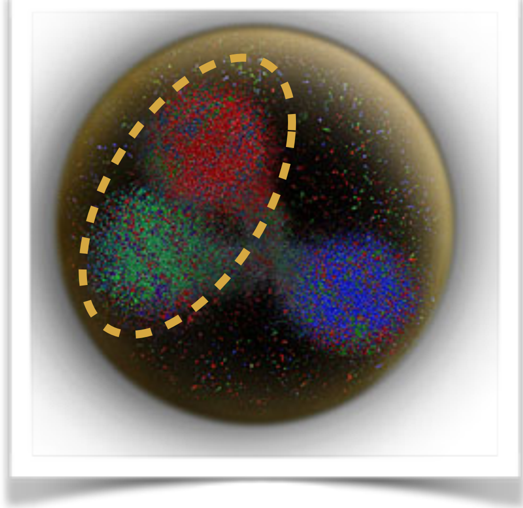
$$f^{||}_{\{uu\}_{1^+}} M_{\{uu\}_{1^+}} \epsilon^\mu_\lambda = \langle 0 | \bar{\psi} \gamma_\mu \psi | \{uu\}_{1^+}(P) \rangle$$

$$f^{||}_{\{uu\}_{1^+}} M_{\{uu\}_{1^+}} = \frac{1}{3} Z_2 \,\mathrm{tr} \int_{dq} \gamma_\lambda \chi^C_{\lambda \{uu\}_{1^+}}(q;P)$$

- Note:
  - * Bethe-Salpeter wave function $\chi^C_D(q;P) = S(q_+)\,\Gamma^C_D(q;P)\,S(q_-)$
  - * Smaller than the corresponding meson decay constant by a factor $\sqrt{3}$.
  - * Purely theoretical object, do not correspond to any physical decay constant of diquarks.

P. Maris, Few Body Syst. 35 (2004) 117-127.

Diquark distribution amplitude (DA)



- Valence-quark twist-two diquark DA

- Scalar diquark

$$\langle 0 | \bar{\psi}(z)\gamma_+\gamma_5\psi(-z) | [ud]_{0+}\rangle$$

$$= i f_{[ud]_{0+}} P_+ \int_0^1 dx e^{i(2x-1)P\cdot z} \varphi_{[ud]_{0+}}(x,\zeta_H)$$

$$\Rightarrow f_{[ud]_{0+}} \varphi_{[ud]_{0+}}(x,\zeta_H)$$

$$= Z_2 \,\mathrm{tr} \int_{dq} \delta_n^x(q_+)\gamma_5\gamma\cdot n\, \chi^C_{[ud]_{0+}}(q;P)$$

L. Chang, I.C. Cloet, J.J. Cobos-Martinez, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 110 (2013) 13, 132001.

- Axial-vector diquark: longitudinal

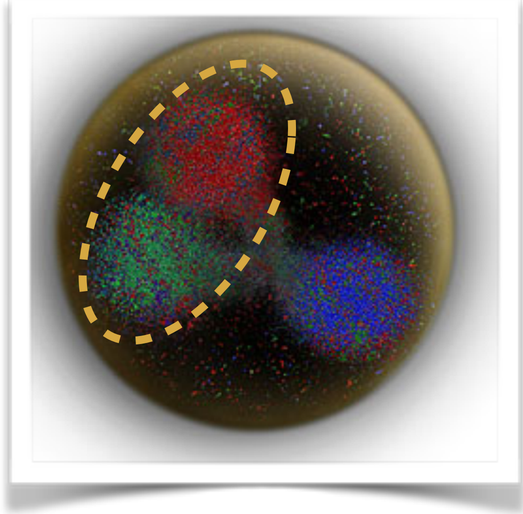
$$\langle 0 | \bar{\psi}(z)\gamma_+\psi(-z) | \{uu\}_{1+}\rangle$$

$$= f^{\parallel}_{\{uu\}_{1+}} M_{\{uu\}_{1+}} P_+ \frac{\epsilon^\lambda\cdot z}{P\cdot z} \int_0^1 dx e^{i(2x-1)P\cdot z} \varphi^{\parallel}_{\{uu\}_{1+}}(x,\zeta_H)$$

$$\Rightarrow f^{\parallel}_{\{uu\}_{1+}} n\cdot P\, \varphi^{\parallel}_{\{uu\}_{1+}}(x,\zeta_H)$$

$$= M_{\{uu\}_{1+}} Z_2 \,\mathrm{tr} \int_{dq} \delta_n^x(q_+)\gamma\cdot n\, n_\lambda \chi^C_{\lambda\{uu\}_{1+}}(q;P)$$

F. Gao, L Chang, Y-X Liu, C. D. Roberts, S. M. Schmidt, Phys. Rev. D 90 (2014) 1, 014011.



- Valence-quark twist-two diquark DF

- Scalar diquark

$$q_{[ud]_{0+}}(x,\zeta_H) = \int \frac{dz_-}{4\pi} e^{-ixP_+z_-} \times \langle [ud]_{0+} | \bar{\psi}(z_-)\gamma_+\psi(0) | [ud]_{0+} \rangle$$

$$q_{[ud]_{0+}}(x;\zeta_H) = N_c \mathrm{tr} \int_{dq} \delta_n^x(q_+)\, \Gamma^C_{[ud]_{0+}}(q_-,P)\, S(q_-) \times \left\{ n\cdot\partial_{q_+} \left[\Gamma^C_{[ud]_{0+}}(q_+,-P) S(q_+) \right] \right\}$$

- Axial-vector diquark:

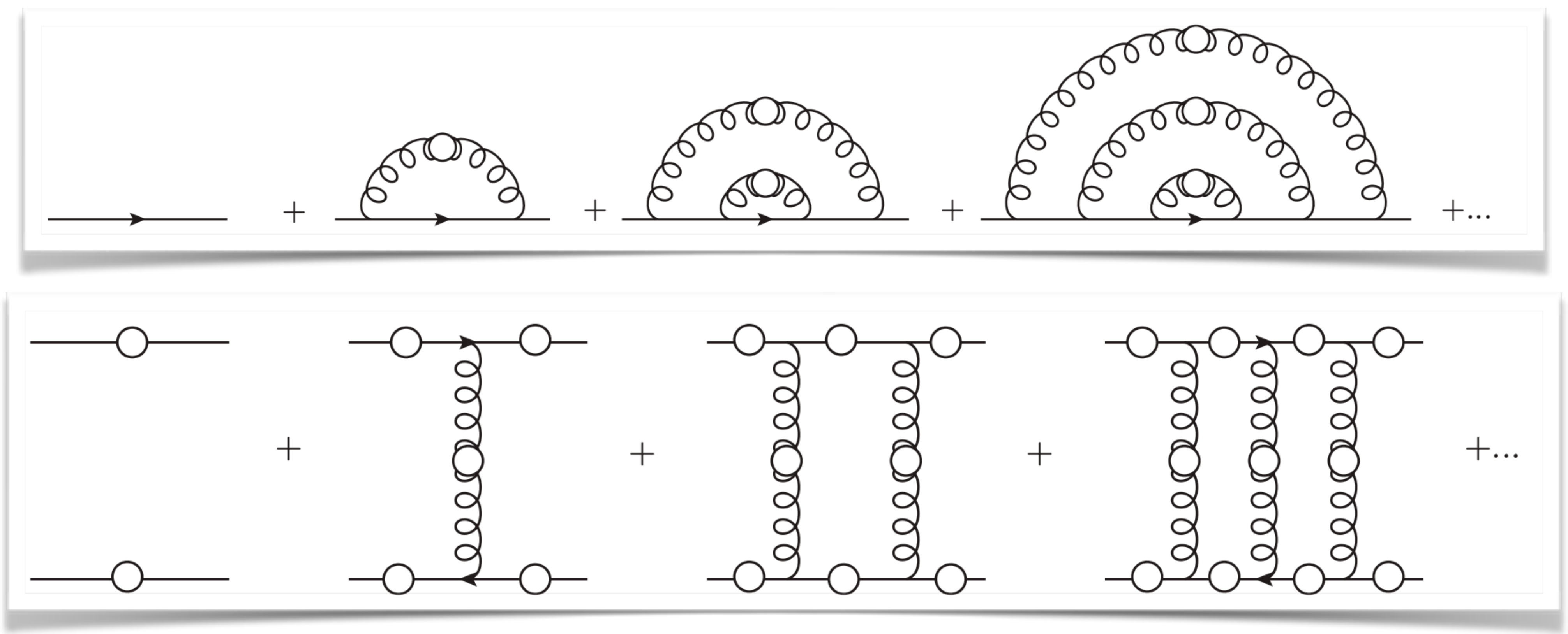
$$q_{\{uu\}_{1+}}(x,\zeta_H) = \int \frac{dz_-}{4\pi} e^{-ixP_+z_-} \times \langle \{uu\}_{1+} | \bar{\psi}(z_-)\gamma_+\psi(0) | \{uu\}_{1+} \rangle$$

$$q_{\{uu\}_{1+}}(x;\zeta_H) = N_c \mathrm{tr} \int_{dq} \delta_n^x(q_+)\, n_\nu \Gamma^C_{\nu\{uu\}_{1+}}(q_-,P)\, S(q_-) \times \left\{ n\cdot\partial_{q_+} \left[n_\mu \Gamma^C_{\mu\{uu\}_{1+}}(q_+,-P) S(q_+) \right] \right\}$$

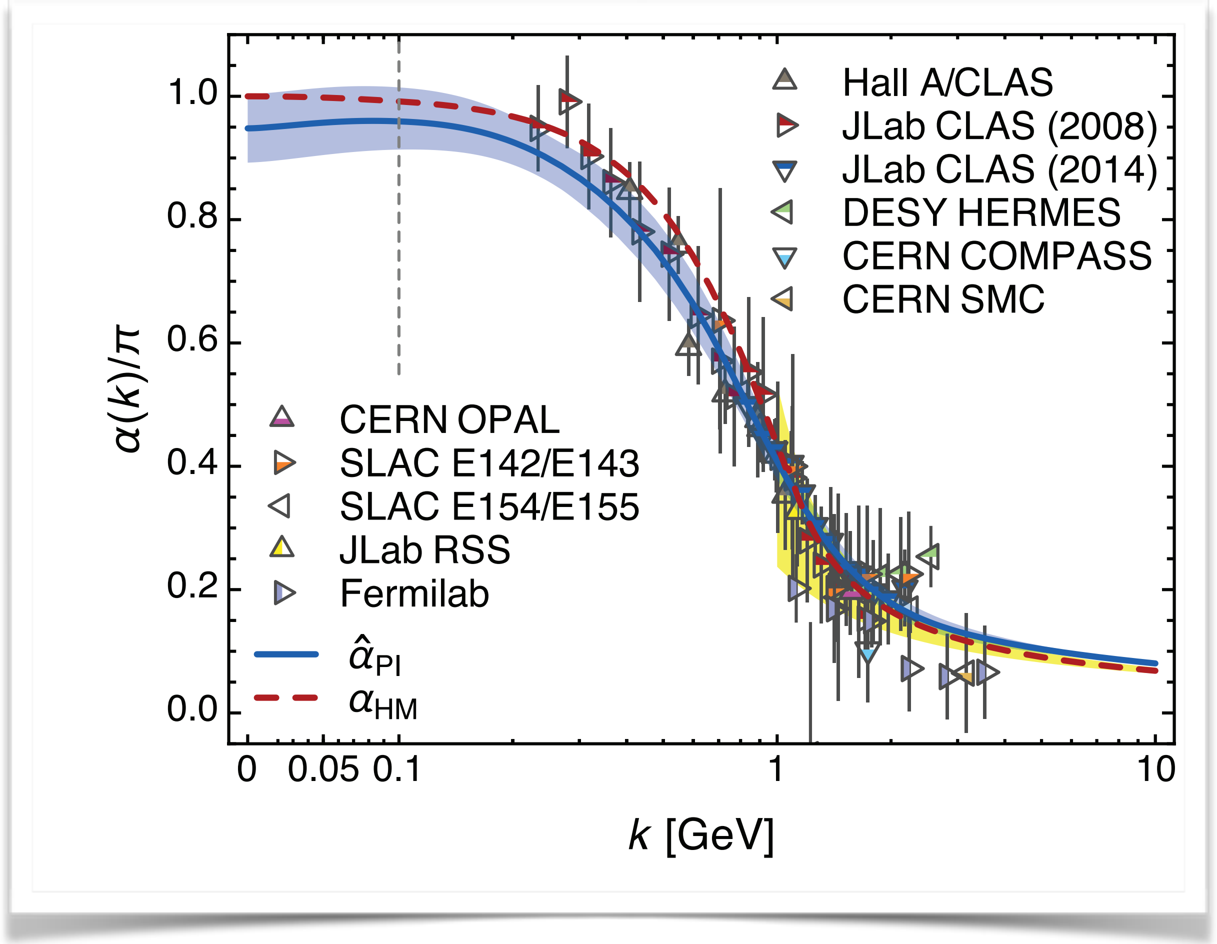
M. Ding, K. Raya, D. Binosi, L. Chang, C. D Roberts, S. M. Schmidt, Phys. Rev. D 101 (2020) 5, 054014.

see talks by D. Binosi and J. Segovia

- Rainbow-Ladder approximation



- Hadronic scale:



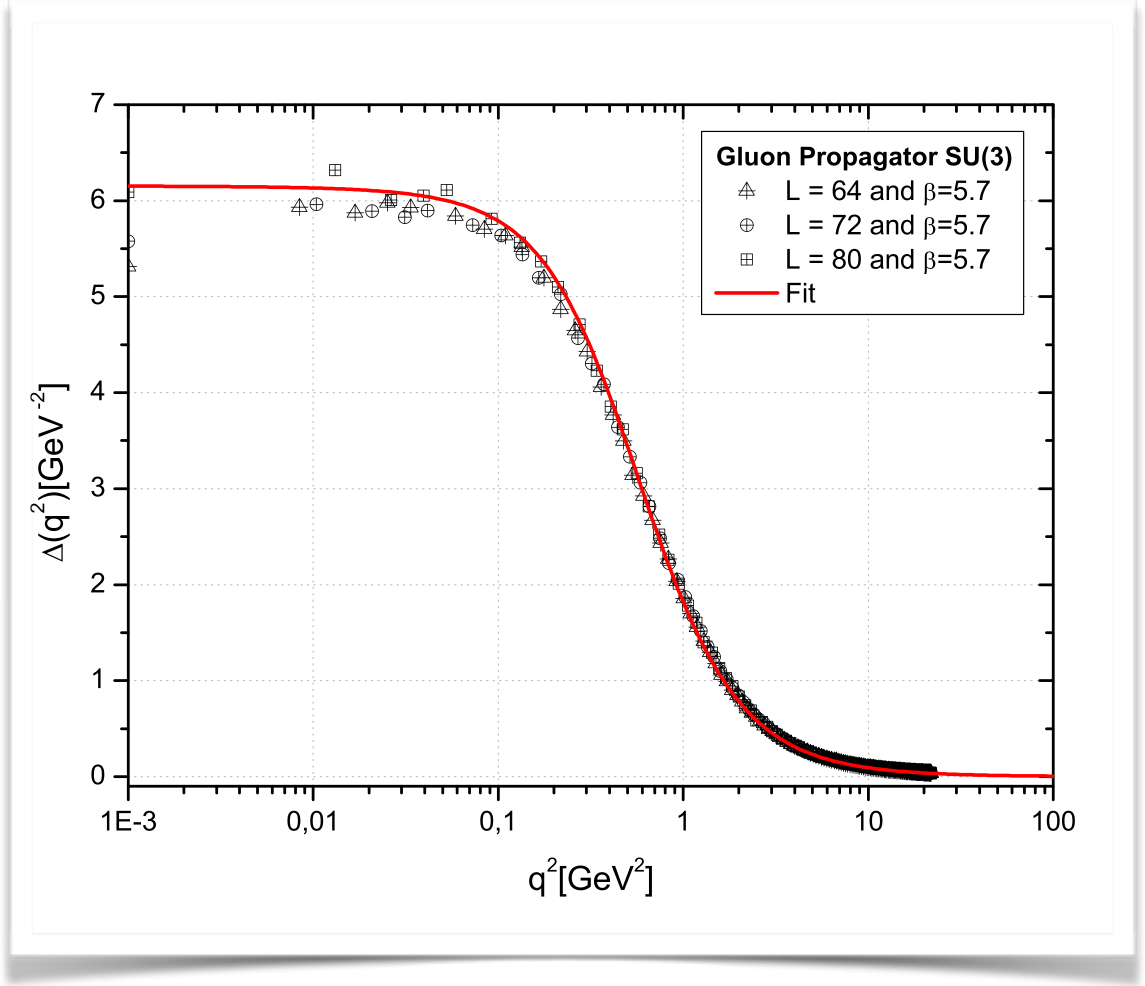
Daniele Binosi et al.. Phys. Rev. D 96, 054026 (2017).

✱ Process-independent running coupling, $\zeta_H = 0.3$GeV.

- Gluon propagator

✱ Finite and non-zero in the infrared

A. C. Aguilar, D. Binosi, and J.Papavassiliou, JHEP 07 (2010) 002.



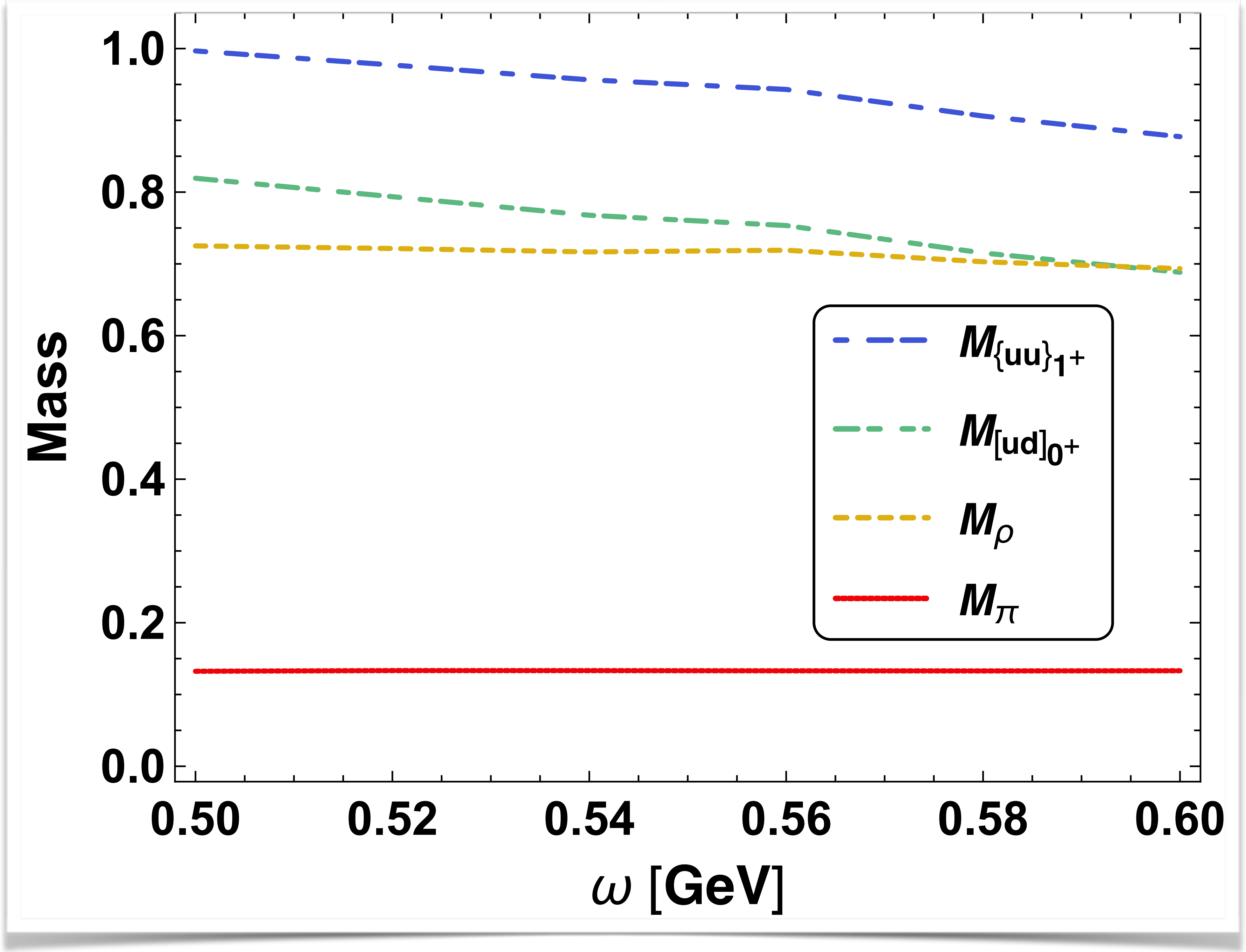
- Mellin moment

$$\langle x_\varphi^m \rangle = \int_0^1 dx x^m \varphi(x) \,, \quad \langle x_q^m \rangle = \int_0^1 dx x^m q(x) \,.$$

✱ DA $\varphi(x)$ and DF $q(x)$ are reconstructed from moments.

Numerical results I: Mass

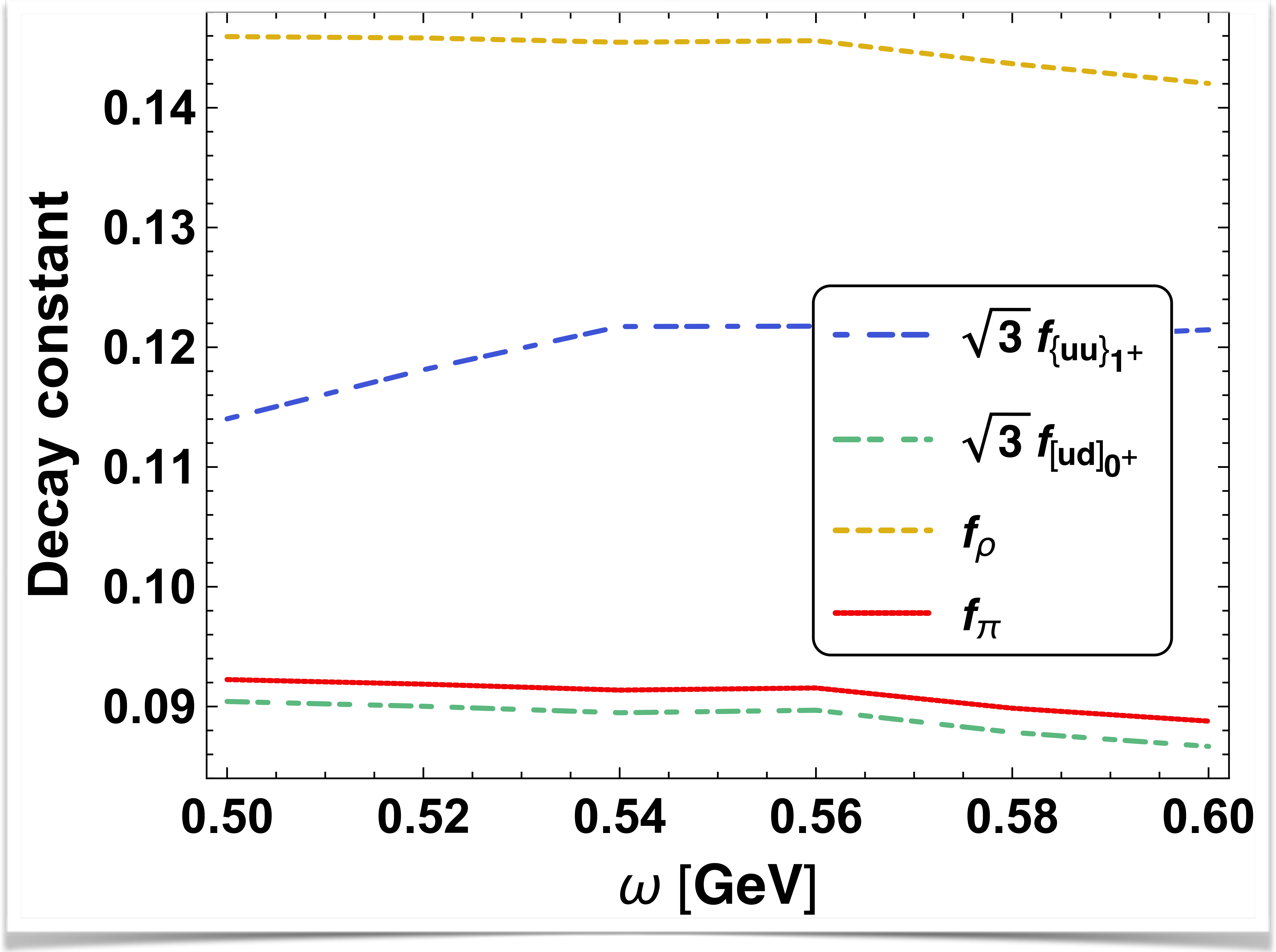
- Four cases:
  - Scalar diquark $[ud]_{0^+}$
  - Axial-vector diquark $\{uu\}_{1^+}$
  - Pseudoscalar meson π
  - Vector meson ρ



- Masses dependence on gluon interaction ω.
  - $\mathfrak{g}_{IR}(k^2) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2}$
  - Meson masses are independent of the gluon interaction.
  - Diquark masses are sensible to the gluon interaction.
- Mass difference $M_{\{uu\}_{1^+}} - M_{[ud]_{0^+}} \approx 0.184(7)$ GeV.
  - Less than the splitting between the Δ-baryon and nucleon, $\delta_{\Delta N} \approx 0.27$ GeV.
  - Meson cloud effect can increase the splitting by $0.05 - 0.10$ GeV.

Numerical results II: Decay constant

- Four cases:
  - Scalar diquark $[ud]_{0^+}$
  - Axial-vector diquark $\{uu\}_{1^+}$
  - Pseudoscalar meson π
  - Vector meson ρ



- Decay constant dependence on gluon interaction ω.
  - Meson decay constant are almost independent of the gluon interaction, deviation are less than 4%.
  - Diquark decay constant are almost independent of the gluon interaction, deviation are less than 6%.
  - Diquark decay constant is smaller than the corresponding meson decay constant by a factor $\sqrt{3}$.
  - Purely theoretical object, do not correspond to any physical decay constant of diquarks.

Numerical results III: BSAs (a)

- BSAs carry rich structure.

  * Scalar diquark $[ud]_{0^+}$, four tensors.

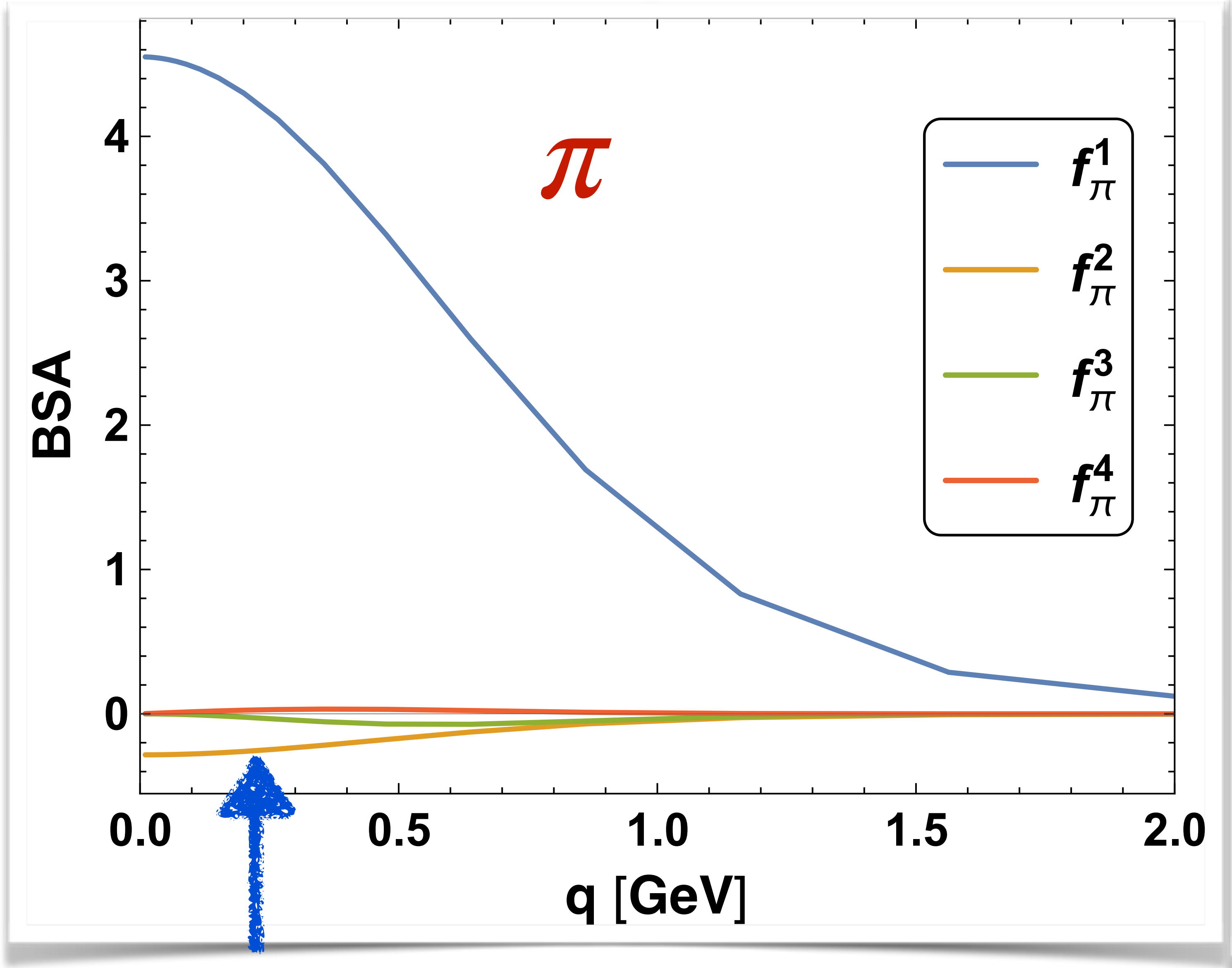
  * Axial-vector diquark $\{uu\}_{1^+}$, eight tensors.

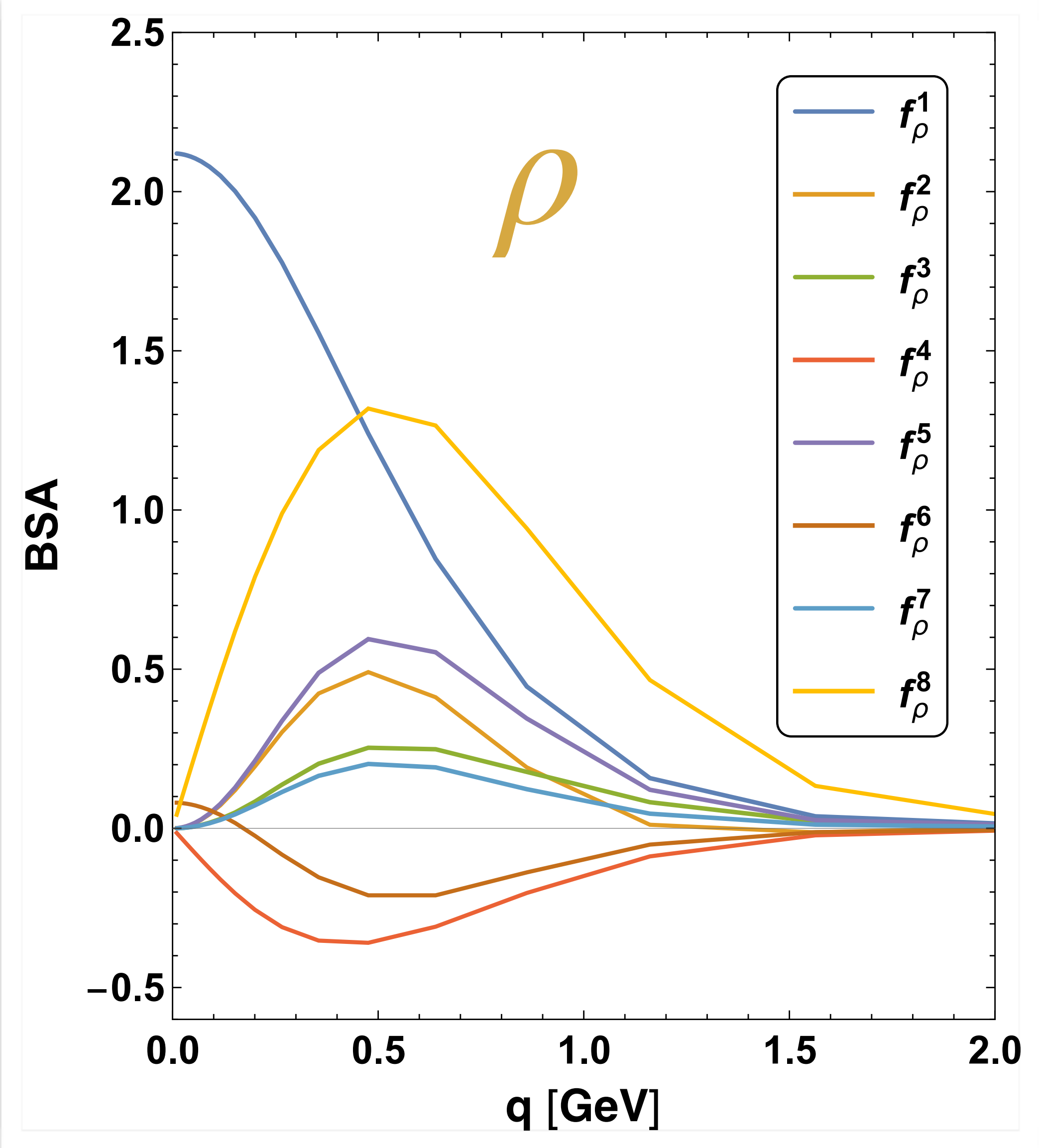
  * Scalar diquark BSAs are different from π BSAs, especially p-wave contributions.

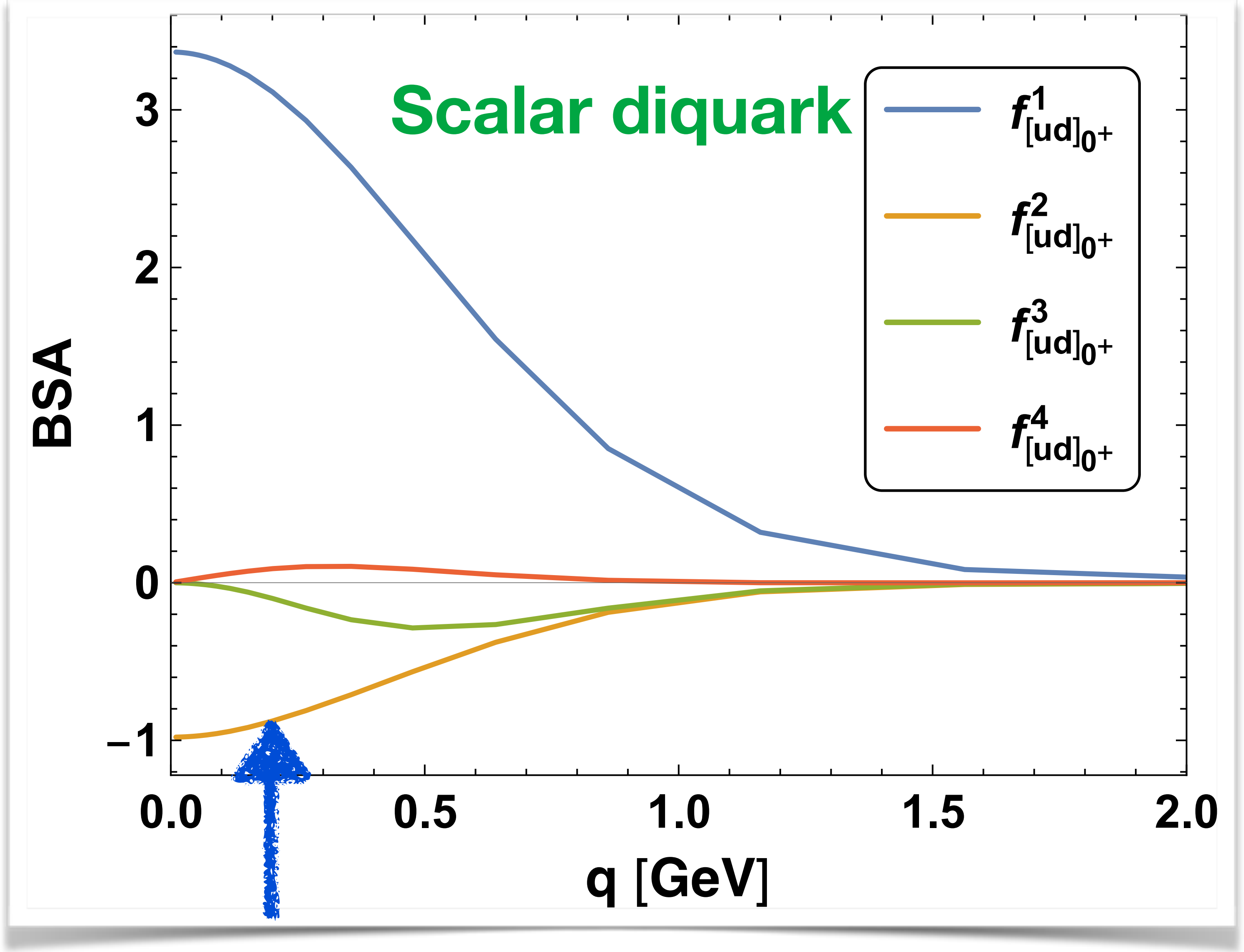
  * Axial-vector diquark BSAs are different from ρ BSAs, especially p-wave and d-wave contributions.

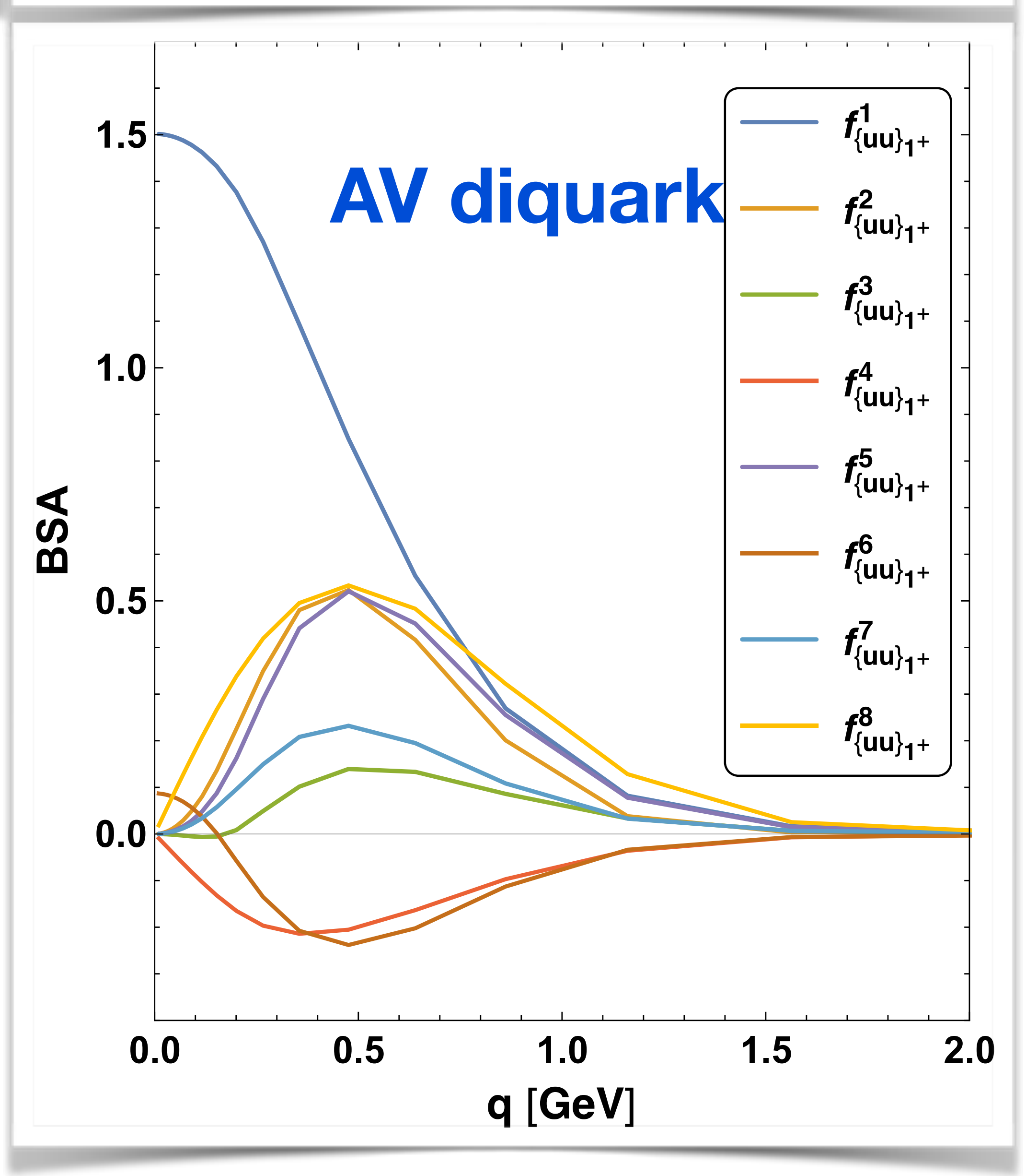
  * $f^1(q;P)$, leading term, dominate.

  * BSAs are the origin of any structure function.

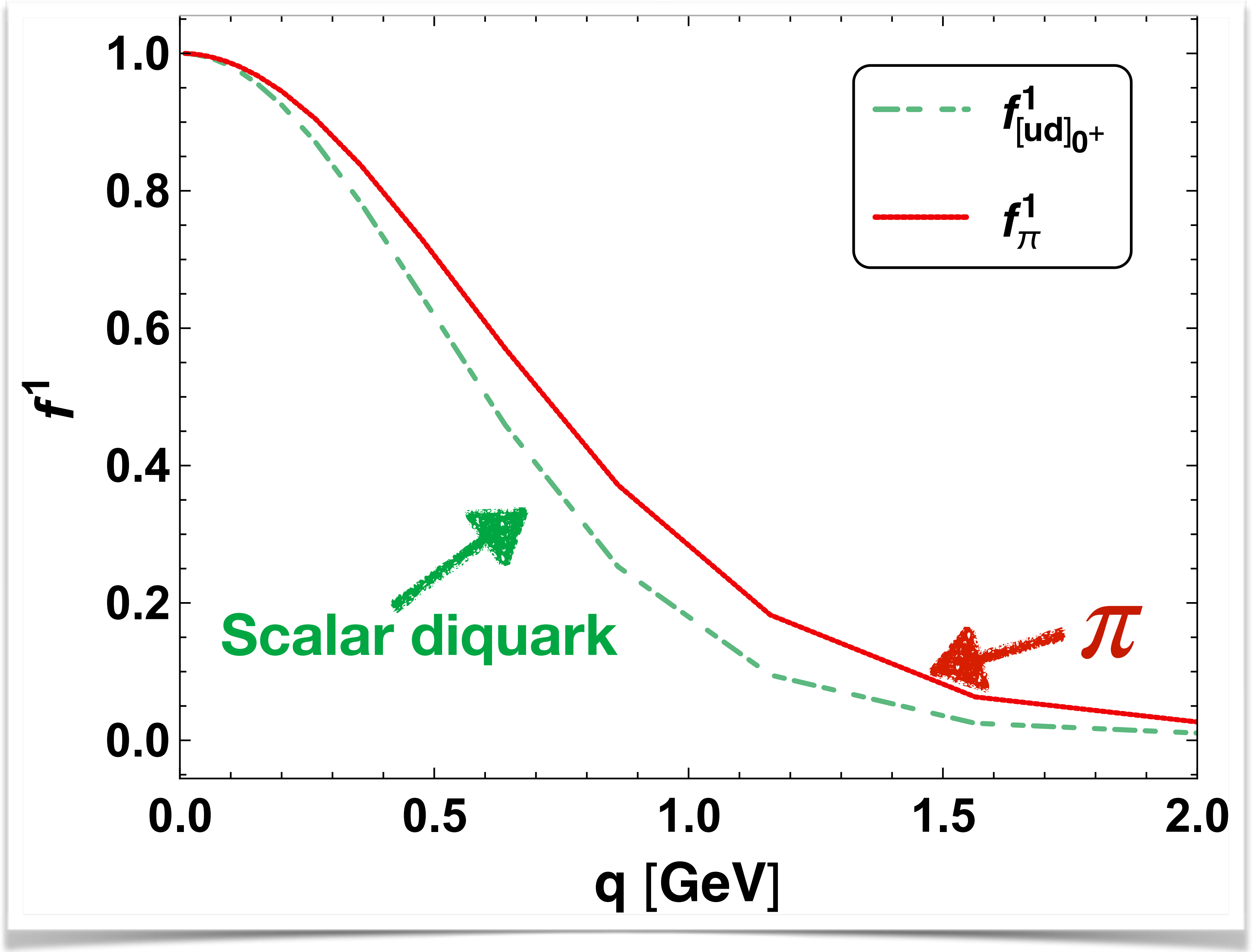


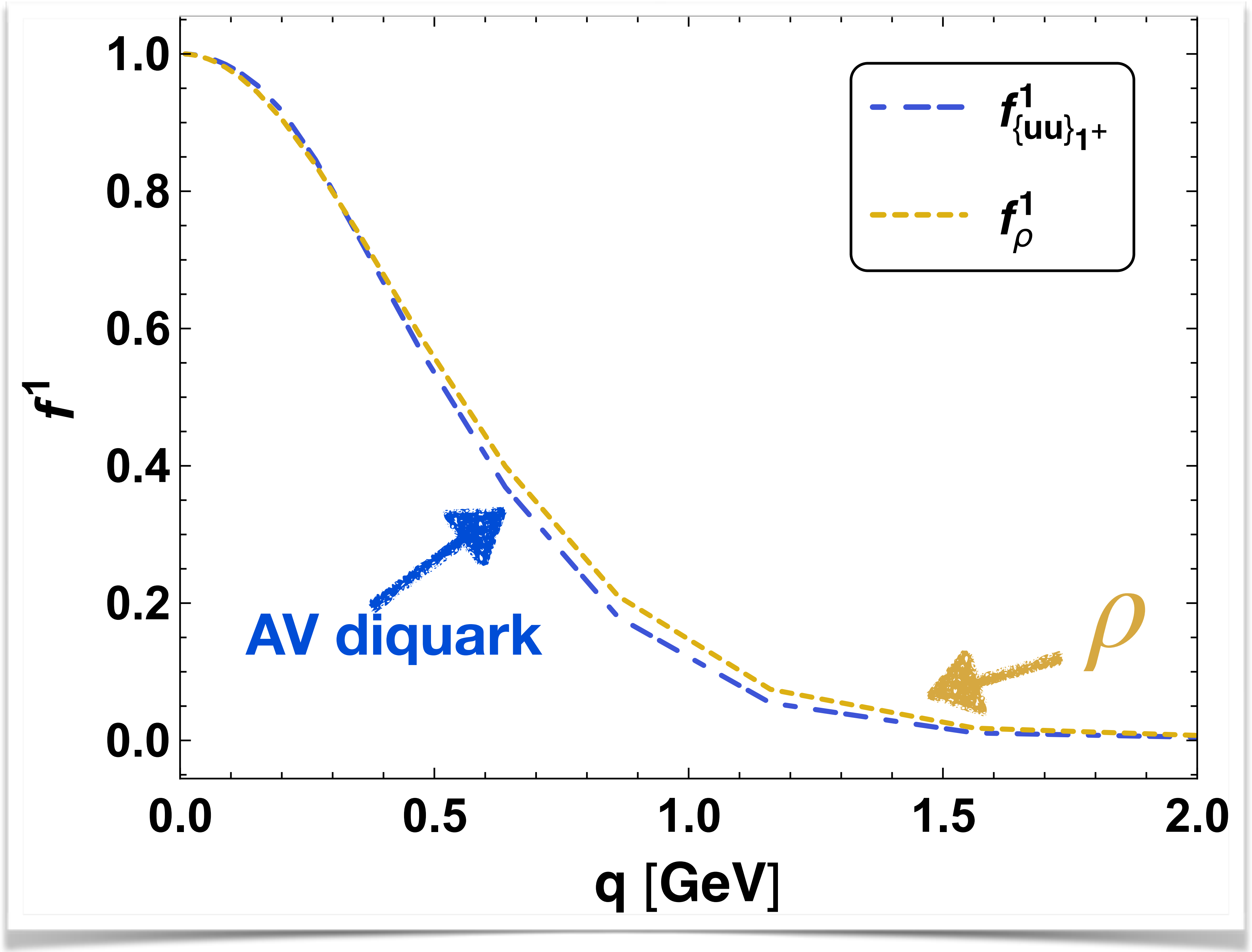






Numerical results III: BSAs (b)



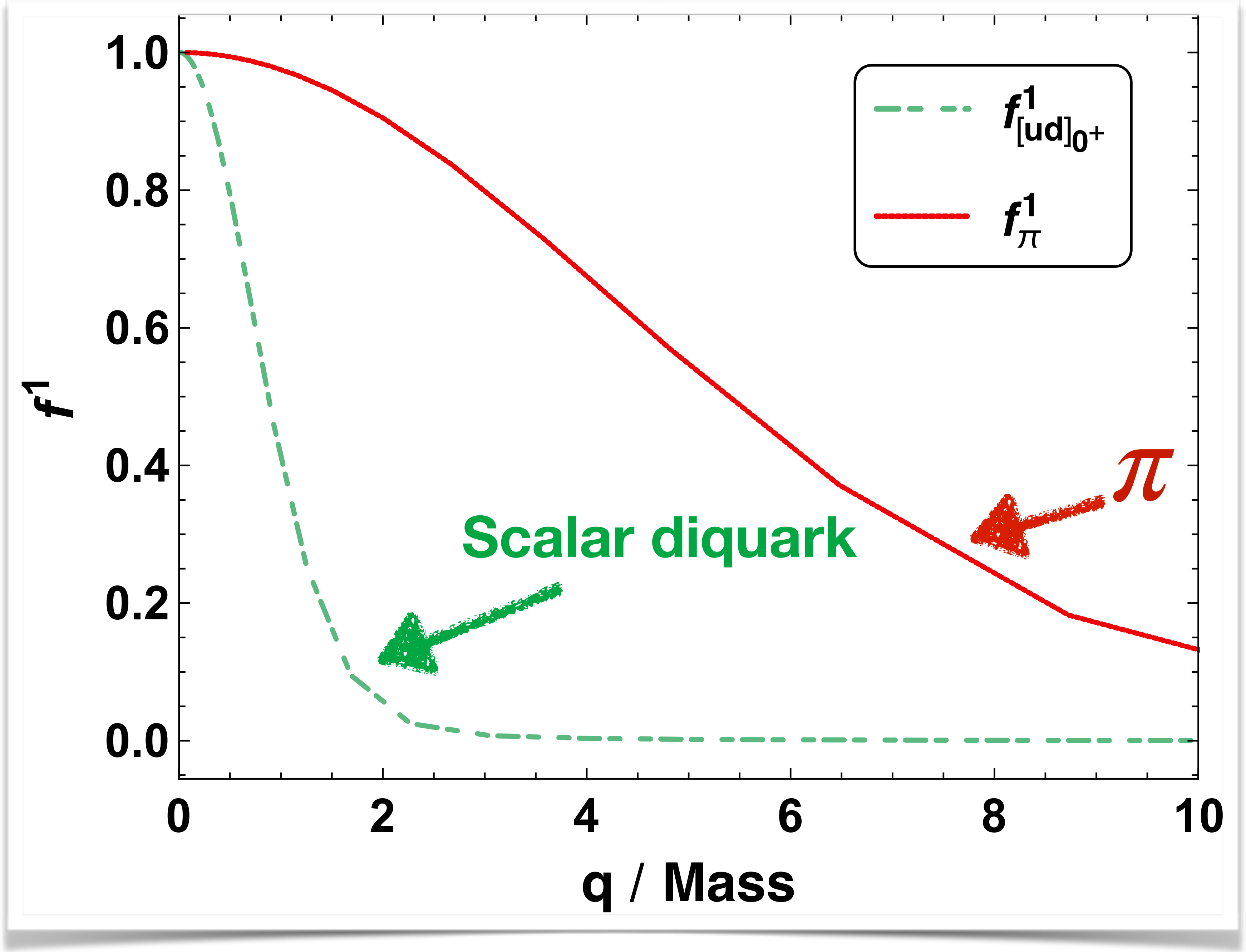


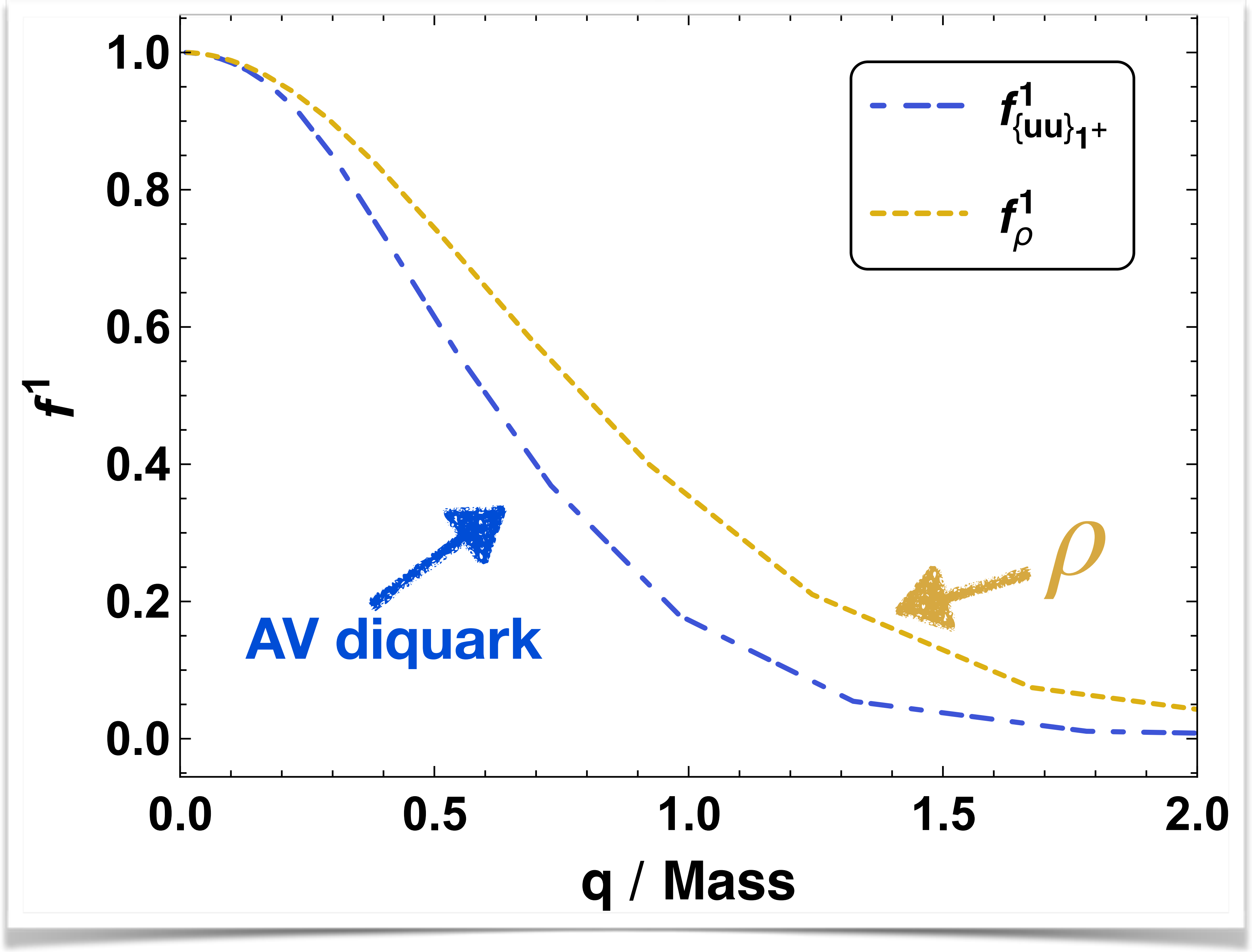
- Scalar diquark $f^1_{[ud]_{0+}}$ in BSAs is narrower than π meson f^1_π in BSAs in momentum space, and thus wider in coordinate space.
- Scalar diquark is less bound than π.
- Charge radius $r_{[ud]_{0+}} > r_\pi$.

P. Maris, Few Body Syst. 32 (2002) 41-52.

- Axial-vector diquark $f^1_{\{uu\}_{1+}}$ in BSAs is narrower than ρ meson f^1_ρ in BSAs in momentum space, and thus wider in coordinate space.
- Axial-vector diquark is less bound than ρ.
- Charge radius $r_{\{uu\}_{1+}} > r_\rho$.

P. Maris, Few Body Syst. 35 (2004) 117-127.



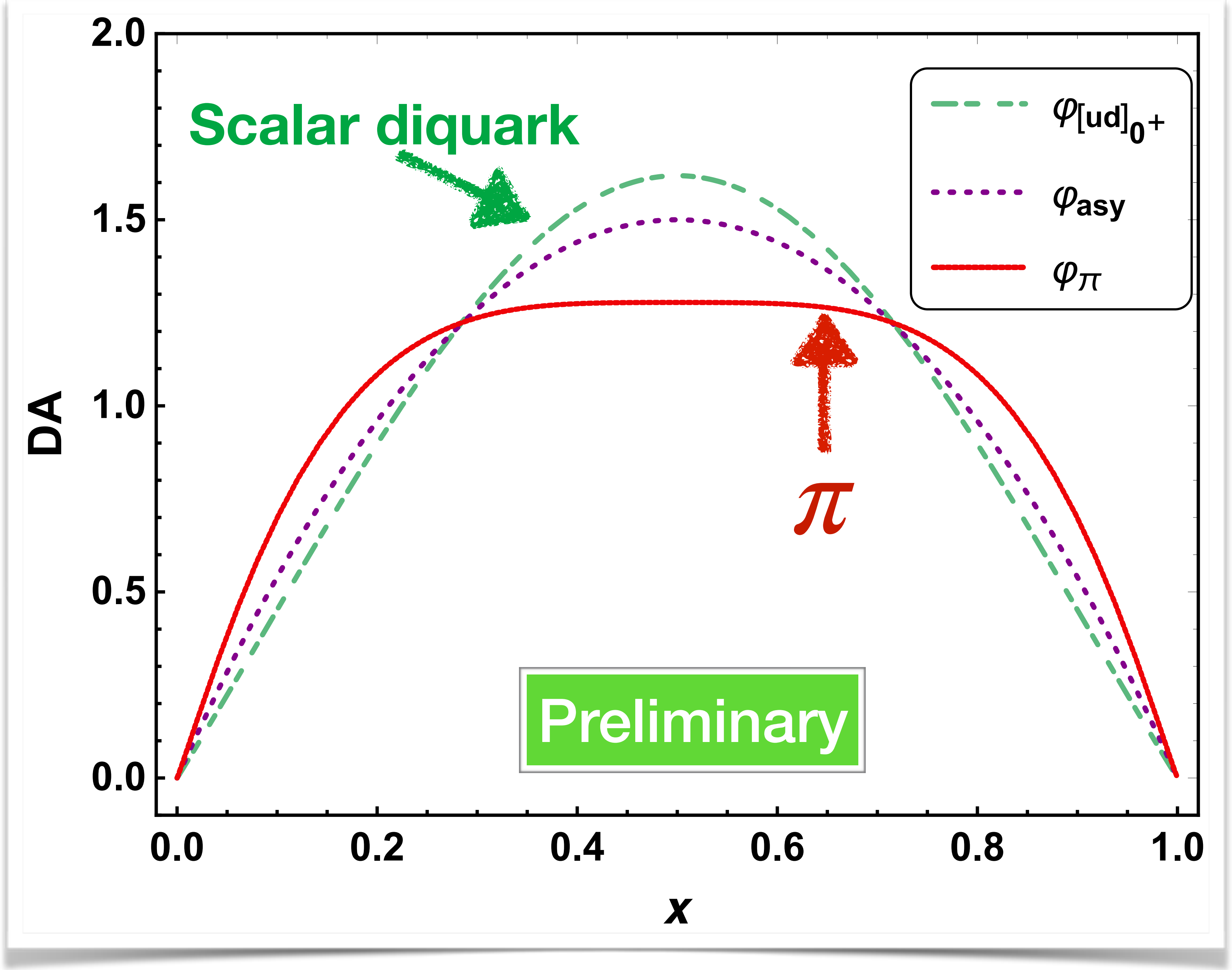


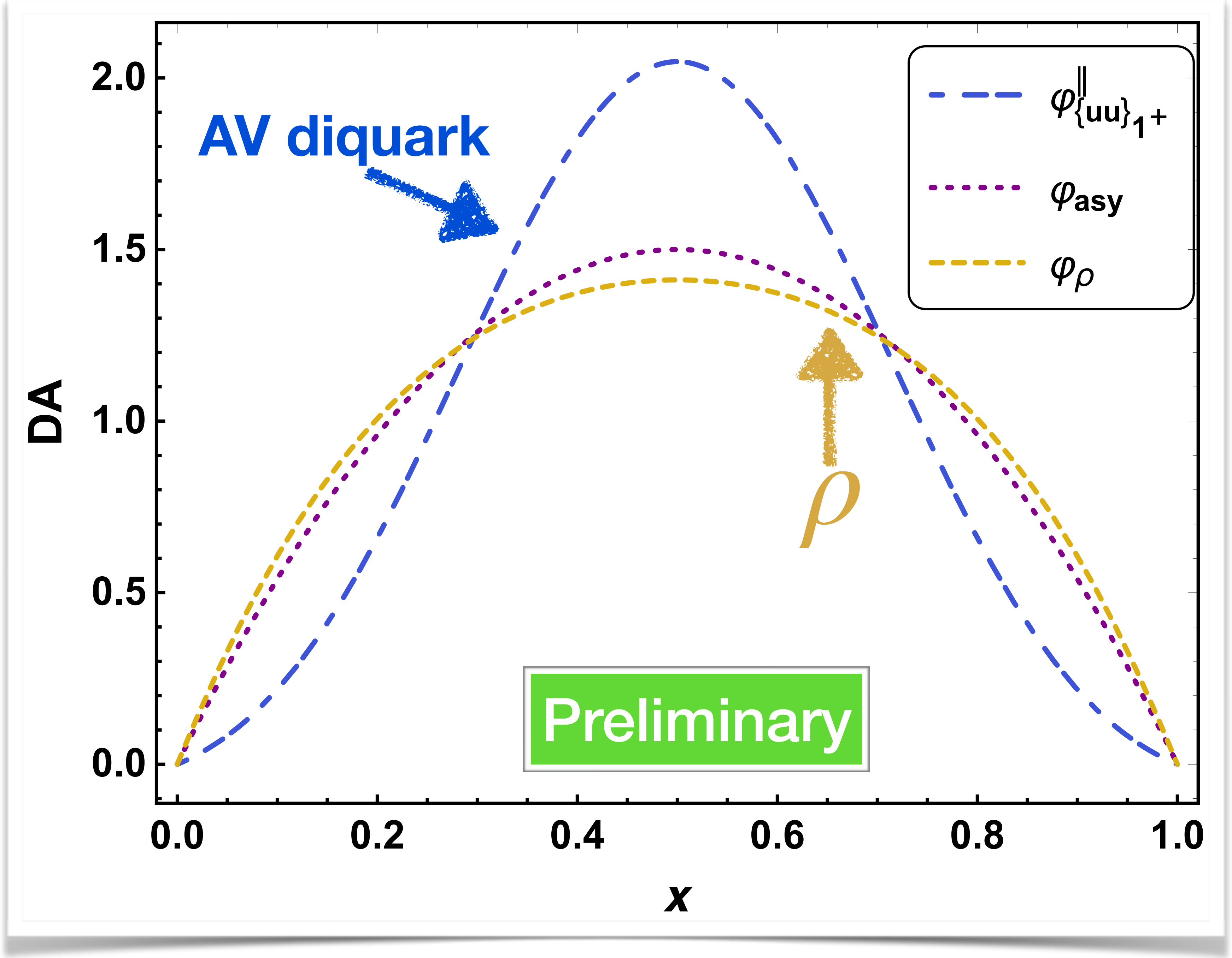
- Charge radius is with mass dimension -1, we can consider $f^1_{[ud]_{0+}}$ versus q [GeV] .
- Light front quantities such as DA and DF with variable x is dimensionless, we can consider $f^1_{[ud]_{0+}}$ versus q / Mass .

➡ * Scalar diquark $f^1_{[ud]_{0+}}$ in BSAs is much narrower than π meson f^1_{π} in BSAs in momentum space.

➡ * Axial-vector diquark $f^1_{\{uu\}_{1+}}$ in BSAs is much narrower than ρ meson f^1_{ρ} in BSAs in momentum space.

Numerical results IV: DAs





- DA $\varphi_{[ud]_{0+}}$ <narrower than $\varphi_{asy} < \varphi_\pi$.
- Unexpected, since they both only consist of light quarks.
- Indicated form BSA analysis.

- DA $\varphi^{\parallel}_{\{uu\}_{1+}} < \varphi_{asy} < \varphi^{\parallel}_{\rho}$.
- Agree with previous studies on π and ρ DAs.

L. Chang, I.C. Cloet, J.J. Cobos-Martinez, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 110 (2013) 13, 132001.

F. Gao, L Chang, Y-X Liu, C. D. Roberts, S. M. Schmidt, Phys. Rev. D 90 (2014) 1, 014011.

Numerical results IV: DAs

- DA $\varphi^{\parallel}_{\{uu\}_{1+}} < \varphi_{[ud]_{0+}} < \varphi_{asy} < \varphi^{\parallel}_{\rho} < \varphi_{\pi}$.

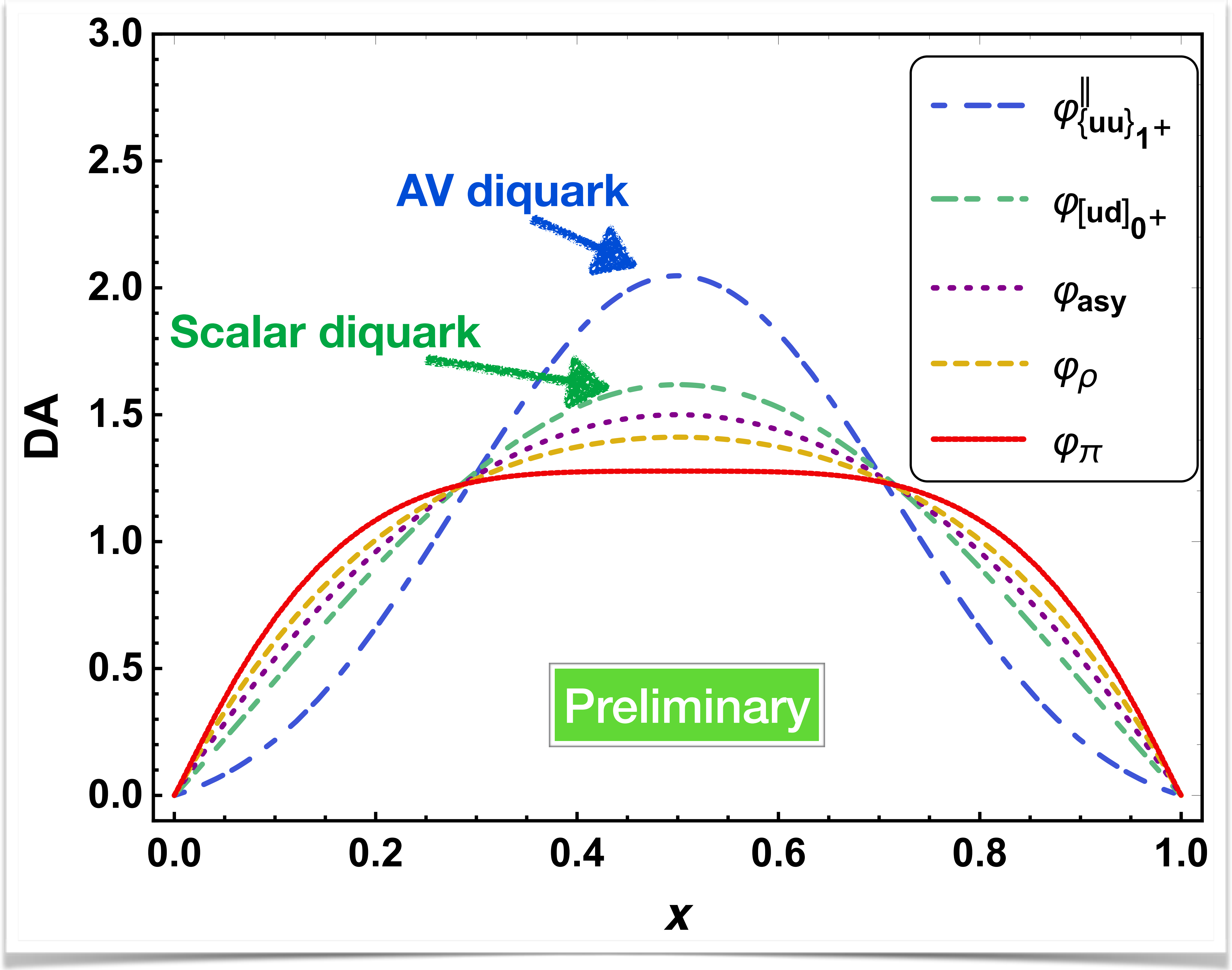
- Order of mass, heavy to light.
- Heavy bound-state is narrower.

see talk by C. Roberts

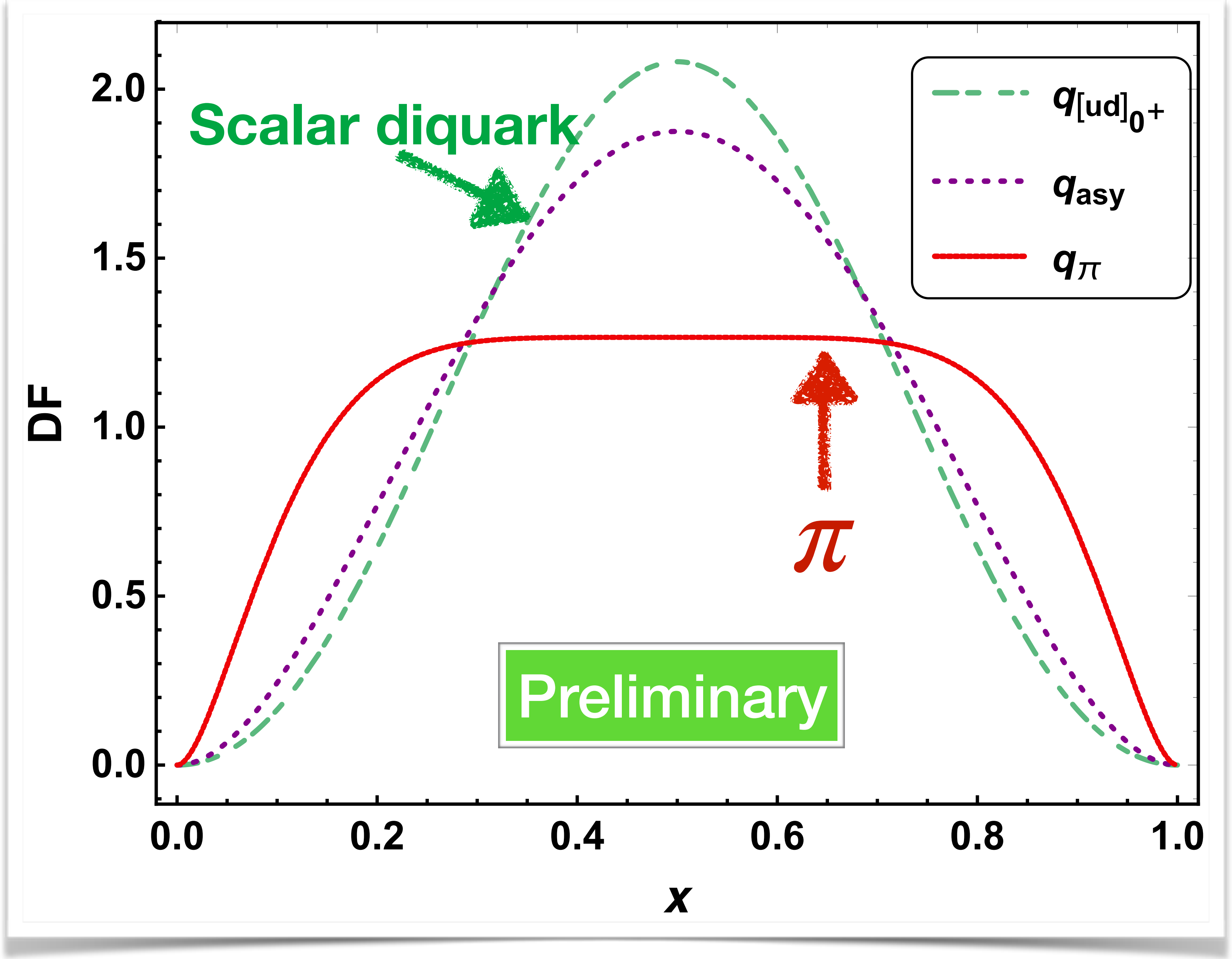
- Diquark is narrower than its corresponding meson.
- Diquark is narrower than the asymptotic distribution φ_{asy}.

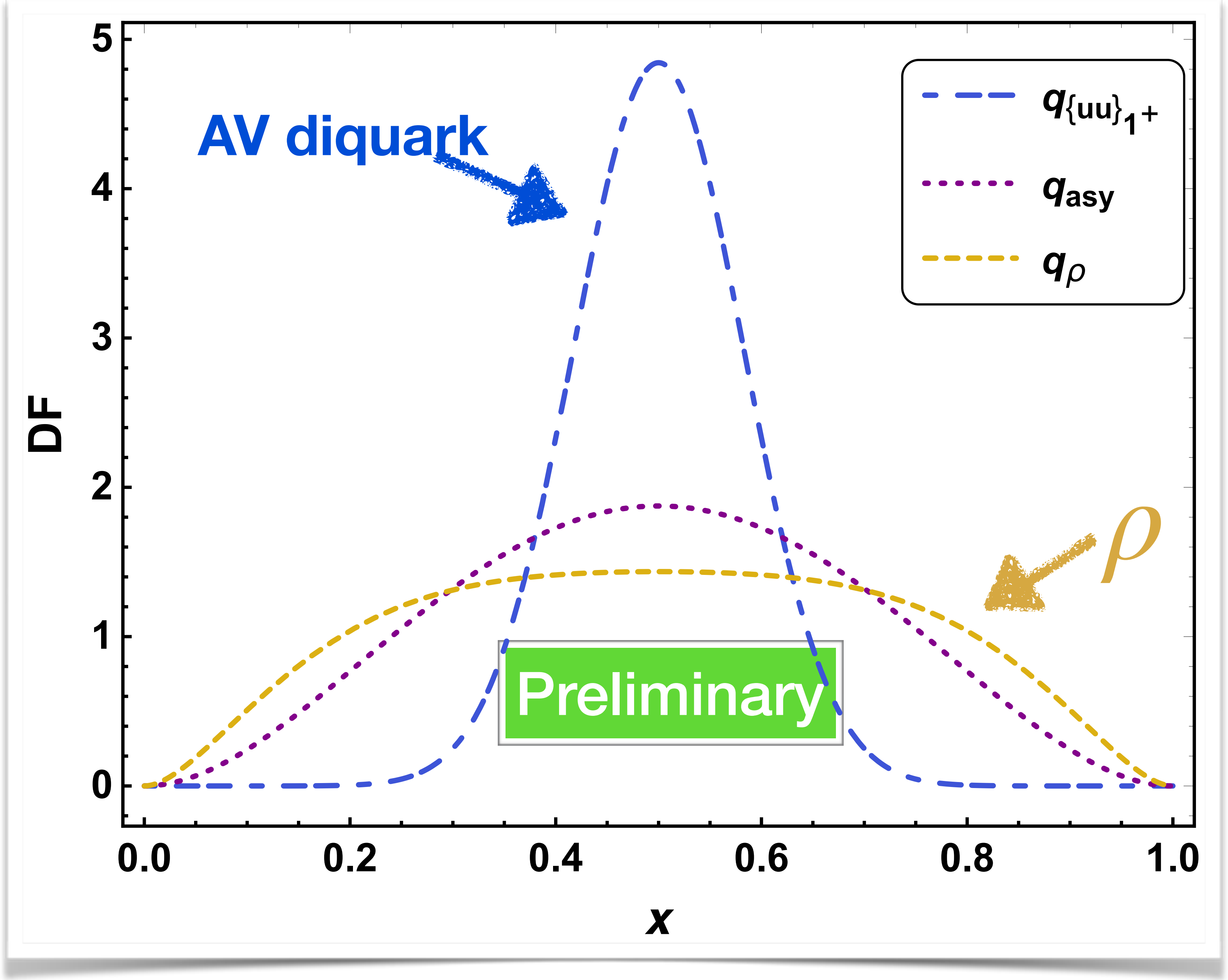
$$\varphi_{[ud]_{0+}} < \varphi_{asy} < \varphi_{\pi}, \qquad \varphi^{\parallel}_{\{uu\}_{1+}} < \varphi_{asy} < \varphi^{\parallel}_{\rho}.$$

- $\varphi^{\perp}_{\rho}$ and $\varphi^{\perp}_{\{uu\}_{1+}}$ are by analogy with $\varphi^{\parallel}_{\rho}$ and $\varphi^{\parallel}_{\{uu\}_{1+}}$.



Numerical results V: DFs





- DF $q_{[ud]_{0+}} < q_{asy} < q_\pi$.
- Agree with previous study on π DF.

M. Ding, K. Raya, D. Binosi, L. Chang, C. D Roberts, S. M. Schmidt, Phys. Rev. D 101 (2020) 5, 054014.

- DF $q_{\{uu\}_{1+}} < q_{asy} < q_\rho$.
- Comparison on ρ DF is welcome.

Numerical results V: DFs

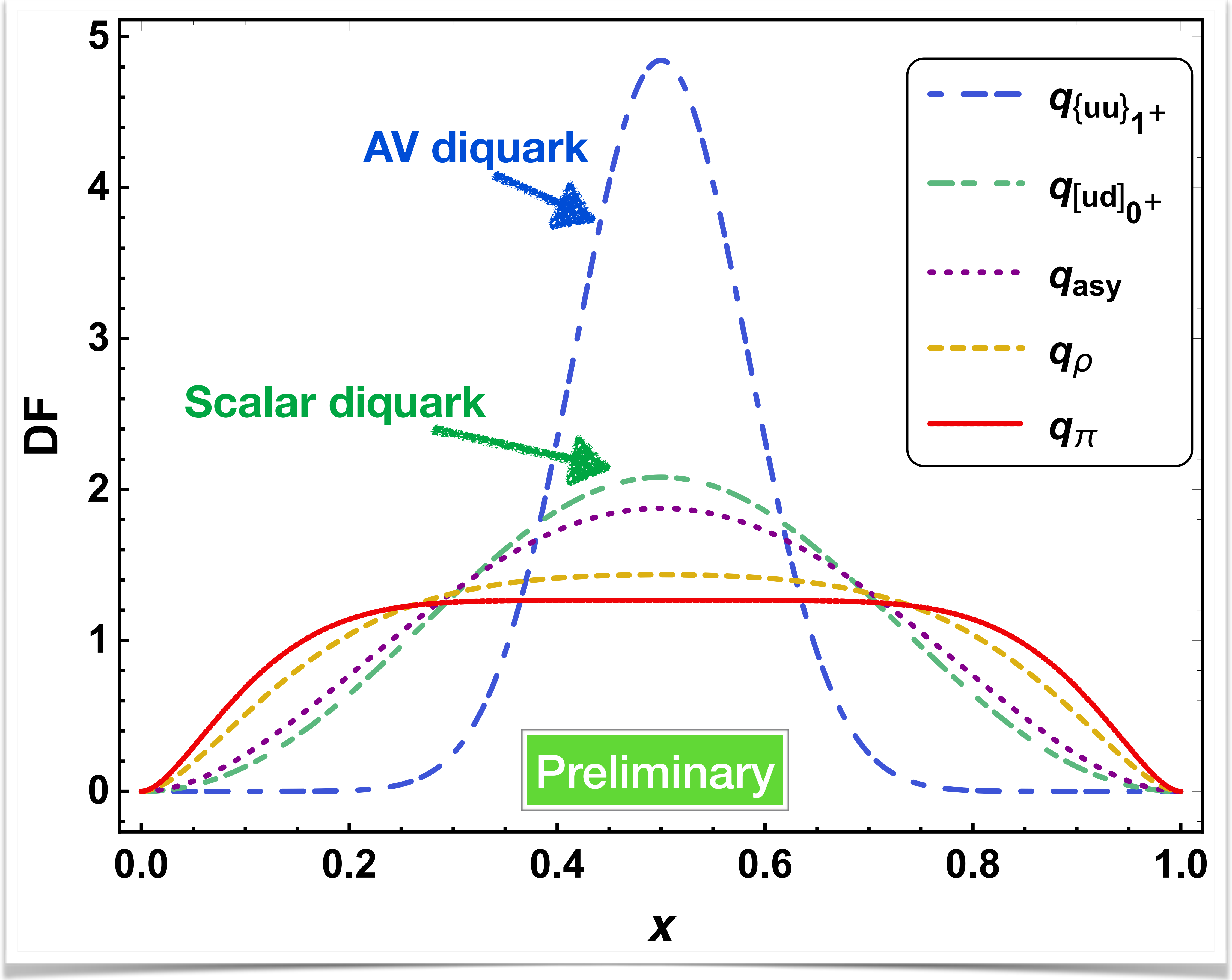
- DF $q_{\{uu\}_{1+}} < q_{[ud]_{0+}} < q_{asy} < q_\rho < q_\pi$.

- Order of mass, heavy to light.
- Heavy bound-state is narrower.

- Diquark is narrower than its corresponding meson.
- Diquark is narrower than the asymptotic distribution q_{asy}.

$$q_{[ud]_{0+}} < q_{asy} < q_\pi, \qquad q_{\{uu\}_{1+}} < q_{asy} < q_\rho.$$

- Generally satisfy $q_{[ud]_{0+}}(x) \approx \varphi^2_{[ud]_{0+}}(x)$.



Summary and outlook

- Summary

  ✱ Diquark distribution amplitudes (DAs) and distribution functions (DFs) using Bethe-Salpeter equation, with rainbow-ladder approximation.

  ✱ Major feature: axial-vector diquark < scalar diquark < asymptotic < rho meson < pion.

  ✱ Emergence of Hadronic Mass (EHM).

- Outlook

  ✱ Nucleon DF.

$$f_{q/P}(x) = \sum_Q \int_0^1 dy \int_0^1 dz\, \delta(x - yz) f_{Q/P}(y) f_{q/Q}(z)$$

  ✱ Nucleon GPDs and TMDs.

Thank you