

# STrEAMlining EFT Matching

Area 5 LPCC EFT WG, Feb 08, 2021

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arXiv: 2011.02484, 2012.07851  
with Timothy Cohen and Zhengkang Zhang

# Outline

- EFT matching with functional methods up to one loop
  - Advantages compared to “amplitude matching”
- A new way of organizing functional supertrace evaluation
  - Log-type and power-type
- **STrEAM** (**S**uper**T**race **E**valuation **A**utomated for **M**atching)
  - Automates supertrace evaluations with CDE

Energy

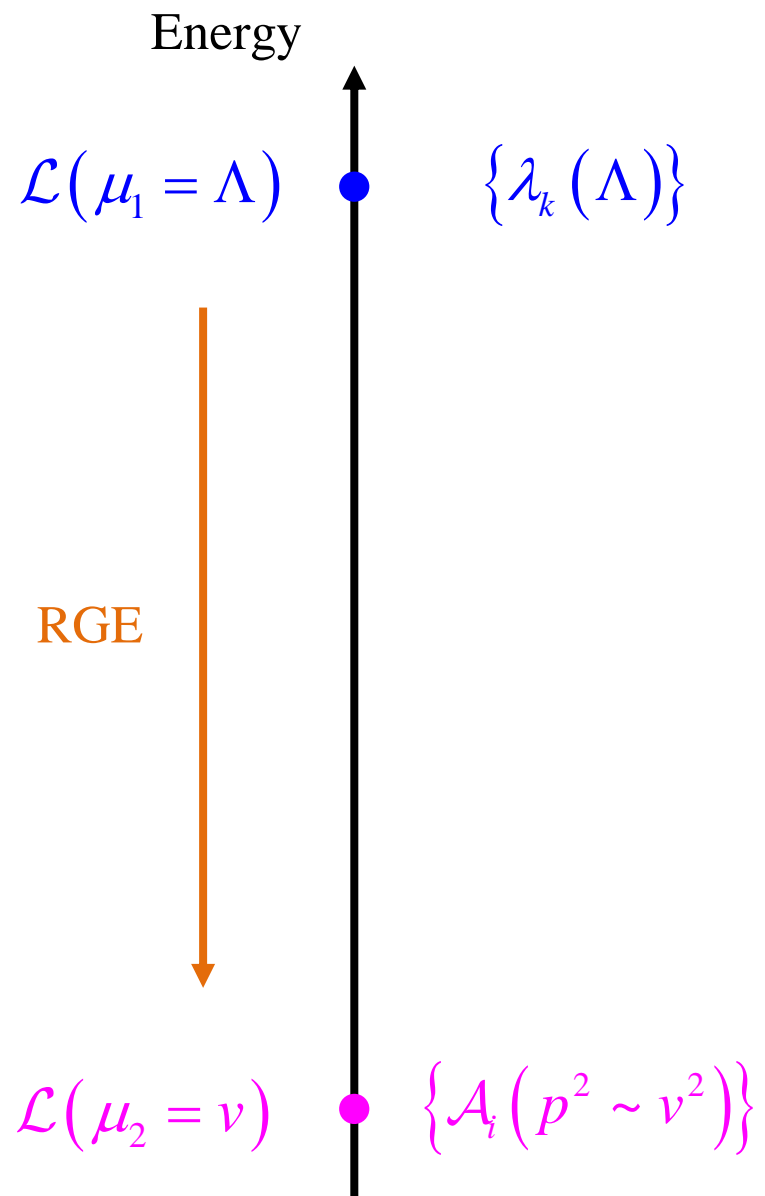
$$\mathcal{L}(\mu_1 = \Lambda)$$

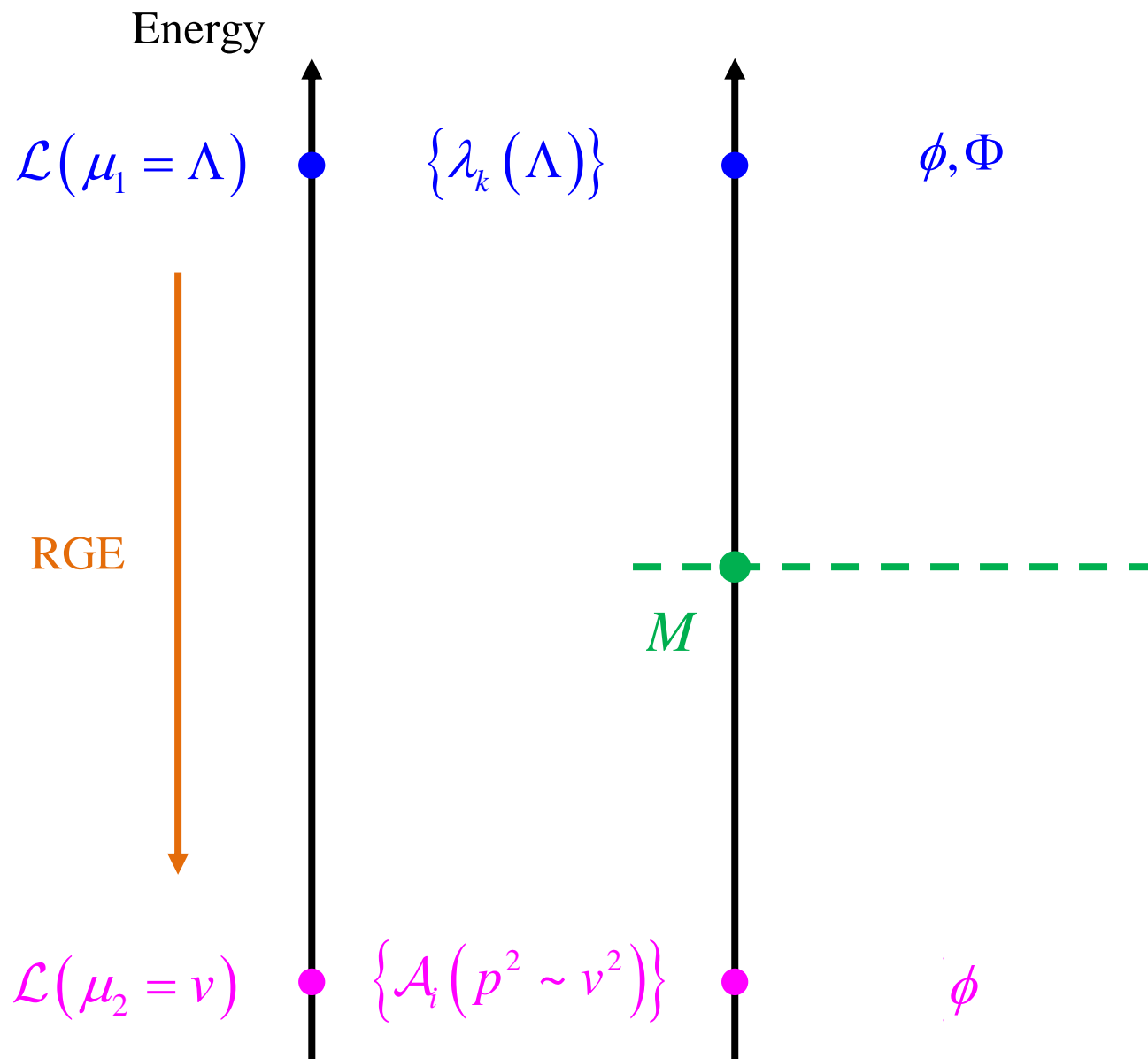


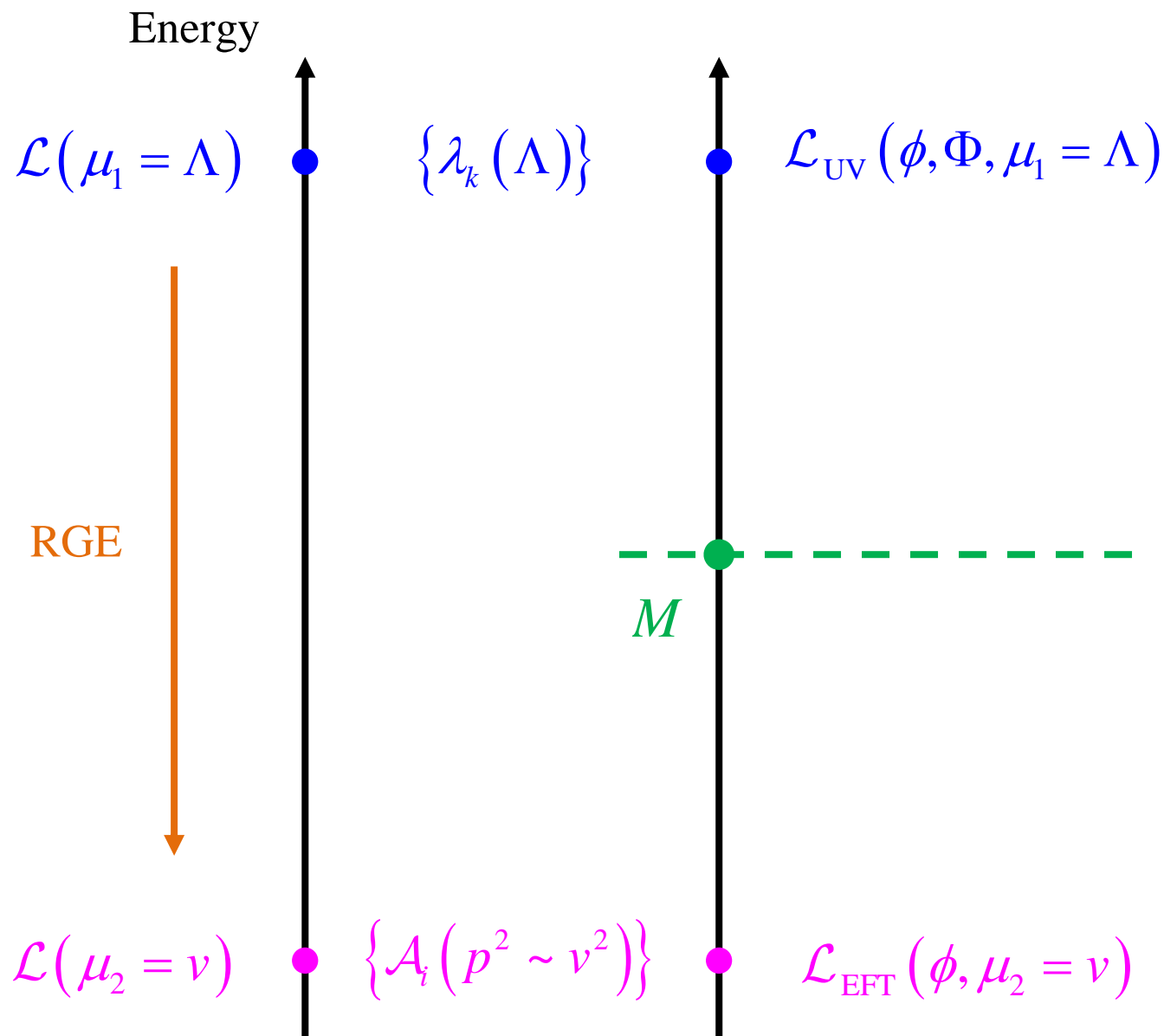
$$\{\lambda_k(\Lambda)\}$$

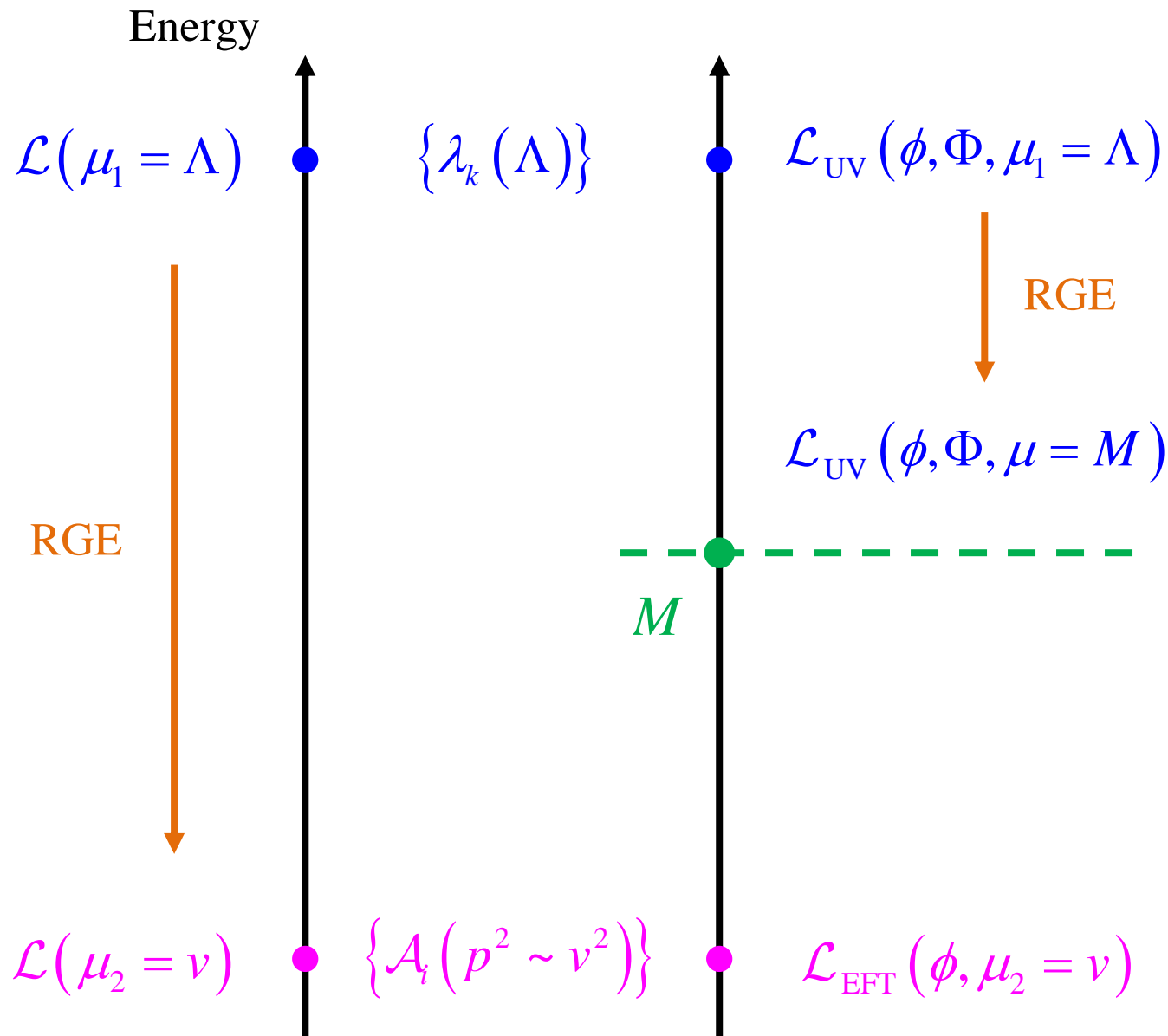


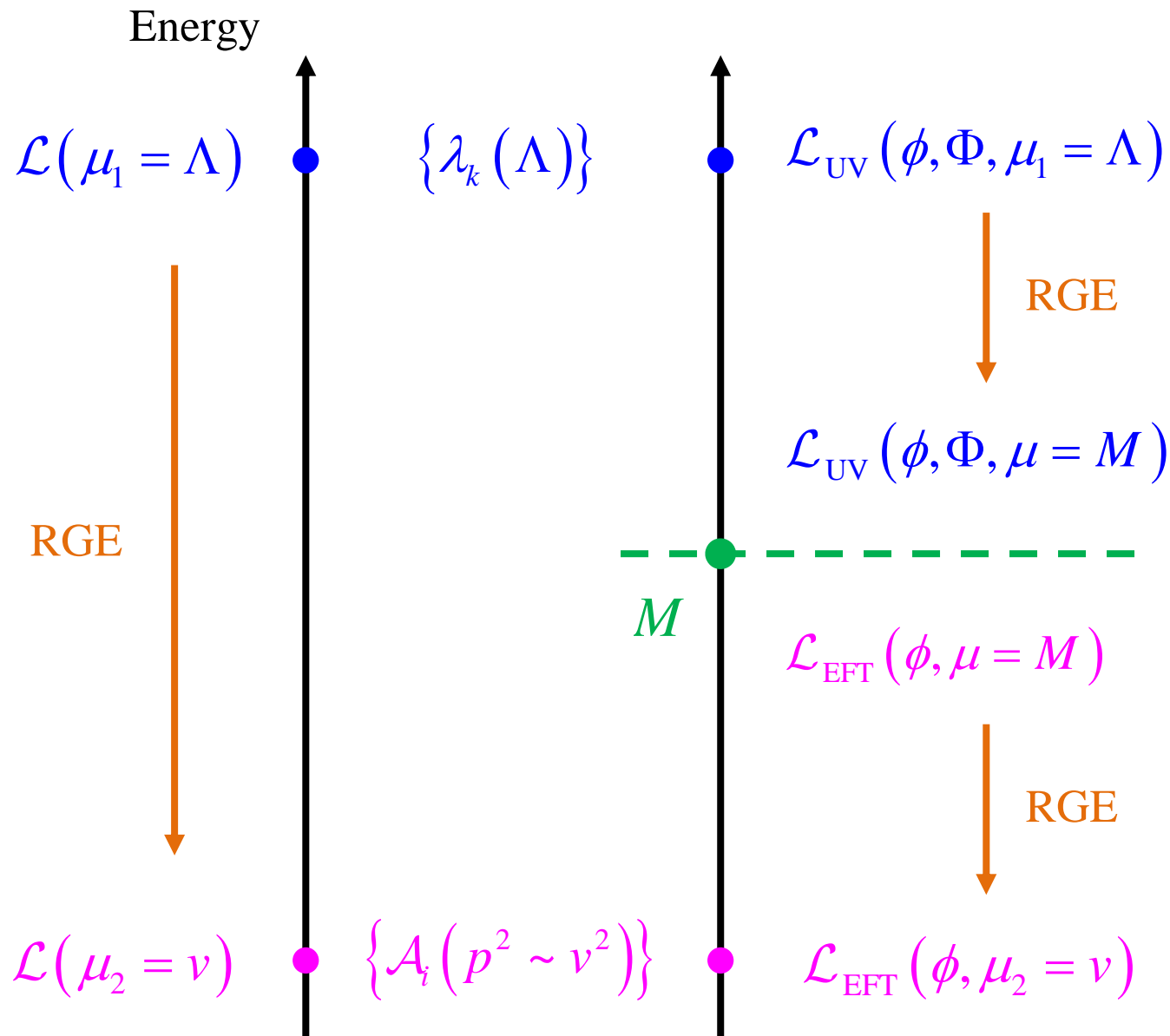
$$\{\mathcal{A}_i(p^2 \sim v^2)\}$$



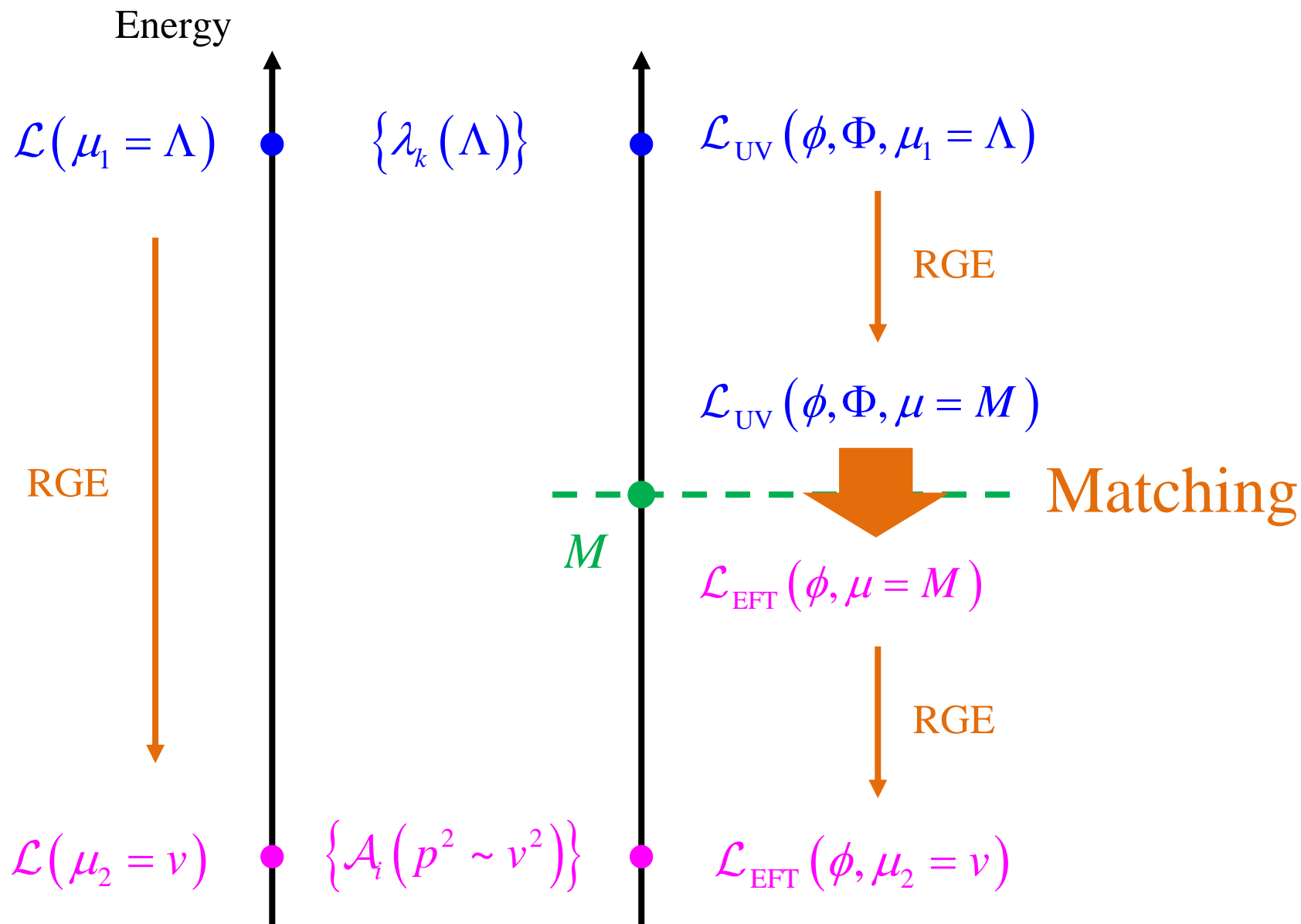






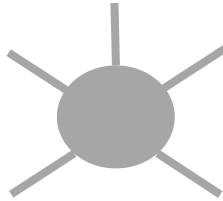






# Matching via Amplitudes

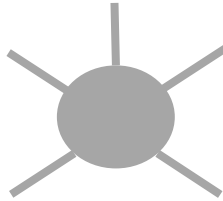
$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}}(\phi_{\text{SM}}, \Phi_{\text{BSM}})$$



$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i C_i \mathcal{O}_i(\phi_{\text{SM}})$$

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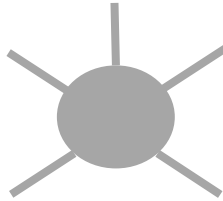
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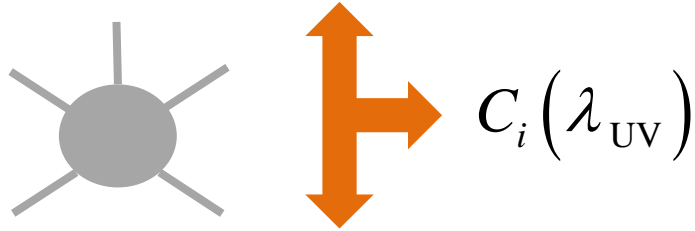
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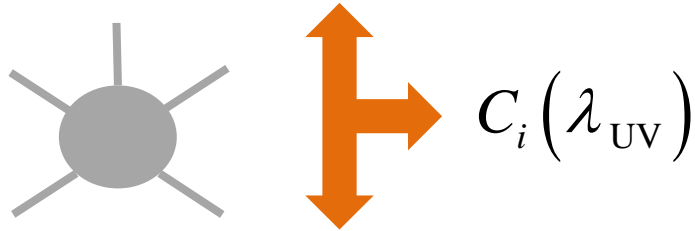
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SM + Singlet

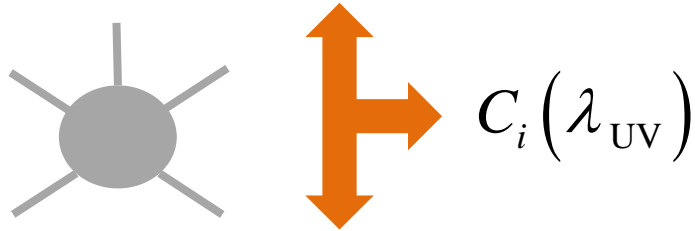
matched onto

SMEFT

- M. Jiang, N. Craig, Y.-Y. Li, and D. Sutherland, arXiv: 1811.08878
- U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini, and A. Weiler, arXiv: 2003.05936

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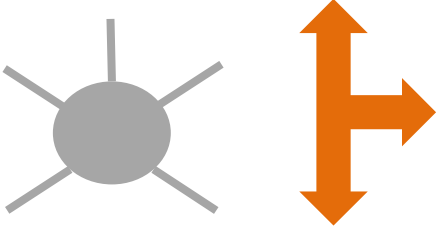
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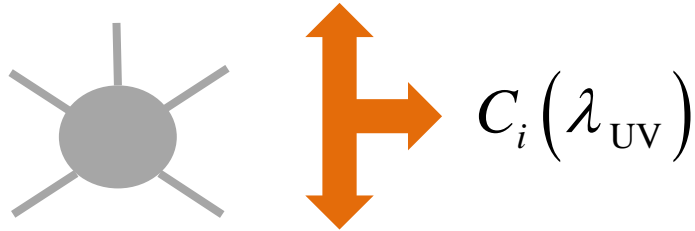
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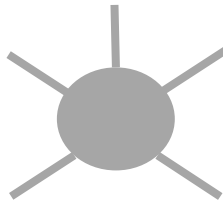
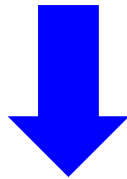
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Ideally



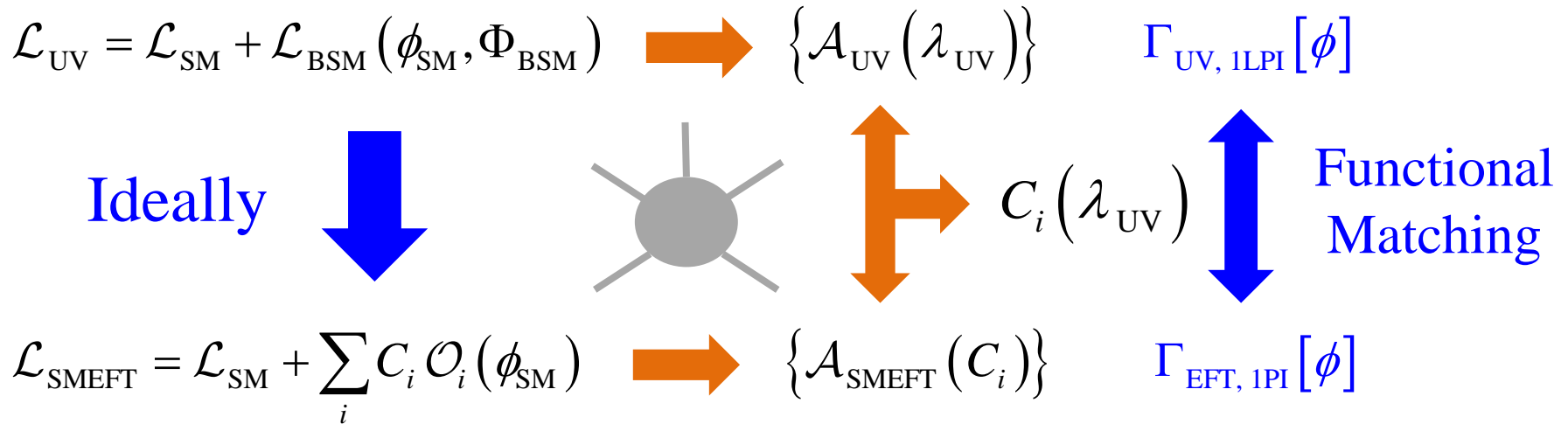
$$C_i(\lambda_{\text{UV}})$$

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$$\mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}(\phi, \Phi = \Phi_c[\phi])$$

$$\left. \frac{\delta \mathcal{S}_{\text{UV}}}{\delta \Phi} \right|_{\Phi = \Phi_c[\phi]} = 0$$

$$\int d^4x \mathcal{L}_{\text{EFT}}^{(1\text{-loop})} = \frac{i}{2} \text{STr} \log \left[ - \frac{\delta^2 \mathcal{S}_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi = \Phi_c} \right] \Big|_{\text{hard}}$$

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## Scope of validity:

- Any spin: scalars, fermions, vector bosons

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## Scope of validity:

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- Derivative interactions in UV
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How to evaluate this functional SuperTrace?

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$$\mathcal{L}_{\text{UV}}(\phi, \Phi) = \frac{1}{2} \phi (-\partial^2 - m^2) \phi - \frac{\lambda}{24} \phi^4 + \frac{1}{2} \Phi (-\partial^2 - M^2) \Phi - \frac{\kappa}{4} \phi^2 \Phi^2 - \frac{\lambda_\Phi}{24} \Phi^4$$

$$(-\partial^2 - M^2) \Phi_c - \frac{\kappa}{2} \phi^2 \Phi_c - \frac{\lambda_\Phi}{6} \Phi_c^3 = 0 \quad \Rightarrow \quad \Phi_c[\phi] = 0$$

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$$i \text{STr} \log [P^2 - M^2 - U(\phi)] = i \int d^d x \int \frac{d^d q}{(2\pi)^d} \log \left[ (\mathbf{P}_\mu - q_\mu)^2 - M^2 - U(\phi) \right], \quad \mathbf{P}_\mu \equiv iD_\mu$$



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Coleman-Weinberg potential:

$$= \int d^4 x \frac{1}{16\pi^2} \text{tr} \left\{ \frac{1}{2} (M^2 + U)^2 \left( \log \frac{\mu^2}{M^2 + U} + \frac{3}{2} \right) \right\} + \mathcal{O}(P_\mu)$$

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Two-derivative generalization: (Cohen, Craig, XL, Sutherland, arXiv: 2008.08597)

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Up to mass dim-6 (UOLEA): (Henning, XL, Murayama, arXiv: 1412.1837)

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$$= \int d^4 x \frac{1}{16\pi^2} \text{tr} \left\{ \frac{1}{2} (M^2 + U)^2 \left( \log \frac{\mu^2}{M^2 + U} + \frac{3}{2} \right) - \frac{1}{12} \frac{1}{M^2 + U} (P_\mu U)^2 \right\} + \mathcal{O}(P_\mu^3)$$

Up to mass dim-6 (UOLEA): (Henning, XL, Murayama, arXiv: 1412.1837)

$$= \int d^4 x \frac{1}{16\pi^2} \text{tr} \left\{ \begin{aligned} & \left[ \frac{1}{2} M^4 \left( \log \frac{\mu^2}{M^2} + \frac{3}{2} \right) + M^2 \left( \log \frac{\mu^2}{M^2} + 1 \right) U + \left( \log \frac{\mu^2}{M^2} \right) \left( \frac{1}{2} U^2 - \frac{1}{12} F_{\mu\nu} F^{\mu\nu} \right) \right. \\ & \left. + \frac{1}{M^2} \left[ \frac{-i}{90} F_\mu^\nu F_\nu^\rho F_\rho^\mu + \frac{1}{60} (P^\mu F_{\mu\nu})(P_\rho F^{\rho\nu}) - \frac{1}{6} U^3 - \frac{1}{12} (P_\mu U)^2 + \frac{1}{12} U F_{\mu\nu} F^{\mu\nu} \right] \right. \\ & \left. + \frac{1}{M^4} \left[ \frac{1}{24} U^4 + \frac{1}{12} U (P_\mu U)^2 + \frac{1}{120} (P^2 U)(P^2 U) \right. \right. \\ & \left. \left. + \frac{i}{60} (P^\mu U)(P^\nu U) F_{\mu\nu} - \frac{1}{60} U F_{\mu\nu} U F^{\mu\nu} - \frac{1}{40} U^2 F_{\mu\nu} F^{\mu\nu} \right] \right. \\ & \left. + \frac{1}{M^6} \left[ -\frac{1}{60} U^5 - \frac{1}{20} U^2 (P_\mu U)^2 - \frac{1}{30} U (P^\mu U) U (P_\mu U) \right] + \frac{1}{M^8} \frac{1}{120} U^6 \right\} + \mathcal{O}(\text{dim-8}) \end{aligned}$$

$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

$$\int d^4x \mathcal{L}_{\text{EFT}}^{(1\text{-loop})} = \frac{i}{2} \text{STr} \log \left[ \frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} \right]_{\text{hard}}$$

Cohen, XL, Zhang,  
arXiv: 2011.02484

$$= \frac{i}{2} \text{STr} \log(K - X) \Big|_{\text{hard}}$$

$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

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Cohen, XL, Zhang,  
arXiv: 2011.02484

$$= \frac{i}{2} \text{STr} \log(K - X) \Big|_{\text{hard}} = \frac{i}{2} \text{STr} \log K \Big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}}$$



$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

$$\int d^4x \mathcal{L}_{\text{EFT}}^{(1\text{-loop})} = \frac{i}{2} \text{STr} \log \left[ \frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} \right]_{\text{hard}}$$

Cohen, XL, Zhang,  
arXiv: 2011.02484

$$= \frac{i}{2} \text{STr} \log(K - X) \Big|_{\text{hard}} = \frac{i}{2} \text{STr} \log K \Big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}}$$

$$-\frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} = \begin{pmatrix} \partial^2 + m^2 + \frac{\lambda}{2} \phi^2 + \frac{\kappa}{2} \Phi_c^2 & \kappa \phi \Phi_c \\ \kappa \phi \Phi_c & \partial^2 + M^2 + \frac{\kappa}{2} \phi^2 + \frac{\lambda_{\Phi}}{2} \Phi_c^2 \end{pmatrix}$$

$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

$$\int d^4x \mathcal{L}_{\text{EFT}}^{(1\text{-loop})} = \frac{i}{2} \text{STr} \log \left[ \frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} \right]_{\text{hard}}$$

Cohen, XL, Zhang,  
arXiv: 2011.02484

$$= \frac{i}{2} \text{STr} \log(K - X) \Big|_{\text{hard}} = \frac{i}{2} \text{STr} \log K \Big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}}$$

Focusing on relativistic EFTs:

$$K_i = \begin{cases} P^2 - m_i^2 & (\text{spin} - 0) \\ P - m_i & (\text{spin} - 1/2) \\ -\eta^{\mu\nu} (P^2 - m_i^2) & (\text{spin} - 1) \end{cases}$$

$$-\frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} = \begin{pmatrix} \partial^2 + m^2 + \frac{\lambda}{2} \phi^2 + \frac{\kappa}{2} \Phi_c^2 & \kappa \phi \Phi_c \\ \kappa \phi \Phi_c & \partial^2 + M^2 + \frac{\kappa}{2} \phi^2 + \frac{\lambda_{\Phi}}{2} \Phi_c^2 \end{pmatrix}$$

$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

$$\int d^4x \mathcal{L}_{\text{EFT}}^{(1\text{-loop})} = \frac{i}{2} \text{STr} \log \left[ \frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} \right]_{\text{hard}}$$

Cohen, XL, Zhang,  
arXiv: 2011.02484

$$= \frac{i}{2} \text{STr} \log(K - X) \Big|_{\text{hard}} = \frac{i}{2} \text{STr} \log K \Big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}}$$

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$$-\frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} = \begin{pmatrix} \partial^2 + m^2 + \frac{\lambda}{2} \phi^2 + \frac{\kappa}{2} \Phi_c^2 & \kappa \phi \Phi_c \\ \kappa \phi \Phi_c & \partial^2 + M^2 + \frac{\kappa}{2} \phi^2 + \frac{\lambda_\Phi}{2} \Phi_c^2 \end{pmatrix}$$

$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

$$\int d^4x \mathcal{L}_{\text{EFT}}^{(1\text{-loop})} = \frac{i}{2} \text{STr} \log \left[ \frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} \right]_{\text{hard}} \quad \text{Log-type}$$

Cohen, XL, Zhang,  
arXiv: 2011.02484

$$= \frac{i}{2} \text{STr} \log(K - X) \Big|_{\text{hard}} = \frac{i}{2} \text{STr} \log K \Big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}}$$

Focusing on relativistic EFTs:

$$K_i = \begin{cases} P^2 - m_i^2 & (\text{spin} - 0) \\ P - m_i & (\text{spin} - 1/2) \\ -\eta^{\mu\nu} (P^2 - m_i^2) & (\text{spin} - 1) \end{cases}, \quad X_{ij} = U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}(P_\mu^2)$$

$$-\frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} = \begin{pmatrix} \partial^2 + m^2 + \frac{\lambda}{2} \phi^2 + \frac{\kappa}{2} \Phi_c^2 & \kappa \phi \Phi_c \\ \kappa \phi \Phi_c & \partial^2 + M^2 + \frac{\kappa}{2} \phi^2 + \frac{\lambda_\Phi}{2} \Phi_c^2 \end{pmatrix}$$

$$i\text{STr} \log(P^2 - M^2 - U) = i\text{STr} \log(P^2 - M^2) - \sum_{n=1}^{\infty} \frac{1}{n} i\text{STr} \left[ \left( \frac{1}{P^2 - M^2} U \right)^n \right]$$

$$\int d^4x \mathcal{L}_{\text{EFT}}^{(1\text{-loop})} = \frac{i}{2} \text{STr} \log \left[ \frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} \right]_{\text{hard}} \quad \text{Log-type} \quad \text{Power-type}$$

Cohen, XL, Zhang,  
arXiv: 2011.02484

$$= \frac{i}{2} \text{STr} \log(K - X) \Big|_{\text{hard}} = \frac{i}{2} \text{STr} \log K \Big|_{\text{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}}$$

Focusing on relativistic EFTs:

$$K_i = \begin{cases} P^2 - m_i^2 & (\text{spin} - 0) \\ P - m_i & (\text{spin} - 1/2) \\ -\eta^{\mu\nu} (P^2 - m_i^2) & (\text{spin} - 1) \end{cases}, \quad X_{ij} = U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}(P_\mu^2)$$

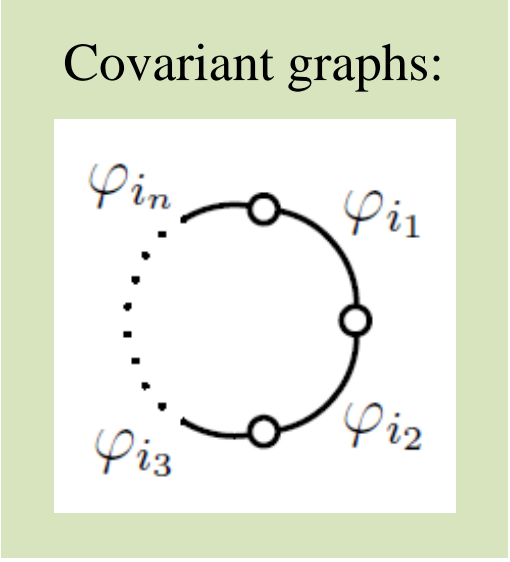
$$-\frac{\delta^2 S_{\text{UV}}}{\delta(\phi, \Phi)^2} \Big|_{\Phi=\Phi_c} = \begin{pmatrix} \partial^2 + m^2 + \frac{\lambda}{2} \phi^2 + \frac{\kappa}{2} \Phi_c^2 & \kappa \phi \Phi_c \\ \kappa \phi \Phi_c & \partial^2 + M^2 + \frac{\kappa}{2} \phi^2 + \frac{\lambda_\Phi}{2} \Phi_c^2 \end{pmatrix}$$

Power-type:

$$-i\text{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Big|_{\text{hard}} = -i\text{STr}\left(\frac{1}{K_{i_1}}X_{i_1i_2} \frac{1}{K_{i_2}}X_{i_2i_3} \cdots \frac{1}{K_{i_n}}X_{i_ni_1}\right)\Big|_{\text{hard}}$$

Power-type:

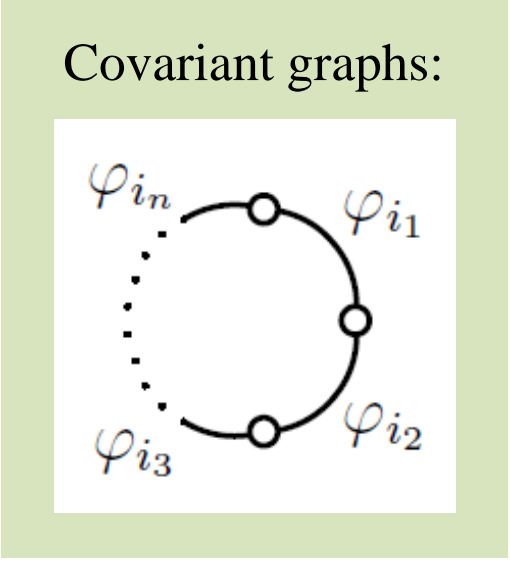
$$-i\text{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\text{hard}} = -i\text{STr}\left(\frac{1}{K_{i_1}}X_{i_1i_2} \frac{1}{K_{i_2}}X_{i_2i_3} \cdots \frac{1}{K_{i_n}}X_{i_ni_1}\right)\Bigg|_{\text{hard}}$$



# Power-type:

$$-i\text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}} = -i\text{STr} \left( \frac{1}{K_{i_1}} X_{i_1 i_2} \frac{1}{K_{i_2}} X_{i_2 i_3} \cdots \frac{1}{K_{i_n}} X_{i_n i_1} \right) \Big|_{\text{hard}}$$

$$\frac{1}{K_i} \sim \begin{cases} \frac{1}{P^2 - m_i^2} \equiv \Delta_i \\ \frac{1}{P - m_i} \equiv \Lambda_i \end{cases}$$





# Power-type:

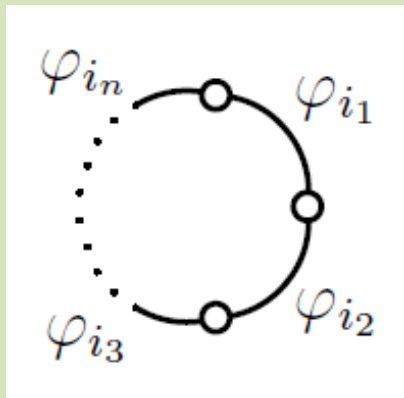
$$-i\text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}} = -i\text{STr} \left( \frac{1}{K_{i_1}} X_{i_1 i_2} \frac{1}{K_{i_2}} X_{i_2 i_3} \cdots \frac{1}{K_{i_n}} X_{i_n i_1} \right) \Big|_{\text{hard}}$$

$$\frac{1}{K_i} \sim \begin{cases} \frac{1}{P^2 - m_i^2} \equiv \Delta_i \\ \frac{1}{P - m_i} \equiv \Lambda_i \end{cases},$$

$$X_{ij} = U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}(P_\mu^2)$$

$$X_{ij} \sim (P_{\mu_1} \cdots P_{\mu_n}) U_k (P_{\nu_1} \cdots P_{\nu_m})$$

## Covariant graphs:



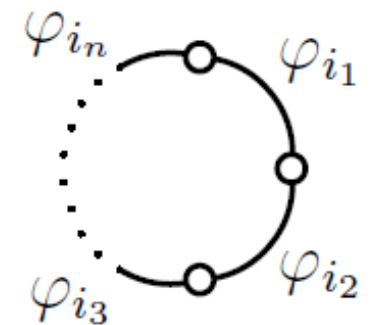
$$-i\text{STr}[f]\Big|_{\text{hard}}, \quad f = \left[ \cdots \left( P_{\mu_1} \cdots P_{\mu_n} \right) (\Delta_i \text{ or } \Lambda_i) \left( P_{\nu_1} \cdots P_{\nu_m} \right) U_k \cdots \right]$$

Power-type:

$$-i\text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}} = -i\text{STr} \left( \frac{1}{K_{i_1}} X_{i_1 i_2} \frac{1}{K_{i_2}} X_{i_2 i_3} \cdots \frac{1}{K_{i_n}} X_{i_n i_1} \right) \Big|_{\text{hard}}$$

$$\frac{1}{K_i} \sim \begin{cases} \frac{1}{P^2 - m_i^2} \equiv \Delta_i \\ \frac{1}{P - m_i} \equiv \Lambda_i \end{cases}, \quad \begin{aligned} X_{ij} &= U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}(P_\mu^2) \\ X_{ij} &\sim \left( P_{\mu_1} \cdots P_{\mu_n} \right) U_k \left( P_{\nu_1} \cdots P_{\nu_m} \right) \end{aligned}$$

Covariant graphs:



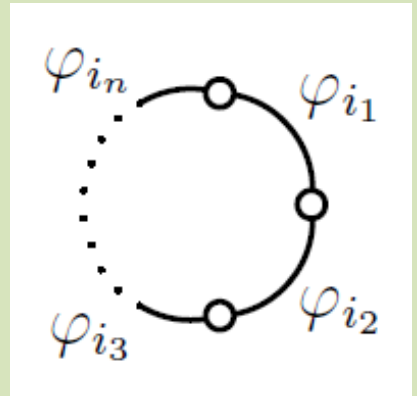
$$-i\text{STr}[f]\Big|_{\text{hard}}, \quad f = \left[ \cdots \left( P_{\mu_1} \cdots P_{\mu_n} \right) (\Delta_i \text{ or } \Lambda_i) \left( P_{\nu_1} \cdots P_{\nu_m} \right) U_k \cdots \right]$$

Power-type:

$$-i\text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}} = -i\text{STr} \left( \frac{1}{K_{i_1}} X_{i_1 i_2} \frac{1}{K_{i_2}} X_{i_2 i_3} \cdots \frac{1}{K_{i_n}} X_{i_n i_1} \right) \Big|_{\text{hard}}$$

$$\frac{1}{K_i} \sim \begin{cases} \frac{1}{P^2 - m_i^2} \equiv \Delta_i \\ \frac{1}{P - m_i} \equiv \Lambda_i \end{cases}, \quad \begin{aligned} X_{ij} &= U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}(P_\mu^2) \\ X_{ij} &\sim \left( P_{\mu_1} \cdots P_{\mu_n} \right) U_k \left( P_{\nu_1} \cdots P_{\nu_m} \right) \end{aligned}$$

Covariant graphs:



Log-type: can be converted into power-type

$$\frac{\partial}{\partial M^2} \left[ i\text{STr} \log(P^2 - M^2) \right] = -i\text{STr} \left( \frac{1}{P^2 - M^2} \right), \quad \frac{\partial}{\partial M} \left[ i\text{STr} \log(P - M) \right] = -i\text{STr} \left( \frac{1}{P - M} \right)$$

$$-i\text{STr}[f]\Big|_{\text{hard}}, \quad f = \left[ \cdots \left( P_{\mu_1} \cdots P_{\mu_n} \right) (\Delta_i \text{ or } \Lambda_i) \left( P_{\nu_1} \cdots P_{\nu_m} \right) U_k \cdots \right]$$

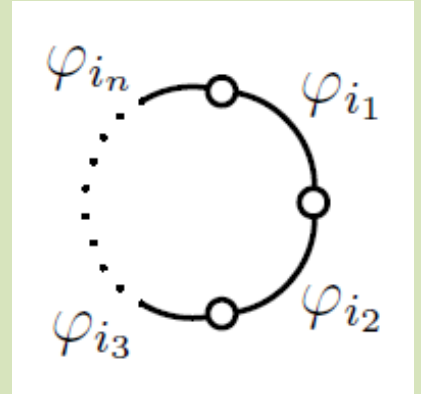
Power-type:

Any functional supertrace can be evaluated with CDE

$$-i\text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}} = -i\text{STr} \left( \frac{1}{K_{i_1}} X_{i_1 i_2} \frac{1}{K_{i_2}} X_{i_2 i_3} \cdots \frac{1}{K_{i_n}} X_{i_n i_1} \right) \Big|_{\text{hard}}$$

$$\frac{1}{K_i} \sim \begin{cases} \frac{1}{P^2 - m_i^2} \equiv \Delta_i \\ \frac{1}{P - m_i} \equiv \Lambda_i \end{cases}, \quad \begin{aligned} X_{ij} &= U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}(P_\mu^2) \\ X_{ij} &\sim (P_{\mu_1} \cdots P_{\mu_n}) U_k (P_{\nu_1} \cdots P_{\nu_m}) \end{aligned}$$

Covariant graphs:



Log-type: can be converted into power-type

$$\frac{\partial}{\partial M^2} \left[ i\text{STr} \log(P^2 - M^2) \right] = -i\text{STr} \left( \frac{1}{P^2 - M^2} \right), \quad \frac{\partial}{\partial M} \left[ i\text{STr} \log(P - M) \right] = -i\text{STr} \left( \frac{1}{P - M} \right)$$

# The scope of STrEAM:

Cohen, XL, Zhang, arXiv: 2012.07851

$$-i\text{STr}[f] \Big|_{\text{hard}}, \quad f = \left[ \cdots \left( P_{\mu_1} \cdots P_{\mu_n} \right) (\Delta_i \text{ or } \Lambda_i) \left( P_{\nu_1} \cdots P_{\nu_m} \right) U_k \cdots \right]$$

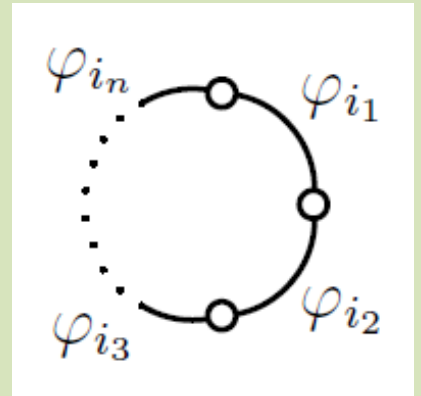
Power-type:

Any functional supertrace can be evaluated with CDE

$$-i\text{STr} \left[ \left( \frac{1}{K} X \right)^n \right] \Big|_{\text{hard}} = -i\text{STr} \left( \frac{1}{K_{i_1}} X_{i_1 i_2} \frac{1}{K_{i_2}} X_{i_2 i_3} \cdots \frac{1}{K_{i_n}} X_{i_n i_1} \right) \Big|_{\text{hard}}$$

$$\frac{1}{K_i} \sim \begin{cases} \frac{1}{P^2 - m_i^2} \equiv \Delta_i \\ \frac{1}{P - m_i} \equiv \Lambda_i \end{cases}, \quad \begin{aligned} X_{ij} &= U_{ij} + P_\mu Z_{ij}^\mu + \bar{Z}_{ij}^\mu P_\mu + \mathcal{O}(P_\mu^2) \\ X_{ij} &\sim (P_{\mu_1} \cdots P_{\mu_n}) U_k (P_{\nu_1} \cdots P_{\nu_m}) \end{aligned}$$

Covariant graphs:



Log-type: can be converted into power-type

$$\frac{\partial}{\partial M^2} \left[ i\text{STr} \log(P^2 - M^2) \right] = -i\text{STr} \left( \frac{1}{P^2 - M^2} \right), \quad \frac{\partial}{\partial M} \left[ i\text{STr} \log(P - M) \right] = -i\text{STr} \left( \frac{1}{P - M} \right)$$

STrEAM.m and STrEAM\_examples.nb  
<https://github.com/EFTMatching/STrEAM>

$$-i\text{STr}\left[\frac{1}{P^2 - m_1^2}U_1^{[2]}\right]\Bigg|_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left[m_1^2\left(1 - \log\frac{m_1^2}{\mu^2}\right)U_1 + \frac{1}{12m_1^2}F_{\mu\nu}F^{\mu\nu}U_1 + \mathcal{O}(\text{dim-8})\right]$$

STrEAM.m and STrEAM\_examples.nb  
<https://github.com/EFTMatching/STrEAM>

$$-i\text{STr}\left[\frac{1}{P^2 - m_1^2} U_1^{[2]}\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left[m_1^2 \left(1 - \log \frac{m_1^2}{\mu^2}\right) U_1 + \frac{1}{12m_1^2} F_{\mu\nu} F^{\mu\nu} U_1 + \mathcal{O}(\text{dim-8})\right]$$

STrEAM.m and STrEAM\_examples.nb  
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$$-i\text{STr}\left[\frac{1}{P^2 - m_1^2} U_1^{[2]}\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left[m_1^2 \left(1 - \log \frac{m_1^2}{\mu^2}\right) U_1 + \frac{1}{12m_1^2} F_{\mu\nu} F^{\mu\nu} U_1 + \mathcal{O}(\text{dim-8})\right]$$

```
In[2]:= SuperTrace[6, {Δ1, U1}, Udimlist → {2}, display → True]
```

$$-i\text{STr}\left[\frac{1}{p^2 - m_1^2} U_1\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left\{ \right.$$

$$- \left(-1 + \text{Log}\left[\frac{m_1^2}{\mu^2}\right]\right) m_1^2 \quad (U_1) \quad (\text{dim-2})$$

$$\frac{1}{12m_1^2} \quad (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_2}) (U_1) \quad (\text{dim-6})$$

$$\left. \right\}$$

```
Out[2]= {{{{ -1 + Log[m12/μ2] } m12 }}, {{U1}}, 2}, {{{{ 1/12 m12 }}, {{Fμ1, μ2}}, {Fμ1, μ2}}, {U1}}, 6}}
```



STrEAM.m and STrEAM\_examples.nb  
<https://github.com/EFTMatching/STrEAM>

$$-i \text{STr} \left[ \frac{1}{P^2 - m_1^2} U_1 \right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr} \left[ m_1^2 \left( 1 - \log \frac{m_1^2}{\mu^2} \right) U_1 + \frac{1}{12m_1^2} F_{\mu\nu} F^{\mu\nu} U_1 + \mathcal{O}(\text{dim-8}) \right]$$

In[2]:= SuperTrace[6, {Δ<sub>1</sub>, U<sub>1</sub>}, Udimlist → {2}, display → True]

$$-i \text{STr} \left[ \frac{1}{p^2 - m_1^2} U_1 \right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr} \{$$

$$- \left( -1 + \text{Log} \left[ \frac{m_1^2}{\mu^2} \right] \right) m_1^2 \quad (U_1) \quad (\text{dim-2})$$

$$\frac{1}{12m_1^2} \quad (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_2}) (U_1) \quad (\text{dim-6})$$

$$\}$$

Out[2]= {{{{ - ( -1 + Log [ m<sub>1</sub><sup>2</sup> / μ<sup>2</sup> ] ) m<sub>1</sub><sup>2</sup> }}, {{U<sub>1</sub>}}, 2}, {{{{ 1 / (12 m<sub>1</sub><sup>2</sup>) }}, {{F<sub>μ<sub>1</sub>, μ<sub>2</sub>}}, {F<sub>μ<sub>1</sub>, μ<sub>2</sub>}}, {U<sub>1</sub>}}, 6}}</sub></sub>

STrEAM.m and STrEAM\_examples.nb  
<https://github.com/EFTMatching/STrEAM>

$$-i\text{STr}\left[\frac{1}{P^2 - m_1^2} U_1^{[2]}\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left[m_1^2 \left(1 - \log \frac{m_1^2}{\mu^2}\right) U_1 + \frac{1}{12m_1^2} F_{\mu\nu} F^{\mu\nu} U_1 + \mathcal{O}(\text{dim-8})\right]$$

```
In[2]:= SuperTrace[6, {Δ1, U1}, Udimlist → {2}, display → True]
```

$$-i\text{STr}\left[\frac{1}{p^2 - m_1^2} U_1\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left\{ \begin{aligned} & - \left(-1 + \text{Log}\left[\frac{m_1^2}{\mu^2}\right]\right) m_1^2 && (U_1) && (\text{dim-2}) \\ & \frac{1}{12m_1^2} && (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_2}) (U_1) && (\text{dim-6}) \end{aligned} \right\}$$

```
Out[2]= {{{{ - (-1 + Log[m12/μ2]) m12 }}, {{U1}}, 2}, {{{{ 1/(12 m12) }}, {{Fμ1, μ2}}, {Fμ1, μ2}}, {U1}}, 6}}
```

STrEAM.m and STrEAM\_examples.nb  
<https://github.com/EFTMatching/STrEAM>

$$-i\text{STr}\left[\frac{1}{P^2 - m_1^2} U_1\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left[m_1^2 \left(1 - \log \frac{m_1^2}{\mu^2}\right) U_1 + \frac{1}{12m_1^2} F_{\mu\nu} F^{\mu\nu} U_1 + \mathcal{O}(\text{dim-8})\right]$$

```
In[2]:= SuperTrace[6, {Δ1, U1}, Udimlist → {2}, display → True]
```

$$-i\text{STr}\left[\frac{1}{p^2 - m_1^2} U_1\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left\{ \begin{aligned} & - \left(-1 + \text{Log}\left[\frac{m_1^2}{\mu^2}\right]\right) m_1^2 && (U_1) && (\text{dim-2}) \\ & \frac{1}{12m_1^2} && (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_2}) (U_1) && (\text{dim-6}) \end{aligned} \right\}$$

```
Out[2]= {{{{-1 + Log[m12/μ2]} m12}}, {{U1}}, 2}, {{{1/(12 m12)}}, {{Fμ1, μ2}}, {Fμ1, μ2}}, {U1}}, 6}}
```

STrEAM.m and STrEAM\_examples.nb  
<https://github.com/EFTMatching/STrEAM>

$$-i\text{STr}\left[\frac{1}{P^2 - m_1^2} U_1^{[2]}\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left[m_1^2 \left(1 - \log \frac{m_1^2}{\mu^2}\right) U_1 + \frac{1}{12m_1^2} F_{\mu\nu} F^{\mu\nu} U_1 + \mathcal{O}(\text{dim-8})\right]$$

dimension  
truncation

In[2]:= SuperTrace[6, {Δ<sub>1</sub>, U<sub>1</sub>}, Udimlist → {2}, display → True]

$$-i\text{STr}\left[\frac{1}{p^2 - m_1^2} U_1\right]_{\text{hard}} = \int d^4x \frac{1}{16\pi^2} \text{tr}\left\{ \begin{aligned} & - \left(-1 + \text{Log}\left[\frac{m_1^2}{\mu^2}\right]\right) m_1^2 && (U_1) && (\text{dim-2}) \\ & \frac{1}{12m_1^2} && (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_2}) (U_1) && (\text{dim-6}) \end{aligned} \right\}$$

Out[2]= {{{{ -1 + Log[m<sub>1</sub><sup>2</sup>/μ<sup>2</sup>] m<sub>1</sub><sup>2</sup> }}, {{U<sub>1</sub>}}, 2}, {{{{ 1/(12 m<sub>1</sub><sup>2</sup>) }}, {{F<sub>μ<sub>1</sub>, μ<sub>2</sub>}}, {F<sub>μ<sub>1</sub>, μ<sub>2</sub>}}, {U<sub>1</sub>}}, 6}}</sub></sub>

## massless (covariant) propagator

$$-i \text{STr} \left[ \frac{1}{P^2 - m_1^2} U_1^{[1]} \frac{1}{P^2 - m_2^2} P_\mu Z^{\mu[1]} \left( \frac{1}{P^2} \right) U_3^{[2]} \frac{1}{P^2 - m_2^2} U_4^{[1]} \right] \Big|_{\text{hard}}$$

In[7]:= SuperTrace[6, {Δ<sub>1</sub>, U<sub>1</sub>, Δ<sub>2</sub>, P<sub>ν</sub>, Z<sub>ν</sub>, Δ<sub>θ</sub>, U<sub>3</sub>, Δ<sub>2</sub>, U<sub>4</sub>}, Udimlist → {1, 1, 2, 1}, display → True];

$$-i \text{STr} \left[ \frac{1}{P^2 - m_1^2} U_1 \frac{1}{P^2 - m_2^2} P_\nu Z_\nu \frac{1}{P^2} U_3 \frac{1}{P^2 - m_2^2} U_4 \right] \Big|_{\text{hard}} = \int d^4x \frac{1}{16 \pi^2} \text{tr} \{$$

$$\frac{3-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{4 m_1^4} (U_1) (Z_{\mu_1}) (P_{\mu_1} U_3) (U_4) \quad (\text{dim-6})$$

$$\frac{3-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{2 m_1^4} (U_1) (P_{\mu_1} Z_{\mu_1}) (U_3) (U_4) \quad (\text{dim-6})$$

$$\frac{5-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{4 m_1^4} (P_{\mu_1} U_1) (Z_{\mu_1}) (U_3) (U_4) \quad (\text{dim-6})$$

}

derivative interaction      massless (covariant) propagator

$$-i \text{STr} \left[ \frac{1}{P^2 - m_1^2} U_1^{[1]} \frac{1}{P^2 - m_2^2} P_\mu Z^{\mu[1]} \frac{1}{P^2} U_3^{[2]} \frac{1}{P^2 - m_2^2} U_4^{[1]} \right] \Big|_{\text{hard}}$$

```
In[7]:= SuperTrace[6, {Δ1, U1, Δ2, Pν, Zν, Δθ, U3, Δ2, U4}, Udimlist → {1, 1, 2, 1}, display → True];
```

$$-i \text{STr} \left[ \frac{1}{P^2 - m_1^2} U_1 \frac{1}{P^2 - m_2^2} P_\nu Z_\nu \frac{1}{P^2} U_3 \frac{1}{P^2 - m_2^2} U_4 \right] \Big|_{\text{hard}} = \int d^4x \frac{1}{16 \pi^2} \text{tr} \{$$

$$\frac{3-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{4 m_1^4} (U_1) (Z_{\mu_1}) (P_{\mu_1} U_3) (U_4) \quad (\text{dim-6})$$

$$\frac{3-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{2 m_1^4} (U_1) (P_{\mu_1} Z_{\mu_1}) (U_3) (U_4) \quad (\text{dim-6})$$

$$\frac{5-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{4 m_1^4} (P_{\mu_1} U_1) (Z_{\mu_1}) (U_3) (U_4) \quad (\text{dim-6})$$

}

## fermionic (covariant) propagator

$$-i \text{STr} \left[ \frac{1}{P^2 - m_1^2} U_1^{[1]} \frac{1}{P^2 - m_2^2} U_2^{[3/2]} \left( \frac{1}{P} \right) U_3^{[3/2]} \frac{1}{P^2 - m_2^2} U_4^{[1]} \right] \Big|_{\text{hard}}$$

```
In[8]:= SuperTrace [6, {Δ1, U1, Δ2, U2, Δ0, U3, Δ2, U4}, Udimlist → {1, 3/2, 3/2, 1}, display → True];
```

$$-i \text{STr} \left[ \frac{1}{p^2 - m_1^2} U_1 \frac{1}{p^2 - m_2^2} U_2 \frac{1}{\text{Pslash}} U_3 \frac{1}{p^2 - m_2^2} U_4 \right] \Big|_{\text{hard}} = \int d^4x \frac{1}{16 \pi^2} \text{tr} \{$$

$$\frac{3-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{4 m_1^4} \quad (U_1) (U_2) (P_{\mu_1} \gamma_{\mu_1} U_3) (U_4) \quad (\text{dim-6})$$

$$\frac{1}{2 m_1^4} \quad (U_1) (P_{\mu_1} U_2) (\gamma_{\mu_1} U_3) (U_4) \quad (\text{dim-6})$$

$$\frac{5-2 \text{Log} \left[ \frac{m_1^2}{\mu^2} \right]}{4 m_1^4} \quad (P_{\mu_1} U_1) (U_2) (\gamma_{\mu_1} U_3) (U_4) \quad (\text{dim-6})$$

}

## Log-type

$$\frac{\partial}{\partial m_1^2} \left[ i \text{STr} \log(P^2 - m_1^2) \right] = -i \text{STr} \left( \frac{1}{P^2 - m_1^2} \right)$$

$$\frac{\partial}{\partial m_1} \left[ i \text{STr} \log(P - m_1) \right] = -i \text{STr} \left( \frac{1}{P - m_1} \right)$$

In[3]:= SuperTrace[6, {Δ<sub>1</sub>}, display → True];

$$-i \text{STr} \left[ \frac{1}{P^2 - m_1^2} \right] |_{\text{hard}} = \int d^4x \frac{1}{16 \pi^2} \text{tr} \left\{ \begin{aligned} & \frac{1}{12 m_1^2} (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_2}) && (\text{dim-4}) \\ & \frac{i}{90 m_1^4} (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_3}) (F_{\mu_2, \mu_3}) && (\text{dim-6}) \\ & \frac{1}{60 m_1^4} (P_{\mu_1} P_{\mu_2} F_{\mu_2, \mu_3}) (F_{\mu_1, \mu_3}) && (\text{dim-6}) \\ & \end{aligned} \right\}$$

In[4]:= SuperTrace[6, {Δ<sub>1</sub>}, NoγinU → True, display → True];

$$-i \text{STr} \left[ \frac{1}{P\text{slash} - m_1} \right] |_{\text{hard}} = \int d^4x \frac{1}{16 \pi^2} \text{tr} \left\{ \begin{aligned} & \frac{1}{6 m_1} (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_2}) && (\text{dim-4}) \\ & -\frac{i}{90 m_1^3} (F_{\mu_1, \mu_2}) (F_{\mu_1, \mu_3}) (F_{\mu_2, \mu_3}) && (\text{dim-6}) \\ & \frac{1}{15 m_1^3} (P_{\mu_1} P_{\mu_2} F_{\mu_2, \mu_3}) (F_{\mu_1, \mu_3}) && (\text{dim-6}) \\ & \end{aligned} \right\}$$



# Summary

- EFT matching is systematically solved by functional methods at tree and one-loop level
- A new way of organizing functional supertrace evaluation
  - step-by-step prescription in Sec. 5 of arXiv: 2011.02484
- A Mathematica package that automates supertrace evaluation
  - Manual in arXiv: 2012.07851
  - STrEAM.m <https://github.com/EFTMatching/STrEAM>