An On-shell Formulation of Chiral Perturbation Theory

Ian Low

Argonne/Northwestern

January 14, 2021

- Chiral Perturbation Theory (ChPT) is a low-energy effective theory of QCD and an integral component of our understanding of many nuclear processes.

- It describes interactions of mesons and baryons at the energy scale ~ 1 GeV or below and, in my view, is the most elegant example employing ”modern” techniques of effective field theories.

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- Modern EFT’s:
  - In most cases, symmetry consideration alone is sufficient to capture the long wavelength dynamics of a physical system.

    For ChPT the symmetry is $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry.
  - Short wavelength fluctuations are encoded in uncalculable “Wilson coefficients.”
  - A power counting scheme must be supplied to organized the relative importance of different effective operators.

    In ChPT the power counting is the “derivative expansion.”

- Ironically, “modern” means more than half-a-century old:

PHYSICAL REVIEW VOLUME 166, NUMBER 5 25 FEBRUARY 1968

Nonlinear Realizations of Chiral Symmetry*

STEVEN WEINBERG†
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 25 September 1967)

We explore possible realizations of chiral symmetry, based on isotopic multiplets of fields whose transformation rules involve only isotopic-spin matrices and the pion field. The transformation rules are unique, up to possible redefinitions of the pion field. Chiral-invariant Lagrangians can be constructed by forming isotopic-spin-conserving functions of a covariant pion derivative, plus other fields and their covariant derivatives. The resulting models are essentially equivalent to those that have been derived by treating

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- ChPT is an EFT about Nambu-Goldstone bosons, which has an even longer history:

PHYSICAL REVIEW VOLUME 117, NUMBER 3 FEBRUARY 1, 1960

Quasi-Particles and Gauge Invariance in the Theory of Superconductivity*

YOICHIRO NAMBU
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received July 23, 1959)

Field Theories with «Superconductor» Solutions.

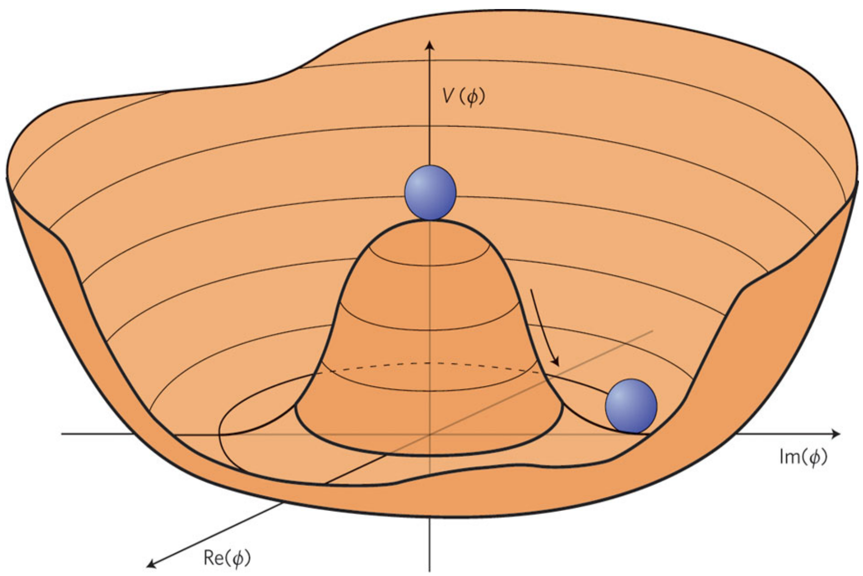
J. GOLDSTONE
CERN - Geneva

(ricevuto l'8 Settembre 1960)

The typical "textbook" example of Nambu-Goldstone bosons starts with the following potential energy:

$$\phi = \phi_1 + i\phi_2$$

$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$



- There is an infinite number of ground state, labelled by the polar angle :

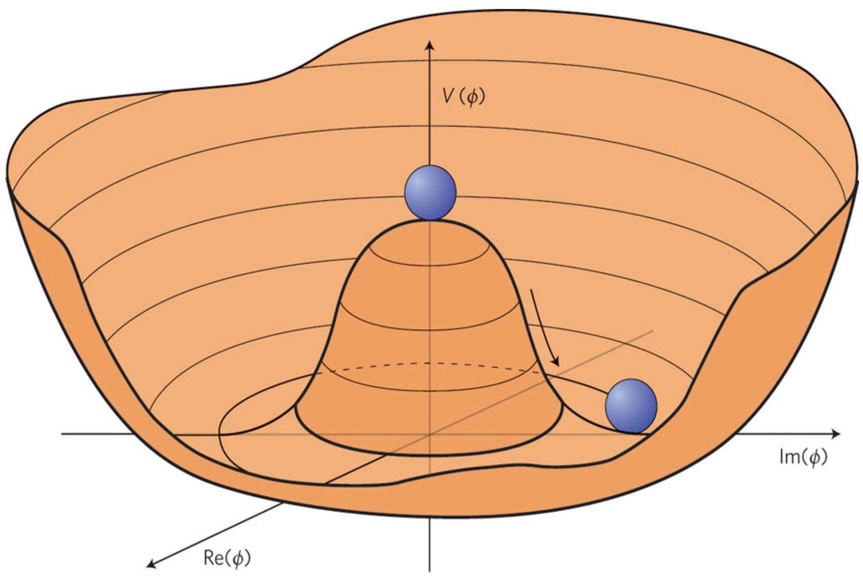
$$|\mathrm{VAC}\rangle = \{|0\rangle_\alpha;\ \alpha \in [0, 2\pi)\}$$

- But once an "alpha" is chosen, the rotational invariance is hidden.

To see the NGB explicitly, let's go to "polar coordinate:"

$$\phi(x) = \frac{\rho(x)}{\sqrt{2}}\ e^{i\frac{1}{\sqrt{2}v}\pi(x)}$$

In this parameterization, the ground state is $\langle\rho\rangle_\alpha = v\ , \quad \langle\pi\rangle_\alpha = \alpha$



Expanding with respect to the ground state:

$$\rho \to \rho + \langle\rho\rangle_\alpha$$
$$\pi \to \pi + \langle\pi\rangle_\alpha$$

Under rotation by theta-angle,

$$\langle\pi\rangle_\alpha \to \langle\pi\rangle_{\alpha+\theta} = \alpha + \theta$$

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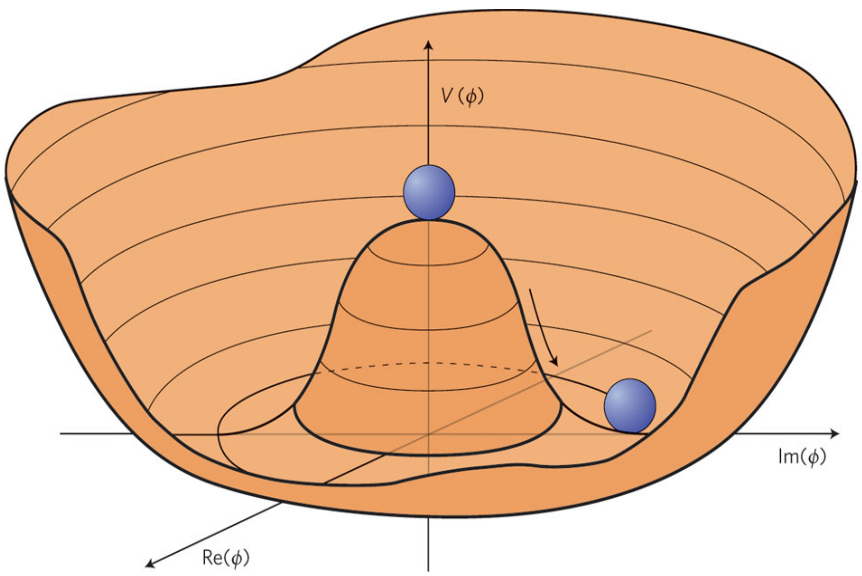

This is the equivalent to shifting the Pi-mode by a constant:

$$\pi \to \pi + \theta$$

Rotational symmetry implies the dynamics must be independent of the constant shift!

This is a "shift symmetry."

More generally, there is a well-defined procedure to write down NGB effective actions for arbitrary symmetry breaking pattern.

(CCWZ: Coleman, Callan, Wess and Zumino, Phys. Rev. 1969.)

One picks a nonlinearly realized group G, and a subgroup H of G that is linearly realized.

We say G is the broken group and H the unbroken group:

$$\xi = e^{i\pi^a X^a}$$

$$g\xi = \xi' U \ , \quad U \in H \ , \quad \xi' = e^{i\pi'^a X^a}$$

The “pions” are the coordinates on the coset manifold G/H.

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The "pions" are the coordinates on the coset manifold G/H.

When $$g = e^{i\varepsilon^a X^a} \quad , \quad \pi'^a = \pi^a + \varepsilon^a + \cdots$$

This is the shift symmetry!

Rarely discussed…

CCWZ looked for objects that have "simple" transformation properties under the action of G.

These are contained in the Cartan-Maurer one-form:

$$\xi^\dagger \partial_\mu \xi \; = \; i\mathcal{D}^a_\mu X^a + i\mathcal{E}^i_\mu T^i \equiv i\mathcal{D}_\mu + i\mathcal{E}_\mu$$

They are the "Goldstone covariant derivative" and the "associated gauge field",

$$\mathcal{D}_\mu \to U\mathcal{D}_\mu U^{-1} \; , \quad \mathcal{E}_\mu \to U\mathcal{E}_\mu U^{-1} - (\partial_\mu U)U^{-1}$$

upon which the complete effective Lagrangian can be built (apart from the topological terms)

$$\mathcal{L}_{eff} = \frac{f^2}{2} \mathrm{Tr} \mathcal{D}_\mu \mathcal{D}^\mu + \cdots$$

- CCWZ has been applied to ChPT to very high orders. Some heroic efforts went into constructing the effective Lagrangian up to O(p^8), which is completed only recently:

The Mesonic Chiral Lagrangian of Order p^6 *

Johan Bijnens

Dept. of Theor. Phys. 2, Lund University,
Sölvegatan 14A, S-22362 Lund, Sweden

Gilberto Colangelo

Inst. Theor. Physik, Univ. Zürich, Winterthurerstr. 190,
CH-8057 Zürich-Irchel, Switzerland

Gerhard Ecker

Inst. Theor. Phys., Univ. Wien, Boltzmanng. 5,
A-1090 Wien, Austria

ABSTRACT: We construct the effective chiral Lagrangian for chiral perturbation theory in the mesonic even-intrinsic-parity sector at order p^6. The Lagrangian contains 112 in principle measurable + 3 contact terms for the general case of n light flavours, 90+4 for three and 53+4 for two flavours. The equivalence between equations of motion and field redefinitions to remove spurious terms in the Lagrangians is shown to all orders in the chiral expansion. We also discuss and implement other methods for reducing the number of terms to a minimal set.

- CCWZ has been applied to ChPT to very high orders. Some heroic efforts went into constructing the effective Lagrangian up to O(p^8), which is completed only recently:

arXiv:1810.06834v2 [hep-ph] 24 Jan 2019

The order p^8 mesonic chiral Lagrangian

Johan Bijnens, Nils Hermansson-Truedsson and Si Wang

Department of Astronomy and Theoretical Physics,
Lund University, Sölvegatan 14A, SE 223-62 Lund, Sweden

Abstract

We derive the chiral Lagrangian at next-to-next-to-next-to-leading order (NNNLO) for a general number N_f of light quark flavours as well as for $N_f = 2, 3$. We enumerate the contact terms separately. We also discuss the cases where some of the external fields are not included. An example of a choice of Lagrangian is given in the supplementary material.

- CCWZ has been applied to ChPT to very high orders. Some heroic efforts went into constructing the effective Lagrangian up to O(p^8), which is completed only recently:

The order p^8 mesonic chiral Lagrangian

	N_f		$N_f = 3$		$N_f = 2$	
	Total	Contact	Total	Contact	Total	Contact
p^2	2	0	2	0	2	0
p^4	13	2	12	2	10	3
p^6	115	3	94	4	56	4
p^8	1862	22	1254	21	475	23

Table 3: Number of monomials in the minimal basis for the case with all external fields included. Also listed is how many of them are contact terms. Our results agree with the known ones for p^2, p^4, p^6.

contact terms separately. We also discuss the cases where some of the external fields are not included. An example of a choice of Lagrangian is given in the supplementary material.

- In fact, ChPT is not just about NGB's –

  We need to incorporate the interaction with fermions (nucleons) and spin-1 bosons (photons). But again, CCWZ gave well-defined prescriptions on how to do it.

- But one thing is clear –

  Since the EFT is based on "symmetry", choosing a different symmetry gives rise to a different EFT.

Conventional wisdom:

NGB interactions depends both on the full symmetry G in the UV and the unbroken symmetry H in the IR.

In spite of the tremendous success of CCWZ in a wide range of topics, there's always something odd about it.

Its best summarized as the following question:

Nambu-Goldstone bosons are long-range degrees of freedom interpolating different vacua, why would its interactions know anything about the "broken group" G in the UV?

In other words, NGB's should be all about the IR physics, not the UV.

Indeed, NGB's have a very peculiar IR property that has been known for (again) more than half-a-century…

The property is about the IR behavior of NGB's:

$$\lim_{p^\mu \to 0} \, {}_\alpha\langle f | i + \pi(p) \rangle_\alpha = 0$$

ie, the S-matrix elements involving a zero-momentum NGB vanish!

This is called the Adler's zero condition since the 1960's and independent of the symmetry breaking pattern G/H.

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This is called the Adler's zero condition since the 1960's and independent of the symmetry breaking pattern G/H.

In fact, one can show that the Adler's zero condition, follows directly from the shift symmetry acting on the NGB:

$$\pi \to \pi + \epsilon + \cdots$$

That is, Adler's zero is the Ward identity of the shift symmetry.

Recall the shift symmetry is an indication of the existence of other degenerate ground states!

It turns out that the Adler's zero condition allows for an entirely on-shell formulation of ChPT.

It all started from an obscure paper by Susskind and Frye from (again) half-a-century ago:

PHYSICAL REVIEW D VOLUME 1, NUMBER 6 15 MARCH 1970

Algebraic Aspects of Pionic Duality Diagrams

LEONARD SUSSKIND* AND GRAHAM FRYE

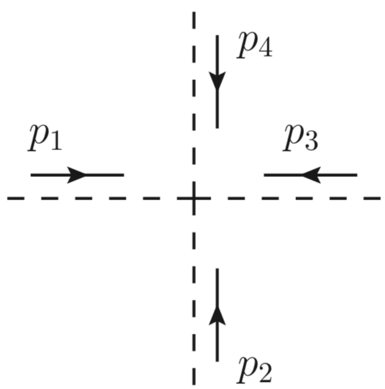
Belfer Graduate School of Science, Yeshiva University, New York, New York 10033

(Received 9 May 1969)

Certain algebraic aspects are abstracted from the duality principle and are incorporated in a simple model of pion n-point functions. An algorithm for constructing the n-point function in the tree-graph approximation is based on the duality assumption and the Adler condition which states that the amplitudes vanishes if any pion four-momentum vanishes, all others remaining on shell. The resulting amplitudes satisfy the constraints of current algebra and partial conservation of axial-vector current for $n=4$, 6, and 8, and (we conjecture) for all n. In addition, duality specifies a definite form for chiral symmetry breaking.

This is what they did, schematically.

Use the Adler's zero condition to fix the 4-pt amplitude of pions,



$$p_i^2 = 0 \; ; \sum_{i=1}^{4} p_i = 0$$

$$M_4(p_1, p_2, p_3, p_4) = c\frac{p_2 \cdot p_4}{f^2} = c\frac{p_1 \cdot p_3}{f^2}$$

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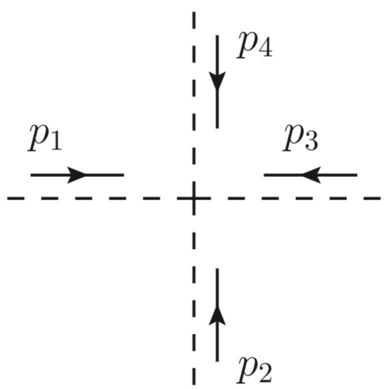

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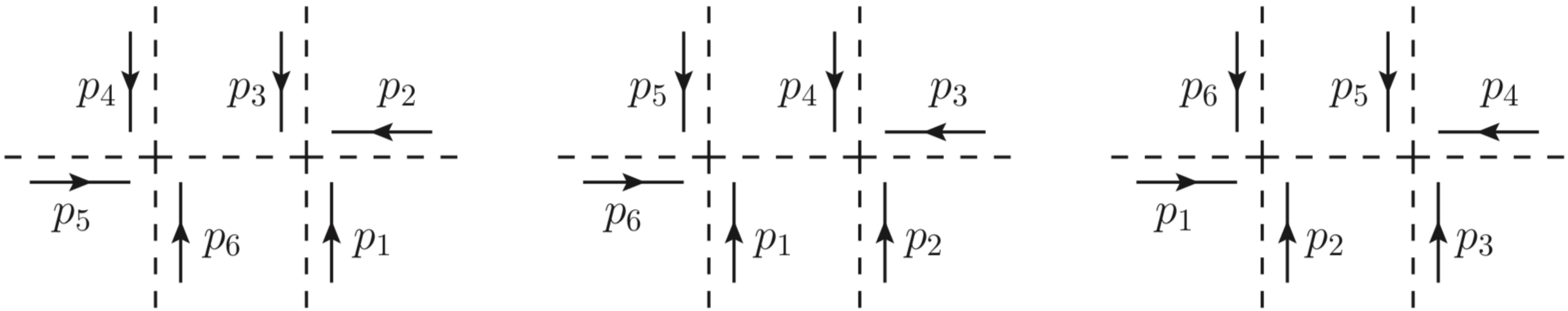
Some comments:

- They worked directly with "flavor-ordered" amplitudes.
- There is no constant term in the amplitude.
- "f" is a dimensionful parameter, while "c" is an arbitrary number, which could be absorbed into the normalization of "f".

Once we have the 4-pt amplitude, we can build up the 6-pt amplitude from the 4-pt amplitude:



$$\frac{1}{f^2}\left(\frac{s_{13}s_{46}}{P_{123}^2}+\frac{s_{24}s_{15}}{P_{234}^2}+\frac{s_{35}s_{26}}{P_{345}^2}\right)$$

$$s_{ij} = (p_i + p_j)^2$$
$$P_{ijk}^2 = (p_i + p_j + p_k)^2$$

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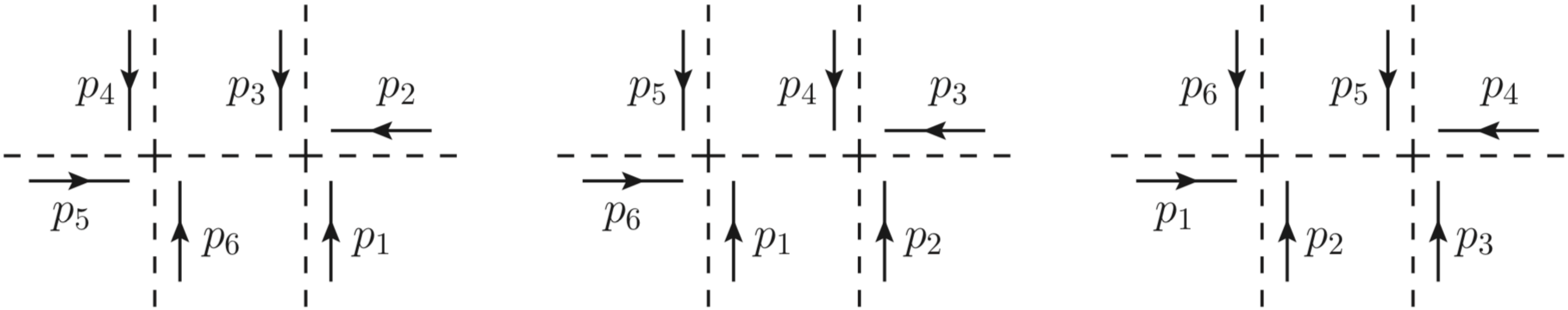

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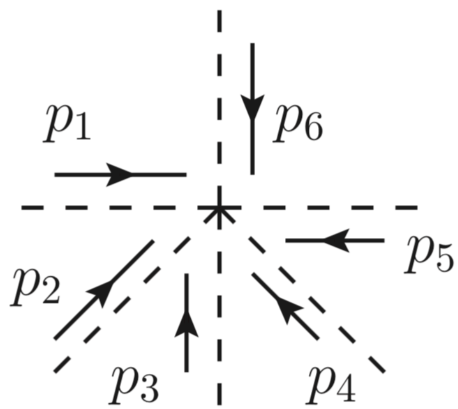
$$s_{ij}=(p_i+p_j)^2$$
$$P_{ijk}^2=(p_i+p_j+p_k)^2$$

This expression doesn't satisfy the Adler's zero condition!

The resolution is to introduce an additional contribution, the "contact interaction,"



It turns out imposing the Adler's condition also uniquely fixes this 6-pt contact interaction:

$$M_6 = \frac{1}{f^2}\left(\frac{s_{13}s_{46}}{P_{123}^2} + \frac{s_{24}s_{15}}{P_{234}^2} + \frac{s_{35}s_{26}}{P_{345}^2}\right) - \frac{1}{f^2}P_{135}^2$$

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This is quite striking because they only invoked

- A notion of “flavor ordering,” which arises due to some discrete quantum numbers, given by the “unbroken group” H.
- The vanishing “soft limit” in the scattering amplitudes.

In other words, only IR data are used.

They made no reference to the group “G” in the UV.
How general is this approach?

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How general is this approach?

The answer is affirmative, but arrived only much later…

- On-shell Soft Bootstrap

What Susskind and Frye did was the precursor to modern S-matrix program, which seeks to define a QFT by on-shell data:

For NGB's, all interaction vertices are determined by recursively requiring Adler's zero on tree-level amplitudes.

In particular, the progress is based on the "soft recursion relation" proposed by Cheung, Kampf, Novotny and Trnka in 1412.4095 and 1509.03309.

An all-leg shift in external momenta:

$$p_i \to \hat{p}_i = (1 - a_i z)\, p_i \qquad\qquad \sum_{i=1}^{n} a_i\, p_i^{\mu} = 0$$

Taking z → $1/a_i$ is equivalent to taking the soft limit of p_i. Then

$$\oint \frac{dz}{z} \frac{\hat{M}_n(z)}{F_n(z)} = 0 \qquad\qquad F_n(z) \equiv \prod_{i=1}^{n} (1 - a_i z)$$

- The integrand vanishes like $1/z^{n-1}$ and the residue at infinity vanishes.
- There is no pole at $z = 1/a_i$ because of Adler's zero condition.
- The only poles are at z = 0 and when the internal propagators go on-shell (ie factorization channel).

Internal propagators go on-shell at

$$\hat{P}_I^2(z_I^{\pm}) = 0 \qquad\qquad \hat{P}_I(z) = \sum_{i \in I} p_i - z \left(\sum_{i \in I} a_i p_i \right)$$

Cauchy's theorem then gives

$$M_n = \hat{M}_n(0) = -\sum_{I,\pm} \frac{1}{P_I^2} \frac{\hat{M}_L^{(I)}(z_I^{\pm}) \hat{M}_R^{(I)}(z_I^{\pm})}{F_n(z_I^{\pm})(1 - z_I^{\pm}/z_I^{\mp})}$$

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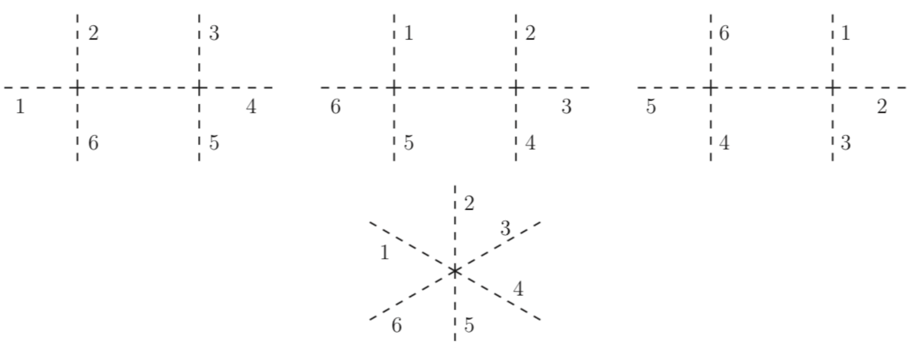

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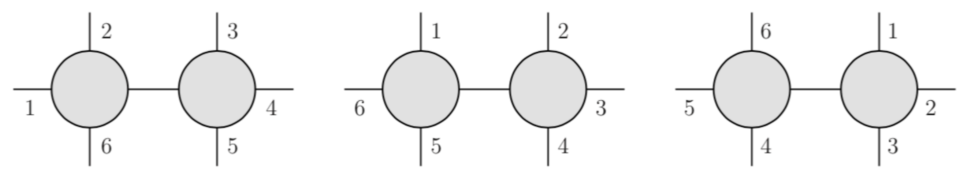

LHS includes contact term.

RHS includes only factorization channel!

A comment on the all-leg-shift:

$$p_i \rightarrow \hat{p}_i = (1 - a_i z)\, p_i \qquad\qquad \sum_{i=1}^{n} a_i\, p_i^{\mu} = 0$$

- Nontrivial solutions for a_i don't always exist.

- For D=4, the number of non-trivial solutions is (n-5), n=number of external momenta.

- The general solution is only defined "projectively" and has a "shift symmetry":

$$\{a_i\} = \sum_{r=1}^{n-5} A^{(r)} \{a_i^{(r)}\} + B$$

$A^{(r)}$ and *B* are arbitrary constants!

When soft-bootstrapping the amplitudes,

$$M_n = \hat{M}_n(0) = -\sum_{I,\pm} \frac{1}{P_I^2} \frac{\hat{M}_L^{(I)}(z_I^\pm)\hat{M}_R^{(I)}(z_I^\pm)}{F_n(z_I^\pm)(1 - z_I^\pm/z_I^\mp)}$$

it is a non-trivial check that the outcome is independent of $A^{(r)}$ and B .

We will see that this doesn't automatically happen. Thus we define a consistent EFT in soft bootstrap when

The amplitude M_n obtained from the soft recursion relation is independent of the arbitrary constants $A^{(r)}$ and B for all n.

Soft-recursion relation allows one to generalize Susskind and Frye, to all orders in the multiplicity “n”.

But the discussion so far has been confined to the leading two-derivative operators.

Soft-recursion relation allows one to generalize Susskind and Frye, to all orders in the multiplicity "n".

But the discussion so far has been confined to the leading two-derivative operators.

However, in a general EFT of NGB's

- There exist higher derivative operators which become increasingly important as the energy becomes higher.
- Each higher derivative operator comes with an uncalculable Wilson coefficient.

How does on-shell soft bootstrap incorporate higher derivative operators?

Introducing "Soft blocks"

- A soft block $\mathcal{S}^{(k)}(p_1, \cdots, p_n)$ is a contact interaction carrying n scalars and k derivatives that satisfies the Adler's zero condition when all external legs are on-shell.

For four-derivative or less, one can show the soft blocks exist only for 4-pt and 5-pt contact terms. (IL and Z. Yin: 1904.12859)

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- Soft blocks are "seeds" for soft recursion relations.
- Soft blocks are in 1-to-1 correspondence with the number of independent operators at a particular order in the derivative expansion.
- Practically speaking, they are the lowest order contact interactions from a particular higher-derivative operator.

4-pt Soft blocks at O(p[4]):

$$\text{Single-trace:}\quad \mathcal{S}_1^{(4)}(1,2,3,4) = \frac{c_1}{\Lambda^2 f^2}\; s_{13}^2\;, \qquad \mathcal{S}_2^{(4)}(1,2,3,4) = \frac{c_2}{\Lambda^2 f^2}\; s_{12}s_{23}\;,$$

$$\text{Double-trace:}\quad \mathcal{S}_1^{(4)}(1,2|3,4) = \frac{d_1}{\Lambda^2 f^2}\; s_{12}^2\;, \qquad \mathcal{S}_2^{(4)}(1,2|3,4) = \frac{d_1}{\Lambda^2 f^2}\; s_{13}s_{23}\;,$$

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All four soft blocks appear in ChPT:

$$\mathcal{L}^{(2)} = \frac{f^2}{2}\mathrm{tr}\left(d_\mu d^\mu\right)$$

$$\mathcal{L}^{(4)} = \sum_{i=1}^{4} L_{4,i}\, O_i$$

$$\begin{aligned} O_1 &= \left[\mathrm{tr}(d_\mu d^\mu)\right]^2\;, \\ O_2 &= \left[\mathrm{tr}(d_\mu d_\nu)\right]^2\;, \\ O_3 &= \mathrm{tr}([d_\mu, d_\nu]^2)\;, \\ O_4 &= \mathrm{tr}(\{d_\mu, d_\nu\}^2)\;, \end{aligned}$$

$$\boxed{c_1 = L_{4,3} + 3L_{4,4}, \quad c_2 = 2(L_{4,3} - L_{4,4}), \quad d_1 = 2L_{4,1} + L_{4,2}, \quad d_2 = 2L_{4,2}.}$$

- As is well-known, construction of independent operators in EFT is notoriously difficult, due to the complicated operator relations such as integration-by-parts, equation-of-motion and etc.

  For example, in ChPT the LO E.O.M. is

$$\nabla_\mu d^\mu = 0 \qquad\qquad \nabla_\mu d_\nu \equiv \partial_\mu d_\nu + i[E_\mu, d_\nu]$$

- Making things worse, there are additional relations imposed by the "symmetry" of the coset in ChPT.

$$\nabla_{[\mu} d_{\nu]} = 0 \ , \qquad E_{\mu\nu} \ \equiv \ -i[\nabla_\mu, \nabla_\nu] = -i[d_\mu, d_\nu]$$

- It turns out that these complicated operator relations manifest themselves trivially in soft blocks.

  a) Integration-by-parts = total momentum conservation
  b) Equation-of-motion = on-shell conditions for external momenta
  c) Symmetry relations are automatically incorporated in soft-bootstrap.

In the end, soft blocks are an efficient way to count the number of independent operators at each order in the derivative expansion!

Moreover, all tree amplitudes can be generated recursively once the soft blocks are obtained.

- We enumerated all soft blocks in ChPT up to $O(p^{10})$, which correspond to pure mesonic operators in ChPT, ie turning off spin-1/2 and spin-1 fields.
- For simplicity we work in general spacetime dimension D and general flavor N_f :

	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$	$\mathcal{O}(p^8)$	$\mathcal{O}(p^{10})$
No. of Soft Blocks	4	19	135	1451

TABLE VI: A summary table for the number of soft blocks, including all trace structures.

Dai, IL, Mehen and Mohapatra: 2009.01819

- The outcome agree with existing literature up to $O(p^8)$ and makes a prediction for $O(p^{10})$.

Concluding Remarks:

- Interactions of NGB’s can be determined (almost) entirely from the IR – using the Adler’s zero condition as the defining property.
- The nonlinearity in the NGB interactions arises entirely from IR physics. What’s being “broken” in the UV is irrelevant, for the most part.
- The complete effective Lagrangian for ChPT can be formulated in an entirely on-shell perspective, when external sources and fermions are neglected.
- What happens when we put back the photon and the nucleon??