

Underdense, Passive Plasma Lens for Focusing Electron Beams

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Introduction

- Plasma lenses are capable of strong, axisymmetric focusing.
- Active plasma lenses use an applied current in a plasma capillary to provide focusing.
- Passive plasma lenses use fields present in the wakes generated by a driving beam
 - Overdense and underdense regimes.
- In this talk:
 - Quick overview of different plasma lens regimes
 - Closer look at underdense passive plasma lens focusing
 - Applications and current experimental plans for underdense passive plasma lenses



Active Plasma Lenses

- APL's use a gas-filled capillary with electrodes on the ends. A current is driven through the plasma and this produces focusing magnetic field.

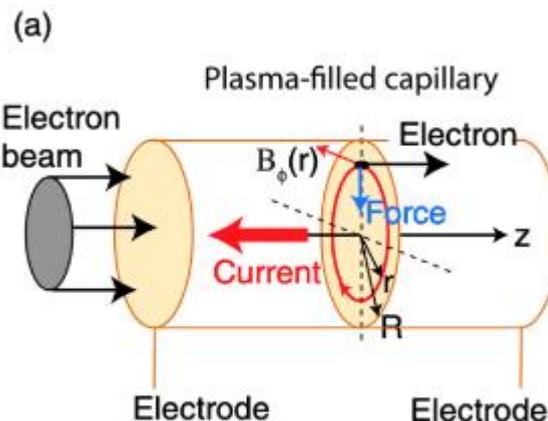
$$g_{APL} = 200 \frac{I[\text{kA}]}{(R[\text{mm}])^2} \text{T/m}$$

Using parameters from the APL in
S. Steinke et. al., Nature **530**, 190-193 (2016):

$$I = 650 \text{A}$$

$$R = 250 \mu\text{m}$$

$$g_{LBNL} = 2080 \text{ T/m}$$



- Even a weak APL is comparable to the strongest PMQ's.
- **Focusing doesn't depend on beam shape.**
- **Works for electrons and positrons.**

J. van Tilborg et. al., PRL **115**, 184802 (2015)



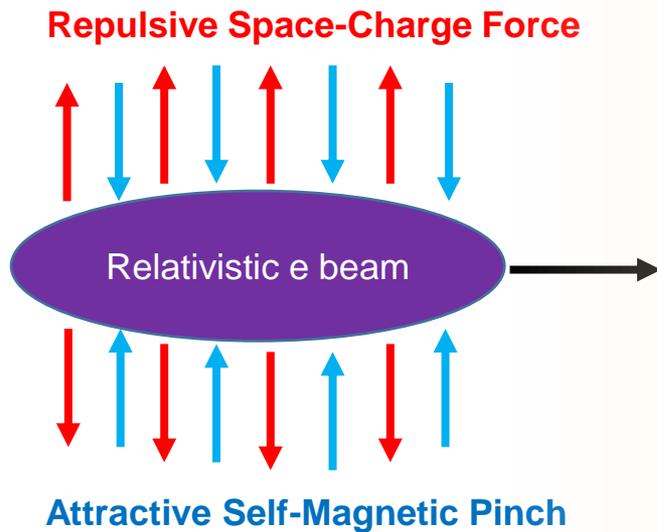
Active Plasma Lenses

- APL's work well in experiment
 - S. Steinke et. al., Nature **530**, 190-193 (2016)
- Preserve emittance growth
 - C. A. Lindstrøm et. al., PRL **121**, 193801 (2018)
- Allow for compact diagnostics
 - S. K. Barber et. al., Appl. Phys. Lett. **116**, 234108 (2020)
- However, requires small beam density compared to plasma.
 - Otherwise a wake is driven in the plasma and this leads to aberrations.
 - Would require a large transverse beam size.
 - C. Lindstrøm, E. Adli, [arXiv:1802.02750] (2018)



Passive Plasma Lens Focusing

- In vacuum, forces on an ideal beam cancel
- Passive plasma effects depend on relative density



$$F_{r,vac} \sim 1/\gamma^2$$

Plasma

Overdense $n_p \gg n_b$

or

Underdense $n_p \lesssim n_b$



Overdense Regime

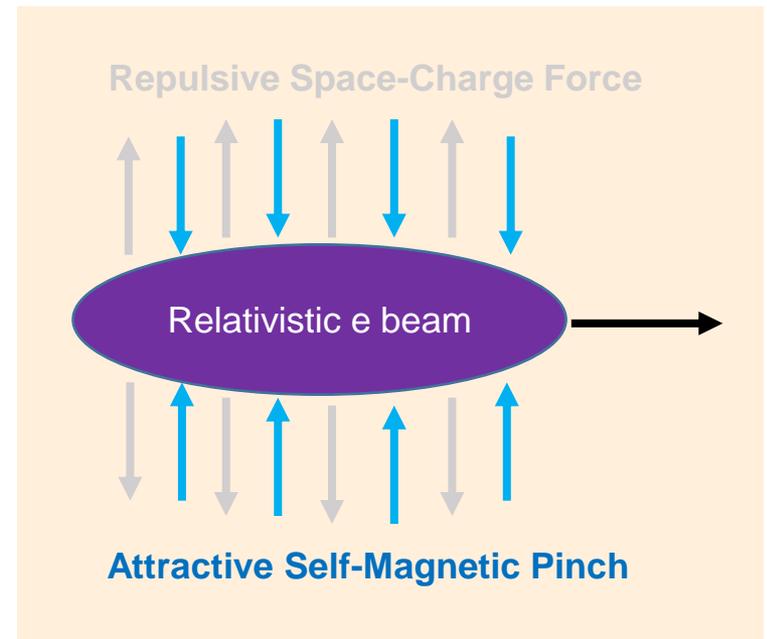
- The beam is a perturbation to the high plasma density.
- Assuming adiabatic motion, plasma e's cancel beam's charge but not its current.
- Ampere's Equation:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 J_{enc} = \mu_0 \pi r^2 v n_b e$$

$$v/c \approx 1$$

$$B_\theta = 2\pi n_b e r$$

$$n_{plasma} \gg n_{beam}$$



$$F_r / r = 2\pi n_b e^2$$



Overdense Plasma Lenses

- The overdense, passive regime was used in early plasma lens experiments due to this regime being more straightforward to access.
- These days, active plasma lenses do a better job in the overdense regime, and the underdense plasma lenses (next slide) do a better job in the passive regime.



Underdense Passive Plasma Lens

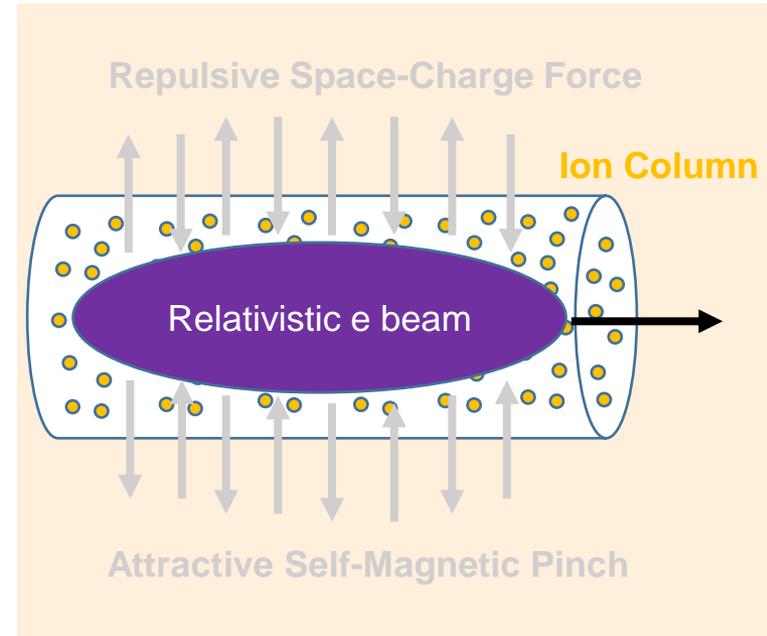
- High density beam blows out plasma electrons, creating an ion column.
- Ion column provides focusing force on beam
- Gaussian cylinder:

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \rho_{enc}$$

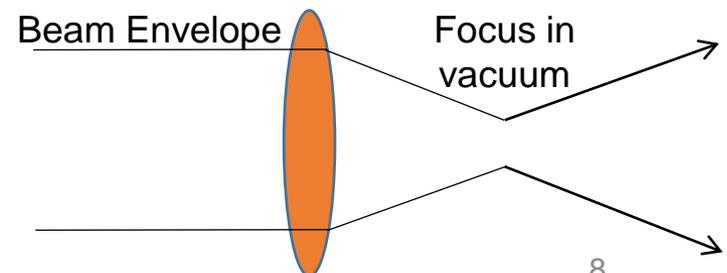
$$E_r = 2\pi n_p e r$$

- Linear focusing force which depends on uniform background plasma density

$$n_{plasma} \lesssim n_{beam}$$



$$F_r / r = 2\pi n_p e^2$$





Beam Dynamics of Thin Plasma Lens

- Consider an x, x' transfer matrix for thick lens

$$M_{thick} = \begin{bmatrix} \cos \sqrt{K}L & \frac{1}{\sqrt{K}} \sin \sqrt{K}L \\ -\sqrt{K} \sin \sqrt{K}L & \cos \sqrt{K}L \end{bmatrix}$$

- Take thin limit:

$$M_{thin} = \begin{bmatrix} 1 & 0 \\ -KL & 1 \end{bmatrix}$$

- A focal length can be derived and interpreted using focusing strength of plasma on e^- beam

$$f = \frac{1}{KL} \simeq \frac{\gamma_L mc^2}{2\pi e^2 n_p} \frac{1}{L} \quad \text{where} \quad K = \frac{2\pi e^2 n_p}{\gamma_L mc^2}$$



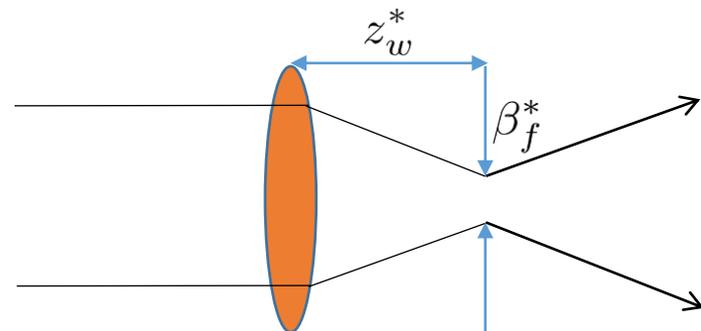
Beam Dynamics of Thin Plasma Lens

- Include the lens and a drift into a transfer matrix for the collective CS parameters:

$$\begin{bmatrix} \beta(z) \\ \alpha(z) \\ \gamma(z) \end{bmatrix} = \begin{bmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & M_{11}M_{22} + M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{bmatrix}$$

- Then we can find the beam size at focus and the location of the focus:

$$\beta_f^* = \frac{1}{\beta_0 K^2 L^2 + 2\alpha_0 K L + \gamma_0}$$
$$z_w^* = \frac{\beta_0 K L + \alpha_0}{\beta_0 K^2 L^2 + 2\alpha_0 K L + \gamma_0}$$



- A similar expression can be found for thick lenses but is slightly less analytically tractable.

C. Doss et. al., PRAB **22**, 111001 (2019)



Applications/Motivations for Plasma Lenses

- Underdense passive plasma lenses are very promising for focusing high-density electron beams.
- Many applications of plasma lenses to be explored at SLAC's FACET-II:
 - Matching beams into Plasma Wakefield Accelerators to preserve emittance.
 - Possibility as a strong final-focus optic and/or tool to study hard synchrotron radiation and the Oide limit.
 - Assist with many other planned experiments at FACET-II and beyond.
 - Who doesn't want smaller beam spot sizes!?



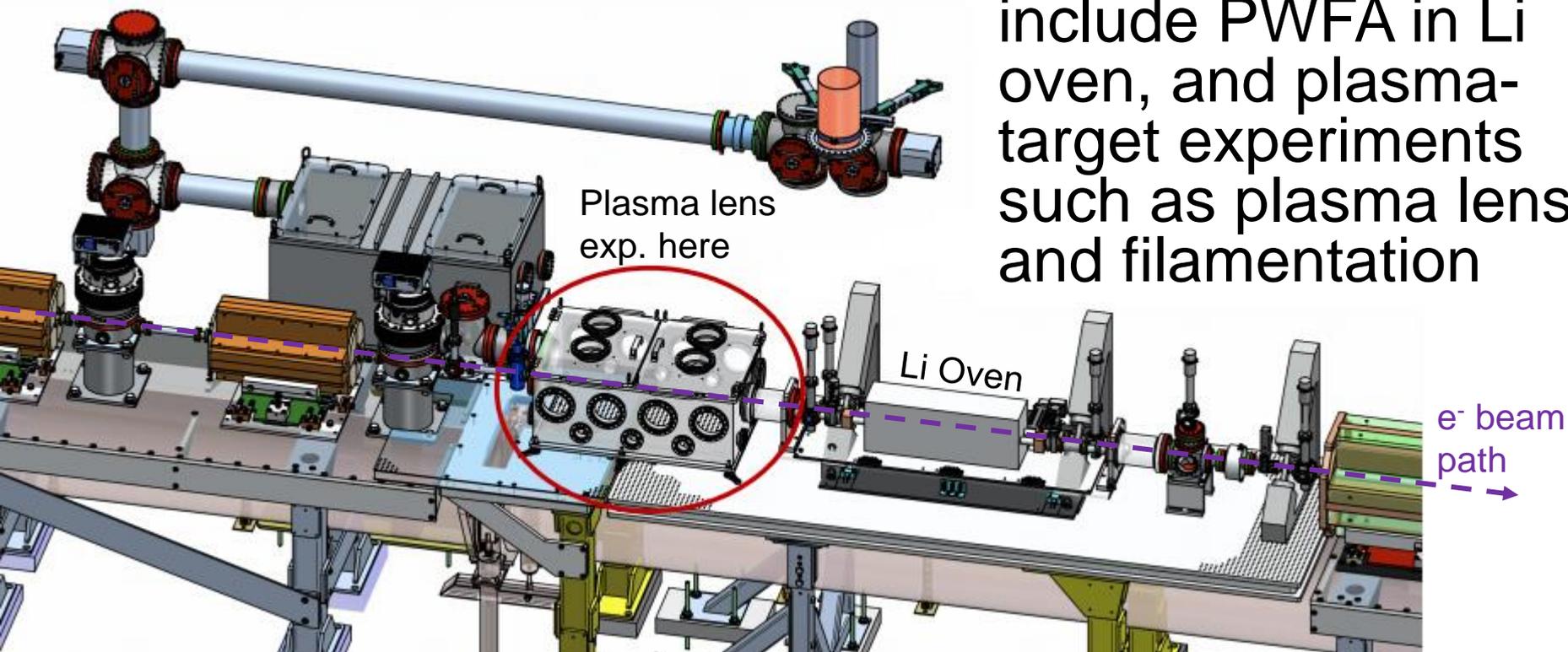
Electron Beam Parameter	Operational Ranges
Final Energy [GeV]	4.0 – 13.5
Charge per Pulse [nC]	0.7 – 5
Repetition Rate [Hz]	1 - 30
Norm. Emittance [$\mu\text{m-rad}$]	3 – 6
Spot Size at Experiments [μm]	5 – 20
Min. Bunch Length rms [μm]	0.7 – 20
Max Peak Current [kA]	10 - 200

- 1 km in length
- Photo-injector generates single- or two-bunch beams
- Imaging spectrometer and other diagnostics in experimental area



- Experimental area features compressed, ≤ 0.6 J, 800 nm laser; compressed probe beams; gas jets; and a bypass-able Lithium vapor oven.

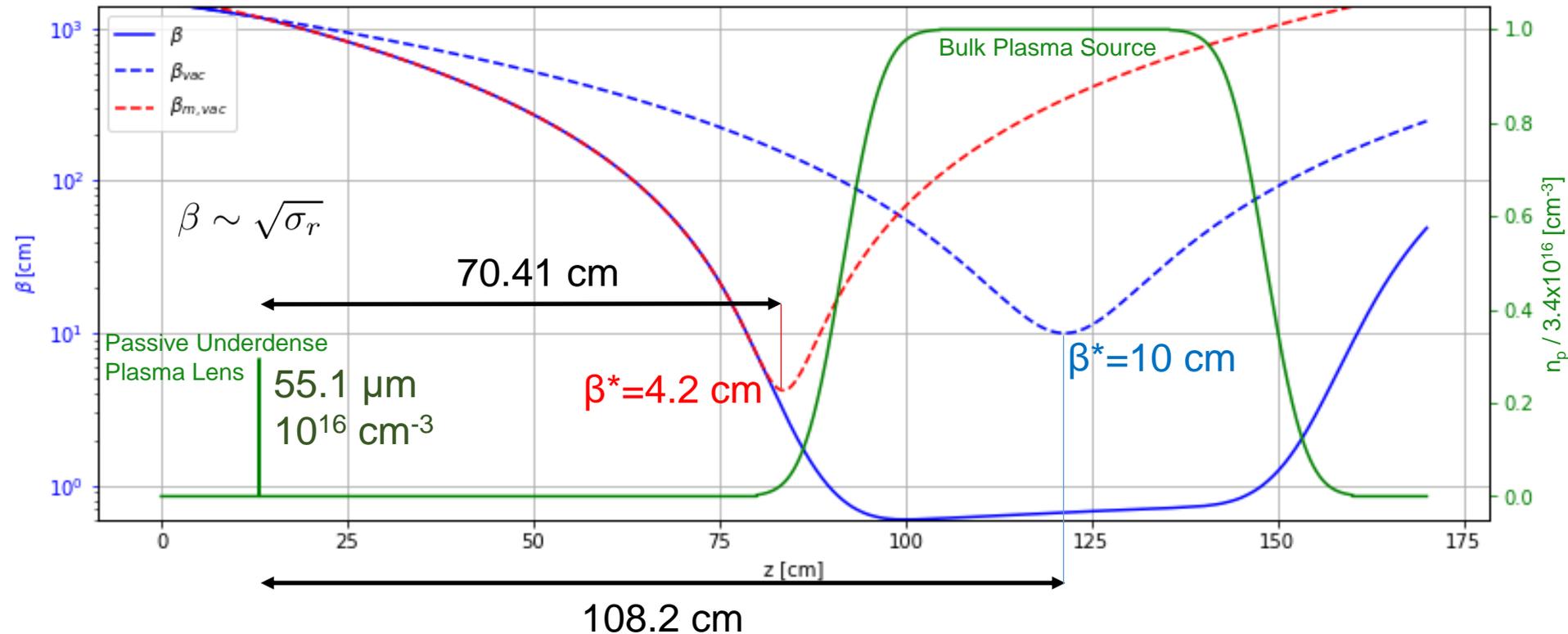
- Early experiments include PWFA in Li oven, and plasma-target experiments such as plasma lens and filamentation





PWFA Matching Example

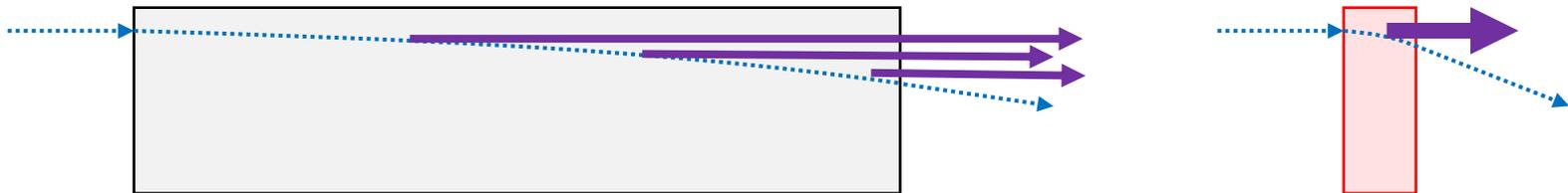
- Matching into a PWFA requires extremely small beam size for emittance preservation.
- Plasma lenses are one option to reach this goal.





Synchrotron Radiation and Oide Limit

- Underdense plasma lenses could be used either as a potential final focus optic or as a proxy for studying Physics present in other strong focusing optics.
- For example: electrons in focusing optics emit synchrotron radiation adiabatically (left) or non-adiabatically (right).



- The non-adiabatic emission of hard photons can significantly affect transverse momentum and final focus spot size.



Theory on Synchrotron Radiation

- Analytic theory for how non-adiabatic emission of synchrotron radiation introduces aberrations at the focus.

- K. Oide, "Synchrotron-Radiation Limit on the Focusing of Electron Beams," Phys. Rev. Lett., vol. 61, no. 15, pp. 1713–1715, Oct. 1988.

$$\sigma^{*2} = \beta^* \epsilon + \frac{110}{3\sqrt{6}\pi} r_e \lambda_e \gamma_L^5 F(\sqrt{K}L, \sqrt{K}l^*) \left(\frac{\epsilon}{\beta^*} \right)^{5/2}$$

Ideal Behavior

New Synch. Rad. Effects

$$F(\sqrt{K}L, \sqrt{K}l^*) \equiv \int_0^{\sqrt{K}L} |\sin \phi + \sqrt{K}l^* \cos \phi|^3 \left[\int_0^\phi (\sin \phi' + \sqrt{K}l^* \cos \phi')^2 d\phi' \right]^2 d\phi$$

$$l^* \equiv f - \frac{1}{2}L$$



Theory on Synchrotron Radiation

- The previous equation can be solved to find a minimum possible spot size for a given focusing optic.

$$\sigma_{min}^* = \left(\frac{7}{5}\right)^{1/2} \left[\frac{275}{3\sqrt{6\pi}} r_e \lambda_e F(\sqrt{K}L, \sqrt{K}l^*) \right]^{1/7} (\epsilon_N)^{5/7}$$

$$\beta_{opt}^* = \left[\frac{275}{3\sqrt{6\pi}} r_e \lambda_e F(\sqrt{K}L, \sqrt{K}l^*) \right]^{2/7} \gamma_L (\epsilon_N)^{3/7}$$

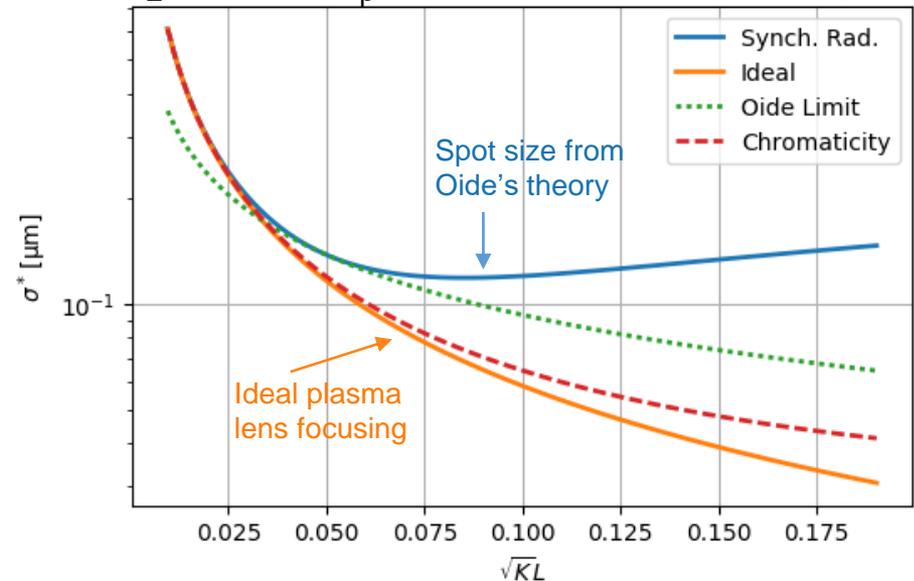
- This minimum spot size is achieved when the beam's betafunction at focus is β_{opt}^*



Synchrotron Radiation and Oide Limit

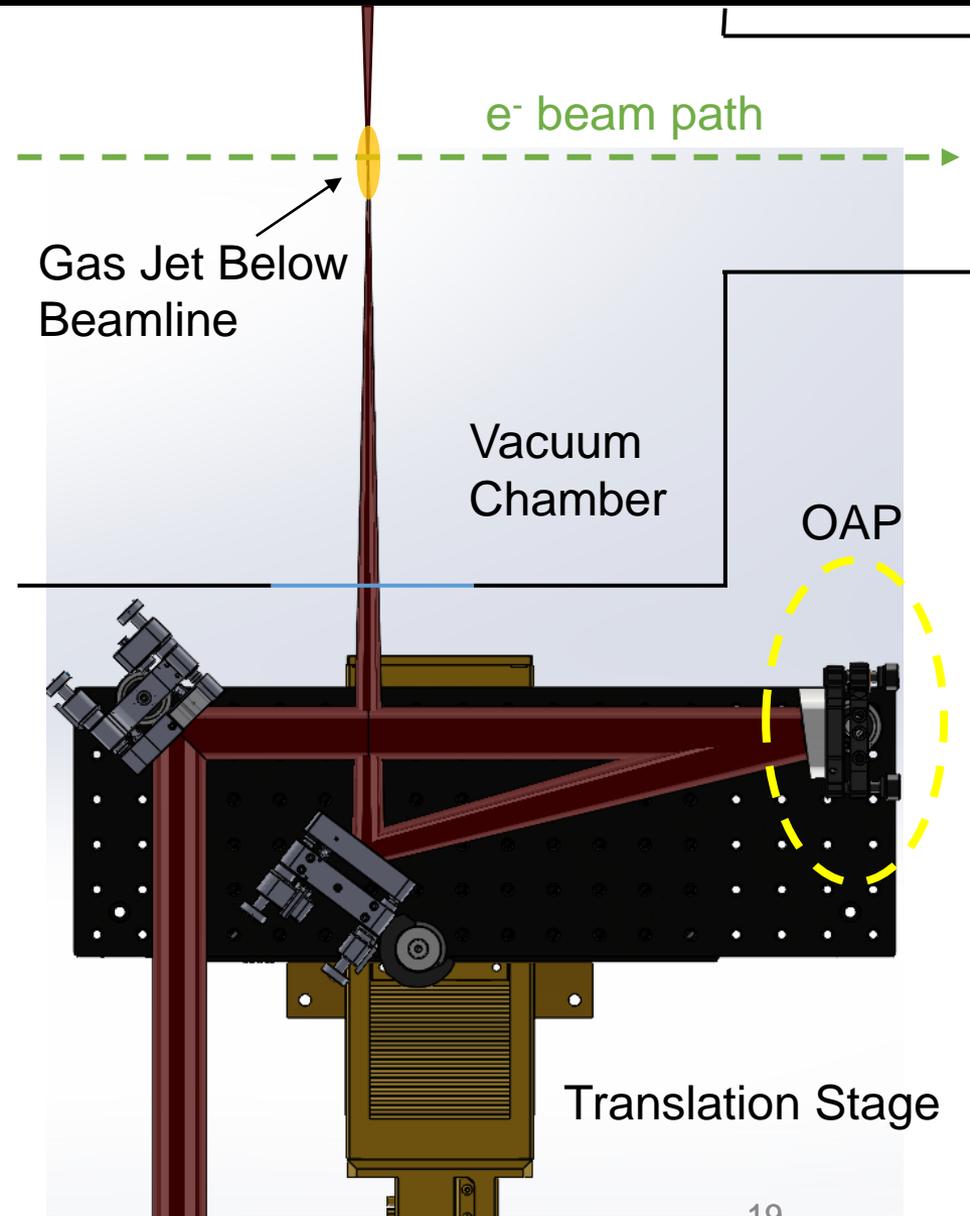
- Understanding the Oide Limit will be important for future colliders, but this limit has yet to be reached experimentally.
- We expect to be able to study this experimentally at FACET-II using underdense plasma lenses.

$E = 10 \text{ GeV}$, $\beta_i = 5 \text{ m}$, $\epsilon_N = 3 \text{ } \mu\text{m-rad}$,
 $\sigma_E = 0.1\%$, $n_p = 10^{18} \text{ cm}^{-3}$, $L = 20\text{-}200 \text{ } \mu\text{m}$





- Hydrogen gas jet in the path of the electron beam.
- Compressed laser pulse of up to 20 mJ ionizes the gas jet at the focus of an OAP.
- Opto-mechanics allow for plasma to be shifted and adjusted.
- Downstream magnetic lattice for imaging the beam's vacuum focus.

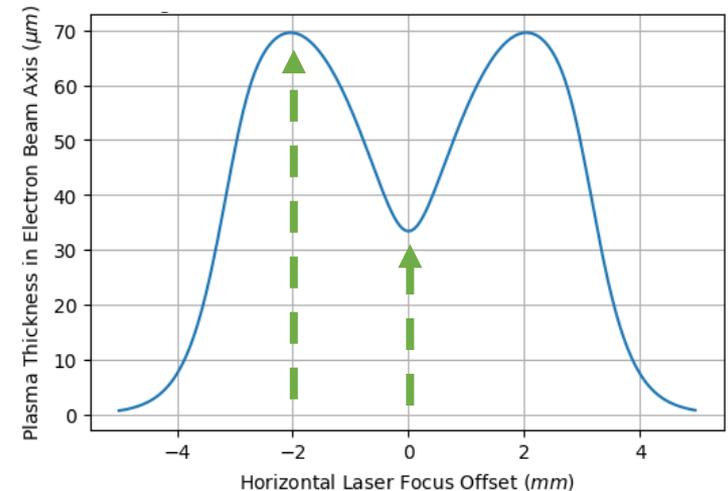
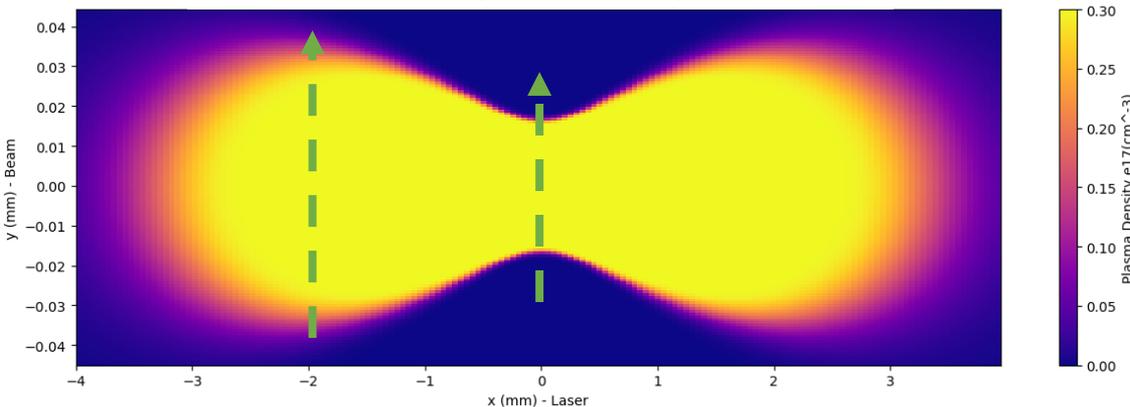




Plasma Lens at FACET-II

- Can scan plasma lens thickness by horizontally shifting the laser focus with respect to the electron beam axis.
- Gas jet pressure can adjust plasma lens density: currently planning for $10^{16} \rightarrow 10^{18} \text{ cm}^{-3}$.
- With 1 mJ, access thickness range $35 \mu\text{m} \rightarrow 70 \mu\text{m}$
- Plasma density to be experimentally measured with shadowgraphy and afterglow techniques.

Plasma Density: $\sim 1/50$ Aspect Ratio

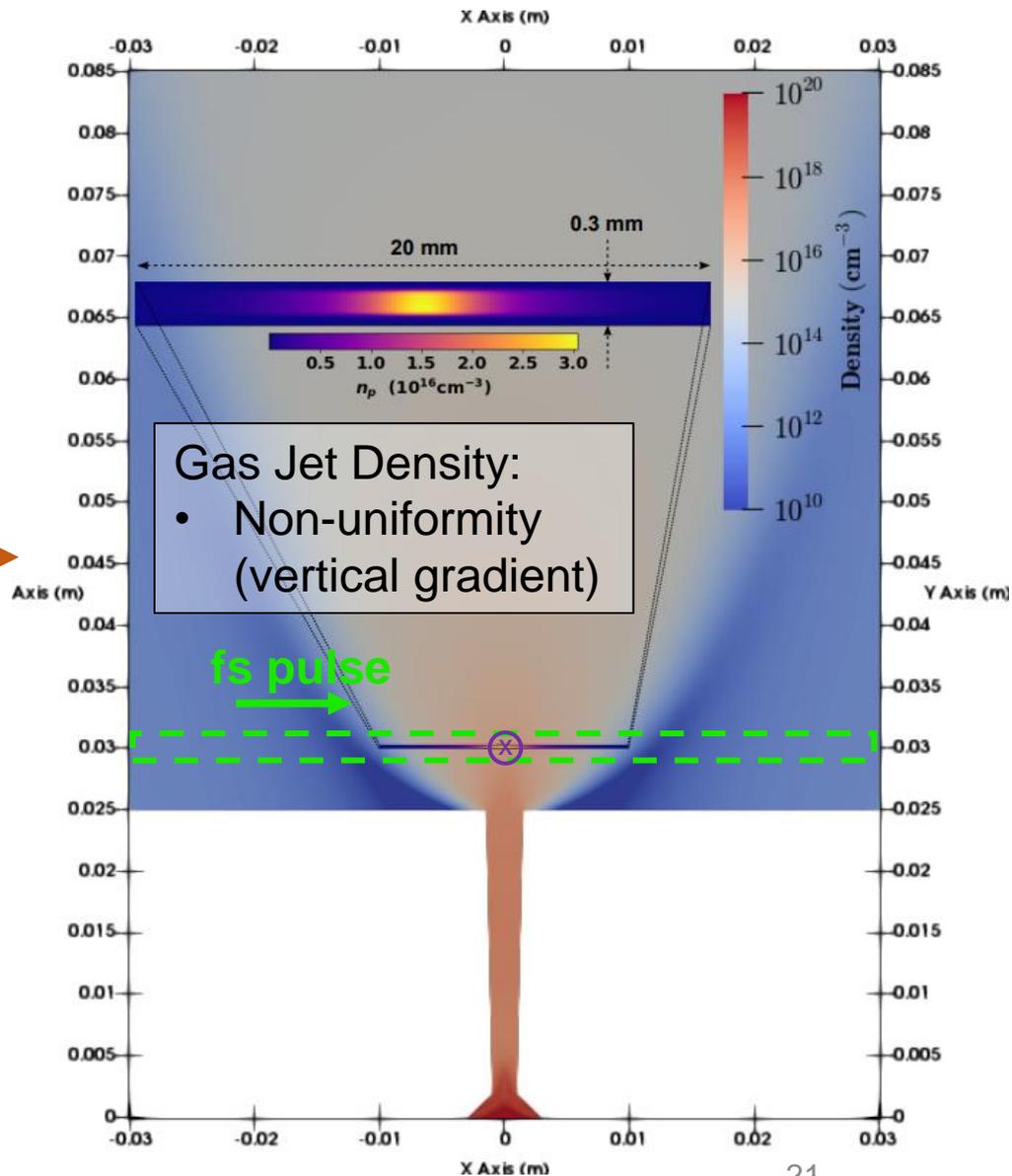
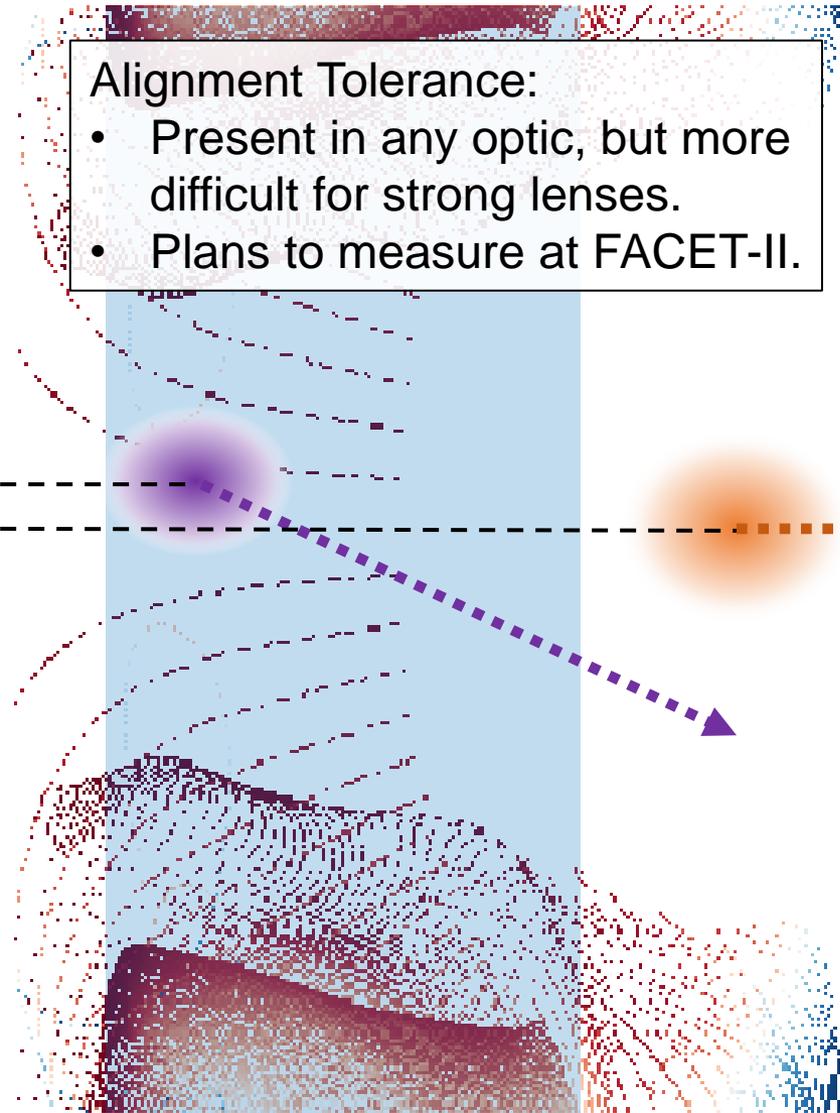




Expected Experimental Challenges

Alignment Tolerance:

- Present in any optic, but more difficult for strong lenses.
- Plans to measure at FACET-II.





Summary and Outlook

- Active plasma lenses have demonstrated their usefulness in experiments, and underdense passive plasma lenses show promise in the future.

Underdense Plasma Lenses at FACET-II:

- Test accuracy of plasma lens model and reliability of electron beam focusing using laser-ionized underdense passive plasma lens.
 - Plasma lens thicknesses from as low as $35 \mu\text{m}$ to as large as $350 \mu\text{m}$
 - Gas densities around $10^{16} - 10^{18} \text{ cm}^{-3}$
- Apply plasma lens towards PWFA matching and studying effects of strong focusing.



Thanks!



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