ON RG FLOWS NEAR A UV AND QUANTUM, 1ST ORDER PHASE TRANSITION - A PROFILE OF A HIGGS MECHANISM

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Talk based on arXiv: 2008.03981 [hep-th], *Nikos Irges and F.K.*

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INTRO

- Ultimate goal is the proposal of a new approach to the Higgs-Hierarchy problem

- The Non-Perturbative Gauge-Higgs Unification (NPGHU) model: An anisotropic, in fifth dimension, lattice with pure SU(2) gauge symmetry in 5d with orbifold boundary conditions generating a 4d boundary on which a U(1) gauge field coupled to a complex scalar survive

- Construction of a 4d continuum effective action for a 5d model originated by the lattice model of NPGHU

- Motivation: The model exhibits, non-pertubatively, spontaneous breaking of its gauge symmetry in infinite fifth dimension (Zero Temperature effect):

  N. Irges and *F. Knechtli,* Nucl. Phys. B **719** (2005) 12; *N. Irges* and *F. Knechtli*, Nucl. Phys. B **775** (2007) 283; *N. Irges, F. Knechtli* and *K. Yoneyama*, Nucl. Phys. B **722** (2013) 378-383; *M. Alberti, N. Irges, F. Knechtli* and *G. Moir*, JHEP **09** (2015) 159

- Three crucial characteristics:

- Even though extra dimensional, no finite-temperature type potential. No Kaluza-Klein states

- Pure bosonic nature of the Higgs mechanism. No need for fermions to trigger the mechanism (no Hosotani mechanism)

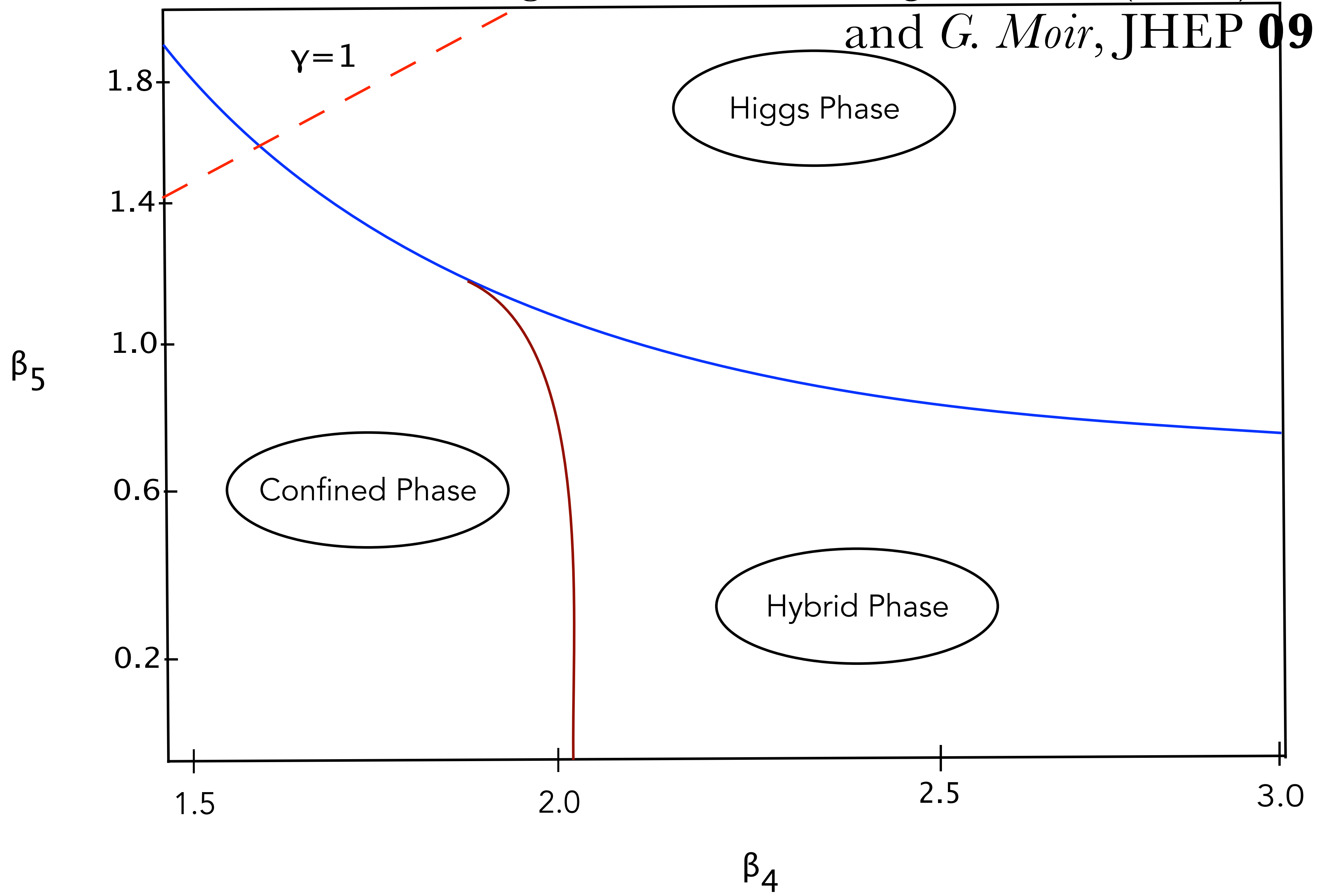
- There should not be any polynomial terms neither in classical nor in the (quantum) effective potential (distinguished from the Coleman-Weinberg (CW) model)

INTRO

- Gain 1. A non-perturbative (NP) new class of Higgs-type mechanisms

- Gain 2. The phase diagram of the lattice model exhibits a Higgs phase separated from two other phases by a1st order ”bulk” or ”zero-temperature” or ”quantum” phase transition:

N. Irges and *F. Knechtli*, JHEP **06** (2014) 070; *M. Alberti*, *N. Irges*, *F. Knechtli* and *G. Moir*, JHEP **09** (2015) 159



The Phase Diagram of the anisotropic orbifold lattice.
N. Irges and *F. Knechtli*, JHEP 06 (2014) 070; *M. Alberti, N. Irges, F. Knechtli* and *G. Moir*, JHEP 09 (2015) 159

INTRO

- The 1st order phase transition implies that a hypothetical continuum effective potential should have a cut-off. If it is low, it may give us a possible resolution to the Higgs mass fine tuning problem
- Questions: What about the continuum? Do the mentioned mechanism and its associated characteristics survive on the perturbative regime?
- Is it possible this new class of Higgs-type mechanisms to give a realist scenario in accordance to our experimental facts, resolving the fine-tuning problem?
- To answer these questions construct the 4d continuum effective action of the 5d NPGHU model and evaluate the associated observables (Scalar and Gauge field mass, etc) as well as the cut-off and the corresponding phase diagram
- We need an algorithm to build an effective action starting with the above lattice construction
- Start from the lattice plaquette action and expand it in small lattice spacing. It is known that this process generates an infinite tower of operators of increasing classical dimension. The ones of dimension larger than d are typically called Higher Dimensional Operators (HDO)
- Truncate the expansion at a given order and then take the naive continuum limit. This gives a classical, continuum effective action that may be quantized

INTRO

- Truncation at LO in lattice spacing expansion is not enough (*N. Irges and F.K.*, Nucl. Phys. B **937** (2018) 135-195)
  Truncate at NLO including the dominant HDO (known as Symanzik's improvement*) which will unlock the physical properties of the Higgs phase

- A continuum action enhanced with HDO for both the 4d boundary and the 5d bulk (a 5d version of the Lee-Wick model**). Renormalize diagrammatically at 1-loop order the boundary and obtain its quantum effective version. Analyze the renormalized scalar potential in order to expose the Higgs mechanism on the 4d boundary. Construct the phase diagram and compare it with the NP one.

- Three crucial facts to keep in mind:

- The boundary effective action, even though naively decoupled from the bulk, carries information of its 5d origin. This is hidden inside its couplings and the constrained way that the RG flows can move on the phase diagram

- The Hybrid phase and the Higgs phase only near the Higgs-Hybrid phase transition are layered (Localization proved NP)

- Connect the lattice parameters (β_4, β_5 or β, γ which consist of a_4, a_5, g_5) with the continuum ones (μ, v, g_4). Use $\mu = \dfrac{F(\beta_4, \beta_5)}{a_4}$

**P. Weisz,* Nucl. Phys. B 212 (1983) 1-17; *M. Luscher, P. Weisz,* Commim. Math. Phys. 97 (1985) 59-77

***B. Grinstein, D. O'Connell and M. B. Wise*, Phys. Rev. D **77** (2008) 025012; *B. Grinstein and D. O'Connell*, Phys. Rev. D **78** (2008) 105005

QUANTIZATION WITH HDO

- What is the action to be quantized? Start from the lattice plaquette action $S^{\rm orb} = S^{\rm b-h} + S^B$
 The boundary action $S^{\rm b-h}$

$$S^{\rm b-h} = \frac{1}{2N}\sum_{n_\mu}\left[\frac{\beta_4}{2}\sum_{\mu<\nu}\mathrm{tr}\left\{1 - U^b_{\mu\nu}(n_\mu,0)\right\} + \beta_5\sum_{\mu}\mathrm{tr}\left\{1 - U^h_{\mu 5}(n_\mu,0)\right\}\right]$$

 The bulk action S^B

$$S^B = \frac{1}{2N}\sum_{n_\mu,n_5}\left[\beta_4\sum_{\mu<\nu}\mathrm{tr}\left\{1 - U_{\mu\nu}(n_\mu,n_5)\right\} + \beta_5\sum_{\mu}\mathrm{tr}\left\{1 - U_{\mu 5}(n_\mu,n_5)\right\}\right]$$

- The parameters of the model $\beta_4 = \frac{4a_5}{g_5^2} = \frac{4}{g_4^2}$, $\beta_5 = \frac{4a_4^2}{a_5 g_5^2} = \frac{4a_4^2}{a_5^2 g_4^2}$, $\gamma = \frac{a_4}{a_5}$, $g_4^2 = \frac{g_5^2}{a_5} = \frac{g_5^2}{a_4}\gamma$

- Expanding w.r.t the lattice spacings and truncate at NLO in the expansion

$$S^{\rm b-h} = \sum_{n_\mu} a_4^4 \sum_{\mu}\left[\sum_{\nu}\left(\frac{1}{4}F^3_{\mu\nu}F^3_{\mu\nu} + \frac{1}{16}a_4^2(\hat{\Delta}_\mu F^3_{\mu\nu})(\hat{\Delta}_\mu F^3_{\mu\nu})\right) + |\hat{D}_\mu\phi|^2 + \frac{a_4^2}{4}|\hat{D}_\mu\hat{D}_\mu\phi|^2\right]$$

- Consider the naive continuum limit and go to Minkowski space with metric $\eta_{\mu\nu} = (+,-,-,-)$ to get the boundary effective action

$$S^{\rm b-h} = \int d^4x\left[-\frac{1}{4}F^3_{\mu\nu}F^{3,\mu\nu} + |D_\mu\phi|^2 + \frac{c_\alpha^{(6)}}{2\mu^2}(\partial^\mu F^3_{\mu\nu})(\partial_\mu F^{3,\mu\nu}) - \frac{c_2^{(6)}}{\mu^2}|D^\mu D_\mu\phi|^2\right]$$

QUANTIZATION WITH HDO

- A^3_μ is the gauge field and $\phi = \dfrac{A^1_5 + iA^2_5}{\sqrt{2}}$ is the scalar field. $\mu, \nu \ldots$ denote the 4d Minkowski index and $\mathbf{A} = 1,2,3$ denote the adjoint index

- $c^{(6)}_\alpha$ and $c^{(6)}_2$ are introduced for the HDO of the gauge and scalar field respectively absorbing the function $F(\beta_4, \beta_5)$ of $\mu = F(\beta_4, \beta_5)/a_4$

- Set $\mu^2 = \Lambda^2 \dfrac{\mu^2}{\Lambda^2}$ and absorb $\dfrac{\mu^2}{\Lambda^2}$ into the couplings. A cut-off for the Effective Field Theory (EFT) is introduced

- In this case Λ is not an external scale that must be introduced by hand. It is rather an internal scale, given by the value of the regulating scale at the phase transition, μ_*, where it assumes its maximum value. HDO are of quantum origin

N. Irges and F.K., Nucl. Phys. B **950** (2020) 114833

- Why we did not just use gauge invariance to get the continuum action?

  1. The anisotropy factor hidden inside the covariant derivative $D_\mu = \partial_\mu - ig_4 A^3_\mu$. This opens a second dimension in the phase diagram, exposing the Higgs phase and a triple point, among others

  2. The above effective action does not have a scalar potential. This is due to the 5d origin of the boundary theory since the lattice does not produce polynomial terms for the boundary effective action at any order in the lattice spacing expansion

- One more step before quantization is to deal with the extra pole instability

QUANTIZATION WITH HDO

- Expanding the gauge fixed action

$$S^{\rm b-h} \equiv \int d^4x \left[-\frac{1}{4}F^3_{\mu\nu}F^{3,\mu\nu} - \frac{1}{2\xi}(\partial^\mu A^3_\mu)^2 + |D_\mu\phi|^2 + \frac{c_\alpha^{(6)}}{2\Lambda^2}(\partial^\mu F^3_{\mu\nu})(\partial_\mu F^{3,\mu\nu}) - \frac{c_2^{(6)}}{\Lambda^2}|D^\mu D_\mu \phi|^2 + \partial^\mu \bar{c}^3 \partial_\mu c^3 \right]$$

$$\begin{aligned} S_0^{\rm b-h} \equiv & \int d^4x \Bigg[-\frac{1}{4}F^3_{\mu\nu,0}F_0^{3,\mu\nu} + \frac{1}{2\xi}A^3_{\mu,0}\partial^\mu\partial_\nu A_0^{3,\nu} - \bar\phi_0\Box\phi_0 - \frac{c^{(6)}_{\alpha,0}}{2\Lambda^2}F^3_{\mu\nu,0}\Box F_0^{3,\mu\nu} - \frac{c^{(6)}_{2,0}}{\Lambda^2}\bar\phi_0\Box^2\phi_0 - \bar{c}^3_0\Box c^3_0 \\ & + ig_0\sqrt{\gamma_0}A^3_{\mu,0}\left(\left\{ \bar\phi_0\partial^\mu\phi_0 - \phi_0\partial^\mu\bar\phi_0 \right\} + \frac{c^{(6)}_{2,0}}{\Lambda^2}\left\{ \Box\bar\phi_0\partial^\mu\phi_0 - \Box\phi_0\partial^\mu\bar\phi_0 \right\} \right) + g_0^2\gamma_0(A^3_{\mu,0})^2\bar\phi_0\phi_0 \\ & + g_0^2\gamma_0\left(\frac{c^{(6)}_{2,0}}{\Lambda^2}\left\{ \phi_0\Box^2\bar\phi_0 + \bar\phi_0\Box^2\phi_0 \right\} - \frac{c^{(6)}_{2,0}}{\Lambda^2}\partial^\mu(A^3_{\mu,0}\bar\phi_0)\partial_\mu(A_0^{3,\mu}\phi_0) \right) \\ & - \frac{ig_0^3\gamma_0^{3/2}c^{(6)}_{2,0}}{\Lambda^2}(A^3_{\rho,0})^2A^3_{\mu,0}\left(\bar\phi_0\partial^\mu\phi_0 - \phi_0\partial^\mu\bar\phi_0 \right) - \frac{g_0^4\gamma_0^2c^{(6)}_{2,0}}{\Lambda^2}(A^3_{\rho,0})^4\bar\phi_0\phi_0 \Bigg] \end{aligned} \quad , \; g^2 = \frac{g_5^2}{a_4}$$

- To deal with the Ostrogradsky ghosts (O-ghosts) perform the most general field redefinition

$$\phi_0 \to \hat\phi_0 = \phi_0 + \frac{x}{\Lambda^2}D^2\phi_0 + \frac{y}{\Lambda^2}(\bar\phi_0\phi_0)\phi_0$$

$$A^3_{\mu,0} \to \hat{A}^3_{\mu,0} = A^3_{\mu,0} + \frac{x_\alpha}{\Lambda^2}(\eta_{\mu\rho}\Box - \partial_\mu\partial_\rho)A_0^{3,\rho}$$

- This introduces the Reparameterization ghosts (R-ghosts) which cancel the O-ghosts pole by pole at classical and quantum level

N. Irges and F.K., Phys. Rev. D 100, 065004 (2019)
N. Irges and F.K., Nucl. Phys. B **950** (2020) 114833

QUANTIZATION WITH HDO

- Fixing $x_\alpha = -c^{(6)}_{\alpha,0}$, $x = -\frac{c^{(6)}_{2,0}}{2}$ and $y = \frac{c^{(6)}_{1,0}}{8}$ gives the bare and redefined boundary action

$$\begin{aligned} S_0^{\mathrm{b-h}} &= \int d^4x \Big[-\frac{1}{4} F^3_{\mu\nu,0} F_0^{3,\mu\nu} + \frac{1}{2\xi} A^3_{\mu,0} \partial^\mu \partial_\nu A_0^{3,\nu} - \bar{\phi}_0 \Box \phi_0 - \frac{c^{(6)}_{1,0}}{4\Lambda^2} (\bar{\phi}_0 \phi_0) \bar{\phi}_0 \Box \phi_0 - \bar{c}_0^3 \Box c_0^3 \\ &+ i g_{4,0} \Big\{ \eta_{\mu\rho} - \frac{\eta_{\mu\rho}\Box - \partial_\mu \partial_\rho}{\Lambda^2} \Big\} A_0^{3,\rho} \Big(\bar{\phi}_0 \partial^\mu \phi_0 - \phi_0 \partial^\mu \bar{\phi}_0 \Big) + g^2_{4,0} (A^3_{\mu,0})^2 \bar{\phi}_0 \phi_0 \\ &+ \frac{g^2_{4,0}}{2\Lambda^2} \Big(A^3_{\mu,0} A^3_{\rho,0} \partial^\rho \bar{\phi}_0 \partial^\mu \phi_0 + A^3_{\mu,0} \partial^\rho A^3_{\rho,0} \partial^\mu (\bar{\phi}_0 \phi_0) \Big) - 2 g^2_{4,0} \frac{A_0^{3,\mu} (\eta_{\mu\rho}\Box - \partial_\mu \partial_\rho) A_0^{3,\rho}}{\Lambda^2} \bar{\phi}_0 \phi_0 \\ &+ i \frac{g_{4,0}\, c^{(6)}_{1,0}}{4\Lambda^2} A^3_{\mu,0} \bar{\phi}_0 \phi_0 \Big(\bar{\phi}_0 \partial^\mu \phi_0 - \phi_0 \partial^\mu \bar{\phi}_0 \Big) + \frac{g^2_{4,0}\, c^{(6)}_{1,0}}{4\Lambda^2} (A^3_{\mu,0})^2 (\bar{\phi}_0 \phi_0)^2 \Big] \end{aligned}$$

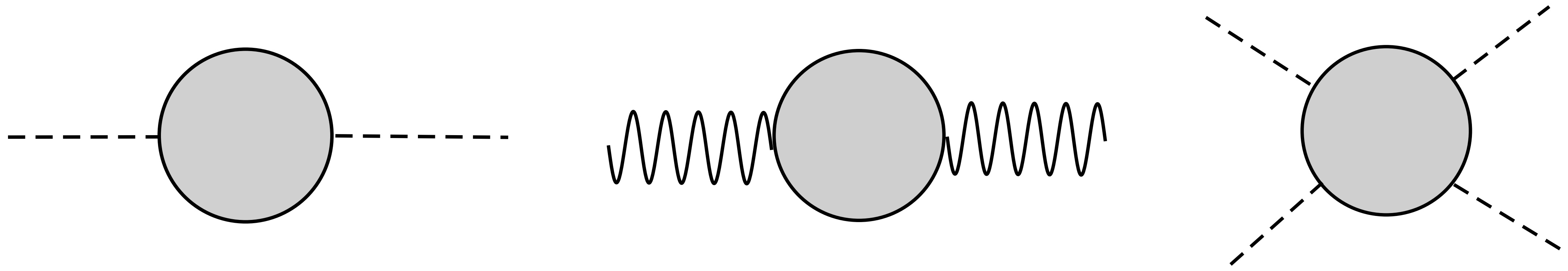
- Now the boundary action is ghost-free and has developed a scalar quartic term $\bar{\phi}\phi \Box \bar{\phi}\phi$ (Recall that these HDO are of quantum nature)

- One coupling in the beginning and two couplings, g_4 and the "quartic coupling" $c_1^{(6)}$ at the end. However is expected to be connected (`a la CW)

- The Feynman rules are straightforward but non-trivial due to the HDO. Ready for the 1-loop level, diagrammatic, renormalization

QUANTIZATION WITH HDO

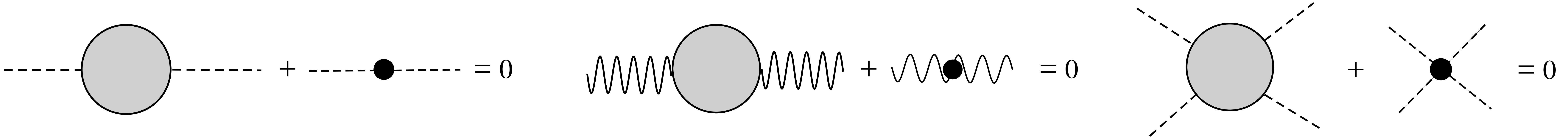
- For the needed parameters g_4, $c_1^{(6)}$, ϕ and A_μ^3 is enough (due to gauge invariance) to renormalize the propagators and the $\bar{\phi}\phi\Box\bar{\phi}\phi$- and $\bar{\phi}\phi A_\mu^3 A_\nu^3$-vertices
- The associated Feynman diagrams are a non-trivial version of the Scalar QED's ones



- The renormalization procedure suggests

$$g_{4,0} = (1+\delta_{g_4})g_4 \text{ or } \alpha_{4,0} = (1+\delta_{\alpha_4})\alpha_4 \text{ with } \alpha_4 = \frac{g_4^2}{16\pi^2} \qquad c_{1,0}^{(6)} = (1+\delta_{c_1^{(6)}})c_1^{(6)} \qquad \phi_0 = \sqrt{1+\delta_\phi}\,\phi \qquad A_{\mu,0}^3 = \sqrt{1+\delta_A}A_\mu^3$$

- The renormalization conditions suggest



QUANTIZATION WITH HDO

- The countqerterms and the associated β-functions of the boundary action are fixed (the off-shell scheme $p_i^2 = \Lambda^2$ is used)

$$\delta_{g_4} = -\frac{1}{2}\delta A,\ \delta g_4 = \frac{1}{16\pi^2}\frac{g_4^3}{\varepsilon} \text{ or } \delta\alpha_4 = 2\frac{\alpha_4^2}{\varepsilon} \qquad \delta c_1^{(6)} = \frac{1}{16\pi^2}\frac{4(c_1^{(6)})^2 + 34g_4^4}{\varepsilon} \qquad \delta\phi = 0$$

$$\beta_{g_4} = \frac{g_4^3}{16\pi^2} \text{ or } \beta_{\alpha_4} = 2\alpha_4^2 \qquad \beta_{c_1^{(6)}} = \frac{4(c_1^{(6)})^2 + 34g_4^4}{16\pi^2}$$

- For completeness apply all the previous steps in the bulk lattice action to get its continuum version (5d Lee-Wick version)

$$\mathcal{L}^B = -\frac{1}{4}F^A_{\mu\nu}F^{A,\mu\nu} + \frac{1}{16\Lambda^2}(D^\mu F^A_{\mu\nu})(D_\mu F^{A,\mu\nu}) - \frac{g_5}{24\Lambda^2}f_{ABC}F^A_{\mu\nu}F^B_{\nu\rho}F^C_{\rho\mu} + (\overline{D_\mu\Phi^A})(D^\mu\Phi^A) - \frac{1}{4\Lambda^2}(\overline{D^2\Phi^A})(D^2\Phi^A)\Big]$$

- The corresponding β-function of g_5 or of its auxiliary coupling $\alpha_5 = \frac{4g_5^2}{16\pi^2}\mu^{-\varepsilon}$ are straightforward in $d = 4 - \varepsilon$

$$\beta_{g_5\mu^{-\varepsilon/2}} = -\frac{\varepsilon}{2}g_5\mu^{-\varepsilon/2} - \frac{125}{6}\frac{g_5^3\mu^{-3\varepsilon/2}}{16\pi^2} \text{ or } \beta_{\alpha_5} = -\varepsilon\alpha_5 - \frac{125}{12}\alpha_5^2$$

THE HIGGS PHASE

- The desired Higgs phase is revealed when a CW procedure is followed

- The algorithm:

  1. Consider the 4d bare potential in momentum space and use the above off-shell scheme

  2. Construct the renormalized and improved effective potential using the scalar field as the running parameter and minimize it to find the non-trivial minimum

  3. Find the relation between the couplings g_4, $c_1^{(6)}$ and then determine the scalar and gauge field masses and from those the scalar-to-gauge mass ratio

- The improved 1-loop effective potential is of a CW type

$$V_{\text{imp.}}(\bar\phi,\phi) = \frac{c_1^{(6)}}{4}(\bar\phi\phi)^2 + \left\{2(c_1^{(6)})^2 + 17g_4^4\right\}\frac{(\bar\phi\phi)^2}{64\pi^2}\left(\ln\frac{\bar\phi\phi}{v^2} - 3\right)$$

or using $\bar\phi\phi = \frac{(A_5^1)^2 + (A_5^2)^2}{2} \equiv \phi_r^2$

$$V_{\text{imp.}}(\phi_r) = \frac{c_1^{(6)}}{4}\phi_r^4 + \left\{2(c_1^{(6)})^2 + 17g_4^4\right\}\frac{\phi_r^4}{64\pi^2}\left(\ln\frac{\phi_r^2}{v^2} - 3\right)$$

- The minimization suggests

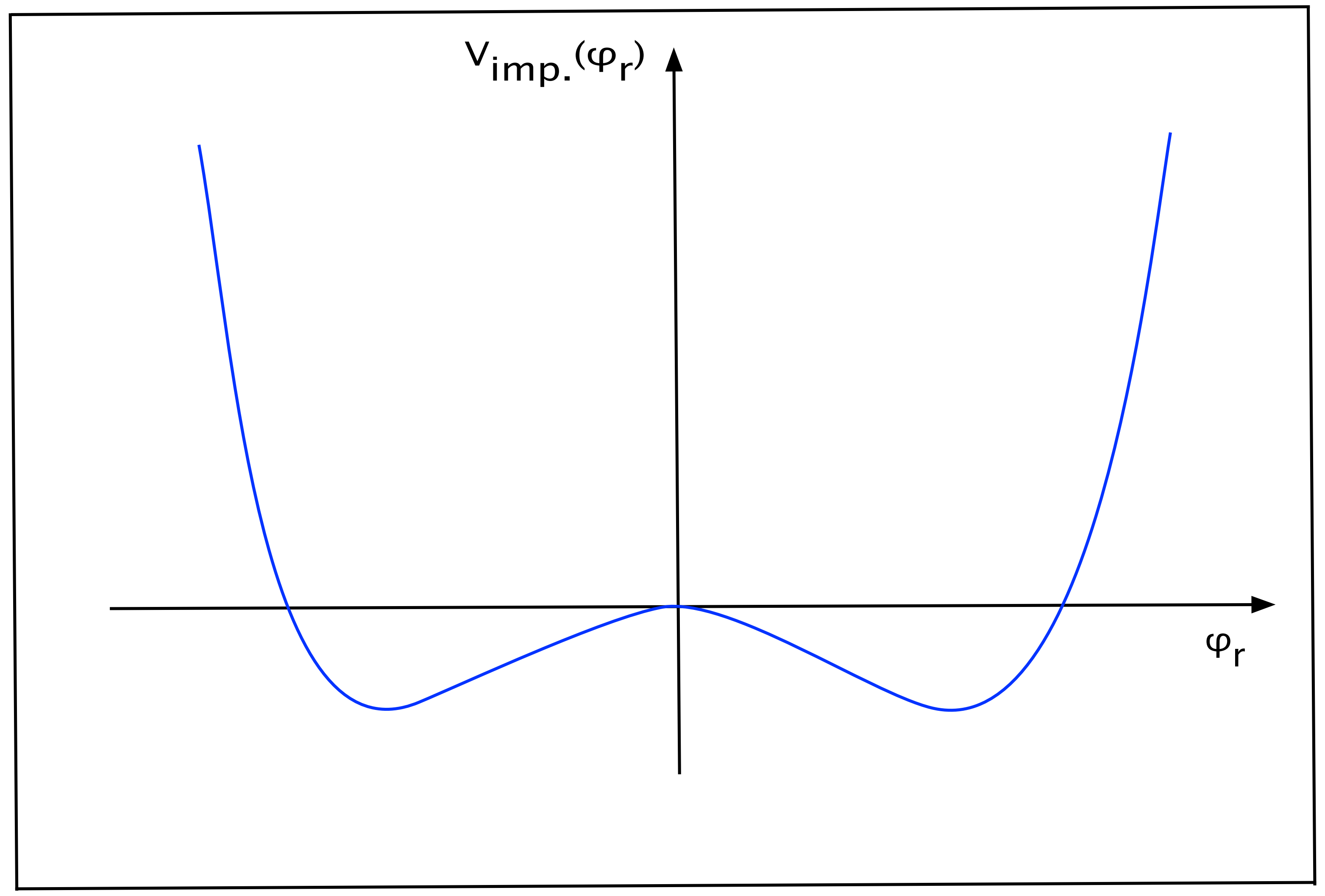
$$\left.\frac{\partial V_{\text{imp.}}(\phi_r)}{\partial\phi_r}\right|_{\phi_r=v} = \frac{-(10(c_1^{(6)})^2 + 85g_4^4 - 32\pi^2 c_1^{(6)})v^3}{32\pi^2} = 0 \Rightarrow$$

$$c_1^{(6)} = \frac{85}{32\pi^2}g_4^4$$

THE HIGGS PHASE

- The expected connection between the couplings is achieved

$$V_{\text{imp.}}(\phi_r) = \frac{17 g_4^4 \phi_r^4}{128\pi^2}\left(2\ln\frac{\phi_r^2}{v^2} - 1\right) + \mathcal{O}(g_4^8)$$



The mexican hat potential $V_{\text{imp.}}(\phi_r)$

- The non-trivial vev ($<\phi_r> = v$) triggers the spontaneous breaking of the gauge symmetry $\phi_r = h + v$

$$m_h^2 \equiv \frac{\partial^2 V(h)}{\partial h^2}\Big|_{h=0} = \frac{210}{8\pi^2} g_4^4 v^2 \qquad m_{A_\mu^3}^2 = g_4^2 v^2 \equiv m_Z^2 \qquad \frac{m_h^2}{m_Z^2} \equiv \rho_{\text{bh}}^2 = \frac{210}{8\pi^2} g_4^2 \Rightarrow \rho_{\text{bh}} = \sqrt{\frac{210}{8\pi^2}}\, g_4 \simeq 1.64\, g_4$$

THE HIGGS PHASE

- Comparison with the CW case in the classical level

$$\rho_{\text{CW}} = \sqrt{\frac{3}{8\pi^2}}e \simeq 0.19\, e \quad \text{and} \quad \lambda = \frac{33}{8\pi^2}e^4$$

- The numerical difference originates from the higher derivative nature of the quartic coupling $c_1^{(6)}$, a crucial point for the model is needed

- At quantum level the solution of the RG equations $\mu \frac{d\, g_4(\mu)}{d\mu} = \beta_{g_4}$ and $\mu \frac{d\, e(\mu)}{d\mu} = \beta_e$ with $\beta_e = \frac{e^3}{48\pi^2}$

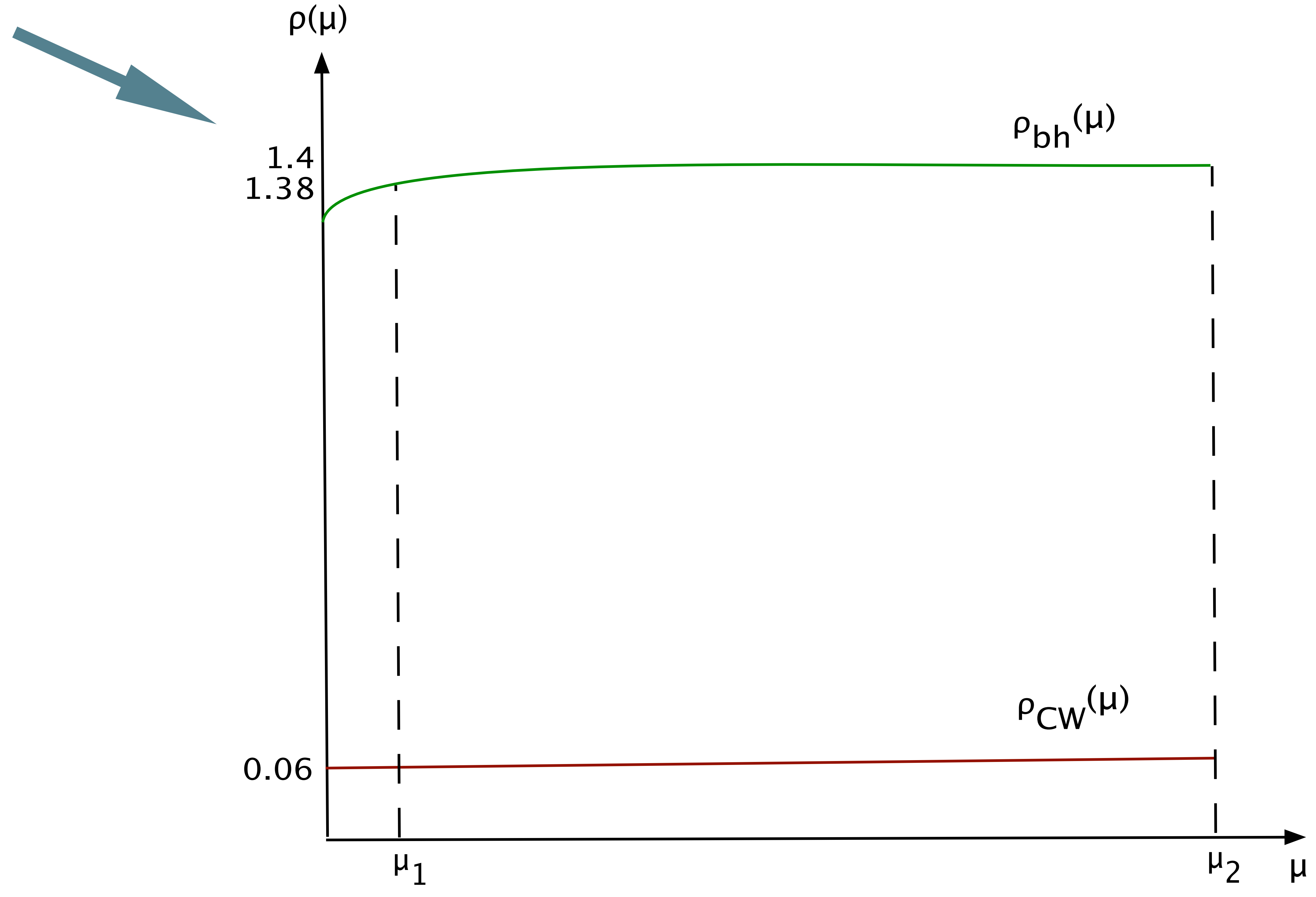
$$g_4(\mu) = \frac{g_{4,R}}{\sqrt{1 - \frac{g_{4,R}^2}{16\pi^2} \ln \frac{\mu^2}{m_R^2}}} \quad \text{or} \quad \alpha_4(\mu) = \frac{\alpha_{4,R}}{1 - \alpha_{4,R} \ln \frac{\mu^2}{m_R^2}} \quad \text{and} \quad e(\mu) = \frac{e_R}{\sqrt{1 - \frac{e_R^2}{48\pi^2} \ln \frac{\mu^2}{M_R^2}}}$$

- The IR boundary conditions are m_R, M_R, $g_4(m_R) = g_{4,R}$ $(\alpha_4(m_R) = \alpha_{4,R})$ and $e(M_R) = e_R$.

- The vet is fixed $v \equiv v_*$

THE HIGGS PHASE

- A premature example: $m_R = M_R = 91.1\,\text{GeV}$, $g_{4,R} = 0.83$, $e_R = 0.31$



The running of the ρ-parameter with respect to the energy scale μ for the Boundary- Hybrid model (green line) and the CW model (red line)

- What about ρ_{bh} and ρ_{CW} are of SM value?

$$g_4 \approx 0.84 \text{ and } e \approx 7.20$$

- What about the quartic coupling?

$$c_1^{(6)} \approx 1.3 \text{ and } \lambda \approx 1123.20$$

THE CONTINUUM PHASE DIAGRAM

- For the construction of the phase diagram recall that the effective boundary action is not decoupled from the 5d bulk and incorporates non-perturbative features which have been observed on the lattice model:

1. Dimensional reduction through localization when the Higgs-Hybrid phase transition (1st order, quantum) is approached. The entire Hybrid phase is layered in the fifth dimension

2. The bulk driven phase transition is reached by the Higgs and Hybrid phase RG flows in the UV simultaneously due to common μ

- From 1. keep in mind that the Hybrid phase contains 4d slices with SU(2) gauge group in the bulk.
- From 2. keep in mind that the RG flow in the Higgs phase is constrained from the one in the Hybrid phase
- The continuum phase diagram is constructed by matching the RG flows of the two phases. The model inherits a finite cut-off
- Does the the above procedure correspond to a viable model?

THE CONTINUUM PHASE DIAGRAM

- The needed ingredients:

  1. The RG evolution of the couplings in both phases

  2. The connection of β_4 and β_5 with the running gauge couplings of Higgs and Hybrid phase

  3. The value of the parameters on the phase transition denoted by *

The parameters

Higgs Phase	Hybrid Phase
$\alpha_4(\mu) = \dfrac{\alpha_{4,R}}{1 - \alpha_{4,R} \ln \frac{\mu^2}{m_R^2}}$	$\alpha_s(\mu) = \dfrac{c'_s}{\ln \frac{\mu}{\Lambda_s}}$ and $\Lambda_s = e^{-\frac{c'_s}{\alpha_{s,R}}} m_R$
$\mu = \exp\left[\dfrac{\alpha_4(\mu) - \alpha_{4,R}}{2\alpha_4(\mu)\,\alpha_{4,R}}\right] m_R$	$\mu = e^{\frac{c'_s}{\alpha_s(\mu)}} \Lambda_s$
$\beta_4(\mu) = \dfrac{1}{4\pi^2 \alpha_4(\mu)}$	$\beta_{4,s}(\mu) = \dfrac{1}{4\pi^2 \alpha_s(\mu)}$
$\beta_5(\mu) = \gamma^2(\mu)\beta_4(\mu)$	$\beta_{5,s}(\mu) = \gamma_s^2(\mu)\beta_{4,s}(\mu)$

THE CONTINUUM PHASE DIAGRAM

- On the Higgs-Hybrid phase transition $\mu = \mu_*$:

$$\alpha_4(\mu_*) = \alpha_s(\mu_*) = \alpha_* \qquad \mu_* = e^{\frac{c_s'}{1+2c_s'}\left[\frac{1}{\alpha_{4,R}}+\frac{2c_s'}{\alpha_{s,R}}\right]}\Lambda_s$$

$$\alpha_* = \frac{\alpha_{4,R}\,\alpha_{s,R}(1+2c_s')}{\alpha_{s,R}+2c_s'\alpha_{4,R}} \qquad m_{h*} = \sqrt{\frac{210}{8\pi^2}}16\pi^2 v_*\alpha_*$$

- The above are controlled by four variables: $\alpha_{4,R}$, $\alpha_{s,R}$, v_*, and Λ_s
- $\alpha_{s,R} = 0.014$ (SM's strong gauge coupling) and $\Lambda_s = m_p = 1000$ MeV (proton mass), fixed by physical motivation
- The first necessary condition for the validity of the effective action is the hierarchy of the scales

$$m_R < m_{h*} < \mu_*$$

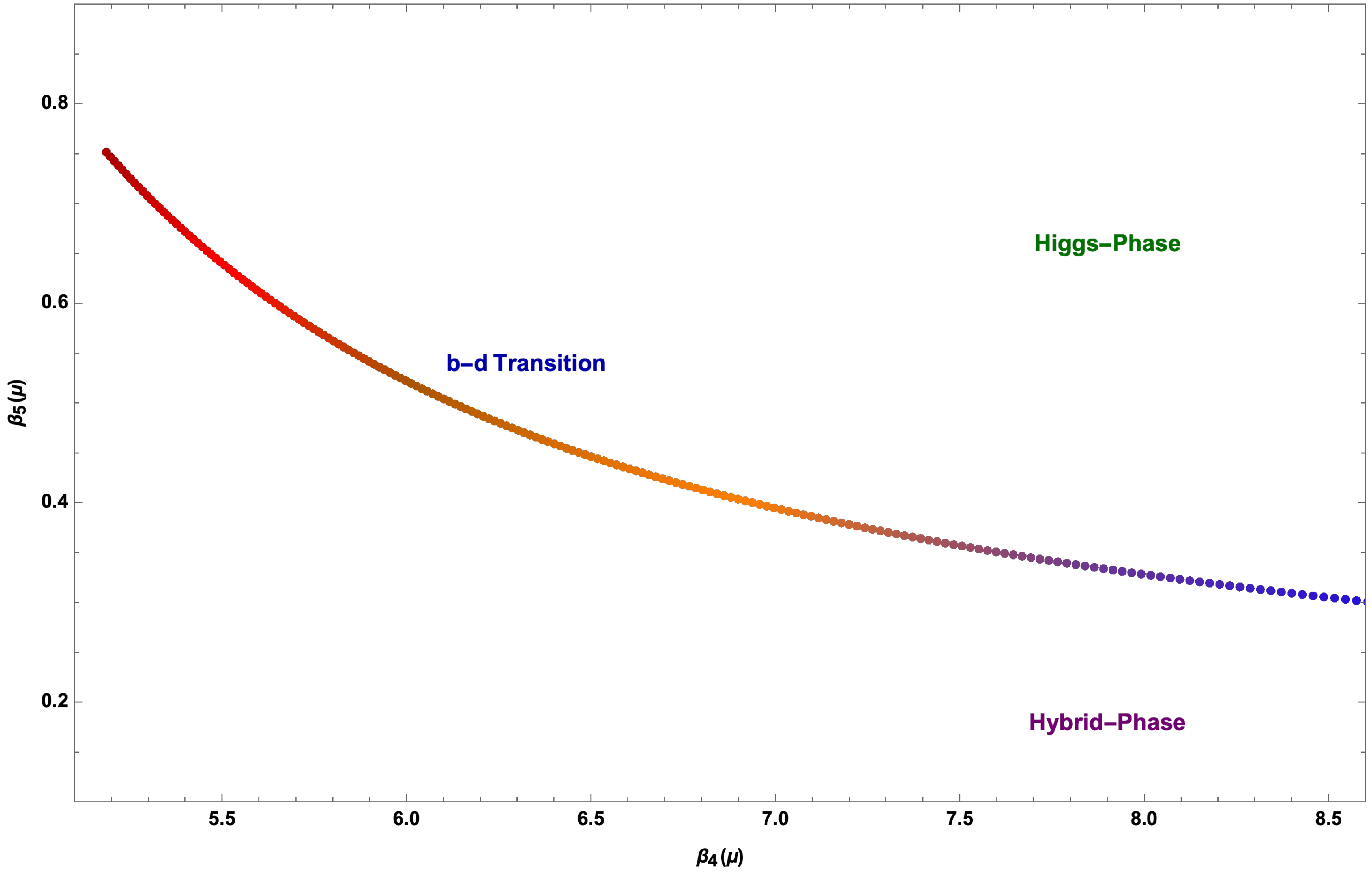
- The second necessary condition is to generate a SM-like spectrum

$$m_{h*} \simeq 125\,\mathrm{GeV} \quad \text{and} \quad \rho_{\mathrm{bh}} > 1$$

- Standard Model spectrum for $\alpha_{4,R} = 0.00435$ and $v_* = 108.2$ GeV → $m_R = 5.55\,\mathrm{GeV}$, $m_{h*} \approx 125.1\,\mathrm{GeV}$, $\mu_* \approx 209\,\mathrm{GeV}$ and $\rho_{\mathrm{bh}} \approx 1.373$

THE CONTINUUM PHASE DIAGRAM

- Keep $\alpha_{s,R} = 0.014$, $\Lambda_s = 1000\,\text{MeV}$ and $v_* = 108.2\,\text{GeV}$. Vary $\alpha_{4,R} \to$ Varies $\mu_* \to$ different pair (β_{4*}, β_{5*})

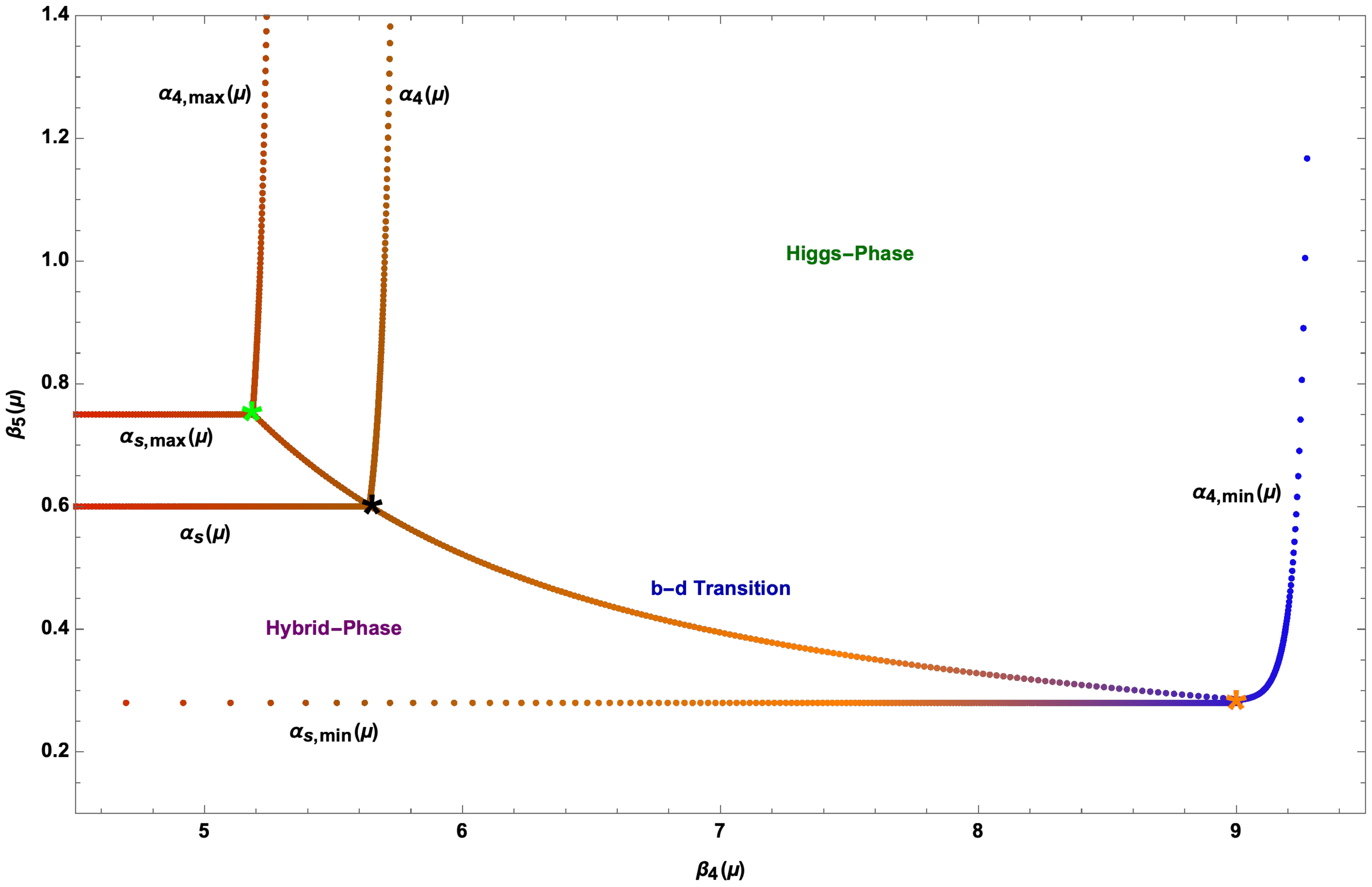


The perturbative phase diagram

THE CONTINUUM PHASE DIAGRAM

- The phase diagram and three RG flows: $\alpha_{4,R} \to$ $(\alpha_{4,R}^{\min} = 0.0027)$, $(\alpha_{4,R} = 0.00435)$, $(\alpha_{4,R}^{\max} = 0.00473)$

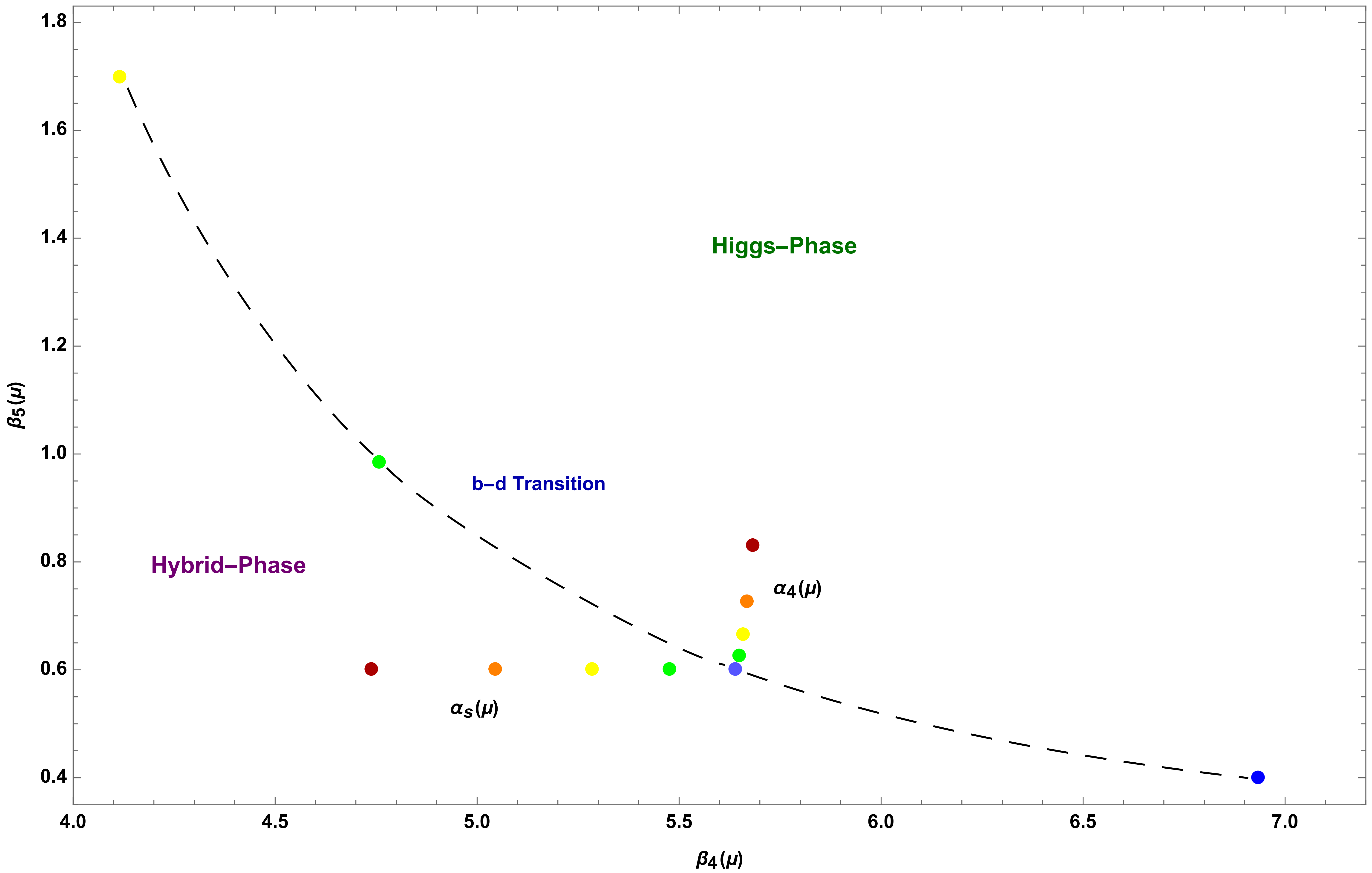
$(m_{h^*} \approx 78\,\text{GeV}, \mu_* \approx 5123\ \text{GeV})$ $(m_{h^*} \approx 125.1\,\text{GeV}, \mu_* \approx 209\ \text{GeV})$ $(m_{h^*} = \mu_* \approx 136.1\,\text{GeV})$



The RG flows and the Phase transition

THE CONTINUUM PHASE DIAGRAM

- A zoomed version of the phase diagram. Numerical analysis shows that the fine tuning of an RG flow that respects the physical constraints is equal or less than $\mathcal{O}(10^2)$. Then the fine tuning in the Higgs mass very small



CONCLUSIONS

- The 1-loop effective action of an SU(2) gauge theory in five dimensions with boundary conditions that leave a U(1)-complex scalar theory on the boundary, located at the origin of a semi-infinite fifth dimension was constructed

- At perturbative level, the boundary theory is a version of the Coleman-Weinberg model where the quartic term is replaced by a dimension-6 derivative operator. A qualitatively similar to the CW model Higgs mechanism is at work but with different coefficients in the scalar mass and the β-functions that change things towards a more realistic direction

- Imposing on the effective action non-perturbative features known from the lattice, the system becomes highly constrained. The picture is that the model possesses a non-trivial phase diagram where the phases are separated by 1st order, quantum phase transitions located in the UV

- In order to use the model as a cartoon of a possible origin of the Standard Model Higgs sector, then it turns out that we have to sit on, or near the interface of the phase transition that separates the Higgs phase and a layered-type of phase, the Hybrid phase

- There, dimensional reduction happens via localization in both phases and the effective action must be constructed with a dynamically generated finite cut-off but also with RG flows that are correlated below and above the phase transition

- Alternative resolution to the Higgs mass hierarchy problem: The fine tuning involved is about one part in a hundred and it is related to the choice of a ”physical RG flow” on the phase diagram while the dynamics do not allow a high cut-off for the effective action. Once such a physical RG flow is picked, there is very little fine tuning that takes place along it

THANK YOU

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