Partial Wave Amplitude Basis and Selection Rules in EFT

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M-Y Jiang, J. Shu., M-L. Xiao, Y-H Zheng, Phys.Rev.Lett. 126 (2021) 1, 011601

Outlook

- Initial start up of partial wave amplitude basis.
- Selecting the channel J.
- Selection rules: Renormalization.
- Selection rules: Vanishing loops.
- Outlook

The Partial Wave Amplitude Basis

Conventional, we use tensor rep in the multi-particle scattering

$$|\Psi\rangle_{\otimes} = |p_1 s_1 \sigma_1, \ldots, p_N s_N \sigma_N\rangle.$$

Can also be decomposed into different j, due to the angular momentum conservation, the S-matrix is block diagonal.

Irreducible rep of definite j: $|\Psi_N\rangle_j = |P, j, \sigma, a, \{n\}\rangle = |\chi^I, \tilde{\chi}^J, a, \{n\}\rangle,$

We define the Poincare CG coefficient as the overlap functions:

$${}_{\otimes}\langle\Psi|\Psi\rangle_j \equiv C^{Pj\sigma,a}_{p_1 s_1 \sigma_1, \ldots, p_N s_N \sigma_N} \delta^{(4)}(P - \sum_i p_i).$$

Projection on a given j

The Partial Wave Amplitude Basis

Convert into a function of helicity spinors:

$$p_i s_i \sigma_i \to (\lambda_i, \tilde{\lambda}_i), \; Pj\sigma \to (\chi, \tilde{\chi}),$$

$$C^{P,j,\sigma,a}_{\{p_i,s_i,\sigma_i\}^N} \equiv \bar{C}^{j,a}_{\{s_i\}^N}(\{\lambda_i, \tilde{\lambda}_i\}^N, \chi, \tilde{\chi}).$$

$$\bar{C}^{j,a}_{\{s_i\}^N} = f^{j,a;\{\alpha_1,\ldots,\alpha_{2j}\}}_{\{s_i\}^N} \chi^{I_1}_{\alpha_1} \cdots \chi^{I_{2j}}_{\alpha_{2j}},$$

Consider the general two massless particle state,
its CG coefficient is like an amplitude for two massless particles
with an "auxiliary" massive spin-j particle:

$$C^j_{h_1,h_2} \sim \frac{[12]^{j+h_1+h_2}}{s^{(3j+h_1+h_2)/2}} (\langle 1\chi\rangle^{j-h_1+h_2} \langle 2\chi\rangle^{j+h_1-h_2}),$$

N. Arkani-Hamed, T.-C. Huang, Y-T, Huang, arxiv:1709.04891

The Partial Wave Amplitude Basis

Partial wave expansion of the N->M amplitude

$$\mathcal{A}(\{p_i,\sigma_i,n_i\}^N;\{p_i',\sigma_i',n_i'\}^M) \equiv {}_{\otimes}\langle\Psi_M|\mathcal{M}|\Psi_N\rangle_{\otimes}$$

$$= \sum_{|\Psi_N\rangle_j,|\Psi_M\rangle_j} {}_{\otimes}\langle\Psi_M|\Psi_M\rangle_j \times {}_j\langle\Psi_M|\mathcal{M}|\Psi_N\rangle_j \times {}_j\langle\Psi_N|\Psi_N\rangle_{\otimes}$$

$$= \sum_{\substack{j,\sigma,a,\\ j',\sigma',b}} C^{Pj'\sigma',b}_{\{p_i's_i'\sigma_i'\}^M} \times \mathcal{M}^j_{ab}(P^2)\delta_{jj'}\delta_{\sigma\sigma'} \times (C^{Pj\sigma,a}_{\{p_is_i\sigma_i\}^N})^*$$

$$= \sum_{j,a,b} \mathcal{M}^j_{ab}(s) \sum_{\sigma} C^{Pj\sigma,b}_{\{p_i's_i'\sigma_i'\}^M} (C^{Pj\sigma,a}_{\{p_is_i\sigma_i\}^N})^*$$

$$= \sum_{j,a,b} \mathcal{M}^j_{ab}(s)\mathcal{B}^{j,a\to b}_{\{s_i\}^N\to\{s_i'\}^M}.$$

Amplitude Basis

Information from external legs for a given j

The Partial Wave Amplitude Basis

$$\mathcal{B}^{j,a}_{N\to M} \equiv \sum_{j_z} C^{j,j_z}_{M} (C^{j,j_z}_{N})^* = f^j(\psi_{1,\dots,M}) \cdot (\chi^I \tilde{\chi}_I)^{2j} \cdot f^j(\psi_{1,\dots,N})^*.$$

$$\mathcal{B}^{j,a\to b}_{\{s_i\}^N \to \{s'_i\}^M} = (-\sqrt{s})^{2j} f^{j,b}_{\{s_i\}^N} (f^{j,a}_{\{s'_i\}^M})^*.$$

For the 2->2 process, in the CoM frame, we get the classic Wigner d-matrices:

$$\mathcal{B}^{j}_{\{h_1,h_2\}\to\{h_3,h_4\}} \sim \sum_i (-1)^{-\Delta+h_3+h_4-i} w_i \times \left[\cos\frac{\theta}{2}\right]^{2j-\Delta+\Delta'-2i} \left[\sin\frac{\theta}{2}\right]^{\Delta-\Delta'+2i} \sim d^j_{\Delta,\Delta'}(\theta), \quad (13)$$

Can give the partial wave operator basis in the amplitude operator correspondence.

Counting the J in a given channel

A systematic method to get the angular momentum of a given channel is through Poincare algebra in the spin-helicity representation:

In QM, we use J^2 to obtain the angular momentum of wave function (non-relativistic)

In relativistic system, however, we need to use the Pauli-Lubanski operator:

$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$$

which induce a Casimir invariant W^2 whose

eigenvalue $-P^2 j(j+1)$

Counting the J in a given channel

Poincare algebra in the spin-helicity representation:

$$M_{\mathcal{I},\alpha\beta} = i\sum_{i\in\mathcal{I}}(\lambda_{i\alpha}\frac{\partial}{\partial\lambda_i^\beta} + \lambda_{i\beta}\frac{\partial}{\partial\lambda_i^\alpha}),$$
$$\tilde{M}_{\mathcal{I},\dot\alpha\dot\beta} = i\sum_{i\in\mathcal{I}}(\tilde\lambda_{i\dot\alpha}\frac{\partial}{\partial\tilde\lambda_i^{\dot\beta}} + \tilde\lambda_{i\dot\beta}\frac{\partial}{\partial\tilde\lambda_i^{\dot\alpha}}).$$

Lorentz generator

$$M_{\mu\nu}\sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} = \epsilon_{\alpha\beta}\tilde{M}_{\dot\alpha\dot\beta} + M_{\alpha\beta}\bar\epsilon_{\dot\alpha\dot\beta}.$$

The Casimir invariant takes the form:

$$W_{\mathcal{I}}^2(\mathcal{B}) = \frac{P_{\mathcal{I}}^2}{8}\left(\mathrm{Tr}\,\tilde{J}_{\mathcal{I}}^2(\mathcal{B}) + \mathrm{Tr}\,J_{\mathcal{I}}^2(\mathcal{B})\right) - \frac{1}{4}\mathrm{Tr}\left(P_{\mathcal{I}}^{\mathsf{T}} J_{\mathcal{I}}(\mathcal{B}) P_{\mathcal{I}} \tilde{J}_{\mathcal{I}}(\mathcal{B})\right)$$

Example: counting j

Consider the dim 5 operator: $\psi_1\phi_2\psi_3\phi_4.$

$\mathcal{B} = \langle 13\rangle$ channel $\{1,2\} \to \{3,4\}$

$\tilde{M}\langle 13\rangle = 0$ Only the 2nd term contribute

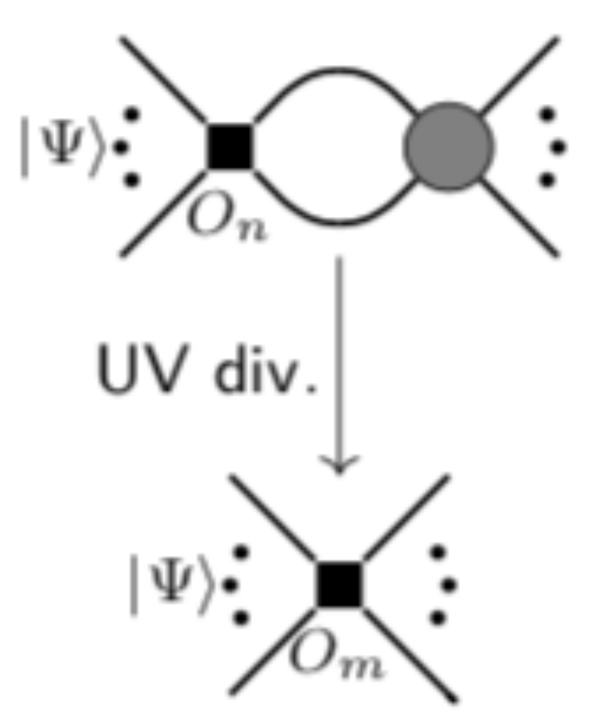
$$\begin{aligned}(M^2_{\{1,2\}})_\alpha{}^\beta\langle 13\rangle &= iM_{\{1,2\},\alpha}{}^\gamma\left(|1\rangle_\gamma\langle 3|^\beta + |3\rangle_\gamma\langle 1|^\beta\right)\\ &= 4|1\rangle_\alpha\langle 3|^\beta - \langle 13\rangle\delta^\beta_\alpha,\end{aligned}$$

$$W^2_{\{1,2\}}\langle 13\rangle = \frac{(p_1+p_2)^2}{8}\left[4\langle 31\rangle - 2\langle 13\rangle\right] = -\frac{3}{4}s\langle 13\rangle.$$

j(j+1): j=1/2

In general one has to act on the amplitude and expand over the eigen-basis.

Selection Rule: renormalization



$$\dot{C}_m = (4\pi)^{-2}\gamma_{mn}C_n$$

$$16\pi^2 \mathcal{A}^{\text{1-loop}}_{\text{UV}} = -\Big(\sum_{m,n}\gamma_{mn}C_n\mathcal{B}_m + \mathcal{A}'\Big)\frac{1}{\epsilon},$$

Further generalized the case by

C. Cheung and C.-H. Shen, Phys. Rev. Lett. **115**, 071601 (2015), arXiv:1505.01844 [hep-ph].

See also the anomalous dimension calculation from partial waves

P. Baratella, C. Fernandez, B. von Harling, A. Pomarol., arxiv: 2010.13809

O_m &
O_n share

Selection Rule

$\lvert\Psi\rangle$	$j=0$	$j=1/2$	$j=1$
F^+F^+	$F^2\phi^2(2,6)$		
$F^+\psi^+$		$F\psi^2\phi(2,6)$	
$F^+\phi$			$F\psi^2\phi(2,6)$ $F^2\phi^2(2,6)$
$\psi^+\psi^+$	$\psi^4(2,6)$ $\psi^2\bar{\psi}^2(4,4)$ $\psi^2\phi^3(4,6)$		$\psi^4(2,6)$ $F\psi^2\phi(2,6)$
$\psi^+\psi^-$			$\psi\bar{\psi}\phi^2D(4,4)$
$\psi^+\phi$		$\psi^2\phi^3(4,6)$ $F\psi^2\phi(2,6)$ $\psi\bar{\psi}\phi^2D(4,4)$	
$\phi\phi$	$\phi^4D^2(4,4)$ $\psi^2\phi^3(4,6)$ $\phi^6(6,6)$		$\phi^4D^2(4,4)$ $\psi\bar{\psi}\phi^2D(4,4)$

the (anti-)holomorphic weights $(w,\bar{w})$

Dim 6 operator: classified by the angular momentum:

Can not renormalize each other

Selection Rule: renormalization

$\lvert\Psi\rangle$	$j=0$	$j=1/2$	$j=1$
F^+F^+	$F^2\phi^2(2,6)$		
$F^+\psi^+$		$F\psi^2\phi(2,6)$	
$F^+\phi$			$F\psi^2\phi(2,6)$ $F^2\phi^2(2,6)$
$\psi^+\psi^+$	$\bar{L}e\bar{Q}u$ $\bar{L}eH\lvert H\rvert^2$		$\bar{L}\tau^i e\bar{Q}\tau^i u$ $W^i_{\mu\nu}\bar{L}\tau^i\sigma^{\mu\nu}eH$
$\psi^+\psi^-$			$\psi\bar{\psi}\phi^2 D(4,4)$
$\psi^+\phi$		$\psi^2\phi^3(4,6)$ $F\psi^2\phi(2,6)$ $\psi\bar{\psi}\phi^2 D(4,4)$	
$\phi\phi$	$H^4D^2(?)$ $\bar{L}eH\lvert H\rvert^2$		$H^4D^2(?)$ $\mathcal{O}^1_{Hl},\mathcal{O}^3_{Hl}$

$$\mathcal{O}^1_{Hl} = (\bar{L}\gamma^\mu L)(iH^\dagger \overleftrightarrow{D}_\mu H), \quad \mathcal{O}^3_{Hl} = (\bar{L}\tau^i\gamma^\mu L)(iH^\dagger\tau^i \overleftrightarrow{D}_\mu H)$$

Further generalized the case by

C. Cheung and C.-H. Shen, Phys. Rev. Lett. **115**, 071601 (2015), arXiv:1505.01844 [hep-ph].

Selection Rule: renormalization

Further generalized when combined with gauge charges (isospin I)

$$\mathcal{L}_{\text{SMEFT}} \supset C_{HD}|H^\dagger D_\mu H|^2 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H),$$
$$\mathcal{A}(H_1, H_2^\dagger, H_3, H_4^\dagger) = [3C_{H\Box}T^{I=0} + (C_{HD} - C_{H\Box})T^{I=1}](\mathcal{B}^{j=0}_{12\to 34} = s_{12})$$
$$+ [(2C_{H\Box} + 2C_{HD})T^{I=0} + 2C_{H\Box}T^{I=1}](\mathcal{B}^{j=1}_{12\to 34} = s_{13} - s_{14}).$$

j=1 channel

$$\dot{C}^1_{Hl} \propto C_{HD} + C_{H\Box}, \quad \dot{C}^3_{Hl} \propto C_{H\Box}.$$

The case for dim 8 operators are in general not diagonal

Simple example: $(\psi, \bar{\psi})$ (allowed by helicity non-renormalization)

$F\bar{F}\psi\bar{\psi}D$ operators have $j = 2$

$\psi\bar{\psi}\phi^4 D$ have $j = 1.$

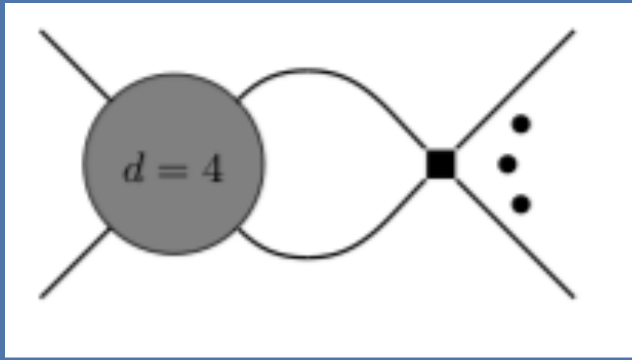
LHS (Δh)	fields in loop	RHS	EFT operators at RHS
F^+F^- (2)	$\phi\phi$	$\phi\phi$	$\phi^4(0)$, $\phi^4 D^2(0, 1)$, $\phi^4 D^4(0, 1, 2)$
		$\psi^+\psi^-$	$\psi\bar{\psi}\phi^2 D^3(1)$, $\psi\bar{\psi}\phi^2 D^3(1, 2)$
		F^+F^+	$F^2\phi^2(0)$, $F^2\phi^2 D^2(0, 1)$
	$\psi\bar{\psi}$	$\phi\phi$	$\psi\bar{\psi}\phi^2 D(1)$, $\psi\bar{\psi}\phi^2 D^3(1, 2)$
		$\psi^+\psi^-$	$\bar{\psi}^2\psi^2(1)$, $\bar{\psi}^2\psi^2 D^2(1, 2)$
		F^+F^+	$F^2\psi\bar{\psi}D(1)$
$F^+\psi^-$ (3/2)	$\bar{\psi}\phi$	$\psi^-\phi$	$\psi\bar{\psi}\phi^2 D(1/2)$, $\psi\bar{\psi}\phi^2 D^3(1/2, 3/2)$
		$F^+\psi^+$	$F\psi^2\phi(1/2)$, $F\psi^2\phi D^2(1/2, 3/2)$
$F^+\phi$ (1)	$\psi\psi$	$\psi^\pm\psi^\pm$	$\bar{\psi}^2\psi^2(0)$, $\psi^4(0,1)$, $\bar{\psi}^2\psi^2 D^2(0,1)$, $\psi^4 D^2(0, 1, 2)$
$\psi^+\psi^-$ (1)	$\phi\phi$	$F^\pm F^\pm$	$F^2\phi^2(0)$, $F^2\phi^2 D^2(0, 1)$
	FF	$\phi\phi$	$F^2\phi^2(0)$, $F^2\phi^2 D^2(0, 1)$
		$F^\pm F^\pm$	$F^2\bar{F}^2(0)$, $F^4(0, 1, 2)$

le: vanishing loops

Along the momentum direction: j >=j_p

$$j \geq j_z = s_z = \Delta h = |h_1 - h_2|$$

Similar arguments applied to Weinberg-Witten theorem



The red one are those j are forbidden

Selection Rule: vanishing loops

Case of dim 6 examples also studied in

"absence of rational term" N. Craig, M. Jiang, Y.-Y. Li, D. Sutherland., arxiv: 2001.00017

Dim 8 examples:

$F^2\phi^2 D^2$ to $\mathcal{A}(\phi\phi F^+ F^-)$ $F^2\bar{F}^2$ to $\mathcal{A}(\psi^+\psi^- F^\pm F^\pm)$.

Selection of particular combinations in the j-basis

H^4D^4 type operators contribute to the amplitude $\mathcal{A}(B^+B^-H^\alpha H^{\dagger\beta})$

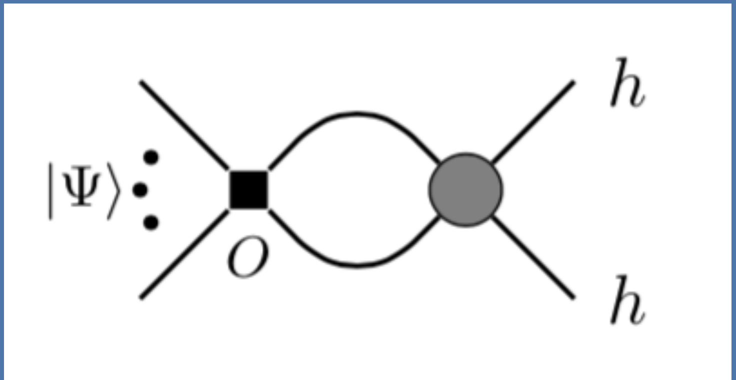
Only j=2, l=0

$$C^{2,0} = \frac{1}{6}(C_1^{H^4D^4} + \frac{1}{3}C_2^{H^4D^4} + C_3^{H^4D^4}),$$

Selection Rule: vanishing loops

For identical particles:

$$C^{Pj\sigma}_{p_1h_1;p_2h_2} \sim [12]^{j+h_1+h_2}(\langle 1\chi\rangle^{j-h_1+h_2}\langle 2\chi\rangle^{j+h_1-h_2})^{\{I_1\ldots I_{2j}\}}$$
$$\overset{h_1=h_2}{\rightarrow} [12]^{j+2h}(\langle 1\chi\rangle^{j}\langle 2\chi\rangle^{j})^{\{I_1\ldots I_{2j}\}}$$



j has to be even

No $\mathcal{A}(\psi^+\psi^-F^+F^+)$ from $\psi\bar{\psi}\phi^2 D$ and $\psi^2\bar{\psi}^2$,

Vanishing Loops for 2 identical particles

Outlook

- Generation to the massive case.
- Explicitly work out the 2->3(N) partial wave basis
- Real loop calculations (anomalous dimension, full results, etc).
- Operator basis classification.
- Application to other pheno related to J.

Backup

Counting the Single Particle State

The “Bridge” method

$$j = \frac{1}{2}\#(\text{bridges}).$$

Only read out the maximal