Introduction to Cosmology

Theory and recent observational results.

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Syllabous

Introduction

References

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- A.R. Liddle & D.H. Lyth: Cosmological Inflation and Large-Scale structure
- S. Dodelson: Modern Cosmology
- P.J.E. Peebles: Principles of physical cosmology, Princeton University Press, 1993
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Physical constants and parameters

Paremeters and units

Reduced Planck constant	h= 1.055×10^{-27} cm^2.g.s^{-1}
Speed of light	c = 2.998×10^{10} cm.s^{-1}
Newton's constant	G = 6.672×10^{-8} cm^3.g^{-1}.s^{-2}
Reduced Planck mass	$M_{Pl} = 4.342 \times 10^{-6}$ g
	$= 2.436 \times 10^{18}$ GeV/c^2
Planck mass	$m_{Pl} = \sqrt{8\pi} M_{Pl} = 2.177 \times 10^{-5}$ g
Reduced Planck length	$L_{Pl} = 8.101 \times 10^{-33}$ cm
Reduced Planck time	$T_{Pl} = 2.702 \times 10^{-43}$ s
Boltzmann constant	$k_B = 1.381 \times 10^{-16}$ erg.K^{-1}
Thomson cross section	$\sigma_T = 6.652 \times 10^{-25}$ cm^2
Electron mass	$m_e = 0.511$ MeV/c^2
Neutron mass	$m_n = 939.6$ MeV/c^2
Proton mass	$m_p = 938.3$ MeV/c^2
Solar mass	$M_\odot = 1.99 \times 10^{33}$ g
Megaparsec	1 Mpc $= 3.086 \times 10^{24}$ cm
1 cm	$= 5.086 \times 10^{13}$ GeV^{-1}.h
1 s	$= 1.519 \times 10^{24}$ GeV^{-1}.h/c
1 g	$= 5.608 \times 10^{25}$ GeV/c^2
1 erg	$= 6.242 \times 10^{2}$ GeV
1 K	$= 8.618 \times 10^{-14}$ GeV/k_B

Parameters

Hubble constant	$H_0 = 100\, h$ km.s^{-1}.Mpc^{-1}
Present Hubble distance	$cH_0^{-1} = 2998 h^{-1}$ Mpc
Present Hubble time	$H_0^{-1} = 9.78\, h^{-1}$ Gyr
Present critical density	$\rho_{c,0} = 1.88\, h^2 \times 10^{-29}$ g.cm^{-3}
	$= 2.775\, h^2 \times 10^{11}\, M_\odot / (\mathrm{Mpc})^3$
	$= \left(3.000 \times 10^{-3} \mathrm{eV}/c^2\right)^4 h^2$
Present photon density	$\Omega_{\gamma,0}\, h^2 = 2.48 \times 10^{-5}$
Present relativistic density	$\Omega_{R,0}\, h^2 = 4.17 \times 10^{-5}$
Baryon-to-photon ratio	$\eta = 2.68 \times 10^{-8}\, \Omega_b\, h^2$
Matter-radiation equality	$1 + z_{\rm eq} = 24000 \Omega_0\, h^2$
Hubble length at equality	$\left(a_{\rm eq} H_{\rm eq}\right)^{-1} = 14 \Omega_0^{-1}\, h^{-2}$ Mpc
Top-hat filter/$10^{12}\, M_\odot$	M(R) = $1.16\, h^{-1} \left(R / 1 h^{-1} \mathrm{Mpc}\right)^3$
Gaussian filter/$10^{12}\, M_\odot$	M(R) = $4.37\, h^{-1} \left(R / 1 h^{-1} \mathrm{Mpc}\right)^3$

Cosmology in a nutshell

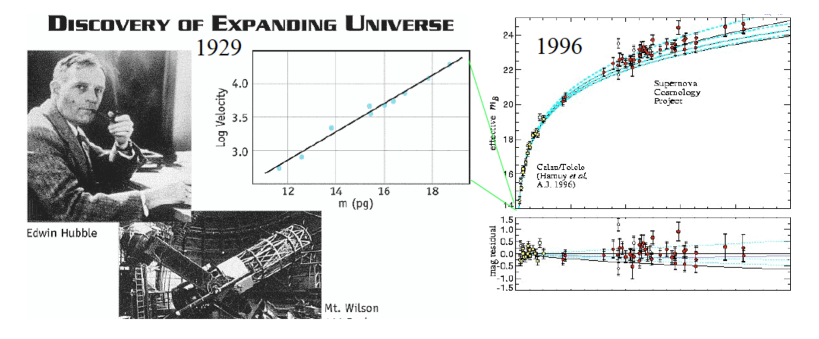
- Cosmology studies the formation and evolution of the universe as a whole in order to explain its origin, its current status and its future.
- Philosophy and religion were originally the main path to the understanding of the universe and their properties.
- Nowadays cosmology studies are mainly based on physical theories: general relativity, quantum physics, statistical physics, quantum field theory, quantum gravity, etc; mathematics: statistical description of fields and data; chemistry and biology: development of life
- Astrophysical observations of our galaxy, other external galaxies, cluster of galaxies and the Cosmic Microwave Background (CMB) are critical to understand our universe

Recent cosmology history

- 1915 Einstein. Theory of general relativity
- 1922-1927 Friedmann-Lemaitre. Expanding universe and Big Bang
- 1929 Hubble. Experimental proof of expansion of the universe
- 1933 Zwicky. First hints of dark matter problems in the Coma cluster
- 1940 Gamow. Prediction of primordial nucleosynthesis and cosmic microwave background
- 1948 Bondi, Gold & Hoyle. Stationary model
- 1965 Penzias & Wilson. CMB discovery
- 1970-1980s. Structure formation models
- 1981 Guth. Inflationary theory
- 1992 COBE satellite measures CMB anisotropies
- 1998 SNIa and accelerated expansion of the universe
- 2000s. Quintessence models for dark energy
- 2001-2018 Precision CMB cosmology with the WMAP and Planck satellites
- 2012 Deceleration of the Universe with SDSS-III BOSS

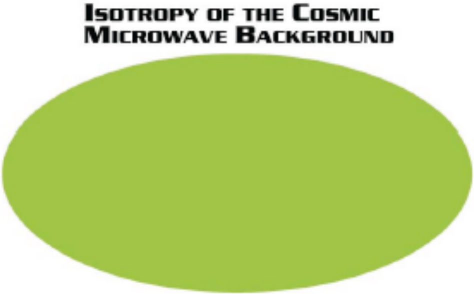
Expanding Universe and dark energy

- Hubble in 1929 measured recession velocity of galaxies and showed that universe was expanding
- In 1998 the study of the luminosity of SN Ia showed the expansion of the universe is now accelerated



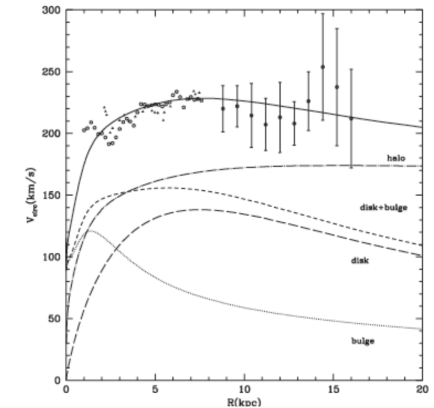
Cosmic Microwave Background

- Penzias & Wilson discovered in 1965 an isotropic and homogeneous radiation with a temperature of about 3 K as predicted by Gamow in 1940
- Te COBE satellite in 1992 showed that the CMB has a black-body spectrum and fluctuations of about 10^{-5}



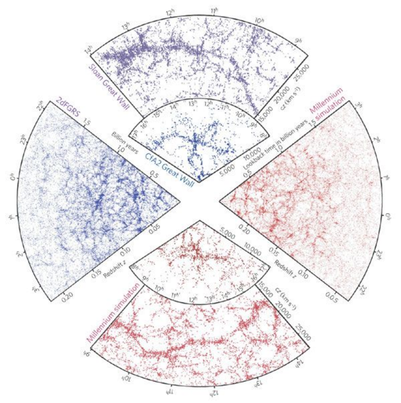
Dark matter

- Mass required to keep rotational curves flat is larger than expected from stars and gas
- In merging galaxy clusters the reconstructed matter distribution doest not peak where gas is observed



Large-scale structure in the Universe

- Galaxy surveys have shown the large-scale structure of the universe which is formed of voids, clusters of galaxies and filaments as expected from simulations
- The universe is homogeneous for scales larger than 100 Mpc



Summary of main cosmological observational facts

- Galaxy distribution
  - the universe is expanding
  - small structures form first and combine to form larger ones
- Supernovae type Ia
  - currently expansion is accelerated: dark energy
- Cosmic Microwave Background (CMB)
  - the universe is isotropic and homogeneous
  - universe fully thermalized
  - density fluctuations of the order of 10^{-5}
- Abundance of light elements
  - Light elements form first from nucleosynthesis
- Dynamics of galaxies and of cluster of galaxies
  - Evidence for extra matter component: dark matter and/or modified gravity theory

Standard Cosmological Model in a nut-shell

The standard cosmological model is based on:

1. Big Bang theory: universe expands from a hot and dense intial point and cool down
2. Λ-CDM model: describes universe energy density
3. Inflation: period of exponential expansion in the early universe

$\Longrightarrow$ primordial nucleosynthesis and CMB emission

$\Longrightarrow$ photons, neutrinos, baryon, cold (warm) dark matter, dark energy

$\Longrightarrow$ produces primordial fluctuations and solves horizon problem

Big Bang theory

FLRW cosmology

The Friedmann-Lemaitre-Robertson-Walker (FRW) cosmology is based on:

1. The cosmological principle: the universe is isotropic and homogeneous on large scales
2. General Relativity (GR) theory:
    - A metric to describe the geometry of space-time: tells matter how to move
    - Einstein field equations: matter tells geometry how to curve
3. Multi-component energy density: photons, neutrinos, baryons, non-relativistic matter, dark energy and curvature

NB: Conceptually it is useful to separate geometry and dynamics to understand alternative cosmologies, e.g.

- Break homogeneity and isotropy assumptions under GR
- Modify gravity theory while keeping the geometry

General Relativity (GR)

Based on the equivalence principle that postulates that the laws of physics take the same form in all reference frames (even those freely falling)

1. Proper time is invariant and defines the metric $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu {}^1, \; x^\mu = (cdt, dx, dy, dz)$$

   The metric defines the curvature of space-time

2. The metric evolves accondingly to Einstein field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

   where $R_{\mu\nu}$ and R are the Ricci tensor and scalar respectively, and G is the gravitational constant

3. $T_{\mu\nu}$ is the stress-energy tensor that evaluates the effect of a given distribution of mass and energy on the space-time curvature

[1] We use here the repeated symbol sum convention $\sum_{\mu=0}^{3} \sum_{\nu=0}^{3}$

Robertson-Walker metric

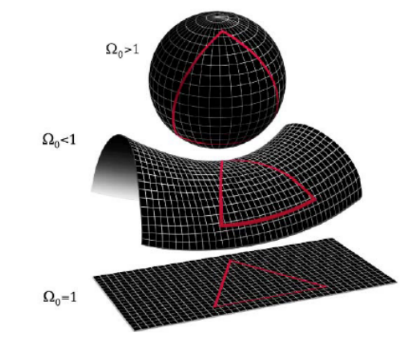
In 1930 Robertson and Walker independently showed that the most possible general metric for describing an expanding universe is

$$ds^2 = (c\ dt)^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right]$$

where (r, θ, ϕ) are spherical comoving coordinates and $a(t)$ is the scale factor

Spatial geometry given by constant curvature:

- $k = 0$ flat geometry universe
- $k = -1$ open universe
- $k = +1$ closed universe



Horizon

- Distance travelled by a photon in the whole lifetime of the universe defines the horizon
- For photons $ds = 0$, wo we have that

$$D_{\mathrm{horizon}}(t) = \int_0^t \frac{dt'}{a(t)} = \eta(t)$$

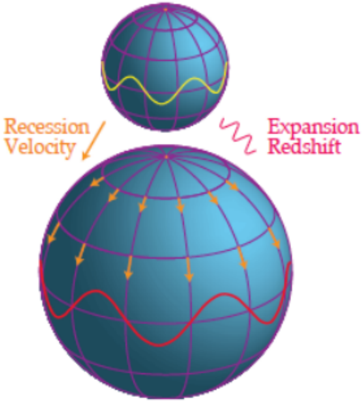
- $\eta(t)$ is also called the conformal time
- Two points in the universe are in casual contact if their distance is smaller than the horizon
- Horizon problem: why is the universe isotropic and homogeneous on large scales? Observable universe is today larger than the horizon

Redshift

- Wavelength of light stretches with the scale factor
- Given a physical rest wavelength at emission λ_o, the observed wavelength today λ is

$$\lambda = \frac{1}{a(t)}\lambda_o \equiv (1+z)\lambda_o$$

- Interpreting redshift as Doppler effect, objects recede in an expanding universe
- Today $z = 0$ and it increases back on time



Deceleration parameter and elapsed time

- Deceleration parameter q_0 is defined by

$$a(t) = a(t_0)\left[1 + H_0(t - t_0) - \frac{1}{2}H_0^2 q_0 (t - t_0)^2 + \ldots\right]$$

- Taylor expanding $a(t)$ we obtain

$$q_0 = -\frac{\ddot{a}(t_0)a(t_0)}{\dot{a}(t_0)}$$

- From above we deduce

$$1 + z = 1 + H_0(t - t_0) + H_0^2(t - t_0)^2\left[1 + \frac{q_0}{2}\right] + \ldots$$

- and inverting

$$t_0 - t = \frac{1}{H_0}\left[z - z^2\left(1 + \frac{q_0}{2}\right) + \ldots\right]$$

Cosmological distances

- Proper distance, time for a photon to go from z to $z + dz$

$$d_{pr} = -c\ dt = -c\frac{da}{\dot{a}}$$

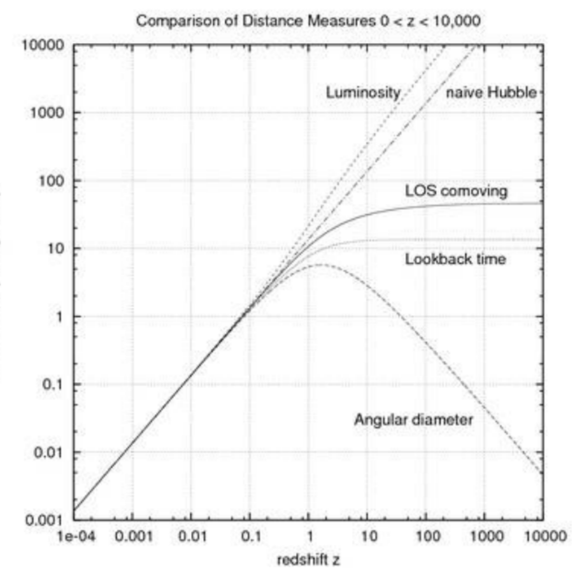
- Comobile distance between observer at z and emitter at $z + dz$

$$d_{com} = -c\ \frac{dt}{a} = -c\frac{da}{\dot{a}a}$$

- Luminosity distance, d_L, such that the observed flux, ℓ of a source of absolute luminosity L is $\ell = \frac{L}{4\pi d_L}^2$
- Diameter angular distance, relates angular size $\Delta\theta$ and physical size, D, of a source

$$d_A = \frac{D}{\Delta\theta} = \frac{d_L}{(1+z)^2}$$

Cosmological distances



Friedmann-Lemaitre equations

Apply the Einstein field equations to the R-W metric

$$G_{\mu\nu} = -8\pi T_{\mu\nu}$$

1. From the LHS we obtain

$$G_0^0 = -\frac{3}{a^2}\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2}\right]$$

$$G_j^i = -\frac{1}{a^2}\left[2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2}\right]$$

2. for the RHS isotropy demands that

$$T_0^0 = \rho$$

$$T_j^i = -p\ \delta_j^i$$

where ρ is the energy density and p is the pressure

Dynamics of the universe

- Finally the FL equations stand

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2} = \frac{8\pi G}{3} a^2 \rho$$

$$2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{R^2} = -8\pi G a^2 p$$

- and can be combined into a single one

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -\frac{4\pi G}{3}\left(\rho + 3p\right) = a\frac{d^2 a}{dt^2}$$

Curvature and critical density

- The first FL equation can be written as

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \rho_k) \equiv \frac{8\pi G}{3}\rho_c$$

- ρ_c is the critical density and its value today is

$$\rho_c(z=0) = \frac{3H_0^2}{8\pi G} = 1.8788 \times 10^{-29}\ h^2 gcm^{-3}$$

- we can define curvature as an effective energy density component

$$\rho_k = -\frac{3}{8\pi G a^2 R^2}$$

Total energy density

- Energy density today can be given as a fraction of the critical density

$$\Omega_{tot} \equiv \frac{\rho}{\rho_c(z=0)}$$

- Note that physical energy density is $\propto \Omega h^2$ (g cm^{-3})
- Likewise the radius of curvature is given by

$$\Omega_k = (1 - \Omega_{tot}) = \frac{1}{H_0^2 R^2} \rightarrow R = \left(H_0 \sqrt{\Omega_{tot} - 1}\right)^{-1}$$

- Ω value defines universe geometry
  - $\Omega_{tot} = 1$, flat universe
  - $\Omega_{tot} > 1$, posetively curved
  - $\Omega_{tot} < 1$, negatively curved

The multi-component universe

- We define the equation of state as $p = w\rho$
- Universe consists of multiple components:
- total energy density summed over all components

1. NR matter $\rho_m = mn_m \propto a^{-3}$, $w_m = 0$
2. R radiation $\rho_r = En_r \propto \nu n_r \propto a^{-4}$, $w_r = 1/3$
3. curvature $\rho_k \propto a^{-2}$, $w_k = -1/3$
4. (cosmological) constant energy density $\rho_\Lambda \propto a^0$, $w_\Lambda = -1$

$$\rho(a) = \sum_i \rho_i(a) = \rho_c(a = 1; z = 0) \sum_i \Omega_i \; a^{-3(1+w_i)}$$

- density evolves as

$$\rho(a) = \rho_c(a = 1; z = 0) \sum_i \Omega_i \; \exp - \int d \log a 3(1 + w_i)$$

- and the Hubble constant as

$$H^2(a) = H_0^2 \sum_i \Omega_i \; \exp - \int d \log a 3(1 + w_i)$$

General solutions of FL equations

- Radiation domination

$$H^2 \propto a^{-4}, \quad a(t) \propto t^{1/2}, \quad H(t) = \frac{1}{2t}, \quad R_H = 2ct$$

- Matter domination

$$H^2 \propto a^{-3}, \quad a(t) \propto t^{2/3}, \quad H(t) = \frac{2}{3t}, \quad R_H = \frac{3}{2}ct$$

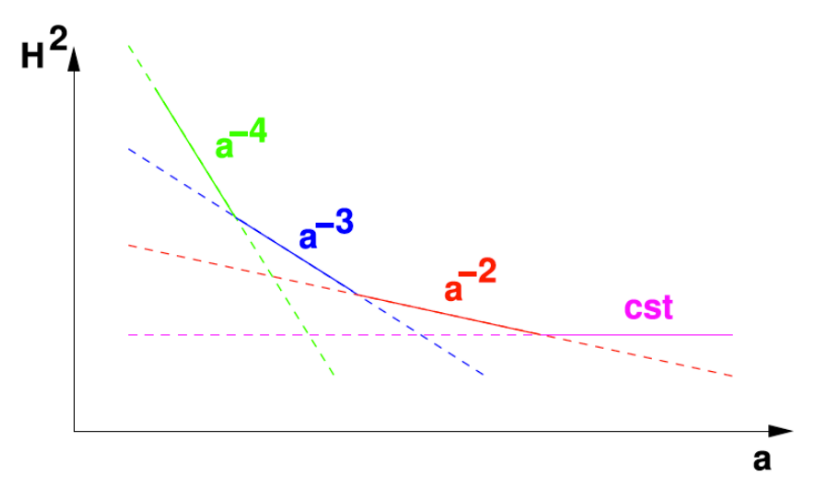
- Curvature domination $k < 0$

$$H^2 \propto a^{-2}, \quad a(t) \propto t, \quad H(t) = \frac{1}{t}, \quad R_H = ct$$

- Dark energy domination

$$H^2 \rightarrow \mathrm{constant}, \quad a(t) \propto \exp \Lambda t/3, \quad H(t) = \frac{c}{R_H} = \sqrt{\Lambda/3}$$

Hubble constant evolution



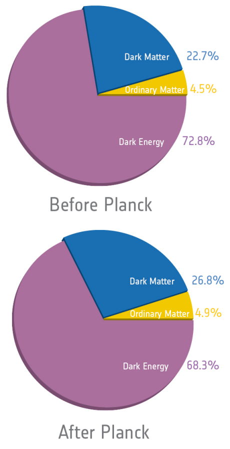
A first set of cosmological parameters and relations

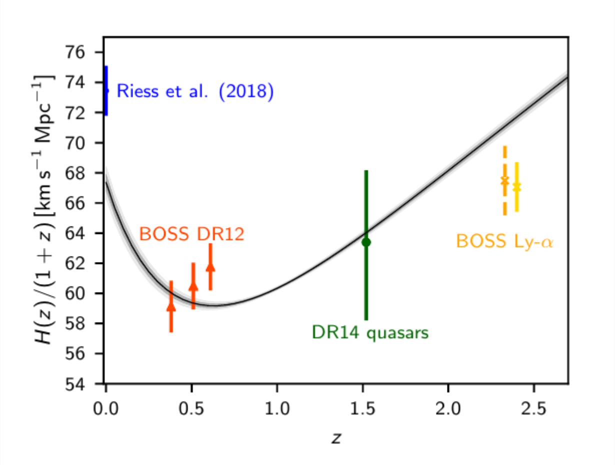
H_0 Hubble constant
Ω_k Curvature energy density
Ω_m Matter density
Ω_Λ Dark energy density
Ω_{CDM} Cold dark matter density
Ω_b Baryonic matter density
Ω_γ Photon density
Ω_ν Neutrino density

- $(1 - \Omega_k) = \Omega_{tot} = \Omega_m + \Omega_\Lambda$
- $\Omega_m = \Omega_{CDM} + \Omega_b + \Omega_\gamma + \Omega_\nu$
- $q_0 = \frac{1}{2}\Omega_m^{NR} - \Omega_\Lambda$

$$H^2(z) = H_0^2 \left(\Omega_m^R(1+z)^4 + \Omega_m^{NR}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right) = H_0^2 E(z)^2$$

First results on the cosmological parameters





Inflation

Motivations for inflation

Inflation was motivated by a set of problems encountered by Big Bang theory

- Flatness problem
  The universe is observed to be flat today to a great accuracy however the flat solution of the FL equations is unstable
- Relic abundances
  Phase transitions in the early universe will lead to relic particles like for example monopoles that are not observed today
- Horizon problem
  CMB temperature is uniform and isotropic all over the sky however regions of the sky separated by more than one degree were not in casual contact at the time of CMB formation
- Origin of cosmological fluctuations
  All observed structures in the universe were formed by the growth up of primordial fluctuations for which we have no explanation

Accelerated expansion

- To solve the horizon, flatness and relics' problems we need

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \Longrightarrow \rho + 3p < 0$$

- So acceleration implies negative pressure $p < 1/3\rho$
- We define the number of e-folds as

$$N = \ln \frac{a_i}{a_f}$$

  where a_i and a_f correspond to the scales factors at the beinning and end of the accelerated expansion period
- Notice that N represents some how the amount expansion
- To solve the horizon, flatness and relics problems we need $N \geq 60$

Scalar fields in cosmology

- For a FRWL universe the dynamics of a scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\Delta^2\phi}{a^2} + V'(\phi) = 0$$

- Assuming $\phi = \phi_0 + \delta\phi$ with $\frac{\delta\phi}{\phi_0} << 1$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{(\Delta\phi)^2}{2a^2} + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{(\Delta\phi)^2}{6a^2} - V(\phi)$$

- And we can write the FL equation for the field as

$$H^2 = \frac{8\pi G}{3}\rho_\phi - \frac{k^2}{2} \sim \frac{8\pi G}{3}\rho_\phi$$

Slow roll dynamics

- We can obtain accelerated expansion of the universe, $p_\phi = -\rho_\phi$, if
  1. We neglect the $\frac{\Delta\phi}{a^2}$ term
  2. We assume $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll 3H\dot{\phi}$
- Thus, the scalar field dynamic equations are:

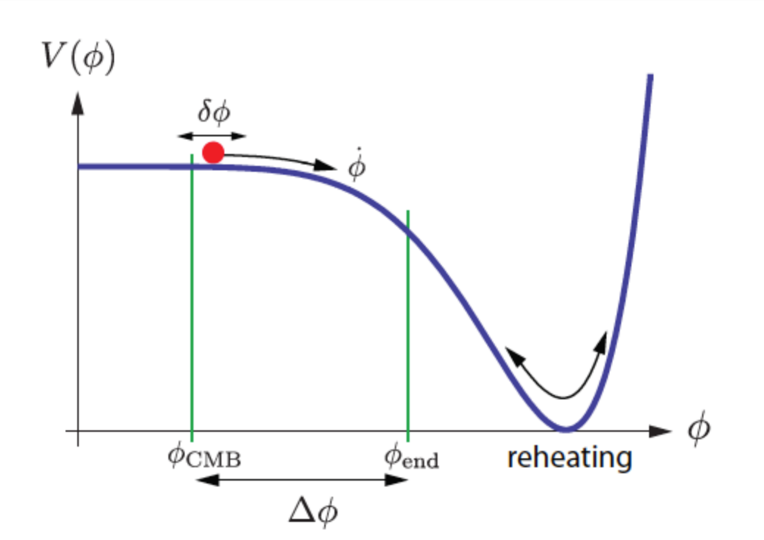
$$H^2 \simeq \frac{8\pi G}{3} V(\phi)$$

$$3H\dot{\phi} + V' \simeq 0$$

- Net energy is dominated by potential energy and thus acts like a cosmological constant $w \to -1$
- We are in the so called slow-roll conditions corresponding to a very flat potential
- We define the slow roll parameters as: $\epsilon = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2$, $\eta = \frac{1}{8\pi G}\left(\frac{V''}{V}\right)$ and $\delta \equiv \epsilon - \eta^2$
- For slow roll conditions ϵ, δ, $|\eta| \ll 1$

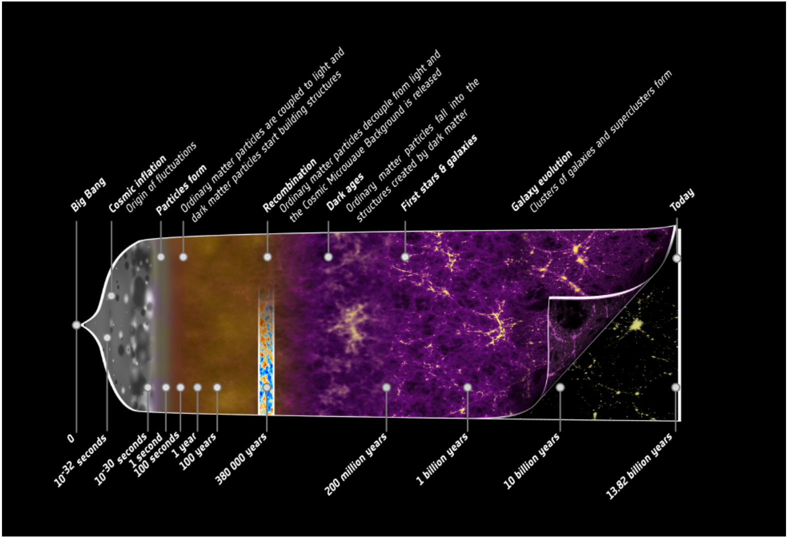
[2] We define the reduced Planck mass as $M_P = \frac{1}{8\pi G}$

Potential slowly rolling down



Thermal history of the Universe

Cartoon thrmal history of the universe



Detailed thermal history of the universe

Event	T(K)	kT (eV)	g_{eff}	z	t
Now	2.76	0.0002	3.43	0	13.6 Gyr
First Galaxies	16	0.001	3.43	6 (?)	$\sim$ 1 Gyr
Recombination	3000	0.3	3.43	1100	380000 yr
M-R equality	9500	0.8	3.43	3500	50000 yr
e^+-e^- pairs	$10^{9.7}$	0.5×10^6	11	$10^{9.5}$	3 s
Nucleosynthesis	10^{10}	10^6	11	10^{10}	1 s
Nucleon pairs	10^{13}	10^9	70	10^{13}	10^{-7} s
E-W unification	$10^{15.5}$	25×10^{10}	100	10^{15}	10^{-12} s
GUT	10^{28}	10^{24}	100 (?)	10^{28}	10^{-38} s
Quantum Gravity	10^{32}	10^{28}	100 (?)	10^{32}	10^{-43} s

Eras: radiation, matter and dark energy

- The energy density of radiation, matter and dark energy (DE) evolves differently

$$\text{radiation: } \rho_R \propto a^{-4}$$
$$\text{matter: } \rho_M \propto a^{-3}$$
$$\text{DE: } \rho_\Lambda = constant$$

- So, the total density of the universe can be written as

$$\rho = \rho_c \left(\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda\right);\ x = 1 + z$$

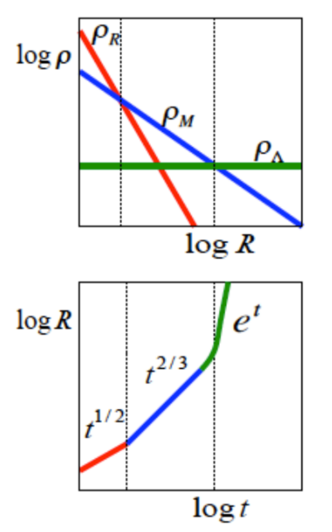
- Matter-radiation equality is obtained when $\rho_M = \rho_R$ at

$$z = \frac{\Omega_M}{\Omega_R} - 1 \sim 3402$$

- Matter-DE equality when $\rho_M = \rho_\Lambda$ at

$$z = \left(\frac{\Omega_\Lambda}{\Omega_M}\right)^{1/3} - 1 \sim 0.29$$

Eras: radiation, matter and dark energy



Big Bang Nucleosynthesis

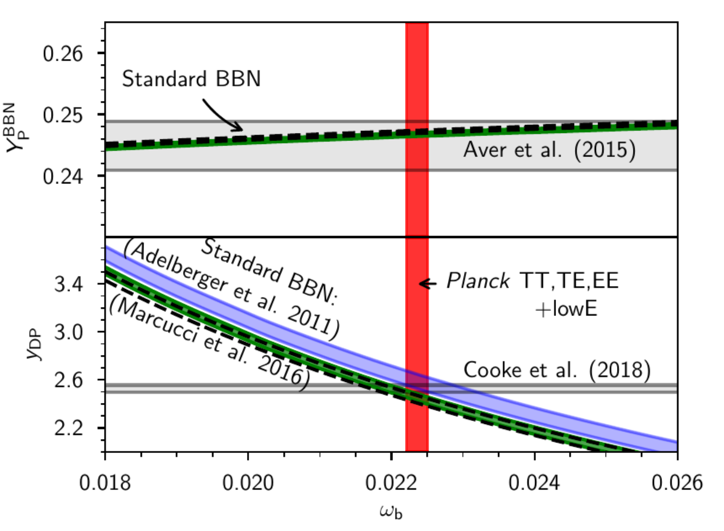
- Primordial nucleosynthesis describes the period in the history of the universe on which light elements are synthesized and then freezeout
- Primordial nucleosynthesis is given by the evolution of the population of neutrons and and protons in the first minutes of the life of the Universe
- Light element abundance depends on the baryon-photon ratio $\eta = \frac{n_B}{n_\gamma} = 2.73 \times 10^{-8} \left(\Omega_B h^2\right)$ as $n_\gamma = 411 \quad cm^3$
- The number density of baryons is well approximated by

$$n_B \sim \frac{\Omega_B \rho_c}{m_p}$$

Nucleosynthesis in a nutshell

- For $T > 1$ MeV neutrons and protons are in thermal equilibrium via the weak interactions
- For $T \sim 1$ MeV protons and neutrons decouple. Neutrons start to decay into protons very fast (in 15 minutes the whole populations would disappear)
- For $T \sim 0.1$ MeV (3 minutes) photo-disociation of D is not efficient and it survives. So neutrons are preserved.
- After D formation a series of nuclear reactions take place producing other light elements: ^{3}He, ^{4}He, ^{6}Li, ^{7}Li
- In about 100 s the chemical composition of the Universe is fixed and nucleosynthesis ends

Nucleosynthesis recent constraints



Recombination

CMB *forms* at the so-called recombination period

- For $T > 1000$ K free electrons and photons are coupled via Thompson scattering
- When temperature drops to $\sim$1000 K it is thermodynamically favorable for the plasma to form neutral atoms via

$$p + e^- \rightarrow H + \gamma$$

- It thermal equilibrium holds then number density for each species is given by

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T} \right)$$

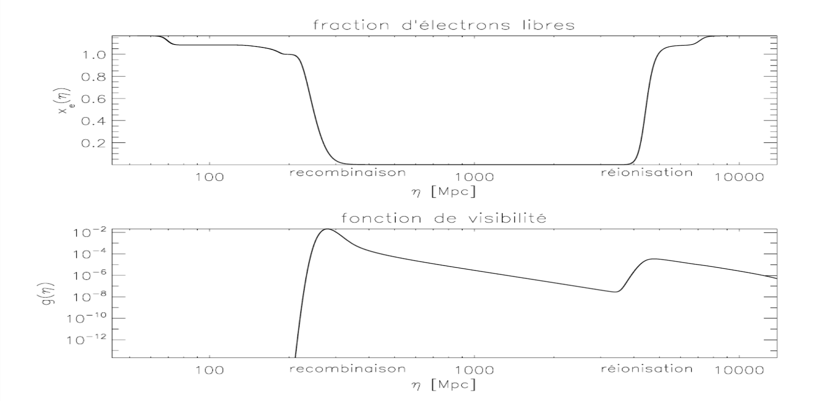
  and chemical equilibrium impose

$$\mu_e + \mu_p = \mu_H$$

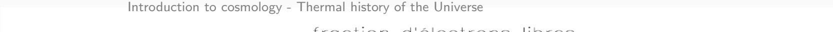
- As $m_H \sim m_p$ and definining $B_H = m_p + m_e - m_H = 13.6$ eV we have

$$n_H = \frac{f_H}{g_p g_e} n_e n_p \left(\frac{m_i T}{2\pi} \right)^{3/2} \exp\left(B_H / T \right)$$

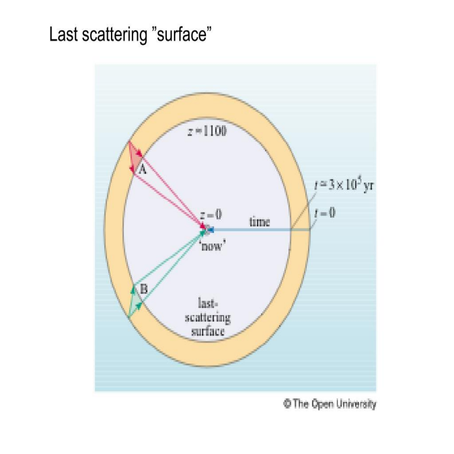
Recombination in a nutshell



- The free electron fraction x_e starts from 1 at high redshift, the universe is opaque
- At recombination, about $z \sim 1080$, x_e decreases sharply and freezes at a very small value
- After recombination the universe is transparent, CMB can be observed
- At reionization all electrons are free again but because of dilution their effect on CMB is much smaller



Last scattering surface

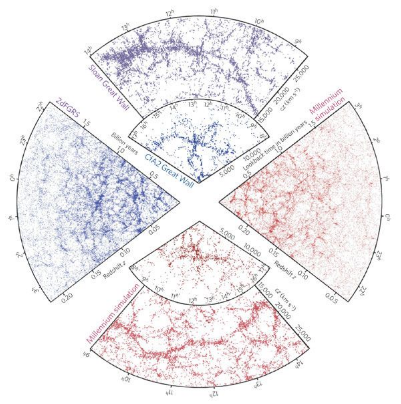


- Interaction between electrons and photons via Thomson scattering before recombination and after reionization
- Angular distribution of radiation is the 3D temperature field projected onto a shell - surface of last scattering
- Integrate along the line of sight in an expanding universe
- Describe radiation as an statistically isotropic temperature field with fluctuations

Large scale structure

Large-scale structure in the Universe

- Galaxy surveys have shown the large-scale structure of the universe which is formed of voids, clusters of galaxies and filaments as expected from simulations
- The universe is homogeneous for scales larger than 100 Mpc



Large scale structure

Linear perturbation theory

Inhomogeneous Universe

- Inflationary theory predicts small curvature and tensor fluctuations in the early universe that will transform in matter-engergy fluctuations
- Inhomogeneities in the matter-energy distribution grow via gravitational instability
- We consider small perturbations to the homogenous Universe: Linear perturbation theory
- As for the homogeneous case, the evolution of the perturbations will be different for the different species

Inhomogeneous fields and the Boltzmann Equation

- As for homogeneous cosmology, a full description of matter is given through the phase space distribution $f(\vec{x}, \vec{q}, t)$ from which $\rho(\vec{x}, t)$ and $p(\vec{x}, t)$ can be defined
- Evolution of density inhomogeneities is governed by the Boltzmann equation that in comoving coordinates and conformal time can be written as

$$f' + \vec{q}' \frac{\partial f}{\partial \vec{q}} + \vec{x}'.\frac{\partial f}{\partial \vec{x}} = C(f)$$

  where $'$ corresponds to derivative with respect to conformal time and $C(f)$ is the collision term
- The collision encoders interaction between particlesm and will be important for photons, baryons and

Summary of homogeneous and isotropic universe results

- We have perfect fluids such that $p = w\rho$
- Energy conservation

$$\dot{\rho}a^3 + 3(\rho + p)\dot{a}a^2 = 0$$

- FL equation

$$H^2 = \frac{8\pi G}{3}\rho$$

- Solution of the FL equations

	w	$\rho(a)$	$a(t)$	$H(t)$
radation	1/3	a^4	$t^{1/2}$	$\frac{1}{2}t^{-1}$
matter	0	a^3	$t^{2/3}$	$\frac{2}{3}t^{-1}$
dark energy	-1+ϵ	H_0	$e^{H_0 t}$	H_0

Linear perturbation theory I

- We assume perturbations are small enough to be in the linear regime so for example
$$\rho(\vec{x},t) = \rho_0(t) + \delta\rho(\vec{x},t)$$
where $\rho_0(t)$ is the background density (homogeneous like)
- We often define constrast quantities as for example the density constrast $\delta_\rho = \frac{\delta\rho(\vec{x},t)}{\rho_0(t)}$
- The evolution of the background is given by the homogeneous solution
- Linear perturbation must be applied to the metric and the stress-energy tensor
$$g_{\mu\nu} = g_{\mu\nu}^{RW}(t) + \delta g_{\mu\nu}(\vec{x},t)$$
$$T_{\mu\nu} = T_{\mu\nu}^{hom}(t) + \delta T_{\mu\nu}(\vec{x},t)$$

Linear perturbation theory II

- Metric perturbations can be written as:

$$ds^2 = a^2(\eta)\left[(1+2\phi)d^2\eta + B_i dx^i d\eta - \{(1-2\psi)\delta_{ij} + h_{ij}\}dx^i dx^j\right]$$

with $\sum h_{ii} = 0$

- Then, for metric perturbations we have 10 degrees of freedom: 4 scalars, 4 vectors and 2 tensors
- For the stress-energy tensor we have 10 degrees of freedom:

$$T_0^0 = -\rho_0 - \delta\rho$$

$$T_i^0 = (\rho_0 + p_0)(v_i - B_i)$$

$$T_0^i = -(\rho_0 + p_0)v_i$$

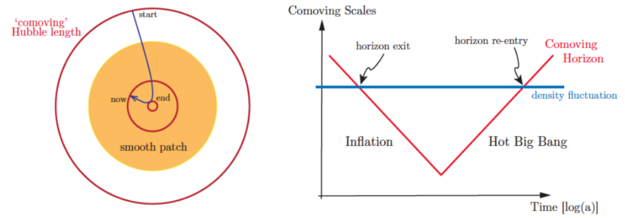
$$T_j^i = (p_0 + \delta p)\delta_j^i + p\Pi_j^i$$

where v_i is a vector velocity and Π_j^i the anisotropic stress

- In general we neglect the anisotropic stress Π_j^i and will not discuss vector perturbations as they decay

Perturbations from inflation

- After decay the inflation will produce scalar, vector and tensor perturbations that transform into matter perturbations
- After inflation perturbations can be outside and might re-enter the horizon later in the history of the universe. While outside the horizon perturbations are frozen
- We define the comoving wavelength and wave number of the perturbation as $\lambda_{com} = \frac{\lambda}{a} = \frac{2\pi}{k}$
- Perturbations inside (outside) the horizon satisfy $k > 2\pi aH$ ($k < 2\pi aH$)



Transfer function

- Perturbations are generally dealt with in the Fourier domain via their power spectrum

$$\langle A(\eta, \vec{k_1}) A^*(\eta, \vec{k_2}) \rangle = \delta(\vec{k_2} - \vec{k_1}) P_A(k)$$

- As physics is linear we can imagine a linear function such that $A(\eta, \vec{k}) = T_A(k, \eta) A(\eta_0, \vec{k}) = T_A(k, \eta) A(\vec{k})$ and so

$$P_A(\eta, k) = T_A(k, \eta)^2 P_A(k)$$

- In the case of adiabatic initial conditions we can set a comon initial perturbation using the Baardeen curvature $\mathcal{R} = \phi \frac{1}{3} \frac{\delta \rho_{tot}}{\rho_{0,tot} + p_{0,tot}}$ such that

$$P_A(\eta, k) = T_{A,\mathcal{R}}(k, \eta)^2 P_{\mathcal{R}}(k) = \frac{2\pi}{k^3} T_{A,\mathcal{R}}(k, \eta)^2 \Delta^2_{\mathcal{R}}(k)$$

- From inflation we have $\Delta^2_{\mathcal{R}}(k) = A_S \left(\frac{k}{k_*}\right)^{ns-1}$, with n_s the scalar spectral index

Large scale structure

Matter power spectrum

Matter power spectrum

- From previous results the matter power spectrum can be written as

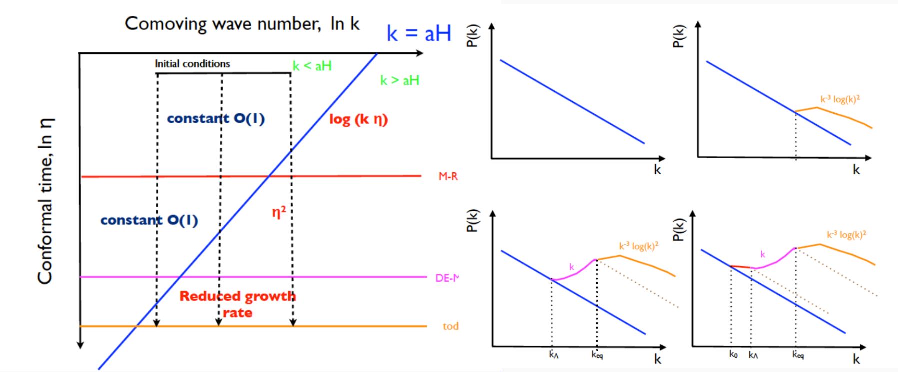
$$P(\eta, k) = \frac{2\pi}{k^3}\left(\frac{k}{k_*}\right)^{ns-1} T_{\delta m}(k,\eta)^2$$

- We can start by assuming CDM only as $\Omega_b << \Omega_{CDM}$ and so $\delta_m \sim \delta_{CDM}$
- To compute the transfer function we need to solve the Boltzmann equation but as CDM is pressureless we only need to consider the (0-0) component of the Einstein equations and the FL equations
- In practice this leads to the Mészáros equation

$$\delta''_{CDM} + \frac{a'}{a}\delta'_{CDM} - \frac{3}{2}\left(\frac{a'}{a}\right)^2 \Omega_{CDM}(a)\delta_{CDM} = 0$$

- We observe that matter clustering rate depends on the expansion rate

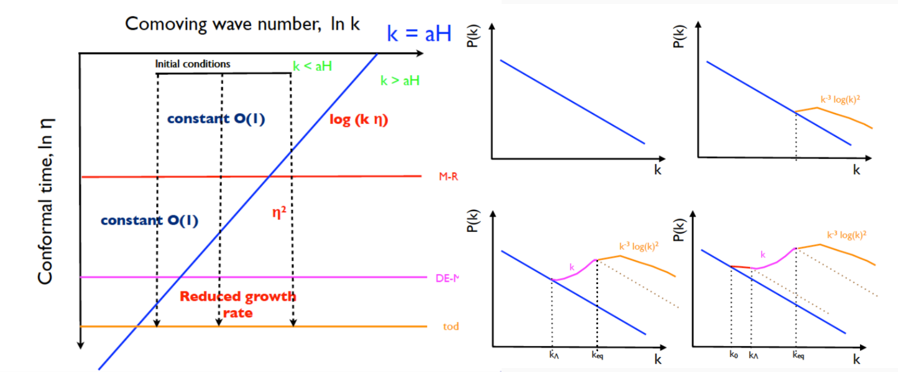
Matter power spectrum evolution



Baryon corrections to the matter power spectrum

- Baryons modify the shape of the power spectrum introducing baryon acoustic oscillations (BAO) and power suppression at $k > k_{eq}$
- BAO are produced by the Thomson interaction of photons and electrons before decoupling. The photon pressure will counter balance gravitational collapse.
- BAOs can be observed both on CMB and Large Scale Structure however the mean time of formation of the oscillations is not the same and so neither their characteristic scale.
- For CMB BAO are frozen at decoupling while for baryons they are frozen at baryon drag (last time baryons interacted)
- Full study of BAOs requires to solve the Boltzmann equation. We will do this for CMB next lecture.

Matter power spectrum evolution



Matter power spectrum measurements

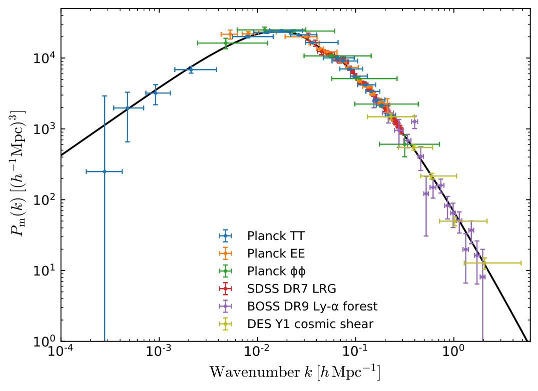
P1 The time of equality determines the peak of the spectrum.

P2 Baryon abundance (relative to CDM) determines suppression at $k > k_{eq}$ and also BAOs features

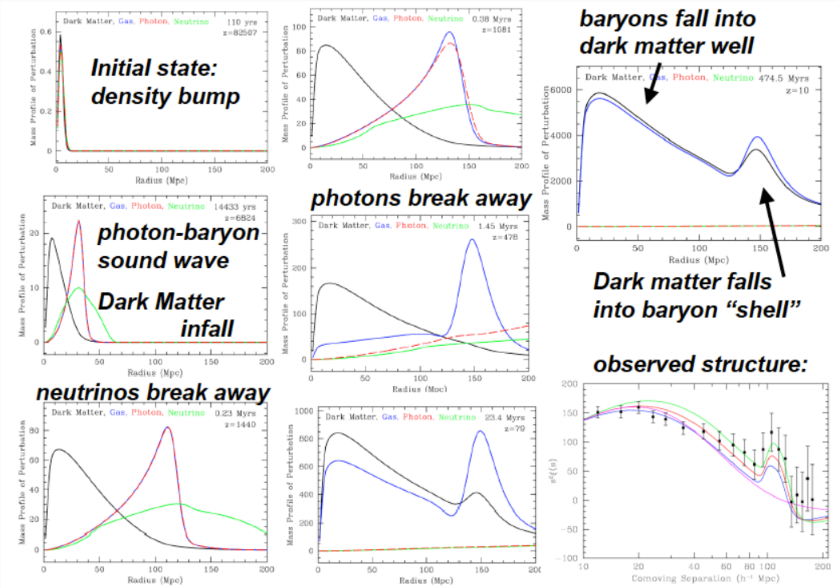
P3 The baryon drag depends mainly on Ω_b

P4 The global amplitude of the spectrum depends on the primordial spectrum amplitude As but also on Ω_Λ because of growth suppression

P5 The global tilt of the spectrum depends on the primordial spectrum tilt, n_s



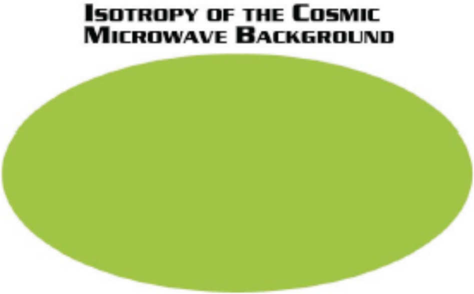
Baryon acoustic oscillations



Cosmic Microwave Background radiation

Cosmic Microwave Background

- Penzias & Wilson discovered in 1965 an isotropic and homogeneous radiation with a temperature of about 3 K as predicted by Gamow in 1940
- Te COBE satellite in 1992 showed that the CMB has a black-body spectrum and fluctuations of about 10^{-5}



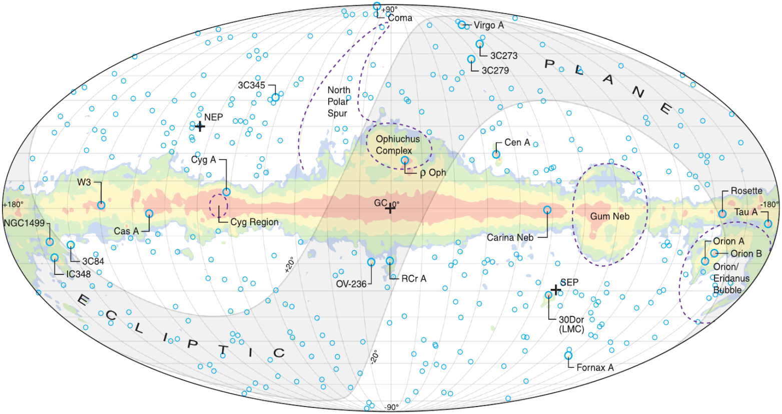
Cosmic Microwave Background radiation

CMB temperature observations

Brief history of CMB observations

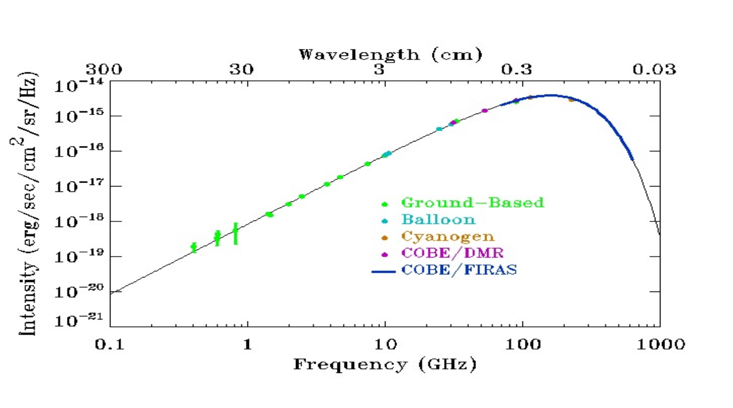
- Penzias & Wilson discovered in 1965 an isotropic and homogeneous radiation with a temperature of about 3 K
- In 1992 the COBE satellite demonstrated that the CMB has a black-body spectrum and fluctuations of about 10^5
- In 1998 Boomerang and Maxima measured the so-called acoustic peaks in the CMB power spectrum
- The WMAP satellite, launched in 2001, provided first CMB polarization precise measurements
- The Planck satellite 2013 results has provided best possible CMB temperature anisotropies measurements and much more (polarization analysis expected in 2014)
- Late 2013 the South Pole telescope and the PolarBear experiment reported first observation of B-lensing modes
- In 2019, release of Planck 2018 results and legacy data

Observing the sky



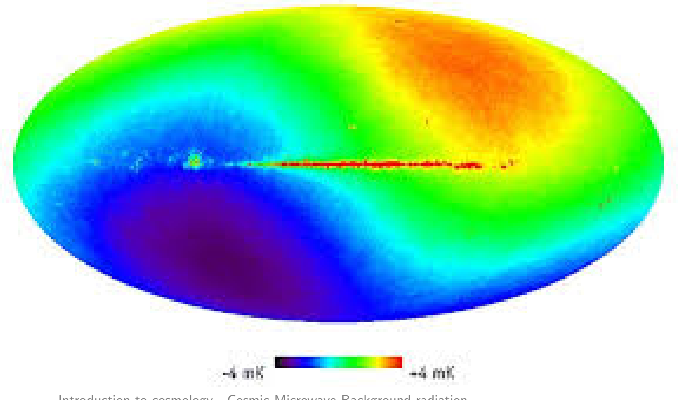
CMB black-body spectrum

- Compton scattering of photons with electrons is very efficient to thermalize photons
- In 1994 the FIRAS spectrograph in the COBE satellite measured the CMB temperature: TCMB $= 2.726 \pm 0.001$ K



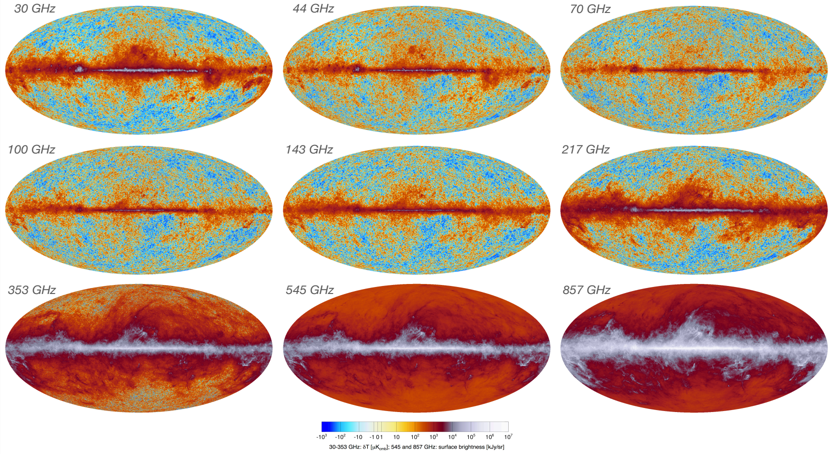
CMB dipole

- Dipole anisotropy induced by Doppler effect (relative motion of the observer with respect to the CMB rest frame)
- First measured by the COBE satellite in 1992 with an amplitude of $3.358 \pm 0.001 \pm 0.023$ mK in the direction of $(l, b) = (264.31 \pm 0.04 \pm 0.16, +48.05 \pm 0.02 \pm 0.09)$ degrees



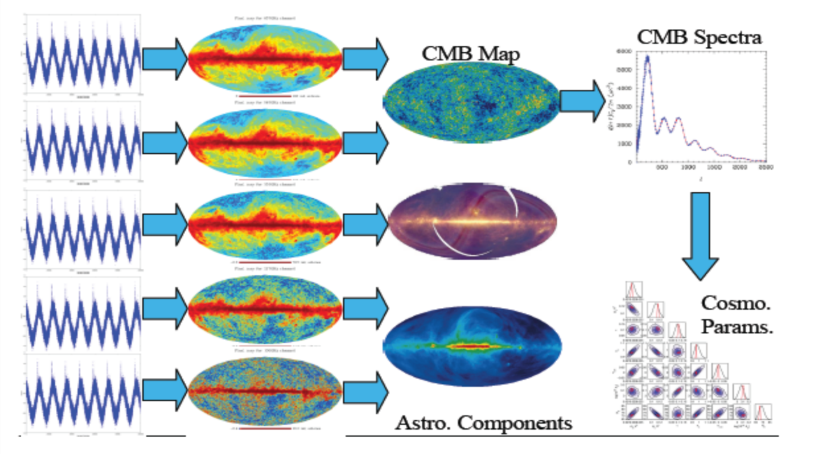
The microwave and mm-sky

- We observe a mixture of components: CMB, galactic thermal dust, synchrotron and free-free emission, extragalactic emission from dusty and radio galaxies



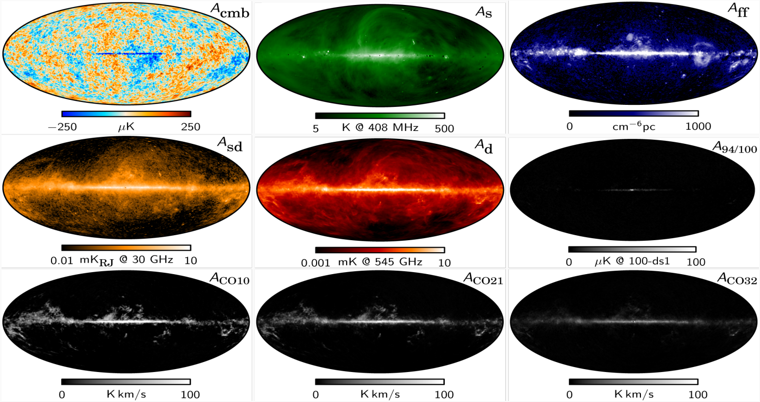
From sky observations to CMB maps

- Component separation algorithms are used to separate CMB emission from foreground emission from the Milky way, distant galaxies, clusters of galaxies, planets



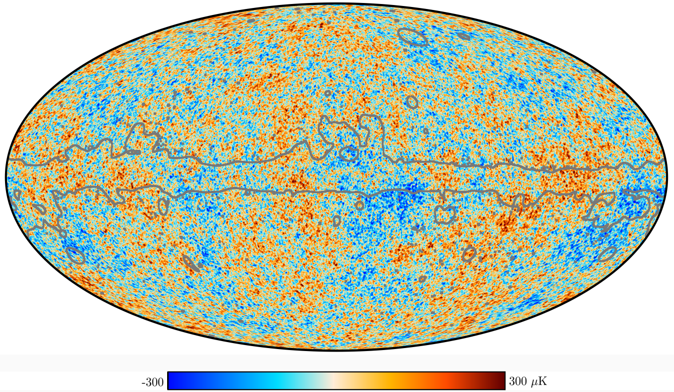
Sky emission

Multiple sky emission components including CMB and foreground emission



CMB temperature anisotropies

- Temperature fluctuations of the order of 10^5
- Planck satellite 2018 results: most precise measurements of the CMB temperature anisotropies



Cosmic Microwave Background radiation

CMB physics and power spectrum

Spherical harmonics and power spectra

- Any scalar field on the sphere, $A(\theta, \phi)$ can be decomposed into spherical harmonics

$$A(\theta, \phi) = \sum_{\ell} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

- We can define the power spectrum as

$$C_\ell = \langle a_{\ell m} a^*_{\ell m} \rangle = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2$$

- And for a Gaussian field like the CMB

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \; \delta_{\ell \ell'} \delta_{m m'}$$

- Furthermore, for Gaussian fields all the physical information is included in the power spectrum

CMB physics in a nutshell

- To describe CMB photons physics we need to solve the Boltzmann equation for the photon space-phase distribution

$$\frac{d}{\eta} f_\gamma(\eta, \vec{x}, \vec{q}) = C\left[f_\gamma(\eta, \vec{x}, \vec{q}), f_e(\eta, \vec{x}, \vec{q})\right]$$

  at first order perturbation $f_\gamma = \bar{f}_\gamma + \delta f_\gamma$ and considering coupling to electrons
- In thermal equilibrium the space-phase photon distribution function behaves as a Bose-Einstein distribution with

$$\bar{f}_\gamma(\eta, \vec{x}, \vec{q}) = \frac{1}{e^{\frac{q}{T(\eta)+\delta T(\eta)}} - 1}$$

  and

$$\delta f_\gamma(\eta, \vec{x}, \vec{q}) = \frac{d\bar{f}_\gamma}{d\log q} \frac{\delta T(\eta, \vec{x})}{\bar{T}(\eta)}$$

- Therefore, we can replace f_γ by the brightness function

$$\Theta(\eta, \vec{x}, \vec{n}) \equiv \frac{\delta T(\eta, \vec{x}, \vec{n})}{\bar{T}(\eta)}$$

  where $\vec{n}$ represents the line-of-sight

Power spectrum of the CMB anisotropies

- We want to compute the power spectrum of the temperature field today as observed from our position, $\vec{x} = \vec{0}$, today $\eta = \eta_0$

$$\frac{\delta T}{\bar{T}}(\vec{n}) = \Theta(\eta_0, \vec{0}, -\vec{n}) = \sum_{\ell} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\vec{n})$$

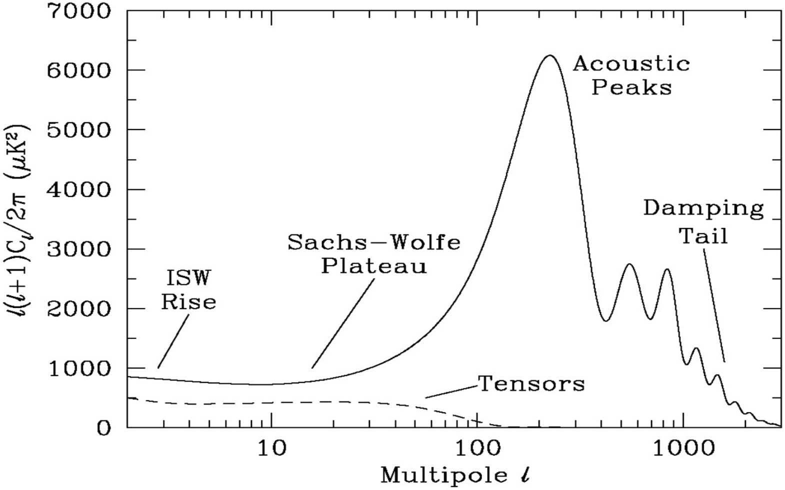
- Using the Legendre polynomials decomposition of the Fourier transform of Θ we can write

$$C_\ell = 4\pi \int_0^\infty \Delta^2_{\Theta_\ell}(\eta_0, k) \frac{dk}{k}$$

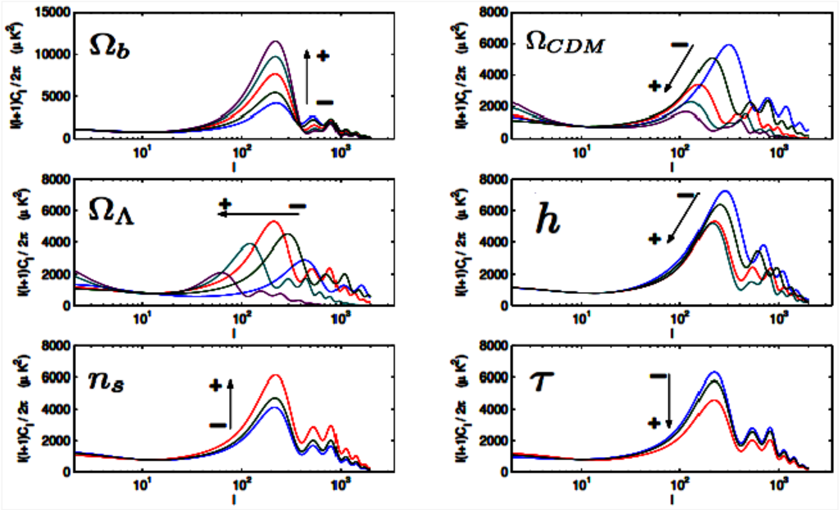
- that we can relate to the curvature fluctuations via a transfer function

$$C_\ell = 4\pi \int_0^\infty T^2_{\Theta_\ell} \Delta^2_R \frac{dk}{k}$$

CMB temperature power spectrum



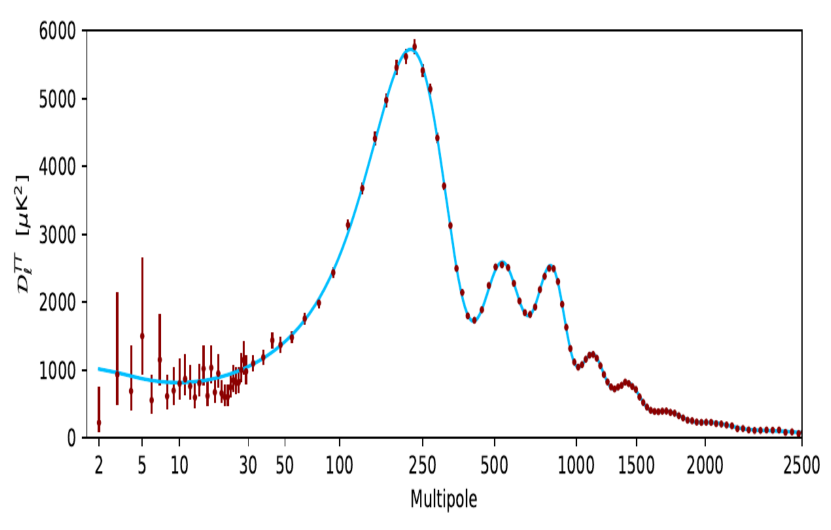
CMB power spectrum and cosmological parameters



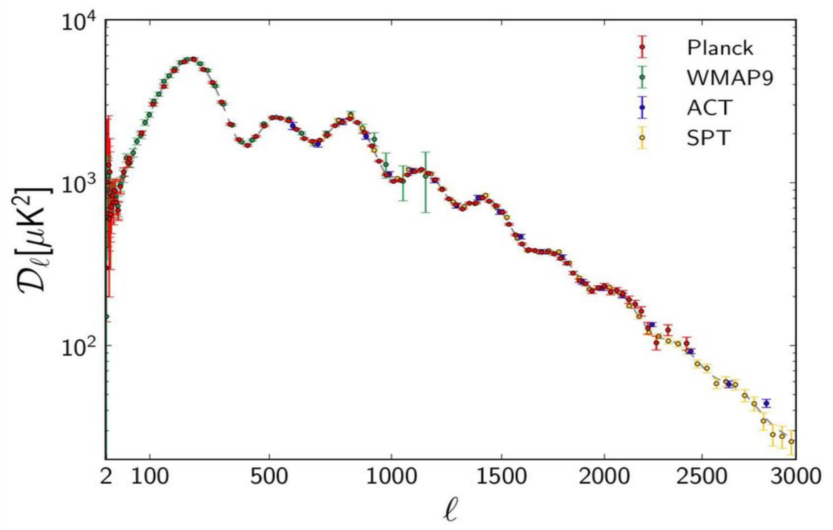
CMB power spectrum and cosmological parameters

(P1)	Peak Scale	Ω_m, Ω_b, Ω_Λ
(P2)	Odd/even peak amplitude ratio	Ω_b
(P3)	Overall peak amplitude	Ω_m
(P4)	Damping envelope	Ω_m, Ω_b, Ω_Λ
(P5)	Global Amplitude	A_s
(P6)	Global tilt	n_s
(P7)	Additional SW plateau tilting via ISW	Ω_Λ
(P8)	Amplitude for $\ell > 40$ only	τ_{reio}

Planck 2018 results CMB temperature power spectrum



CMB temperature power spectrum at small angular scales



Cosmic Microwave Background radiation

CMB polarization

Stokes parameters I

- Polarised light can be described using Stokes parameters
- For a light beam propagating on the z direction, the polarization plane is defined by x - y plane
- The electric field can be decomposed as

$$E(t,z) = E_x(t,z)\vec{e_x} + E_y(t,z)\vec{e_y}$$

where $E_{x,y}(t,z)$ are plane waves defined as

$$E_{x,y}(t,z) = A_{x,y} e^{\phi_{x,y}} e^{i(kz-wt)}$$

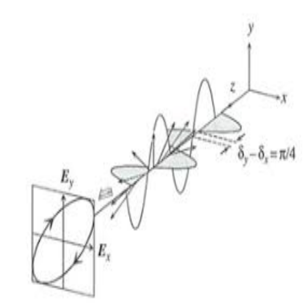
- Stokes para-meters can be defined as

$$I = \langle E_x E_x^* + E_y E_y^* \rangle = A_x^2 + A_y^2$$

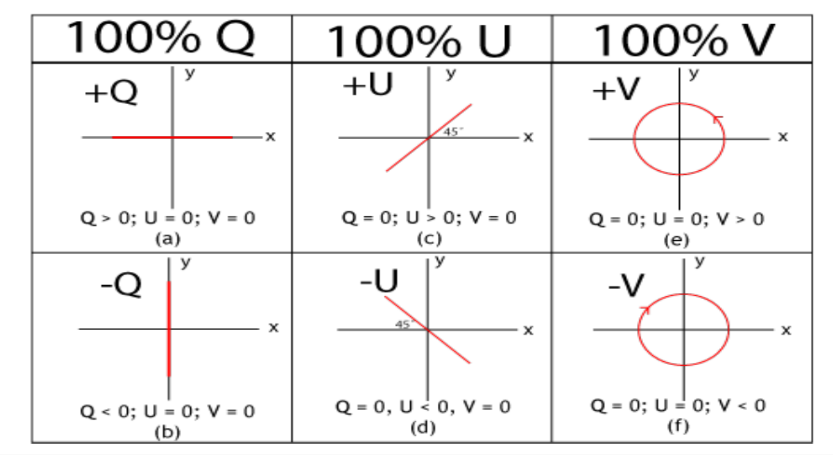
$$Q = \langle E_x E_x^* - E_y E_y^* \rangle = A_x^2 - A_y^2$$

$$U = \langle E_x E_y^* + E_y E_x^* \rangle = 2A_x A_y \cos(\phi_y - \phi_x)$$

$$V = -i\langle E_x E_y^* + E_y E_x^* \rangle = 2A_x A_y \sin\phi_y - \phi_x$$



Stokes parameters II



Linear polarization and spherical harmonics

- In the case of linearly polarised light a change of reference frame modify the Stokes parameters as follows

$$I' = I$$

$$Q' = Q\cos(2\theta) + U\sin(2\theta)$$
$$U' = -Q\sin(2\theta) + U\cos(2\theta)$$

- So we can form a spin ± 2 object $Q \pm iU$ that transforms as

$$Q' \pm iU' = e^{\mp 2i\theta}\left[Q \pm iU\right]$$

- Thus, Stokes parameters on the sphere can be decomposed as

$$T(\vec{n}) = \sum_{\ell m} a^T_{\ell m} Y_{\ell m}(\vec{n})$$

$$[Q \pm iU] = \sum_{\ell m} \left[a^E_{\ell m} \pm i a^B_{\ell m}\right] \ {}_{\pm 2}Y_{\ell m}(\vec{n})$$

Polarization power spectra

- We can define three scalar fields T, E, B which are independents of the chosen reference frame
- Using those we can form 3 auto-power spectra

$$C_\ell^{TT} = \frac{1}{2\ell+1}\sum_m |a_{\ell m}^T|^2$$

$$C_\ell^{EE} = \frac{1}{2\ell+1}\sum_m |a_{\ell m}^E|^2$$

$$C_\ell^{BB} = \frac{1}{2\ell+1}\sum_m |a_{\ell m}^B|^2$$

- and 3 cross-spectra

$$C_\ell^{TE} = \frac{1}{2\ell+1}\sum_m (a_{\ell m}^T a_{\ell m}^{E*})$$

$$C_\ell^{TB} = \frac{1}{2\ell+1}\sum_m (a_{\ell m}^T a_{\ell m}^{B*})$$

$$C_\ell^{EB} = \frac{1}{2\ell+1}\sum_m (a_{\ell m}^E a_{\ell m}^{B*})$$

- C^{TB} and C^{EB} vanish if parity is conserved

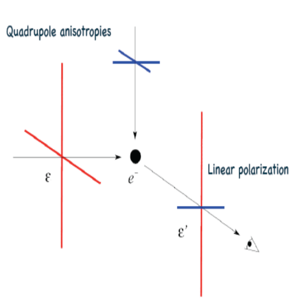
Thomson scattering and CMB polarization

CMB radiation is polarized via Thomson scattering between CMB photons and free electrons

- The differential cross section of Thomson scattering is given by

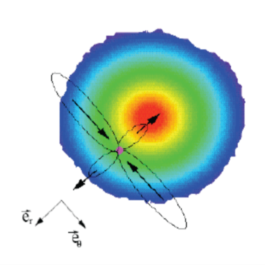
$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi}|\vec{E}'\cdot\vec{E}|^2$$

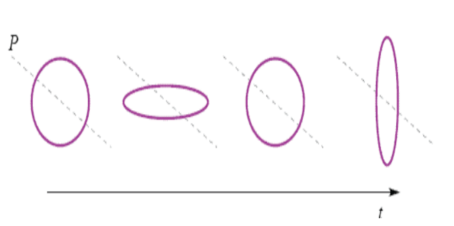
  where $\vec{E}$ and $\vec{E}'$ are the incoming and outgoing directions of the electric field
- Summing over all possible directions, we observe only quadripole anisotropies generate polarization



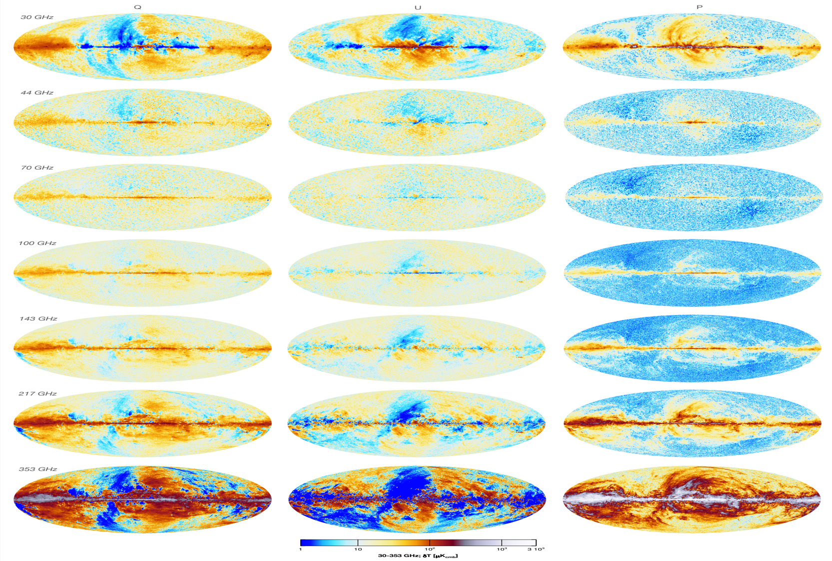
Local quadrupole perturbations and CMB polarization

- In hot and cold spots electrons observes local quadrupoles
- Density, scalar, perturbations produce Qr polarisation corresponding to E modes
- Gravitational waves distort the polarization pattern and induce also Ur polarization which corresponds to E and B modes

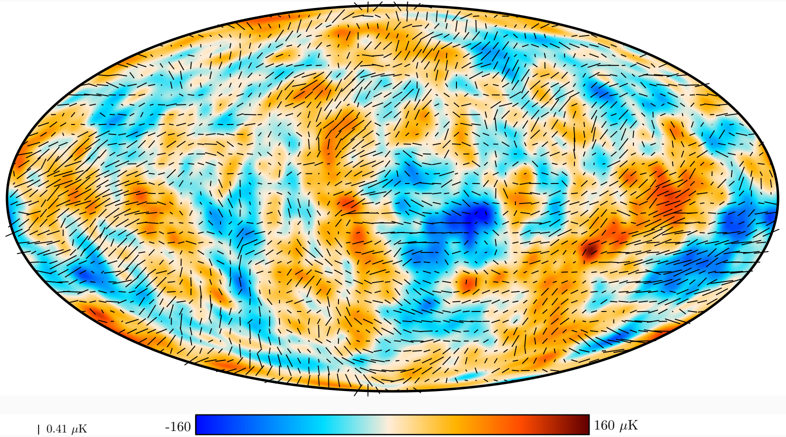




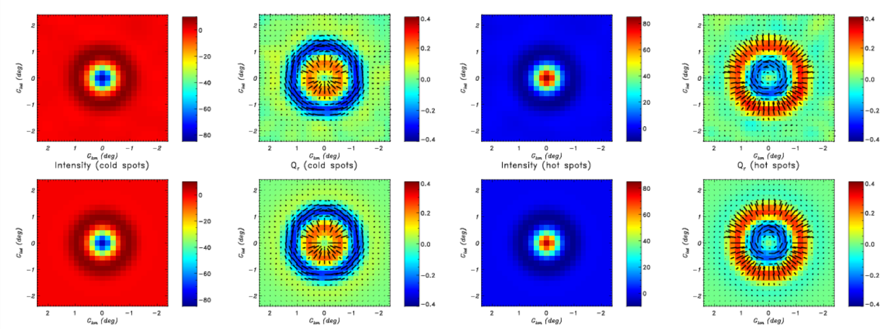
Sky polarization maps



CMB polarization map

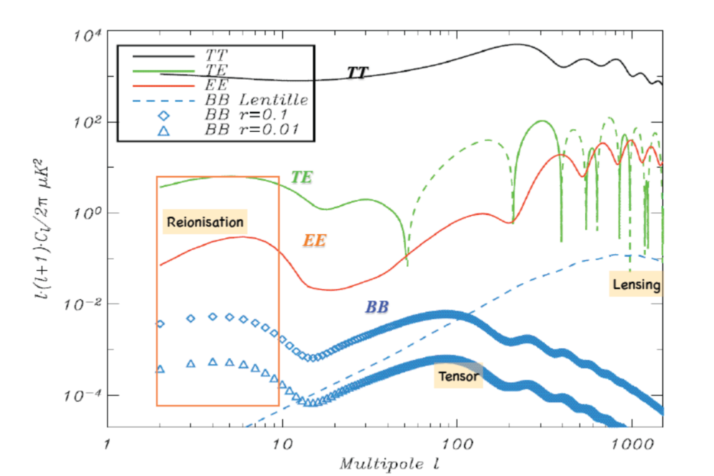


Local quadrupole perturbations and CMB polarization



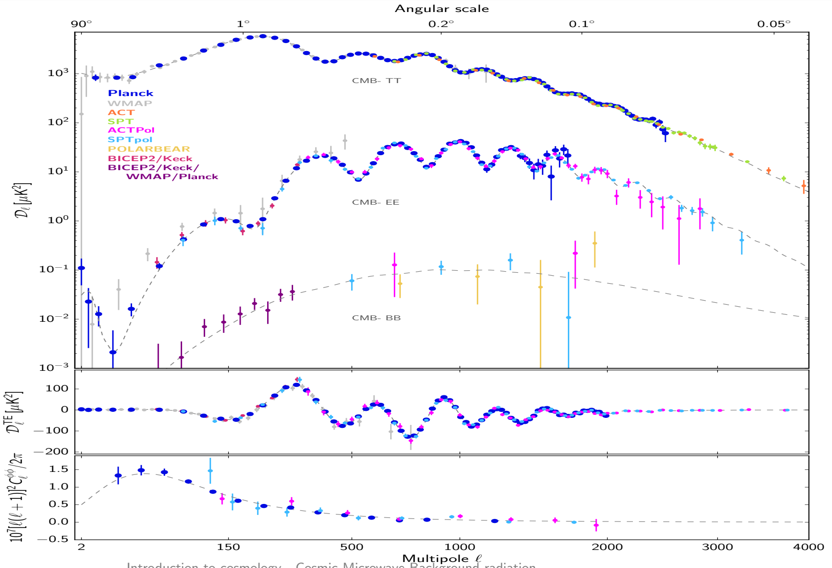
CMB polarization is dominated by E-modes

Expected CMB temperature and polarization power spectra



CMB polarization carries extra physical information that can be used to further constrain cosmological parameters

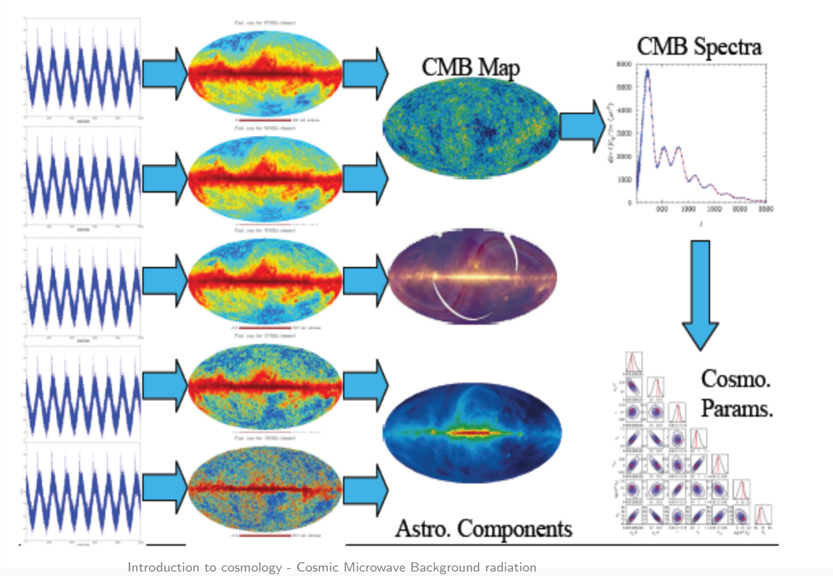
Measured CMB temperature and polarization power spectra



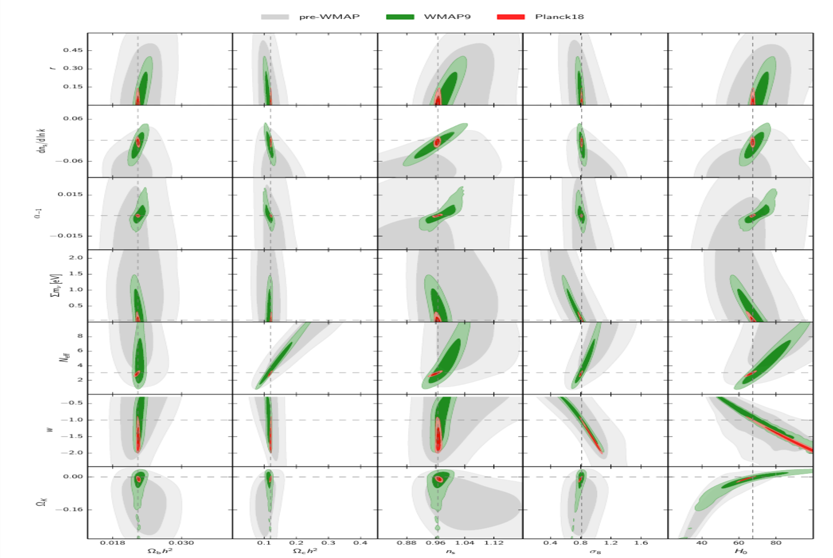
Cosmic Microwave Background radiation

Constraints on cosmological parameters

Likelihood analysis



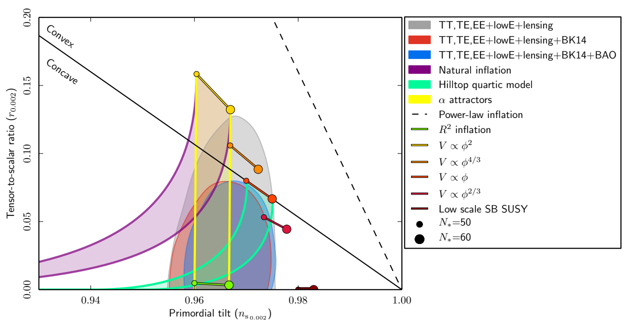
Recent cosmological parameter constraints I



Recent cosmological parameter constraints II

Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8111 ± 0.0060	0.8102 ± 0.0060
$\sigma_8(\Omega_m/0.3)^{0.5}$ . . .	0.832 ± 0.013	0.825 ± 0.011
z_{re}	7.67 ± 0.73	7.82 ± 0.71
Age[Gyr]	13.797 ± 0.023	13.787 ± 0.020
r_*[Mpc]	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04110 ± 0.00031	1.04119 ± 0.00029
r_{drag}[Mpc]	147.09 ± 0.26	147.57 ± 0.22
z_{eq}	3402 ± 26	3387 ± 21
k_{eq}[Mpc^{-1}]	0.010384 ± 0.000081	0.010339 ± 0.000063
Ω_K	-0.0096 ± 0.0061	0.0007 ± 0.0019
Σm_ν [eV]	< 0.241	< 0.120
N_{eff}	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
$r_{0.002}$	< 0.101	< 0.106

Inflation constraints



- The tensor to scalar ratio, r, is a direct measurement of inflation
- However, only direct detection of primordial B-modes would allow us to measure r. Waiting for next generation of experiments and scientists (maybe you) !!

Cosmic Microwave Background radiation

Secondary CMB anisotropies

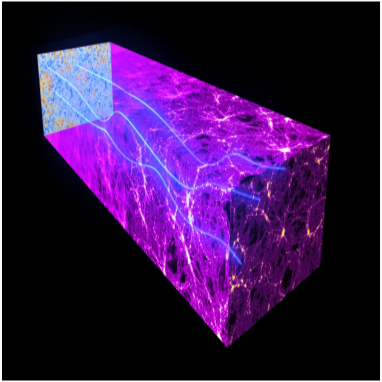
Main secondary temperature and polarisation anisotropies

We call secondary CMB anisotropies those that are generated after recombination either by gravitational effects of interaction of photons with electrons:

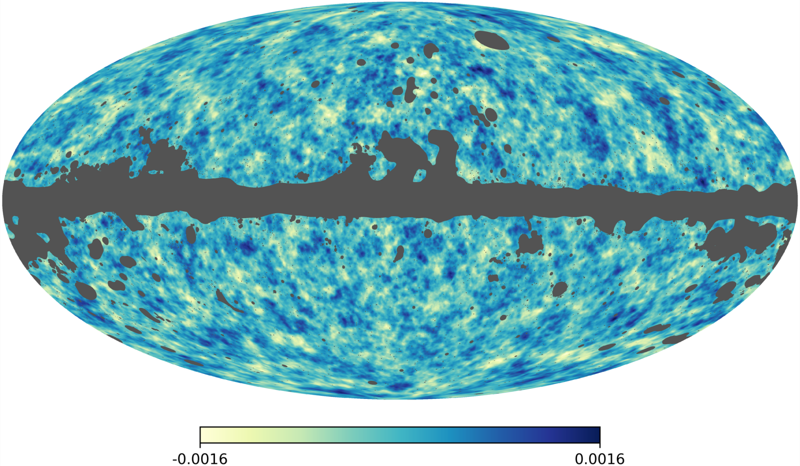
- Integrated Sachs Wolfe (ISW) effect: Sachs-Wolfe effect originated by changes in the gravitational potentials along the line-of-sight. The non-linear contribution is generally called Vishniac effect.
- Gravitational Lensing: gravitational lensing induced by mass distribution along the line-of-sight
- Sunyaev-Zeldovich effect: Compton inverse between CMB photons and hot free electrons on clusters of galaxies
- Reionization: Thomson interaction of CMB photons with free electrons at the time global reionization of the universe when first star form.

CMB lensing

- Gravitational potentials along the line of sight $\vec{n}$ to some source at comoving distance Ds gravitationally lens the image
- In the case of CMB lensing we are in the weak lensing regime and we expect small distortions of the image
- In particular we can observe that the convergence is simply the projected mass
- CMB lensing allows us to reconstruct the effective gravitational potential of all structures in the Universe

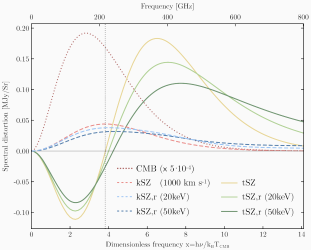


Planck lensing potential



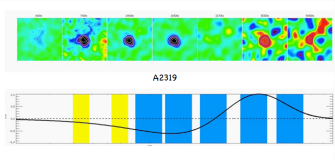
Sunyaev-Zeldovich (SZ) effect

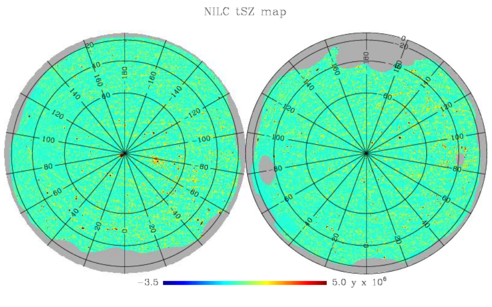
- Thermal (t)SZ effect corresponds to a small spectral distortion of the CMB spectrum induced by the Compton inverse interaction of CMB photons with hot electrons in clusters of galaxies
- Kinetic (k)SZ effect: If clusters are moving with respect to the CMB frame there is an additional spectral distortion due to the Doppler effect of the cluster bulk velocity on the scattered CMB photons.
- Relativistic (t)SZ effect: For very high temperature clusters relativistic corrections to the tSZ and kSZ effects are needed



Cluster detection via the (SZ) effect

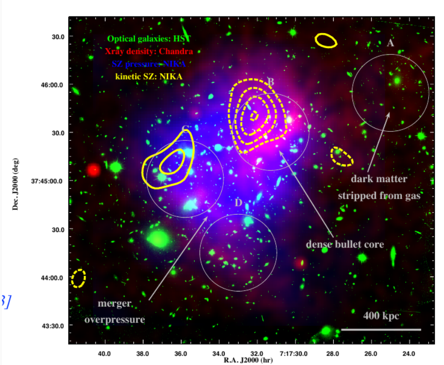
The SZ effect is an ideal tool to detect and study clusters of galaxies.





The SZ effect combined to other observables allows a detailed characterization of the physics on clusters of galaxies

Multivalength map of the cluster MACS0717 with NIKA



Velocity map from kSZ effect