

# Astroparticles - Tutorial

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Menu:

*A classical problem based on the Heitler model of shower development.*

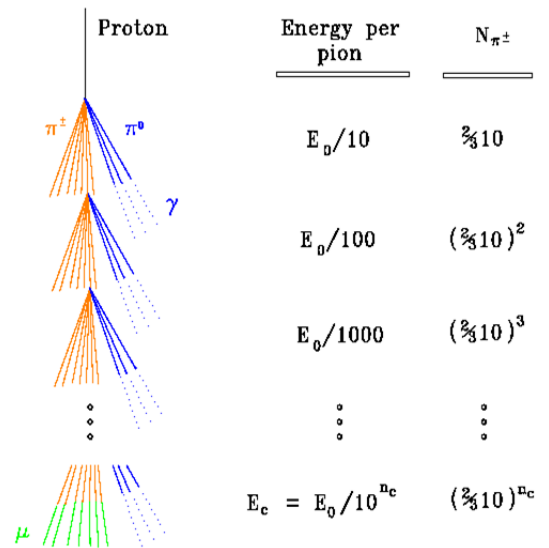
## 1 Heitler model for EM showers

Using a simplified model à la Heitler, we want to model the electromagnetic (EM) and the muonic component of a shower induced by a proton with an energy  $E_0 = 10^{18}$ eV and to study its longitudinal development.

One gives the critical energy to be (fixed)  $\varepsilon_\pi = 100$  GeV and  $\lambda_\pi = 120$  g/cm<sup>2</sup> is the pion interaction length.

We will do the following hypotheses:

1. At each fixed interaction length  $\lambda$ , all the non decayed hadrons with energy  $> \varepsilon_\pi$ , interact with a nucleus of the atmosphere.
2. Each interaction produces secondary hadrons (we assume here only pions) with multiplicity  $m$ ,  $2/3$  of them are  $\pi^\pm$  and  $1/3$  are  $\pi^0$ . One assume that the multiplicity is fixed and its value is  $m = 10$ .
3. Each of the  $m$  pions produced takes away a fraction of the parent energy  $1/m$ .
4.  $\pi^0$  decay immediately in two  $\gamma$  that will feed the EM component.
5. When charged pions reach the critical energy  $\varepsilon_\pi$ , they all decay and produce each one muon that propagates to the ground. We will assume that the muons take away  $1/2$  of the critical energy.
6. Both the hadronic and the EM component reach their respective critical energy at about the same depth. After this point, charged pions will decay before they interact and the hadronic shower development will stop. Charged pions decay and produce muons. As for the EM component, ionisation losses of  $e^+$ ,  $e^-$  start to dominate over the radiation processes and the development will also stop. The overall shower will decrease in terms of number of particle. The depth at which both components reach critical energy is also where the number of particles in the shower reaches maximum. This depth is called  $X_p^{max}$  and the number of particles at that point is called  $N^{max}$  (also called the shower size).



*Remark about the EM component development:* the EM component, which is fed at each step of the hadronic shower by the  $\pi_0 \rightarrow \gamma\gamma$  decays, is going to develop as well on its own (but instead with multiplicity 2 and interaction length  $X_0$ ). As  $\lambda_\pi \approx 4 \times X_0$ , after every  $\lambda_\pi$  step of roughly  $4 \times X_0$  the EM components energy is divided into  $2^4 = 16$  parts which is not very different than the assumed pion multiplicity. Hence, we will make the assumption here that the hadronic and

EM components reach their respective critical energy at about the same depth. Note that this coincidence can also be described by the following numerical equivalence:

$$\frac{\lambda_\pi}{X_0} \approx \frac{\ln(10)}{\ln(2)}.$$

Try to answer the following questions:

1. Find the number of interaction lengths before the hadronic component reaches its critical energy and give an approximate value using the given parameters.
2. Show that

$$X^{max} = \lambda_\pi \log_{10} \left( \frac{E_0}{\varepsilon_\pi} \right).$$

3. By how much does the depth of maximum  $X_p^{max}$  change when the primary energy  $E_0$  changes by a  $\times 10$ ?

Let's assume that the superposition principle holds, namely that a shower initiated by a nucleus with energy  $E_0$  consisting of  $A$  nucleons can be described as the superposition of  $A \times$  proton-induced showers each having an initial energy  $E_0/A$ .

4. Give the expression for the depth of the maximum development  $X_A^{max}$  of a shower initiated by a nucleus of mass  $A$  as a function of the interaction length  $\lambda_\pi$  and the depth of maximum for a proton shower  $X_p^{max}$ .
5. What would be the statistical difference between the depth of maximum of iron showers and of proton showers?
6. Given that the resolution on the  $X^{max}$  measured by the Pierre Auger Observatory using the fluorescence telescopes techniques reaches  $\Delta X^{max} = 20 \text{ g/cm}^2$ , what is the resolution achieved on  $\ln(A)$  ?

Note that in reality, the  $X^{max}$  measurements are affected by very large shower to shower fluctuations (mostly due to the broad distribution of the depths of first-interactions) that spoil the resolution on  $\ln(A)$ .

Let now study the energy balance un muons and in the EM component.

7. Compute what is the fraction of the total energy transferred to the EM component after  $n_c$  steps.
8. Compute the number of muons produced at the end of the development. What fraction of energy do they take away.
9. Compare the EM, muonic and initial energy. Do you observed missing energy? What is taking this missing energy away?
10. Suppose now that the incident particle is an iron nucleus with the same total energy than the proton. Assuming superposition principle holds, what is now the muons energy fraction?

Conclude if this muons energy fraction can be used to deduce the nature of the incident particle.

A few useful numerical values and formulae:

$$(2/3)^7 \approx 6 \times 10^{-2}$$

$$\log_{10}(x) = \ln(x) / \ln(10)$$

$$\log_2(x) = \ln(x) / \ln(2)$$

$$\ln(10) \approx 2.3$$

$$\ln(2) \approx 0.7$$

$$\log_{10}(56) \approx 1.75$$

$$\log_2(56) \approx 5.8$$

$$\sum_{k=1}^n r^k = r \frac{1 - r^n}{1 - r}$$