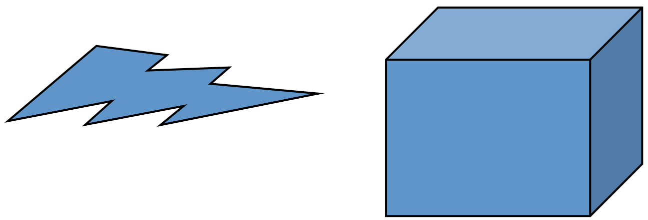
esipap...
European School of Instrumentation
in Particle & Astroparticle Physics

INTENSIVE PROGRAM
FOR MASTER & PH.D. STUDENTS
AND PROFESSIONALS

PRACTICAL LABORATORY SESSIONS
AT CERN

ACCREDITED BY
PARTNER UNIVERSITIES (ECTS)

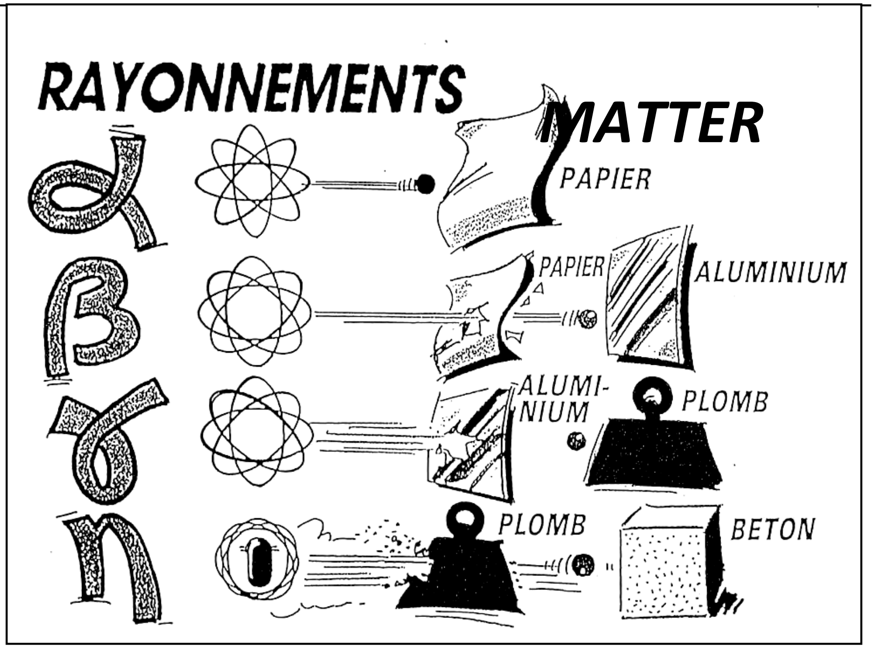
Interactions of Particles/Radiation with Matter



ESIPAP : European School in Instrumentation for Particle and Astroparticle Physics

Non-exhaustive list of « Particles/Radiation » and « Matter »

PARTICLES	RADIATION
• $^{4}_{2}He$	α radiation
• $e^{\pm}$	$\beta^{\pm}$ radiation
• γ	e.m, X, γ radiation
• μ, γ, $e^{\pm}$, π, ν ,p …	cosmic radiation

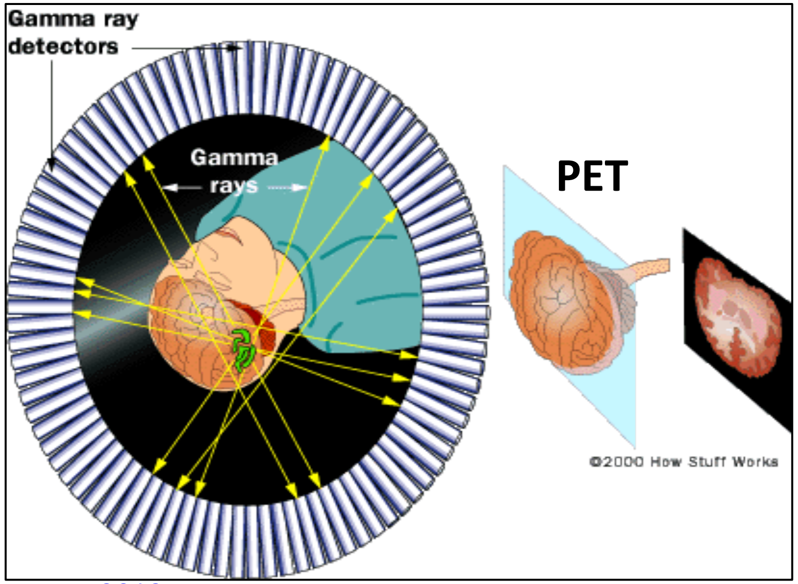


PARTICLES <--> RADIATION

2 aspects of the same « entity »

De Broglie relation $\lambda = h/p$

(h = Planck constant)



MATTER

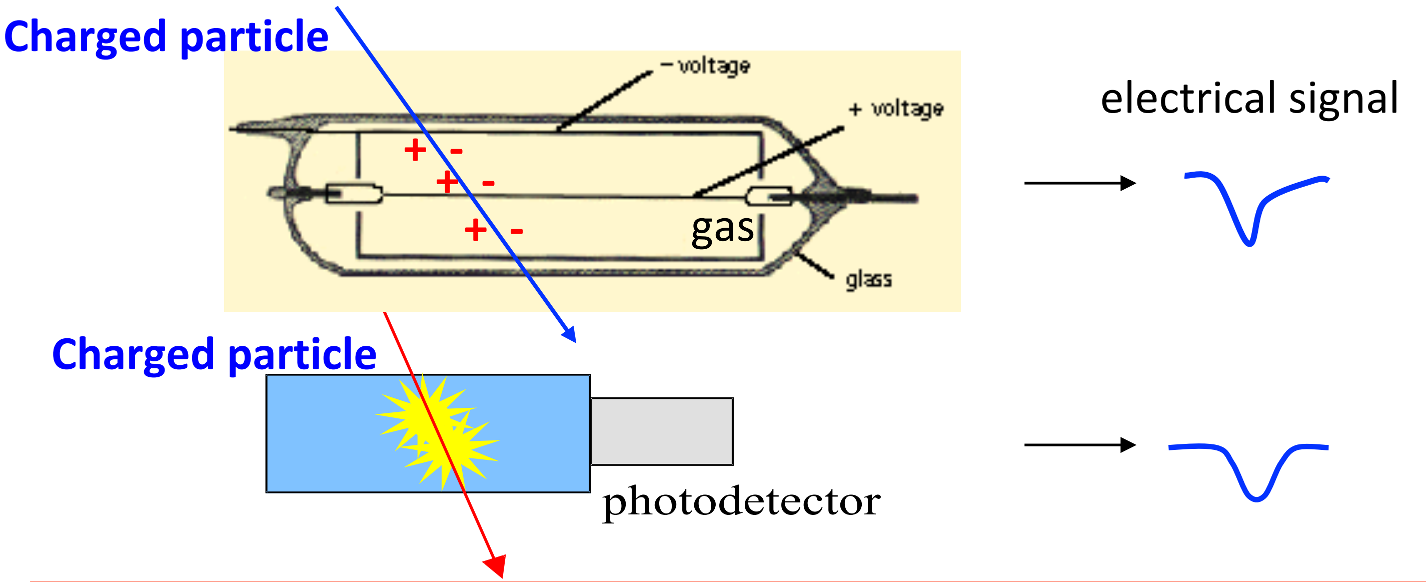
- detectors (research, medical app.,..)
- humain tissus/body (medical app.)
- electronic circuits
- Louvre paintings
- beauty cream, potatos, …

Motivation

- The interaction between **particles & matter** is at the base of several human activities
- Plenty of applications **not only in research** and not **only in Particle & Astroparticle**

Very important for particle detection !

- In order **to detect a particle**, the latter must interact with the material of the detector, and produce 'a (detectable) signal'

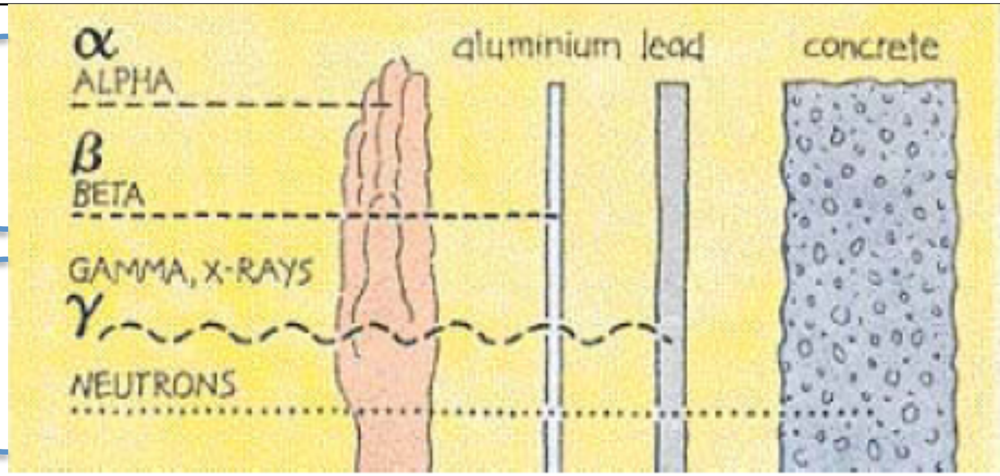


The understanding of **particle detection** requires the knowledge of the
Interactions of particles & matter

Brief outline and bibliography

Two lectures + two tutorials

- **Interaction of charged particles**
  « heavy » ($m_{Pa} >> m_e$)
  « light » ($m_{Pa} \sim m_e$)
- **Interaction of neutral particles**
  Photons
  Neutral Hadrons: n, π^0, …



- **Radiation detection and measurement**, G.F. Knoll, **J. Wiley & Sons**
- **Experimental Techniques in High Energy Nuclear and Particle Physics**, T. Ferbel, **World Scientific**
- **Introduction to experimental particle physics,** R. Fernow, **Cambridge University Press**
- **Techniques for Nuclear and Particle Physics Experiments ,** W.R. Leo, **Springer-Verlag**
- **Detectors for Particle radiation,** K. Kleinknecht, **Cambridge University Press**
- **Particle detectors**, C. Grupen, **Cambridge monographs on particle physics**
- **Principles of Radiation Interaction in Matter and Detection,** C. Leroy, P.G. Rancoita, **World Scientific**

- **Nuclei and particles**, Emilio Segré, **W.A. Benjamin**
- **High-Energy Particles**, Bruno Rossi, **Prentice-Hall**

← "**The** classic "

Also: **Particle Data Group**
http://pdg.lbl.gov/2019/reviews/rpp2019-rev-passage-particles-matter.pdf

For ‘**professionals**’(*) : **GEANT4 (for GEometry ANd Tracking)**
(Platform for the **simulation** of the **passage of the particles through the matter**
Using Monte Carlo simulation, Open software)
https://www.sciencedirect.com/science/article/pii/S0168900203013688

My slides have been inspired by :

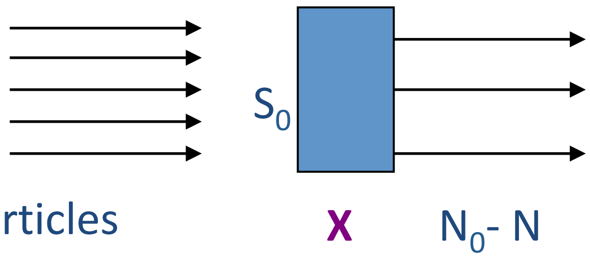
Hans Christian Schultz-Coulon’s lectures

Johann Collot @ ESIPAP 2014

(*) more exists:
Fluka
Garfield (simulation gas detectors)

Interaction Cross Section (σ) definition

σ characterises the **probability** of a given **interaction** process



N have **interacted**

N_0 particles on area S_0

(thin) target

Target parameters:

- **n** $\equiv$ number of target particles
- M = target mass
- A_{mol} = molar mass
- ρ = target density

$N_A = 6.022\ 10^{23}\ mol^{-1}$ (Avogadro number)

$$\sigma \equiv \frac{\text{Number of interactions per number of target particles in unit time}}{\text{Incident flux}}$$

Number of **interactions** per number of **target particles** in unit time = **(1/n) * dN/dt**

Incident flux = **(1/S_0) * dN_0 /dt**

Interaction probability

$$\sigma \equiv [(1/n) * dN/dt] / [1/S_0 * dN_0/dt] = dN/dN_0 * (S_0 / n)$$

where $n = (M/A_{mol})\ N_A = (\rho\ V)\ (N_A/A_{mol}) = \rho\ (S_0\ X)\ (N_A/A_{mol})$

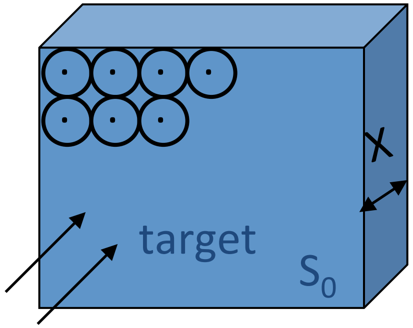
σ doesn't depend from S_0

Cross section (σ)

$\sigma \equiv$ (interaction probability) * (S_0 /n)

n ≡ number of target particles

$\sigma \equiv$ area of a small disk around a target particle



$[\,\sigma\,] = [\,l\,]^2$ --> σ is measured in **m²** or **barn**

1 barn = 10^{-28} m² = 10^{-24} cm²

1 mbarn = 10^{-27} cm

Oder of magnitude of cross sections :

'strong interaction'

Neutron of ~ 1 eV on $^{48}_{113}Cd$ σ = 100 barn = 10^{-22} cm²

'weak interaction'

Neutrino of ~ 1 GeV on p $\sigma = 10^{-38}$ cm²

See also Marco Delmastro lectures

Mean free path λ (*)

λ = ** Average distance traveled between **two consecutive interactions in matter

Another way of expressing the probability of a given process

$$\lambda \equiv \frac{1}{\sigma\, n_v}$$

σ total interaction cross-section

n_v number of scattering centers per unit volume $n_v = (\rho N_A)/A_{mol}$

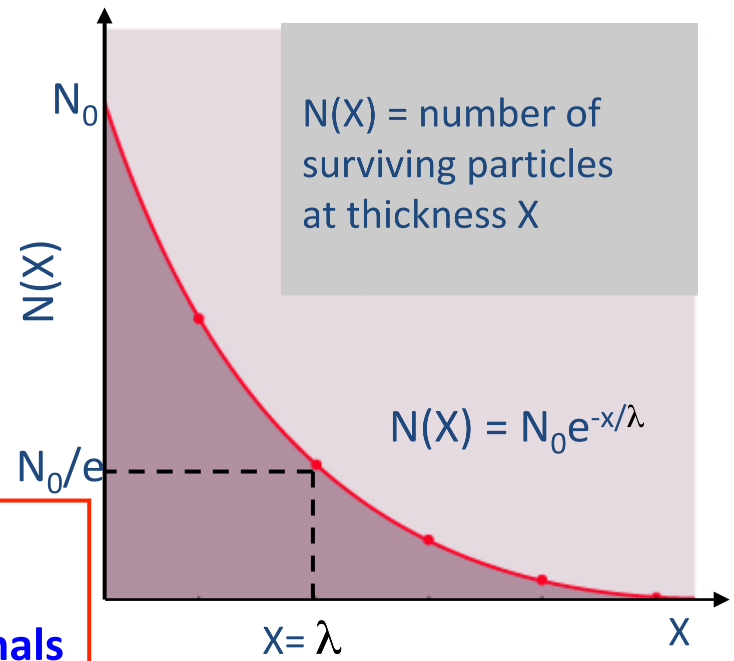
Order of magnitudes:

Electromagnetic interaction : $\lambda < \sim 1\ \mu m$

Strong interaction : $\lambda > \sim 1\ cm$

Weak interaction : $\lambda > \sim 10^{15}\ m$

A practical signal (> 100 interactions or 'hits') results from the electromagnetic interaction

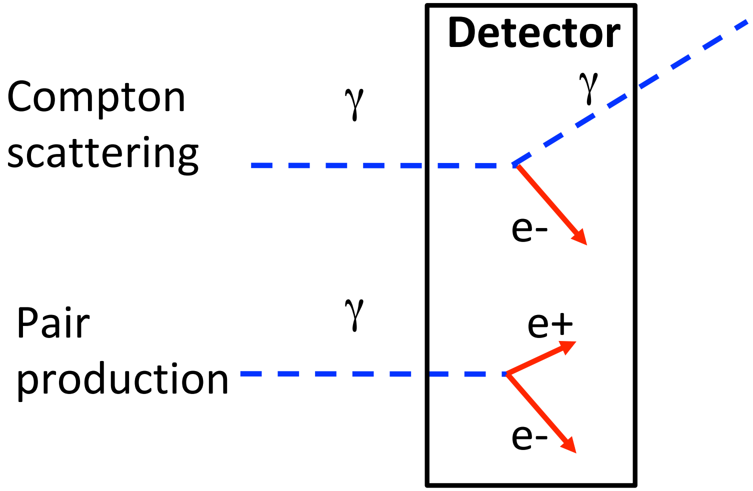


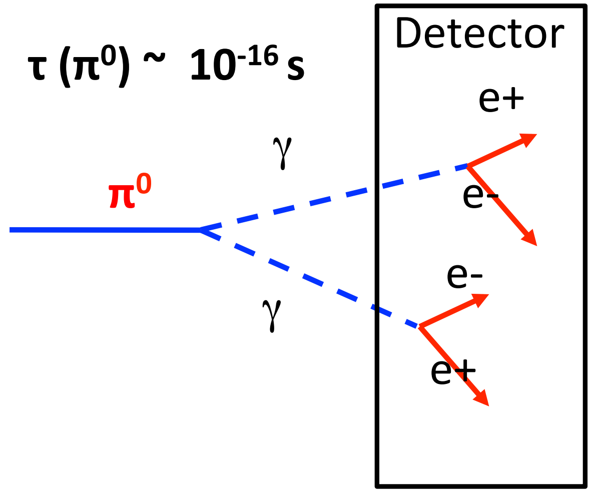
Particle detection proceeds in two steps :
1) primary interaction
2) charged particle interaction producing the signals

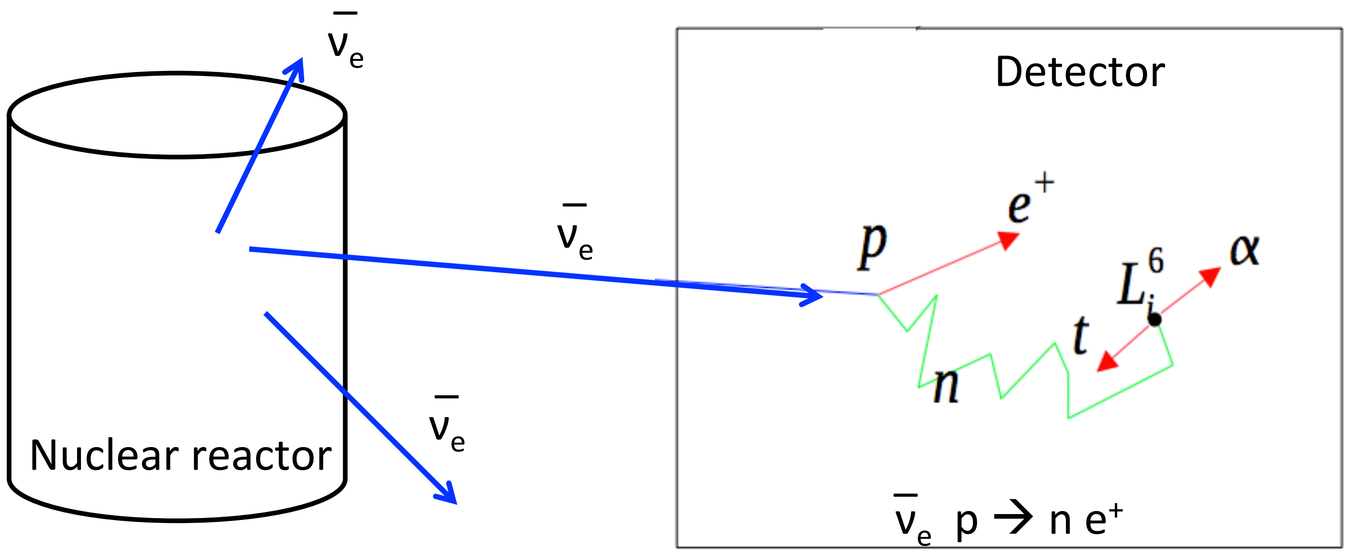
(*) Also: λ = absorption length, interaction length, attenuation length, …then σ is the cross-section for the corresponding process (see later)

Examples: detection of photons(γ), π^0(2γ), neutrons(n), neutrinos(ν)

Signals are induced **by e.m. interactions of charged particles** in detectors







Useful relations of relativistic Kinematics and HEP units

- $\vec{p} = m_0 \gamma \vec{v}$ $\quad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$ $\quad \beta \equiv v/c$

$m_0 \equiv$ rest mass
$\gamma \equiv$ Lorentz boost
$m \equiv m_0 \gamma$

- Kinetic energy $E_k = (\gamma-1)\, m_0\, c^2$
- Total energy $E = \sqrt{(pc)^2 + (m_0 c^2)^2}$
- Total energy $E = E_k + m_0 c^2 = m_0 \gamma c^2 = m c^2$ $\quad \gamma = E/(m_0 c^2)$

$E = m c^2$ « equivalence mass & energy »

Units :

$[E] = eV \quad [m] = eV/c^2 \quad [p] = eV/c$

See Marco Delmastro lectures

« Natural units » $\hbar = 1$, $c = 1$

$[c] = \frac{[l]}{[t]} \quad [l] = [t]$

$[E] = [p] = [m] = [v] = [t]^{-1}$

Outline: main interaction processes

2nd lecture

1rst lecture

- **Charged particle interactions**
  - 1) **Ionization:** inelastic collision with **electrons** of the atoms
  - 2) **Bremsstrahlung:** photon radiation emission by an accelerated charge
  - 3) **Multiple Scattering:** elastic collision with **nucleus**
  - 4) **Cerenkov & transition radiation effects:** photon emission
  - (5) **Nuclear interactions (p, π, K):** processes mediated by strong interactions)

  Items 1–4: **e.m. interactions**

- **Neutral particle interactions**
  - Photons :
    - **photoelectric and Compton** effects, **$e^+ e^-$ pair production** — **e.m. interactions**
  - High energy neutral hadrons with > $\tau \sim 10^{-10}$ s (n, K^0, ..) :
    - **nuclear interactions**
  - Moderate/low energy neutrons :
    - **scattering** (moderation), **absorption, fission**
  - Neutrinos :
    - processes mediated by **weak interactions** — **not treated here**

After the interaction the particles loose their energy and/or change direction or 'disappear'

Neutral particle interactions

- Photons
- High energy neutral hadrons with > $\tau \sim 10^{-10}$ s (n, K^0, ..) :
- Moderate/low energy neutrons

Interactions of photons (γ)

γ : particles with $m_\gamma = 0$, $q_\gamma = 0$, $J^{PC}(\gamma) = 1^{--}$

Since $q_\gamma = 0$, the photons are **indirectly** detected : in their interactions they produce **electrons** and/or **positrons** which subsequently interact (**e.m.**) with matter.

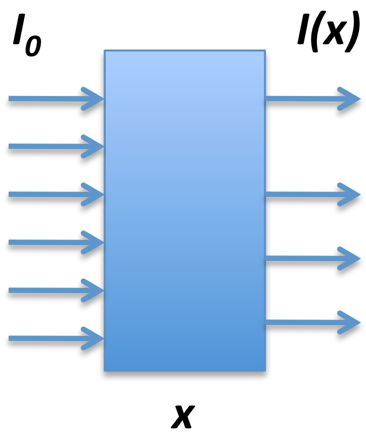
Main processes :

1. Photoelectric effect
2. Compton scattering
3. e+ e- pairs production

↓ $E\gamma$

Photons may be **absorbed** (photoelectric effect or e+e- pair creation) or **scattered** (Compton scattering) through large deflection angles.

→ difficult to define a mean range → an attenuation law is introduced :



$$I(x) = I_0 e^{-\mu x}$$

$$\mu = N\sigma = \frac{N_A}{A}\rho\,\sigma \equiv \frac{1}{\lambda}$$

See also slide 8

- μ **absorption coefficient**
- N atoms/m^3
- A masse molaire
- N_A nombre Avogadro
- ρ density
- σ **Photon cross section**
- λ Mean free path or absorption lenght

γ Absorption lenght ($\lambda' \equiv 1/\mu'$)

$$I(x) = I_0 e^{-\mu x} \qquad I(x) = I_0 e^{-\mu' x'}$$

$x' \equiv x\,\rho$

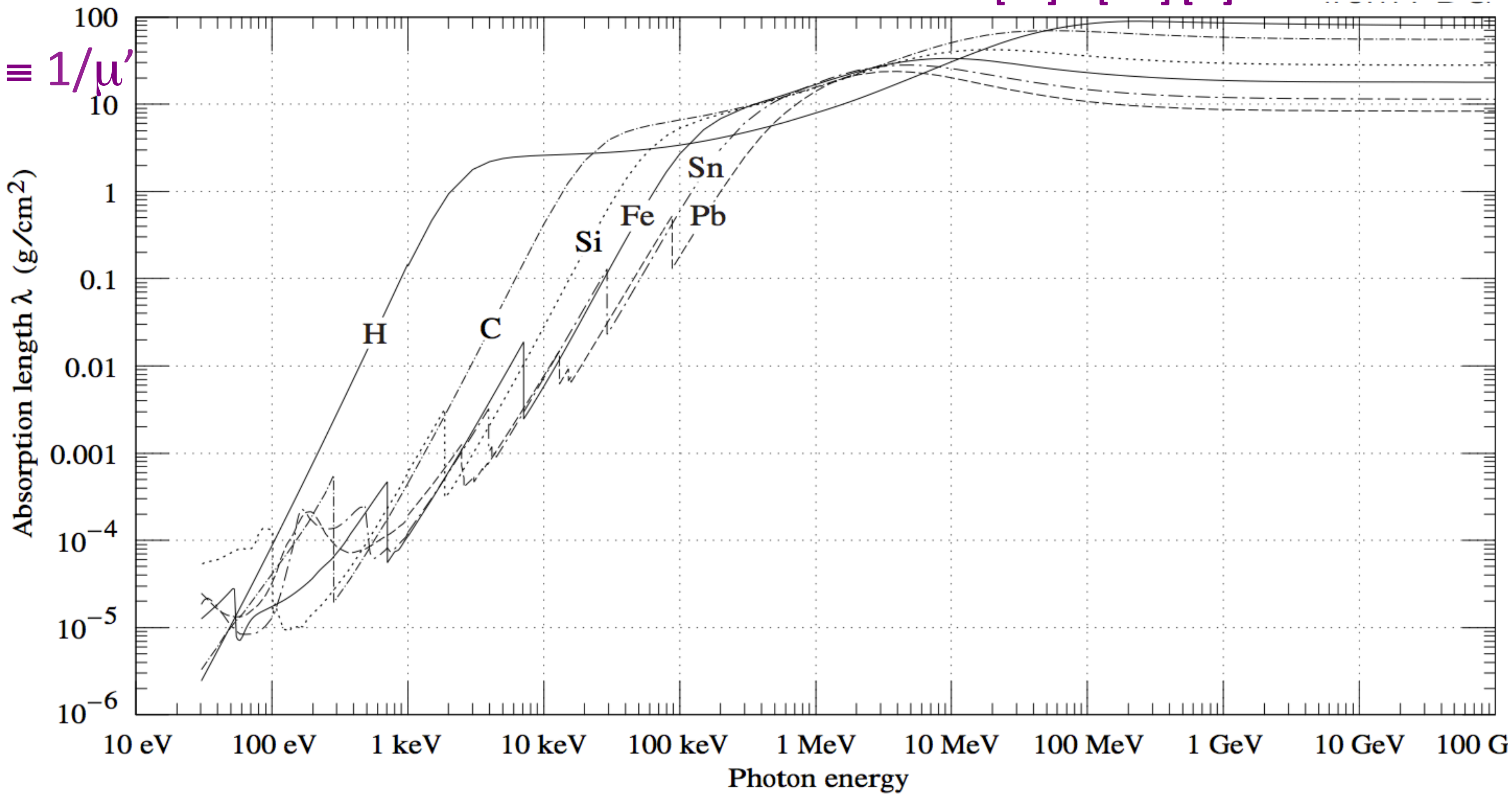
$\mu' \equiv \mu/\rho$

$[\,x'\,] = [m]\,[\,l\,]^{-2}$

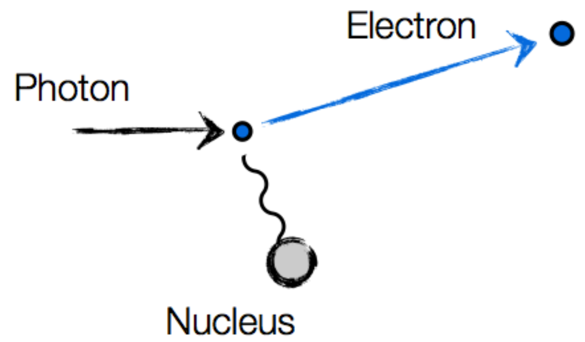
$[\,\mu'\,] = [m]^{-1}\,[\,l\,]^{2}$

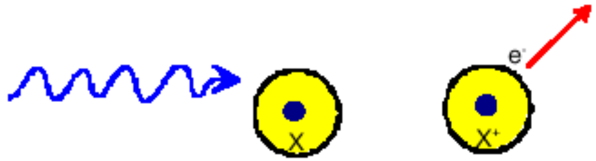
$[\,\lambda\,] = [\,m\,]\,[\,l\,]^{-2}$

$\lambda' \equiv 1/\mu'$



1.Photoelectric effect



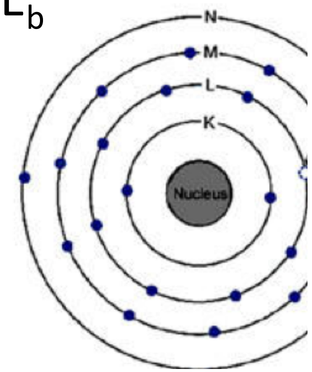


$$\gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

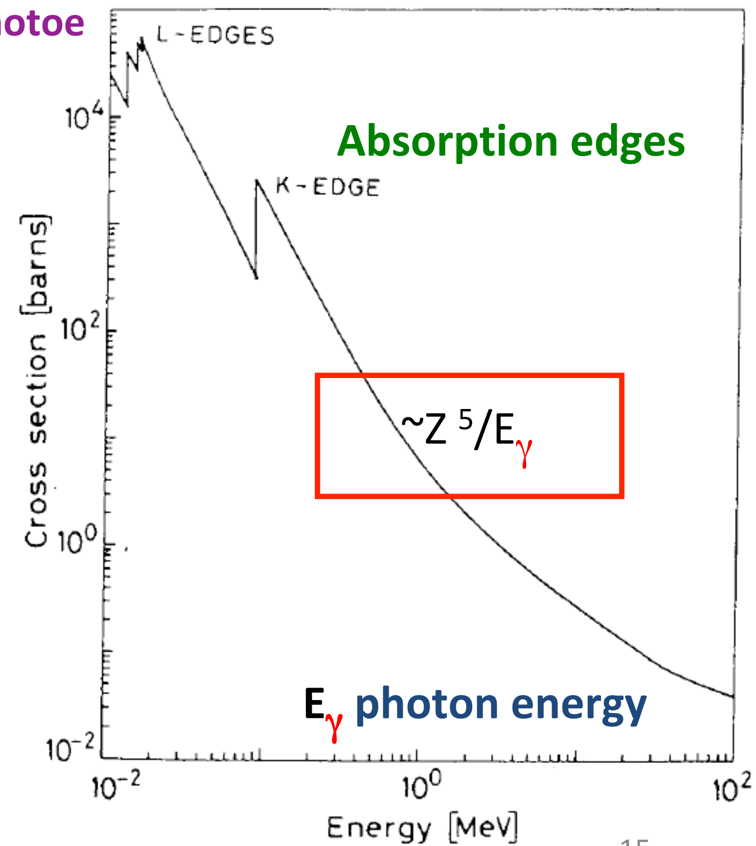
- The energy of the γ is transferred to the electron and the γ disappeares
- Energy of the final electron:

$E_e = E_\gamma - E_{\text{electron binding energy}} = h\nu - E_b$

$E_b = E_K$ or E_L or E_M etc…



σ_{photoe}



This effect can take place only on **bounded** electrons since the process (on 'free' electrons)

γ e --> e

cannot conserve the momentum and energy

1.Photoelectric effect

- At « low » energy ($I_0 << E_\gamma << m_e c^2$):

$$\sigma_{ph} = \alpha\pi\, a_B\, Z^5\, (I_0/E_\gamma)^{7/2}$$

- At « high » energy ($E_\gamma >> m_e c^2$):

$$\sigma_{ph} = 2\pi r_e^2\, \alpha^4\, Z^5\, (mc^2)/E_\gamma$$

I_0 = ionisation potential

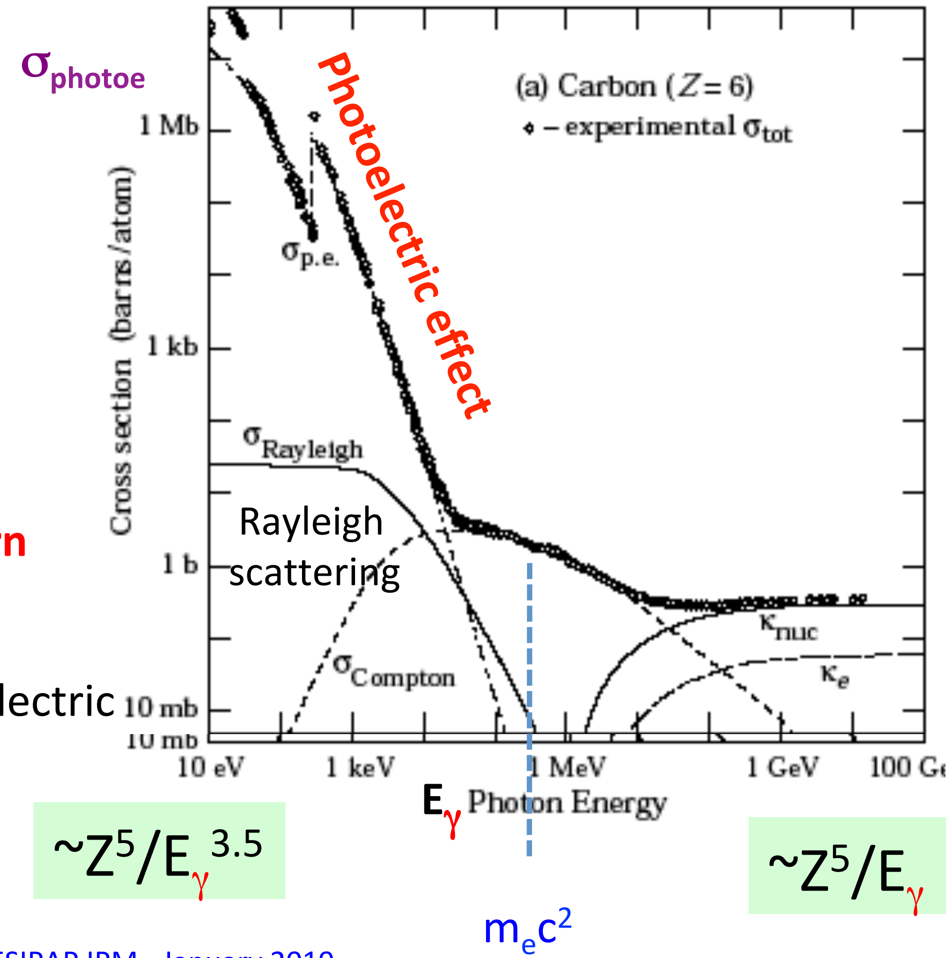
α fine structure constant

$r_e = \alpha/(m_e c^2)$= classical electron radius

Example:

$a_B = 0.53\ 10^{-10}$ m
I_0 = 13.6 eV

For E_γ = 100 KeV **σ (Fe) = 29 barn**
σ (Pb) = 100 barn



- At low energy (E_γ <100 keV) , the photoelectric effect dominates the total photon cross section

Atom de-excitation (after photoelectric effect)

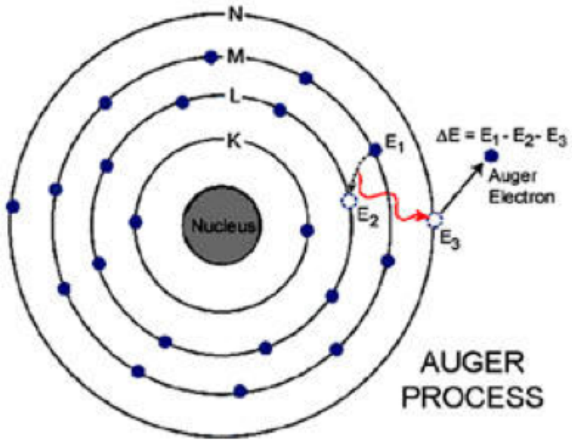
Following the emission of a "photoelectron", the atom is in an excited state

De-excitation occurs via two effects (time scale: ~10^{-16} s)

- Fluorescence: Atom^{*+} → Atom $^{*+}$ + γ → **X rays**
- Auger effect Atom^{*+} → Atom $^{*++}$ + e- → **Auger electron**

Observed first by Lisa Meitner

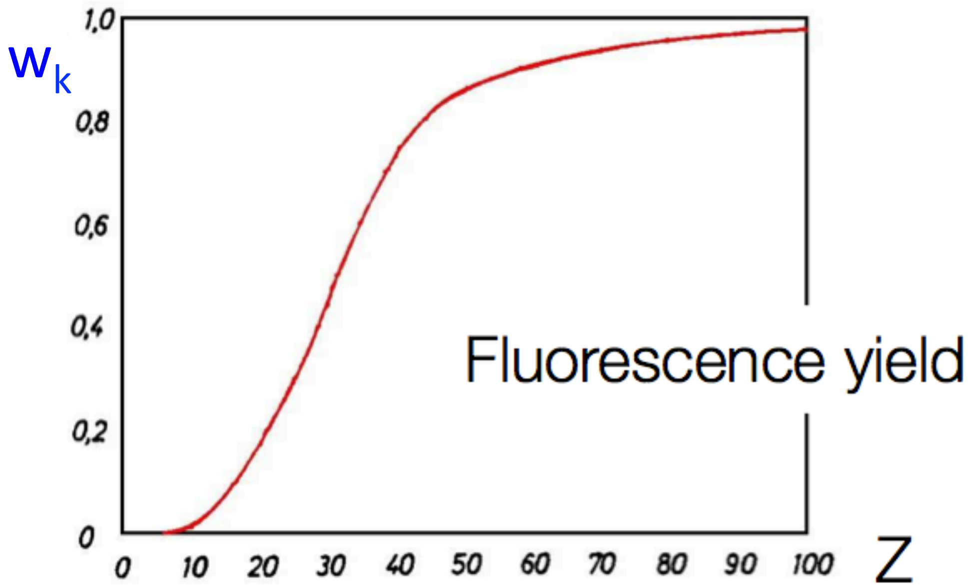
Used for surface Spectroscopy (AES)



Auger electrons deposit energy locally (small energy < ~ 10 KeV)

Fluorescence yield :

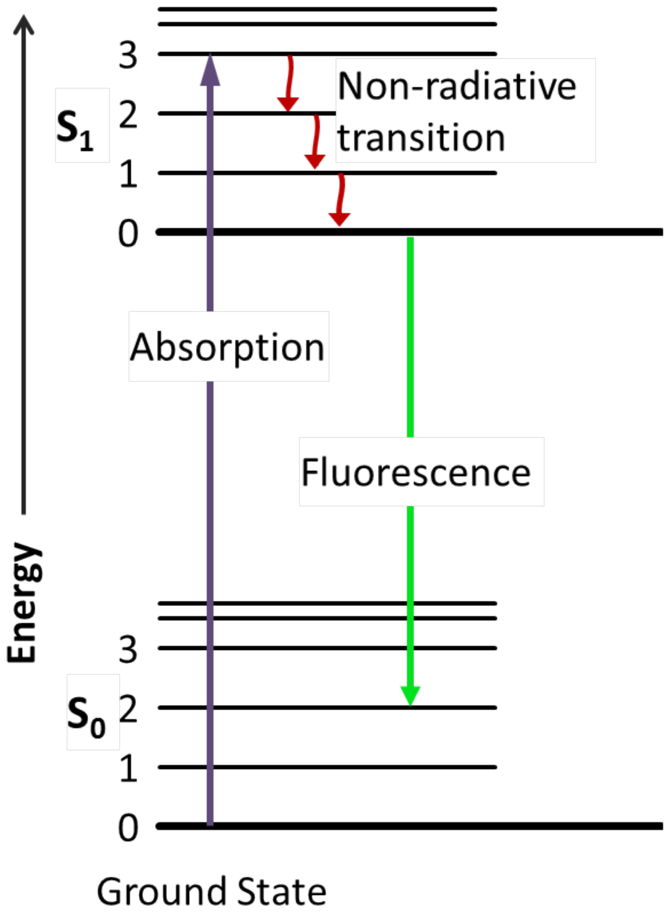
$$w_k = \frac{\text{Prob (fluorescence)}}{\text{Prob (florescence) + Prob (Auger)}}$$



General definition of fluorescence

Emission of light (UV to near infrared) by an atom, molecule that has absorbed light or other electromagnetic radiation, within the range of 0.5 to 20 nanoseconds

Energy levels in a molecule :

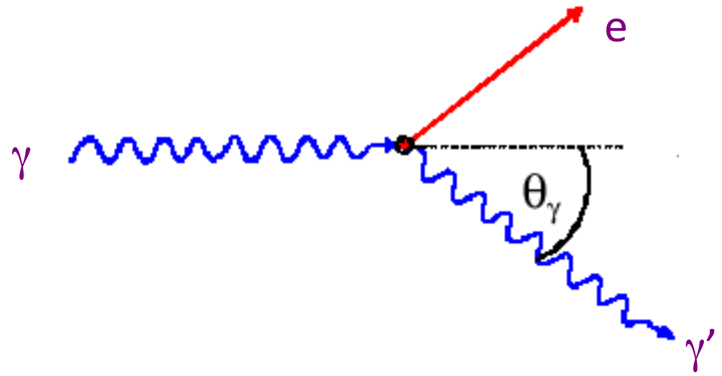


Important for scintillators !

2. Compton scattering

Scattering of γ on « **free** » electrons $\quad\quad$ **$\gamma + e \to \gamma' + e$**

In the matter electrons are bounded. When the γ energy, E_γ >> binding electron energy the electron can be considered as free.



$$E_{\gamma'} = \frac{E_\gamma}{1 + (E_\gamma/m_e c^2)(1 - \cos\theta_\gamma)}$$

Kinetic energy of the outgoing electron E_k^e :

$$E_k^e = E_\gamma - E_{\gamma'} = E_\gamma \frac{(1-\cos\theta)(E_\gamma/m_e c^2)}{1 + (E_\gamma/m_e c^2)(1 - \cos\theta_\gamma)}$$

γ Forward scattering $\quad \theta_\gamma = 0 \to \quad E_{\gamma'} = E_{\gamma' max} = E_\gamma \quad$ **$E_e = 0$**

Initial photon can give all its energy to final photon

γ Backward scattering $\quad \theta_\gamma = \pi \to \quad$ **$E_{\gamma'} = E_{\gamma' min}$** $\rightarrow$ **E_k^e is max**

but not to the e → The photon cannot be completely absorbed

Compton Edges in the final e spectrum

- γ Backward scattering $\theta_\gamma = \pi$

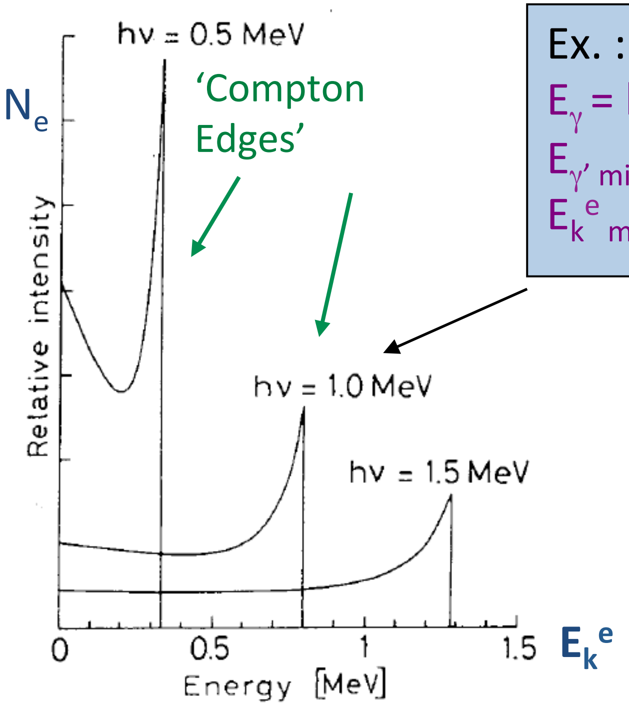
$\gamma + e \dashrightarrow \gamma' + e$

$E_{\gamma'\ min} \rightarrow E_k^e{}_{max}$

$E_\gamma = E_{\gamma'} + E_k^e$

$$E_{\gamma'\ min} = \frac{E_\gamma}{1 + 2E_\gamma/m_e c^2} \rightarrow E_k^e{}_{max} = E_\gamma \frac{2E_\gamma/m_e c^2}{1 + 2E_\gamma/m_e c^2}$$

Transfer of complete γ energy to e via Compton scattering is not possible



Ex. :
$E_\gamma = h\nu = 1$ MeV
$E_{\gamma'\ min} = 0.2$ MeV
$E_k^e{}_{max} = 0.8$ MeV

Important **for single photon detection**: the photon cannot be completely absorbed and the scattered electron misses a small amount of initial energy

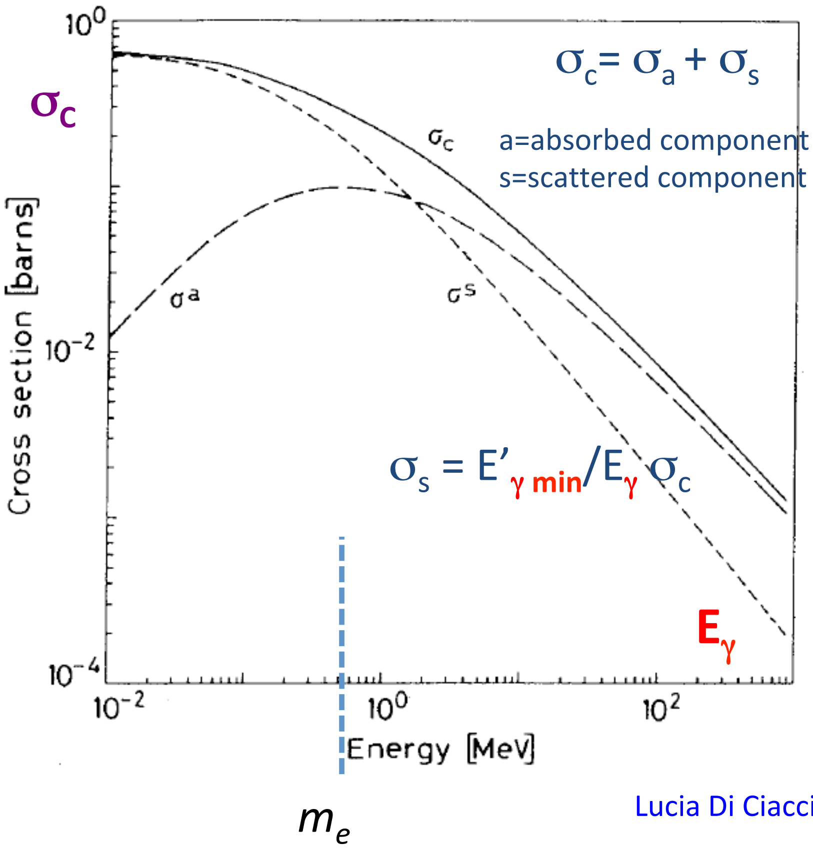
Compton Cross Section

Klein-Nishina Formula (LO QED):

$$\frac{d\sigma_c^e}{d\Omega} = \frac{r_e^2}{2}\frac{1+\cos^2\theta_\gamma}{(1+\epsilon(1-\cos\theta_\gamma))^2}\left(1+\frac{\epsilon^2(1-\cos\theta_\gamma)^2}{(1+\cos^2\theta_\gamma)(1+\epsilon(1-\cos\theta_\gamma))}\right) \quad (\textit{per electron})$$

$$\sigma_c^e = 2\pi r_e^2\left(\left(\frac{1+\epsilon}{\epsilon^2}\right)\left\{\frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon}\ln(1+2\epsilon)\right\} + \frac{1}{2\epsilon}\ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2}\right) \quad (\textit{per electron})$$

$$\epsilon = \frac{E_\gamma}{m_e}$$



@ Small photon energy ($E_\gamma << m_e c^2$)

$\sigma_C = \sigma_{Th}$ (1- $E_\gamma/(mc^2)$)

$\sigma_{Th} = 8\pi/3\ r_e^2 = 0.66$ barn
(σ_{Th}) = Thomson σ)

@ Large photon energy ($E_\gamma >> m_e c^2$)

$\sigma_C \sim (\ln E_\gamma)/E_\gamma$

Cross section per atom:

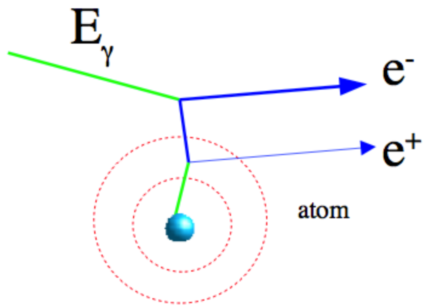
$$\sigma_c^{atom.} = Z\ \sigma_c^e$$

$r_e = \alpha/(m_e c^2)$= classical electron radius

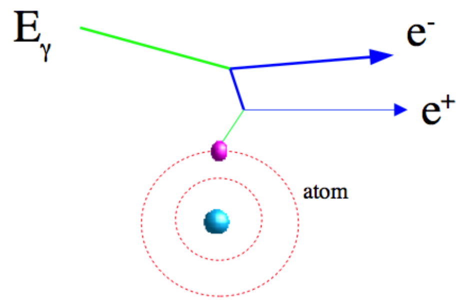
3. Pair production: $\gamma \rightarrow$ e+ e-

Called also photon conversion

For energy-momentum conservation this process cannot take place in 'vacuum', an interaction with an electromagnetic field is necessary



Pair production in the field of the **nucleus**



Pair production in the field of **an electron** (smaller probability ~ 1/Z)

Threshold process : $\mathbf{E_\gamma} > 2\, m_e c^2\, (1 + m_e/m_x)$

$m_x = m_N$ or $m_x = m_e$

Kinetic energy transferred to the "target" (nucleus or electrons)

First experimental observation of a positron

direction of the
high-energy photon

Pb plate

Production of an electron-positron pair by a high-energy photon in a Pb plate

$e^+ e^-$ pair production cross-section

$$\epsilon = \frac{E_\gamma}{m_e}$$

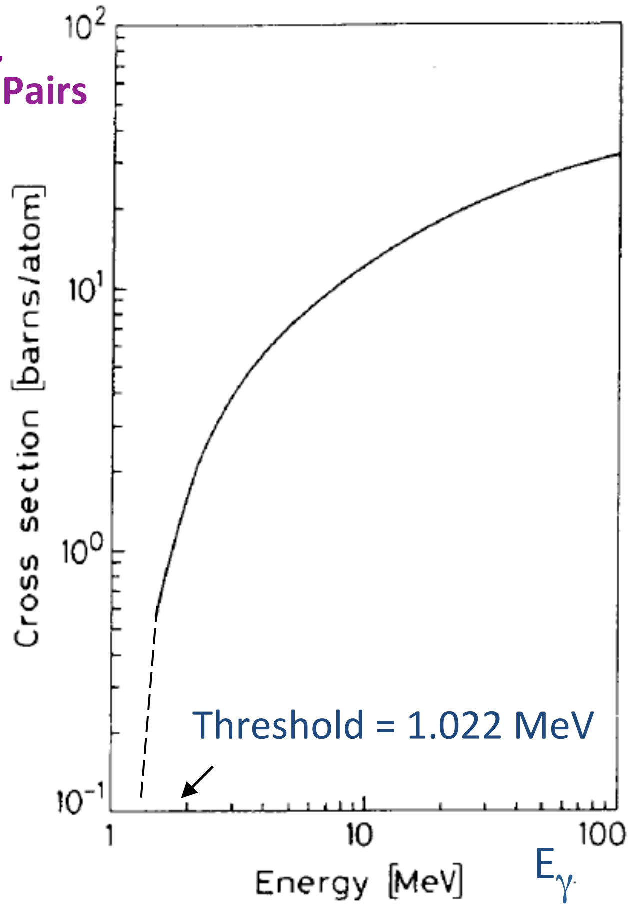
$$1 \ll \epsilon < \frac{1}{\alpha Z^{1/3}} \qquad \sigma^{atom.}_{pair} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9}\ln(2\epsilon) - \frac{109}{54}\right)$$

$$\epsilon \gg \frac{1}{\alpha Z^{1/3}} \qquad \sigma^{atom.}_{pair} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9}\ln\left(\frac{183}{Z^{1/3}}\right) - \frac{1}{54}\right)$$

In the high energy regime ($E_\gamma \to \infty$)

$$\sigma^{atom.}_{pair} \simeq \frac{7}{9}\frac{A}{N_A}\frac{1}{X_0}$$

accurate to within a few percent down to energies as low as 1 GeV, particularly for high-Z materials.



X_0 = radiation lenght

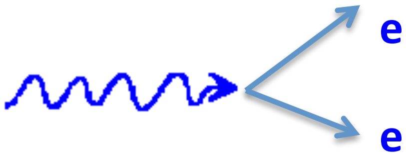
	ρ [g/cm³]	X_0 [cm]
H_2 [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Air	$1.2\cdot10^{-3}$	$30\cdot10^{3}$

Pair production is the leading effect at high energy

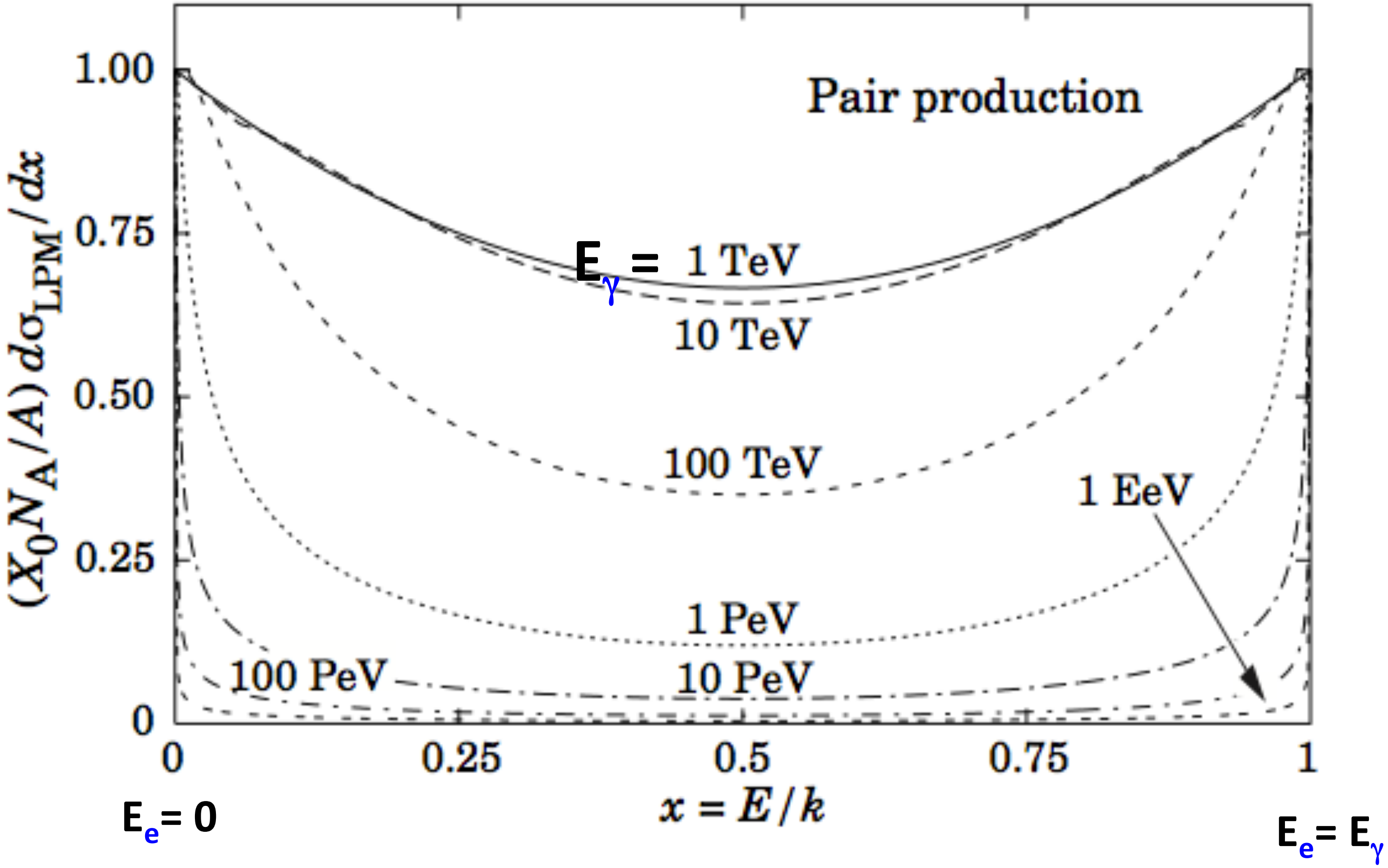
- Rises above threshold
- reaches saturation for large Eγ [screening effect]

Normalized $e^+ e^-$ pair production cross section

LPM =Landau–Pomeranchuk–Migdal cross section.

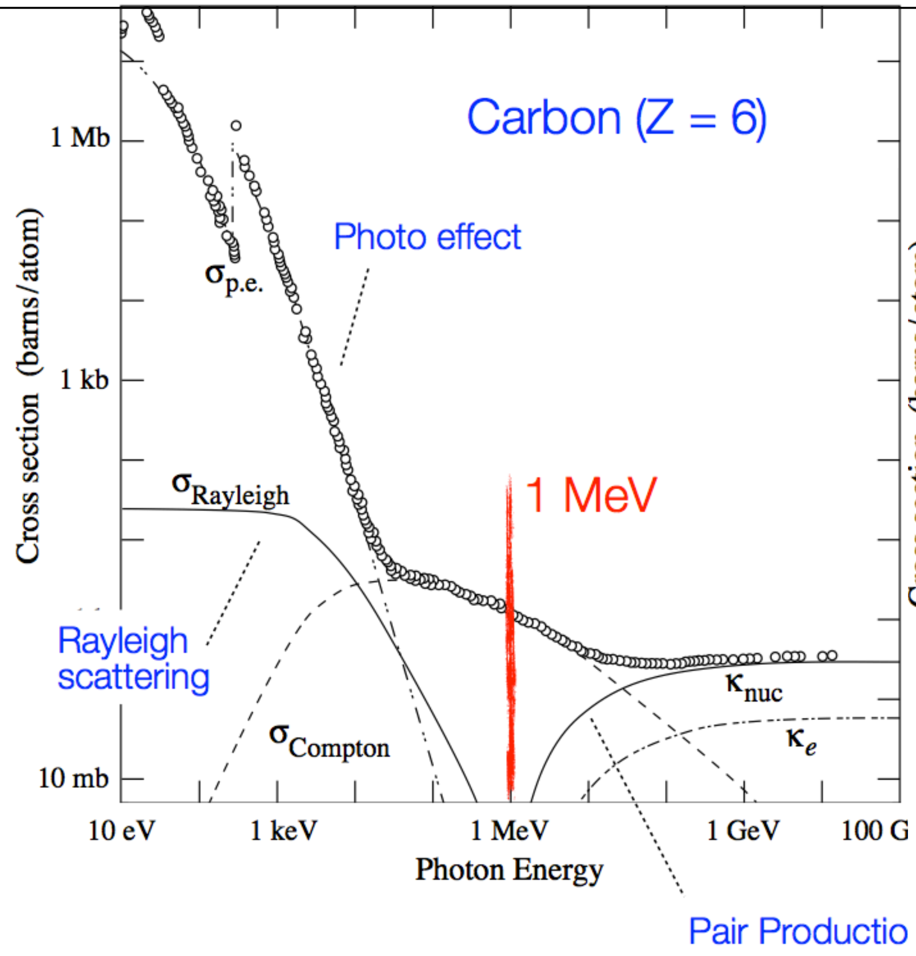


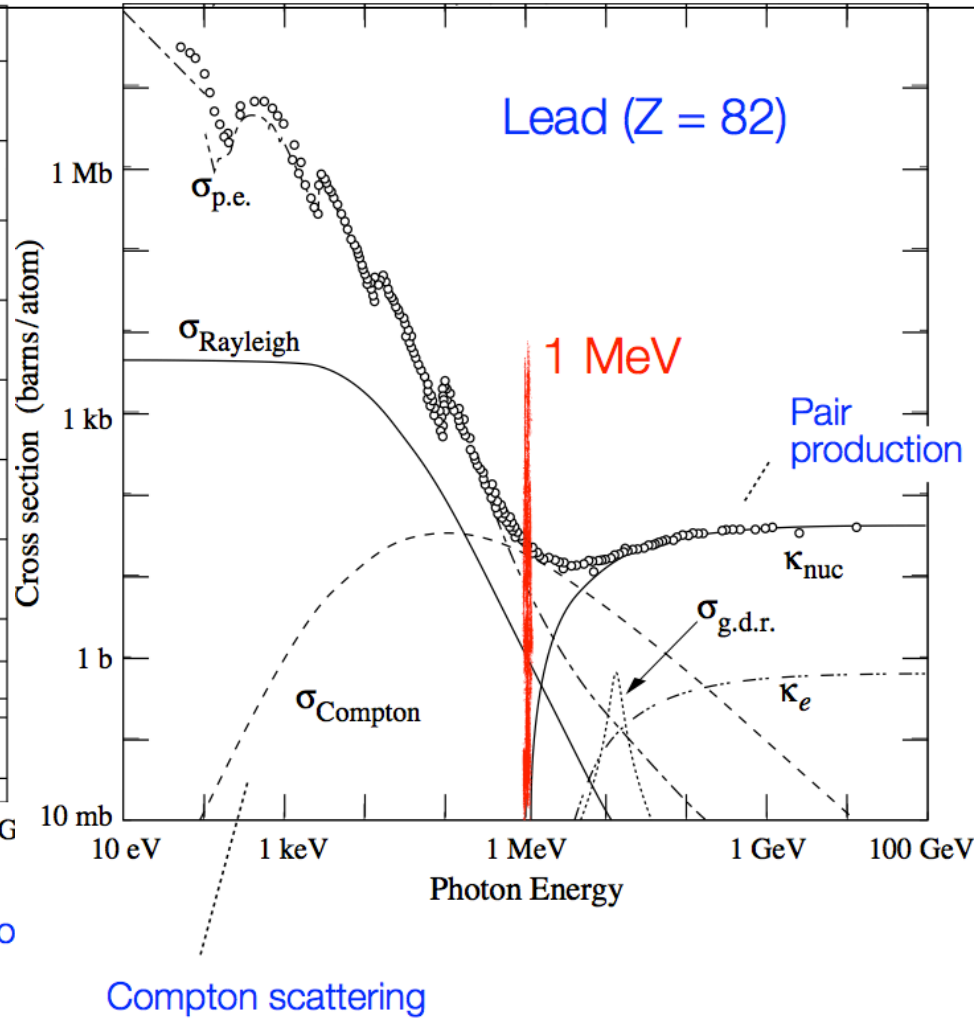
$k = E_\gamma$ $E = E_e$



fractional electron energy **$x = E/k = E_e/E_\gamma$**

γ total cross section



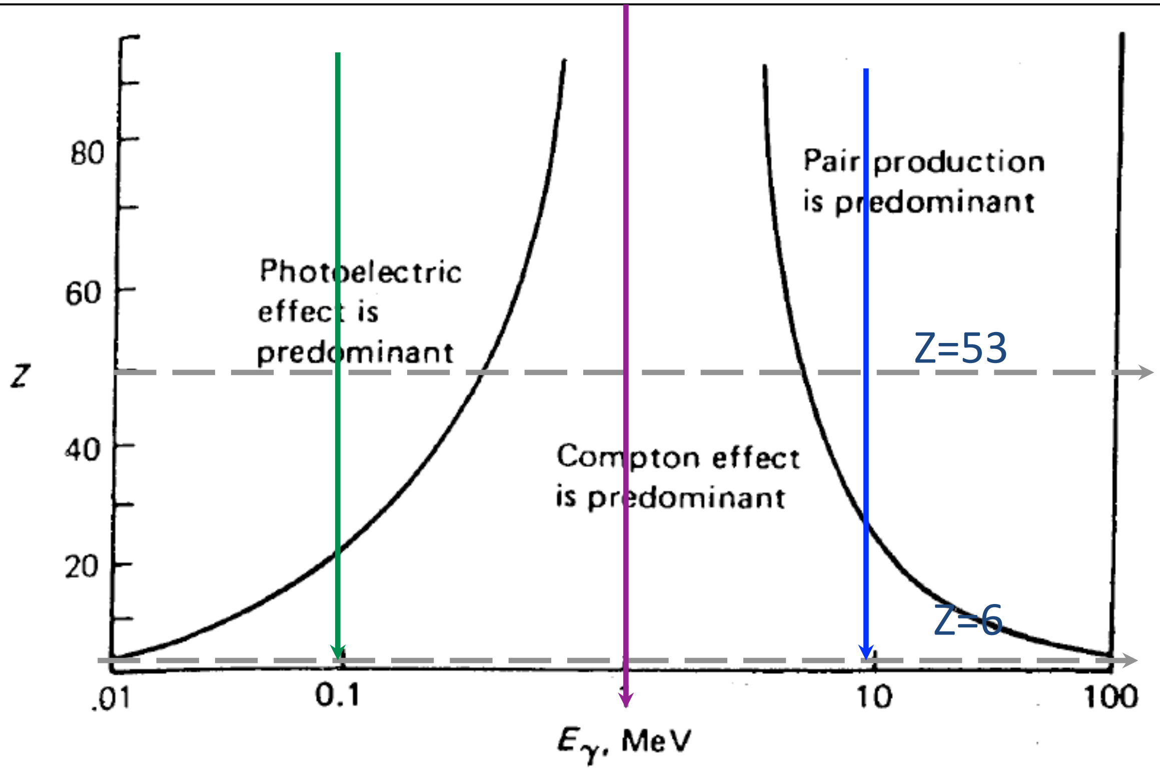


Several other effects take place (not discussed here):

Rayleigh Scattering (scattering on atmosphere particles, blue sky)

Photo Nuclear Interactions (Giant Dipole Resonance, collective excitation of atomic nuclei).

Dependence on Z et on E



E_γ = 0.1 MeV in C (Z=6) Compton effect is dominant

in I (Z=53) Photoelectric effect is dominant

E_γ = 1 MeV Compton effect is dominant

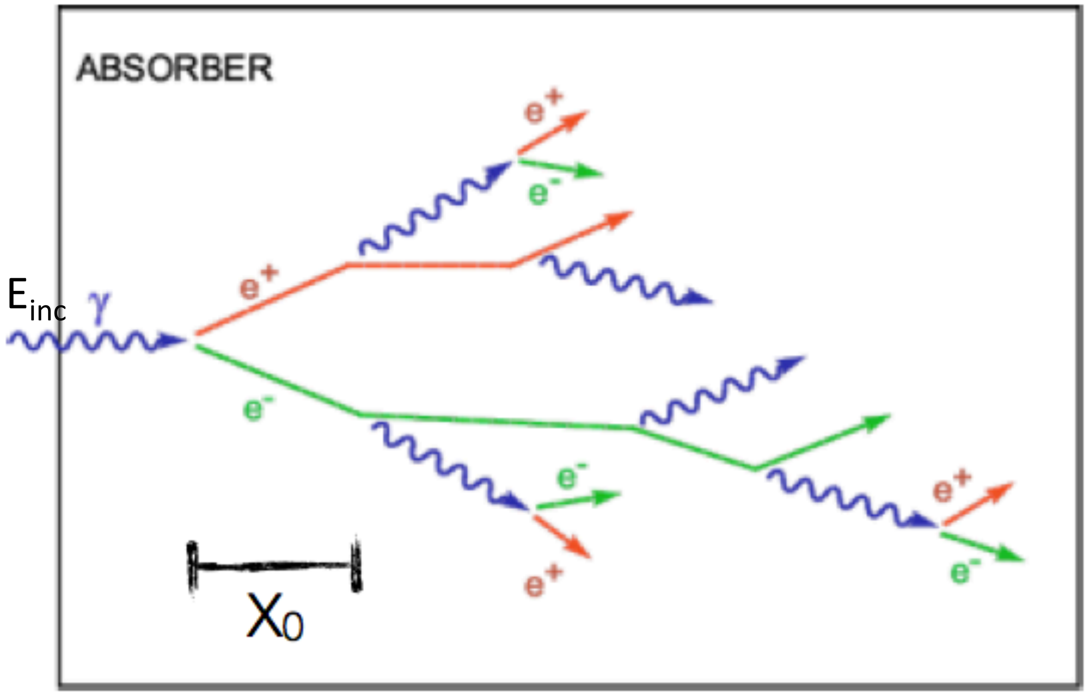
E_γ = 10 MeV in C (Z=6) Compton effect is dominant

in I (Z=53) pair production is dominant

Electromagnetic showers

See M. Delmastro lectures

Dominant processes for photons (and electrons) at very high energies



$$t_{max} = \ln \frac{E_{inc}}{E_c} - \begin{cases} 1 & e- \\ 0.5 & \text{gamma} \end{cases}$$

$$L\,95\% \simeq t_{max} + 0.08\,Z + 9.6\ [X0]$$

L 95% = longitudinal shower containment

t_{max} =depth in radiation length units, where the max energy is deposited

E_{in}= incoming photon energy

E_c= critical energy

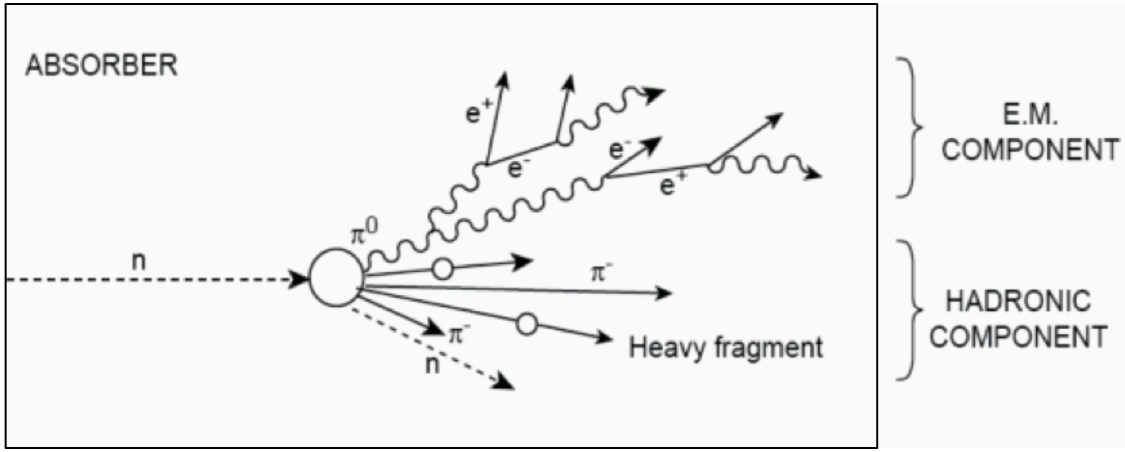
Also electrons can start e.m. showers

Hadron collisions and interaction lengths

The total cross section for **very high energy hadrons** is expressed as:

$\sigma_T = \sigma_{elastic} + \sigma_{inelastic}$

The inelastic part of the total cross-section is susceptible to induce a **hadron shower** (increase of particles multiplicity)



Two mean-lengths are introduced:

- nuclear collision length

$$\lambda_T = \frac{A}{N_A \sigma_T} \text{g cm}^{-2}$$

- nuclear interaction length

$$\lambda_I = \frac{A}{N_A \sigma_{inelastic}} \text{g cm}^{-2}$$

See M. Delmastro slides for more details

95% containment of a hadronic shower is for a material thickness of :

L 95%(in units of λ_I) ≃ 1+1.35 ln (E(GeV))

→ ~ **10 interaction lengths** are needed to contain a **1 TeV hadronic shower**

In high A materials $\boldsymbol{\lambda_I > X_0}$ This explains why hadron calorimeters are after installed electromagnetic

6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

Table 6.1. Abridged from `pdg.lbl.gov/AtomicNuclearProperties` by D. E. Groom (2007). Quantities in parentheses are for NTP (20° C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values $\gg$1 in brackets are for $(n-1)\times 10^6$ (gases).

Material	Z	A	$\langle Z/A\rangle$	Nucl.coll. length λ_T {g cm^{-2}}	Nucl.inter. length λ_I {g cm^{-2}}	Rad.len. X_0 {g cm^{-2}}	$dE/dx\|_{\min}$ { MeV g^{-1}cm^2}	Density {g cm^{-3}} ({gℓ^{-1}})	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
H_2	1	1.00794(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
D_2	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
Li	3	6.941(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
Be	4	9.012182(3)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			
N_2	7	14.0067(2)	0.49976	61.1	89.7	37.99	(1.825)	0.807(1.165)	63.15	77.29	1.20[298.]
O_2	8	15.9994(3)	0.50002	61.3	90.2	34.24	(1.801)	1.141(1.332)	54.36	90.20	1.22[271.]
F_2	9	18.9984032(5)	0.47372	65.0	97.4	32.93	(1.676)	1.507(1.580)	53.53	85.03	[195.]
Ne	10	20.1797(6)	0.49555	65.7	99.0	28.93	(1.724)	1.204(0.839)	24.56	27.07	1.09[67.1]
Al	13	26.9815386(8)	0.48181	69.7	107.2	24.01	1.615	2.699	933.5	2792.	
Si	14	28.0855(3)	0.49848	70.2	108.4	21.82	1.664	2.329	1687.	3538.	3.95
Cl_2	17	35.453(2)	0.47951	73.8	115.7	19.28	(1.630)	1.574(2.980)	171.6	239.1	[773.]
Ar	18	39.948(1)	0.45059	75.7	119.7	19.55	(1.519)	1.396(1.662)	83.81	87.26	1.23[281.]
Ti	22	47.867(1)	0.45961	78.8	126.2	16.16	1.477	4.540	1941.	3560.	
Fe	26	55.845(2)	0.46557	81.7	132.1	13.84	1.451	7.874	1811.	3134.	
Cu	29	63.546(3)	0.45636	84.2	137.3	12.86	1.403	8.960	1358.	2835.	
Ge	32	72.64(1)	0.44053	86.9	143.0	12.25	1.370	5.323	1211.	3106.	
Sn	50	118.710(7)	0.42119	98.2	166.7	8.82	1.263	7.310	505.1	2875.	
Xe	54	131.293(6)	0.41129	100.8	172.1	8.48	(1.255)	2.953(5.483)	161.4	165.1	1.39[701.]
W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Au	79	196.966569(4)	0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
Pb	82	207.2(1)	0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	

Material	Z	A	$\langle Z/A \rangle$	Nucl.coll. length λ_T {g cm^{-2}}	Nucl.inter. length λ_I {g cm^{-2}}	Rad.len. X_0 {g cm^{-2}}	$dE/dx\|_{min}$ { MeV g^{-1}cm^2}	Density {g cm^{-3}} ({gℓ^{-1}})	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
Air (dry, 1 atm)			0.49919	61.3	90.1	36.62	(1.815)	(1.205)		78.80	
Shielding concrete			0.50274	65.1	97.5	26.57	1.711	2.300			
Borosilicate glass (Pyrex)			0.49707	64.6	96.5	28.17	1.696	2.230			
Lead glass			0.42101	95.9	158.0	7.87	1.255	6.220			
Standard rock			0.50000	66.8	101.3	26.54	1.688	2.650			
Methane (CH_4)			0.62334	54.0	73.8	46.47	(2.417)	(0.667)	90.68	111.7	[444.]
Ethane (C_2H_6)			0.59861	55.0	75.9	45.66	(2.304)	(1.263)	90.36	184.5	
Butane (C_4H_{10})			0.59497	55.5	77.1	45.23	(2.278)	(2.489)	134.9	272.6	
Octane (C_8H_{18})			0.57778	55.8	77.8	45.00	2.123	0.703	214.4	398.8	
Paraffin ($CH_3(CH_2)_{n\approx23}CH_3$)			0.57275	56.0	78.3	44.85	2.088	0.930			
Nylon (type 6, 6/6)			0.54790	57.5	81.6	41.92	1.973	1.18			
Polycarbonate (Lexan)			0.52697	58.3	83.6	41.50	1.886	1.20			
Polyethylene ($[CH_2CH_2]_n$)			0.57034	56.1	78.5	44.77	2.079	0.89			
Polyethylene terephthalate (Mylar)			0.52037	58.9	84.9	39.95	1.848	1.40			
Polymethylmethacrylate (acrylic)			0.53937	58.1	82.8	40.55	1.929	1.19			1.49
Polypropylene			0.55998	56.1	78.5	44.77	2.041	0.90			
Polystyrene ($[C_6H_5CHCH_2]_n$)			0.53768	57.5	81.7	43.79	1.936	1.06			1.59
Polytetrafluoroethylene (Teflon)			0.47992	63.5	94.4	34.84	1.671	2.20			
Polyvinyltoluene			0.54141	57.3	81.3	43.90	1.956	1.03			1.58
Aluminum oxide (sapphire)			0.49038	65.5	98.4	27.94	1.647	3.970	2327.	3273.	1.77
Barium flouride (BaF_2)			0.42207	90.8	149.0	9.91	1.303	4.893	1641.	2533.	1.47
Carbon dioxide gas (CO_2)			0.49989	60.7	88.9	36.20	1.819	(1.842)			[449.]
Solid carbon dioxide (dry ice)			0.49989	60.7	88.9	36.20	1.787	1.563	Sublimes at 194.7 K		
Cesium iodide (CsI)			0.41569	100.6	171.5	8.39	1.243	4.510	894.2	1553.	1.79
Lithium fluoride (LiF)			0.46262	61.0	88.7	39.26	1.614	2.635	1121.	1946.	1.39
Lithium hydride (LiH)			0.50321	50.8	68.1	79.62	1.897	0.820	965.		
Lead tungstate ($PbWO_4$)			0.41315	100.6	168.3	7.39	1.229	8.300	1403.		2.20
Silicon dioxide (SiO_2, fused quartz)			0.49930	65.2	97.8	27.05	1.699	2.200	1986.	3223.	1.46
Sodium chloride (NaCl)			0.55509	71.2	110.1	21.91	1.847	2.170	1075.	1738.	1.54
Sodium iodide (NaI)			0.42697	93.1	154.6	9.49	1.305	3.667	933.2	1577.	1.77
Water (H_2O)			0.55509	58.5	83.3	36.08	1.992	1.000(0.756)	273.1	373.1	1.33
Silica aerogel			0.50093	65.0	97.3	27.25	1.740	0.200	(0.03 H_2O, 0.97 SiO_2)		

Neutron interactions

Electric charge of the neutron n : $q_n = 0$

➡ The n interacts via « **strong interaction** » with nuclei (short range force ~ 10^{-13} cm)

Classification of neutrons:

Cold or ultracold neutrons	$E_n < 0.025$ eV
Thermal or slow neutrons	$E_n \sim 0.025$ eV
Intermediate neutrons	$E_n \sim 0.025$ eV ÷ 0.1 MeV
Fast neutrons	$E_n \sim$ 0.1 ÷ 10-20 MeV
High energy neutrons	$E_n > 20$ MeV

Alternative classification:

Slow neutrons (absorbed)	$E_n <\sim 0.5$ MeV	
Fast neutrons	$E_n >\sim 0.5$ MeV	E = 0.5 MeV = 'cadmium cutoff'

Main interaction processes of **n**: scattering (elastic and inelastic), absorption, fission hadron shower production depending on the neutron energy

Neutron interactions

Scattering with nuclei : $n + {}^{A}_{Z}X \rightarrow {}^{A}_{Z}X^{(*)} + n$

Elastic → important for **moderation**

Inelastic

Absorption & **Nuclear reactions:**

$n + {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + p$

$n + {}^{A}_{Z}X \rightarrow {}^{A-3}_{Z-2}Y + {}^{4}_{2}H_e$

$n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X + \gamma$ **radiative capture of n**

$n + {}^{A}_{Z}X \rightarrow {}^{A-1}_{Z}X + 2n$

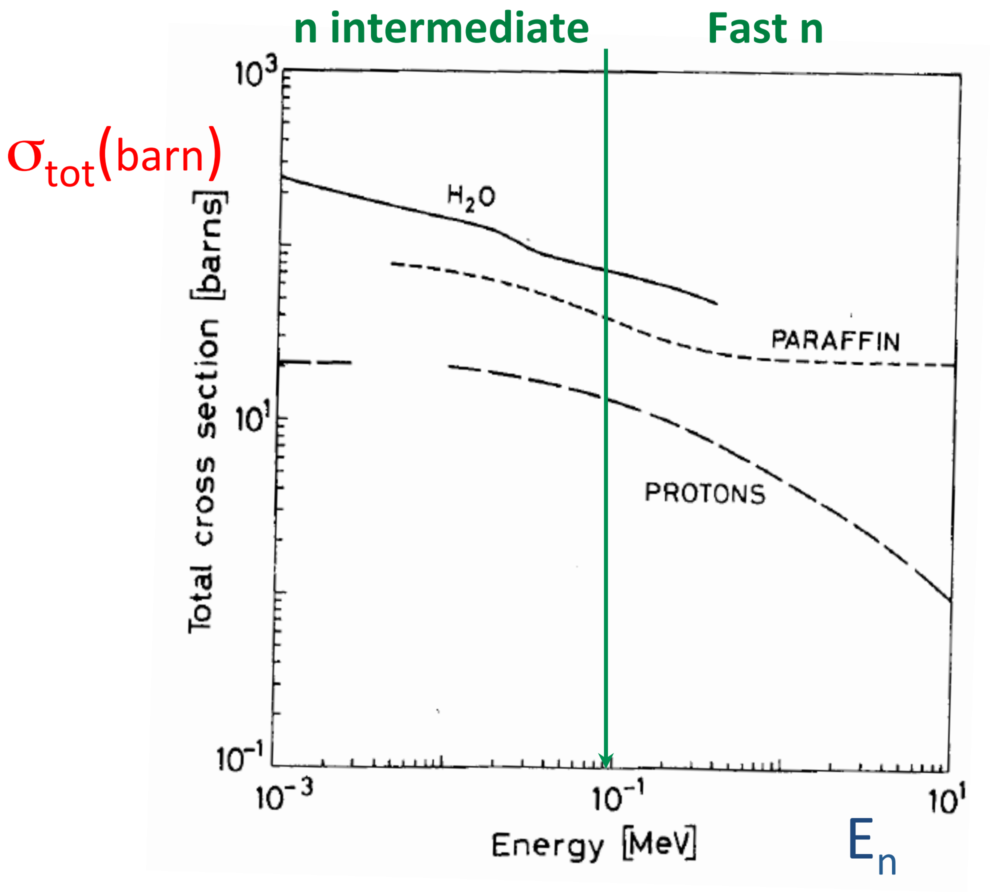
Fission: $n + {}^{A}_{Z}X \rightarrow {}^{A1}_{Z2}Y + {}^{A2}_{Z2}Y + n + n + \ldots$

Cross section $\approx 1/v_n$ (more probable for low energy) + resonant peaks

Hadron showers $E_n > \sim 100$ MeV

Cross section of low energy neutrons (n)

Neutron cross section on H_2O, paraffine and protons



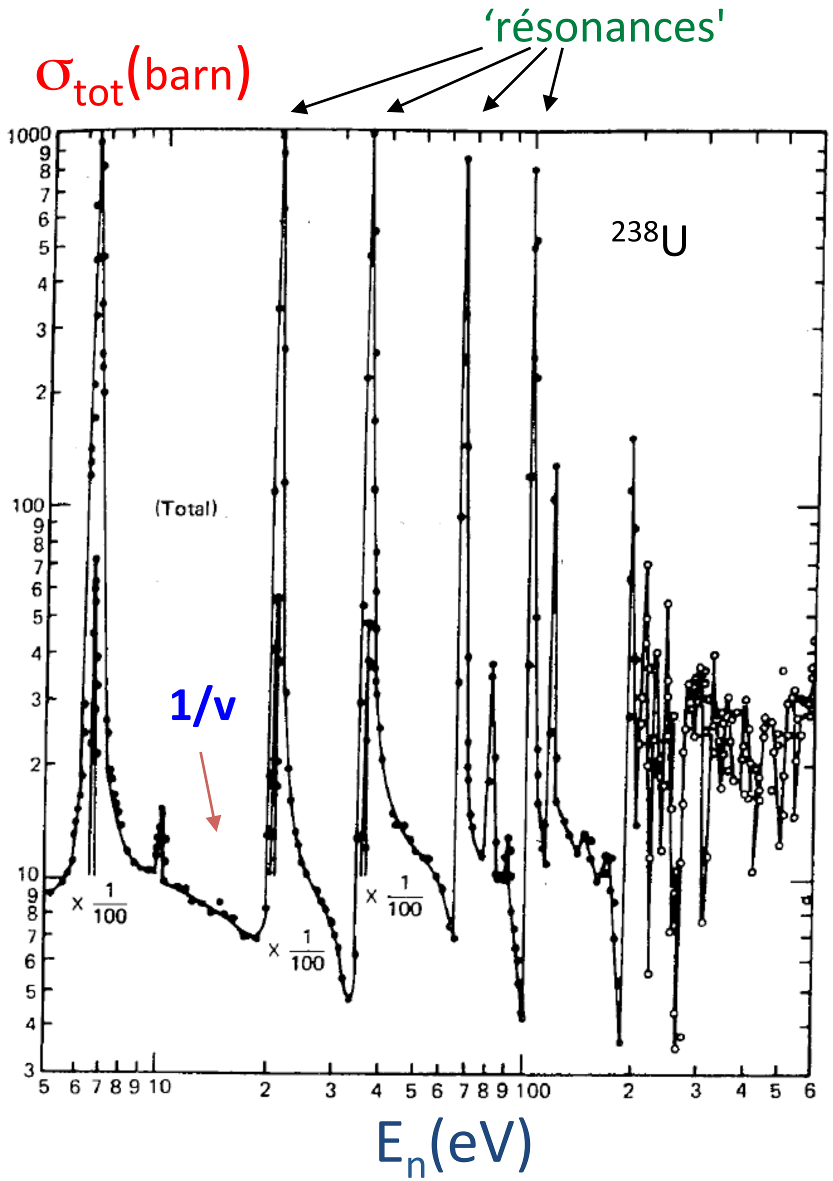
1 barn = 10^{-24} cm^2 = 10^{-28} m^2

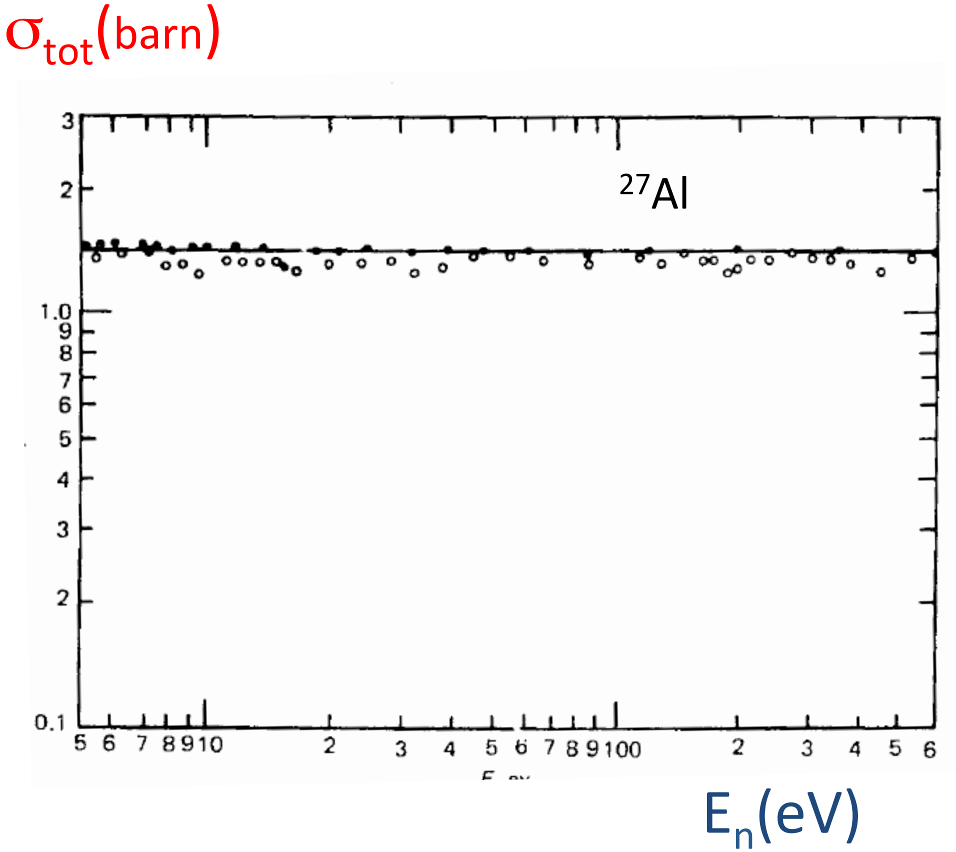
Radius of the nucleon R ≈ $10^{-15} \div 10^{-14}$ m

1 barn ≈ R^2

Low energy neutron (n) cross section

NB : Very different scales on the vertical axis





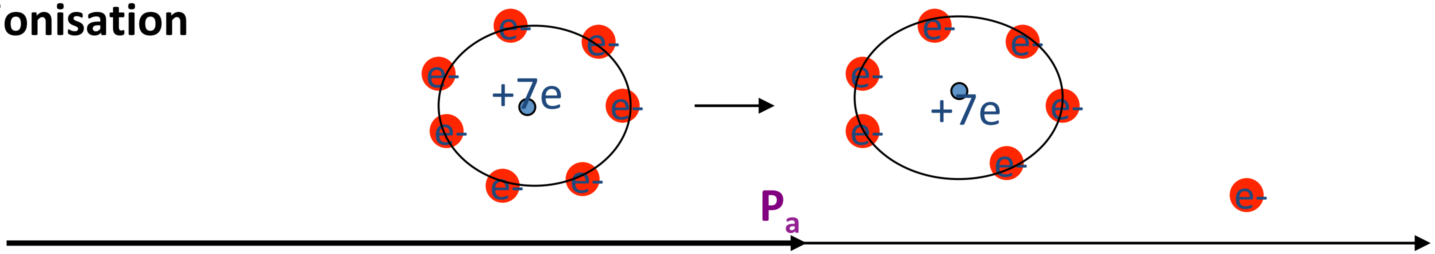
Charged particle interactions

- 1) **Ionization:** inelastic collision with **electrons** of the atoms
- 2) **Bremsstrahlung:** photon radiation emission by an accelerated charge
- 3) **Multiple Scattering:** elastic collision with **nucleus**
- 4) **Cerenkov & transition radiation effects:** photon emission
- (5) **Nuclear interactions (p, π, K):** processes mediated by strong interactions)

1) Inelastic collision with electrons of the atoms

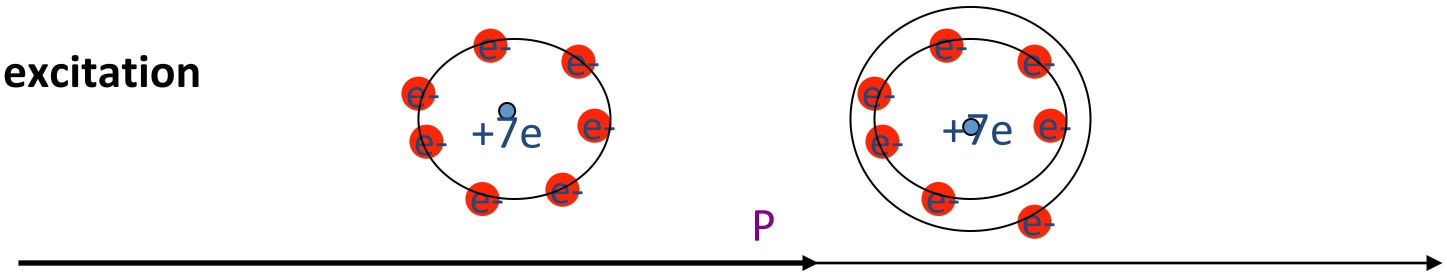
Main e.m. process for heavy (**M_{Pa} >> m_e**) **charged particles P_a** (ex. μ)

- **ionisation**



P_a + atom ---> atom$^+$ + e$^-$ + P_a

- **excitation**

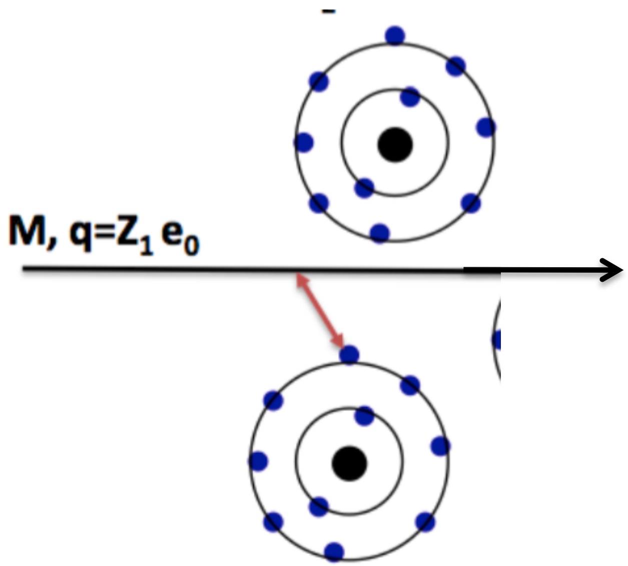


P_a + atom ---> atom* + P_a

↳ atom + γ

- Both processes together (**ionization & excitation**) can also happen
- **Inelastic collisions on nucleus(N)** are much less frequent (since the **energy transfer** depends inversely on the target mass and m_N >> m_e)
- **The particle P_a looses a bit of its energy (in each of the many collisions), its directions is ~ unchanged.**

Average energy loss per unit of lenght (- dE/dx) of P_a due to inelastic collisions with electrons of the atom

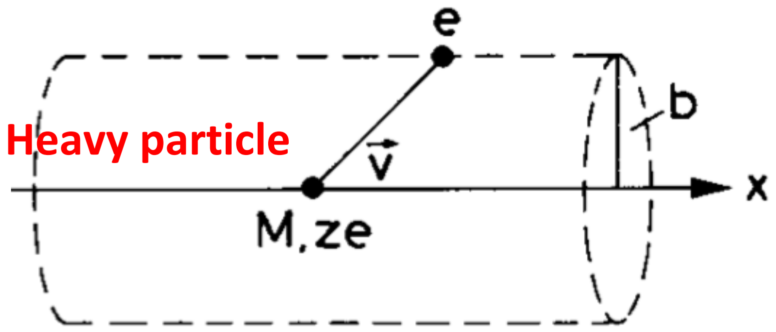


Analytic formula: **Bethe & Bloch (B&B) formula**

let'us derive here a simplified 'semi-relativistic' expression for **- dE/dx**

Simple computation of the average energy loss of particle P_a

(derivation of the B&B formula)



$\vec{E}$ = electric field generated by P_a

Assumptions:

- **e** considered free and initially at rest
- **e** moving slightly during interaction
- Heavy particle undeflected (v ~ const)
- Electric force **acting on e** (during dt = dx/v):

$$\vec{F} = \frac{d\vec{p}}{dt}$$

(*) $\Delta p_\perp = \int F_\perp \, dt = e \int E_\perp \, dt = e \int E_\perp \, dx/v$

$E_\perp$ from Gauss law: $\Phi_S(\vec{E}) = 4\pi ze$

$$\int E_\perp \, 2\pi b \, dx = 4\pi ze \qquad \int E_\perp \, dx = 2ze/b$$

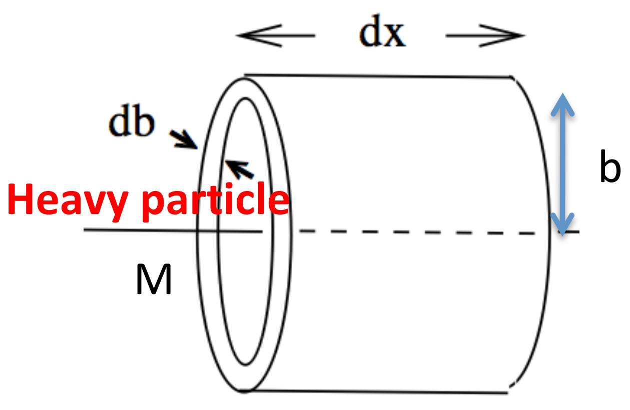
Momentum transferred to **one** electron: $\Delta p = 2z e^2 / (b v)$

Kin. Energy transferred to **one** electron: $\Delta E = \Delta p^2/(2 m_e) = \dfrac{2 z^2 e^4}{m_e v^2 b^2}$ (non-rel.)

(*) w.r.t the particle direction

$\Delta p_{//}$ effects average to 0 (symmetry)

Simple computation of the average energy loss



dV ≈ 2 π b db dx

$$\Delta E = \frac{2\, z^2 e^4}{m_e v^2 b^2}$$

- Effect of the interaction of P_a with the electrons **in dV** (energy loss by P_a):

$$- dE(b) = \Delta E\ N_c\ dV = \frac{4\pi z^2 e^4}{m_e v^2} N_c \frac{db}{b} dx$$

$N_c = (\rho N_A Z)/A_{mol}$= number of electrons per unit of volume

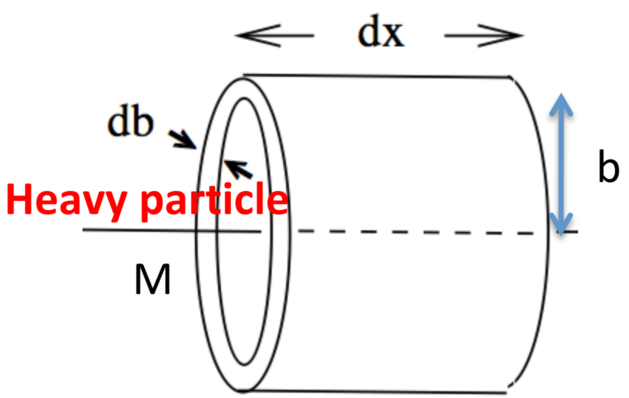
$$- \frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_c \ln \frac{b_{max}}{b_{min}}$$

De Broglie wavelength of electron

(after an head-on collision v_e ≈ particle velocity)

b_{min} from De Broglie wavelenght $b_{min} = \lambda_e = h/p_{emax} \sim h/(m_e \gamma v)$

Simple computation of the average energy loss



$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_c \ln \frac{b_{max}}{b_{min}}$$

b_{max} from "adiabatic invariance" :
the perturbation should occur in a time short compared to the revolution period τ of the bound electron

"Collision time"

Electron period in atom

$b/(\gamma v) < \tau$ where $\tau = 1/\nu_e$

Particle velocity

$b_{max} = (\gamma v)/\nu_e$

Orbital frequency

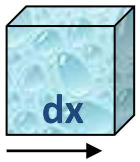
$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_c \ln \frac{m_e c^2 \beta^2 \gamma^2}{h \nu_e}$$

$I = h \nu_e$

Close to the Bethe&Bloch formula (within a factor ~ 2)

Average energy loss by a charged particle ($m_{Pa} >> m_e$) in matter

Incident charged
'**heavy**' particle P_a of energy ***E, M***



matter (e.x. gaz of a detector)

Bethe-Bloch formula (B & B)

$$-\frac{dE}{dx} = K\rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

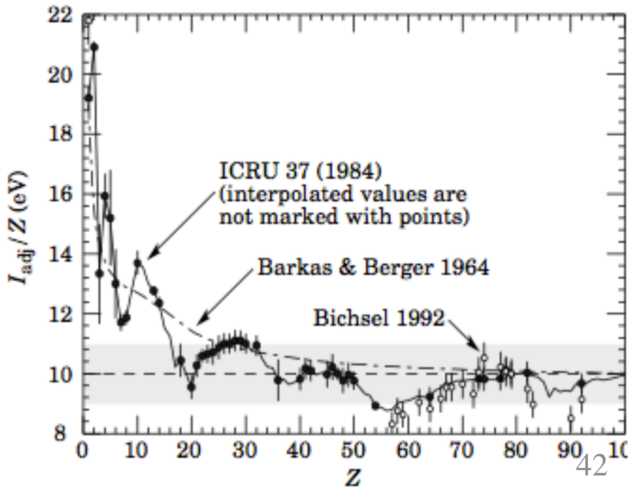
$2K = 4\pi N_A r_e^2 m_e c^2 = 0.307\ MeV\ g^{-1}\ cm^2$

r_e = classic radius of electron = $\alpha/(m_e c^2)$ = 2.8 fm
m_e = electron mass = 511 KeV
z = charge of incident particle in unit of e
β = particle speed in unit of c
$\gamma = 1/\sqrt{1-\beta^2}$
T_{max} = **maximum** Kin **energy** transferred in a collision)
ρ = density of the matter
Z, A = atomic number, atomic weight of the matter
I = effective excitation potential of the matter
Difficult to compute --> obtained from ***dE/dx***

I (eV) = (12+ 7/Z) Z (Z ≤ 12)
I (eV) = (9.76 + 58.8 $Z^{-1.19}$) Z (Z ≥ 12)

$$T_{max} = E_e^{max} - m_e = \frac{2 m_e \beta^2 \gamma^2}{(E_{CM}/M)^2}$$

$T_{max} \sim 2 m_e c^2 \beta^2 \gamma^2$ for $\gamma << m_{Pa} / (2 m_e)$



Shell (C) and Density(δ) effect corrections

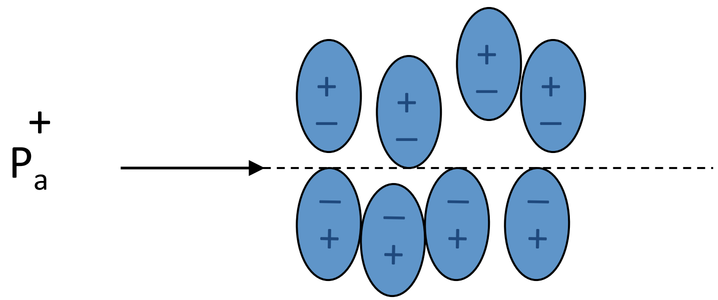
C = **Relevant at low energy**. Small correction. The particle velocity ~ orbital velocity of **e**
→ the assumption that atomic electrons initially are at rest breaks.
Takes into account binding energy. The energy loss is reduced.
The capture process of the particle is possible

δ = **"Density effect". Relevant at high energy.**
The electric field of the particle polarise the atoms of the matter
→ The energy loss is reduced since shielding of electrical field far from the particle path → **moderation of the relativistic rise**
It depends on the particle speed and on the matter density

Density effect leads to "saturation" at high energy

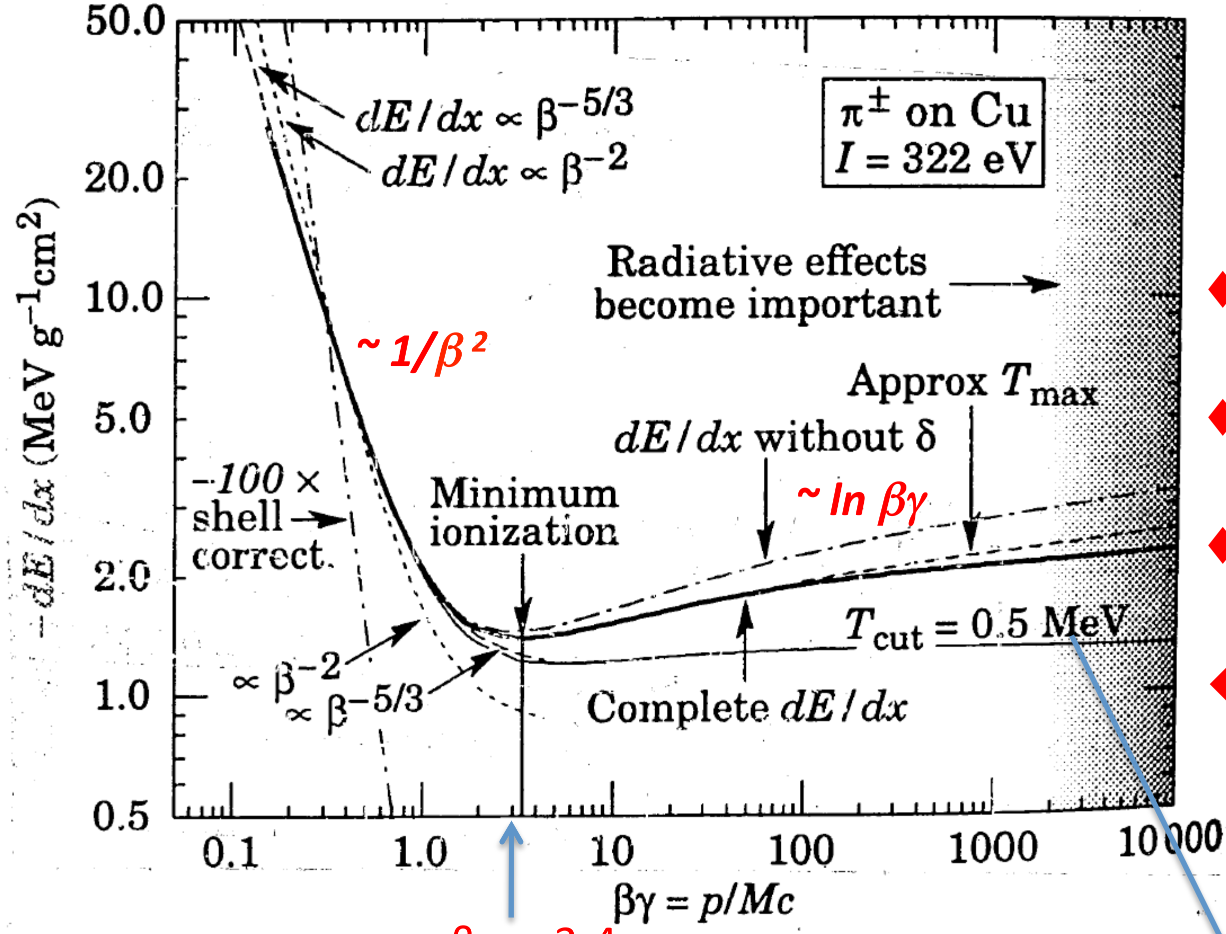
For high energy: $\delta/2 \to \ln(\hbar\omega_p/I) + \ln\beta\gamma - 1/2$

$\hbar\omega_p$ ="Plasma energy" = $\sqrt{\rho\,\langle Z/A\rangle} \times 28.816$ eV (ρ in g cm^{-3})



Polarisation effect (correction δ)
→ reduction of the energy loss

Stopping power or mean specific energy loss = dE/dx ($M_{Pa} >> m_e$)



β γ = 3-4
(Minimum Ionizing Particle= MIP)

Few remarks:

- **dE/dx = f(β)**
- **dE/dx doesn't depend on M_{Pa}**
- **dE/dx ∝ z^2 (particle charge)**
- On vertical l'axis **$-dE/(\rho\, dx)$ (MeV cm²)/g**

Why?

Tcut
'restricted energy loss'
discussed later

See Marco Delmastro lectures for explanation of $1/\beta^2$ and ln $\beta\gamma$

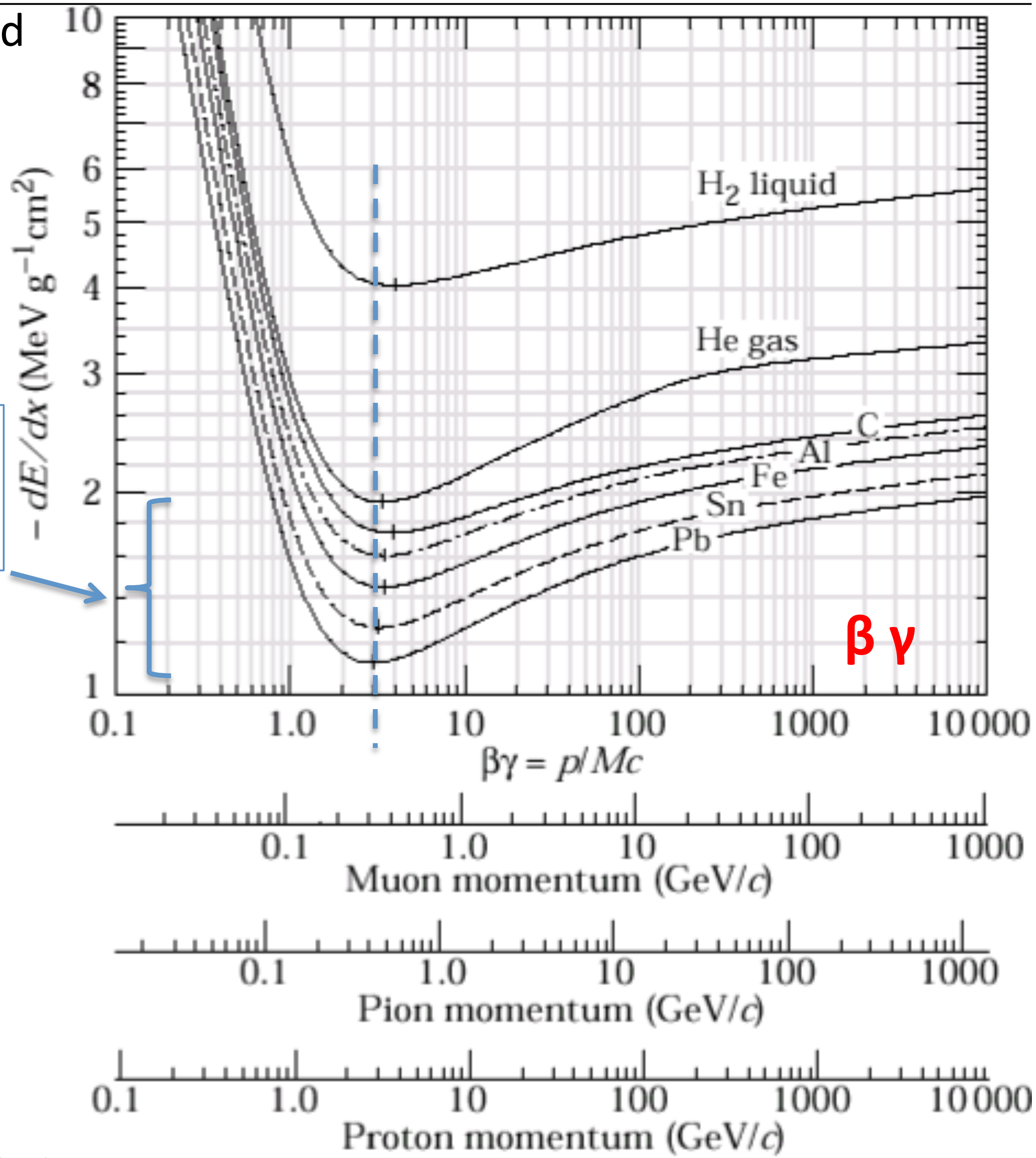
Stopping power

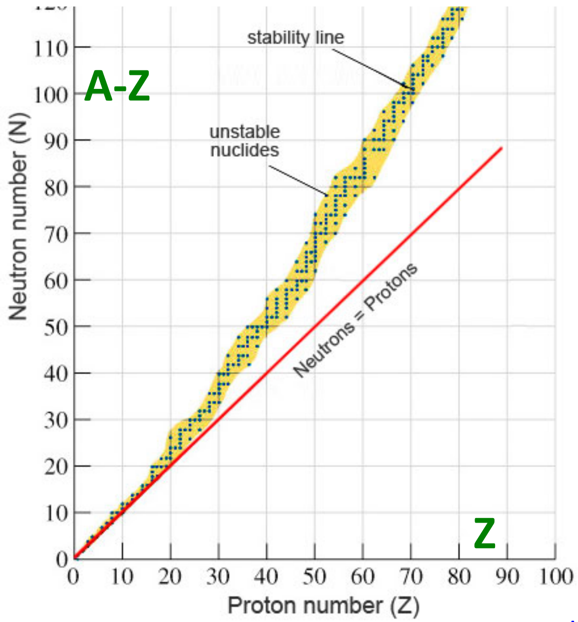
* If material thickness is measured in ρ **dx** **(g/cm²)**

→ on vertical l'axis
-dE/(ρ dx) **(MeV cm²)/g**

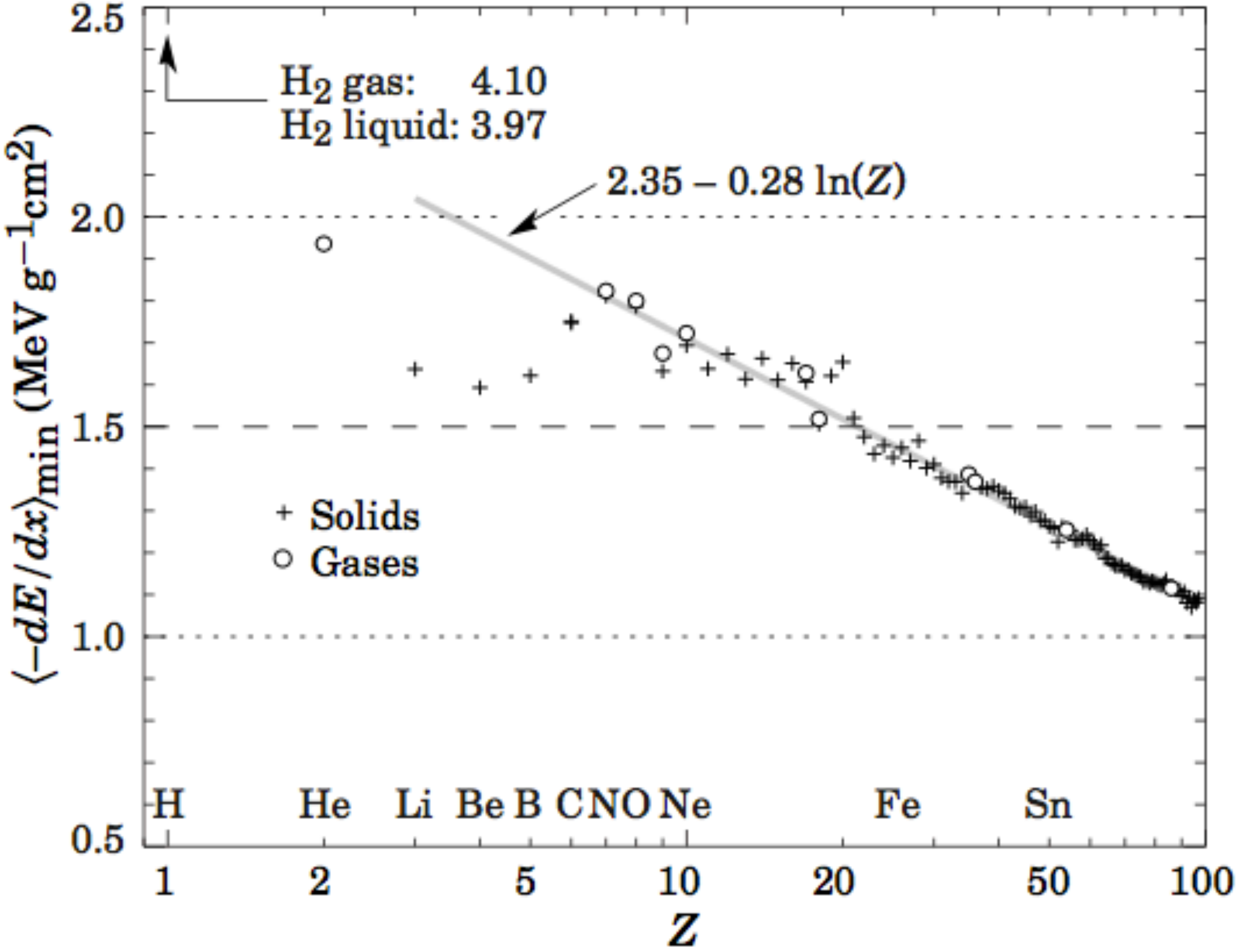
→ the dependence on the material is reduced
(**Z/A** ~ 0.5)

1-2 MeV g^{-1}cm^{2}
« minimum of ionisation »





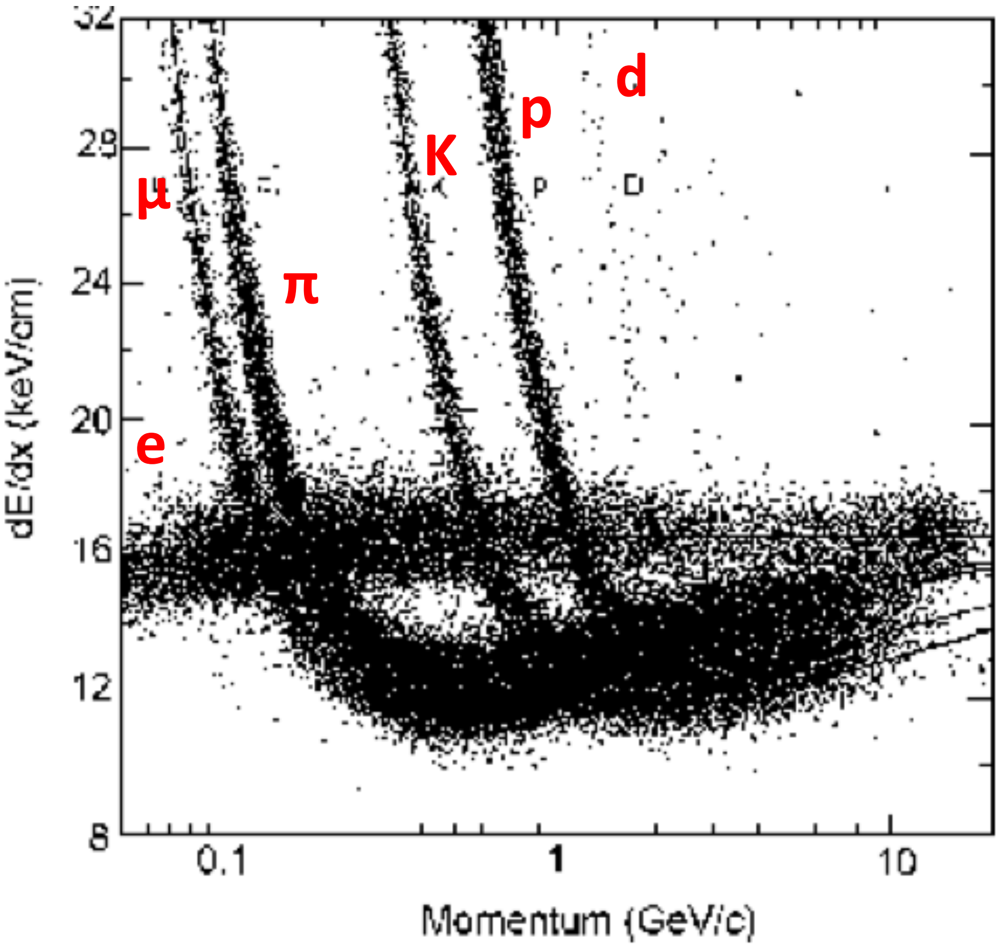
Stopping power at the *minimum of ionization* in greater detail



Use of dE/dx for particle identification

- $\vec{p} = m \gamma c \vec{\beta}$ $\qquad \gamma \equiv \dfrac{1}{\sqrt{1-\beta^2}}$

Measuring **independently** **p** and $\gamma\beta$ one can extract **m** → particle identification



Plot from
PEP4-9
Time Projection Chamber (TPC)
@SLAC (late '70)

Knock-on electrons or delta(δ) rays or secondary electrons

High energy transfers generates **secondary electrons (from delta rays)**

Distribution (prob.) of **δ** with **kinetic energies** $T \gg I$ **:**

$$\frac{d^2N}{dTdx} = \frac{1}{2} K z^2 \frac{Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2} \quad MeV^{-1} cm^2 g^{-1}$$

K *= 0.307*

F(T) **= Spin** dependent factor

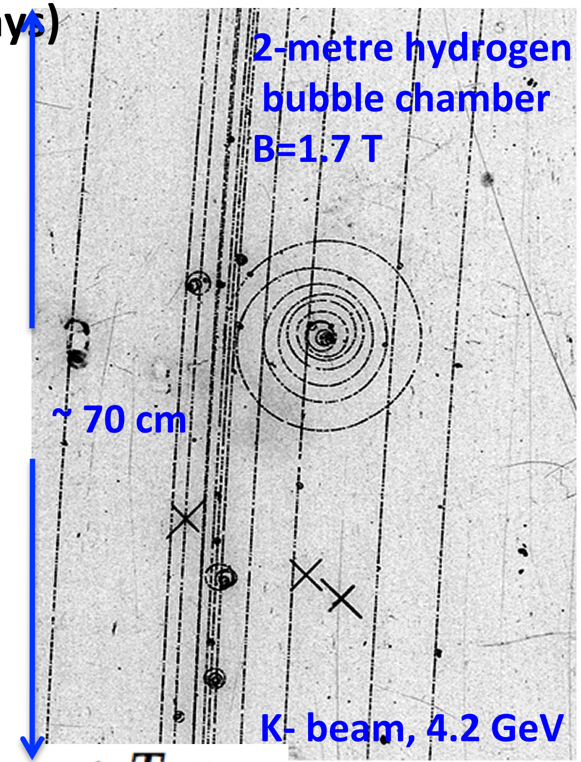
β, m_{Pa} = speed and mass of **primary** particle

x = "**mass** thickness" (ρ*t)

Spin 0 $$F(T) = F_0(T) = \left(1 - \beta^2 \frac{T}{T_{max}}\right)$$

Spin 1/2 $$F(T) = F_{1/2}(T) = F_0(T) + \frac{1}{2}\left(\frac{T}{E}\right)^2$$

Spin 1 $$F(T) = F_1(T) = F_0(T)\left(1 + \frac{1}{3}\frac{T m_e}{m^2_{Pa}}\right) + \frac{1}{3}\left(\frac{T}{E}\right)^2\left(1 + \frac{1}{2}\frac{T m_e}{m^2_{Pa}}\right)$$



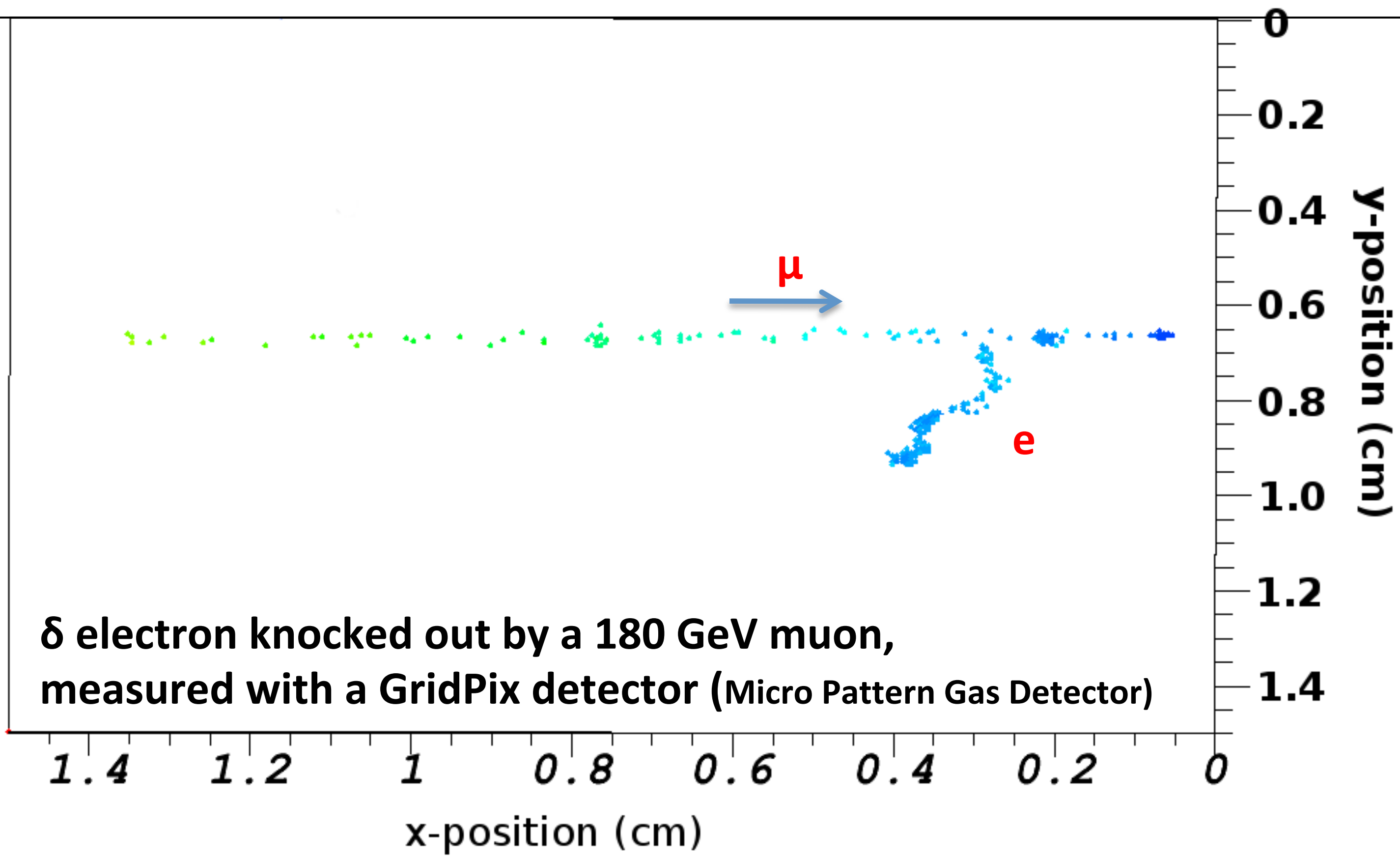
For $T \ll T_{max}$ & $T \ll m_{Pa}^2/m_e$ & $F(T) = 1$:

approximate probability to generate a **δ** with $T > T_s$

in a thin absorber of thickness x:

$$w(Ts, E, x) \simeq 0.3071\, x \frac{z^2 Z}{A(g)\beta^2} \frac{1}{T_s}$$

Delta(δ) rays in Micro Pattern Gas Detector



δ electron knocked out by a 180 GeV muon, measured with a GridPix detector (Micro Pattern Gas Detector)

δ rays produce ionization. This is called secondary to distinguish from the primary (impinging particle)

For a β ≈ 1 particle, on average **one** collision with **T > 10 keV** along a path length of **90 cm** of **Ar** gas

δ rays are ~ rare, why to care?

Restricted energy loss

- **δ rays** may escape the detector if it is too thin
  → The average energy deposits are very often much smaller than predicted by **Bethe & Bloch**

If the energy transferred is restricted to $T \leq T_{\rm cut} \leq T_{\rm max}$ → **"restricted energy loss"**

$$-\frac{dE}{dx}\bigg|_{T<T_{\rm cut}} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\rm cut}}{I^2} - \frac{\beta^2}{2}\left(1+\frac{T_{\rm cut}}{T_{\rm max}}\right) - \frac{\delta}{2}\right]$$

The difference between the **restricted energy loss** formula and the **B & B** is given by the contribution of the (escaping) **δ rays**

At very high energies (**$\beta\gamma > 10^{S1}$**, ***S1*** ~ 2-5) the stopping power reaches a constant called "**Fermi plateau**":

$$-\left(\frac{dE}{dx}\right)\left[\frac{MeV}{g/cm^2}\right]=0.3071\,\frac{z^2 Z}{2.A(g)}\ln\left(\frac{2m_e T_{cut}}{(h\nu_p)^2}\right)$$

$h\nu_p$ = $\hbar\omega_p$ = "Plasma energy" = $\sqrt{\rho\,\langle Z/A\rangle}\times 28.816$ eV (ρ in g cm^{-3})

S1 , $h\nu_p$ = "**density effect" parameters depending on the material**

Density effect parameters

Table 2.1 Values of Z, Z/A, I, ρ in units of g/cm^3, $h\nu_p$ and density-effect parameters S_0, S_1, a, md, and δ_0 for elemental substances.

El.	Z	Z/A	I eV	ρ	$h\nu_p$ eV	S_0	S_1	a	md	δ_0
He	2	0.500	41.8	1.66 10^{-4}	0.26	2.202	3.612	0.134	5.835	0.00
Li	3	0.432	40.0	0.53	13.84	0.130	1.640	0.951	2.500	0.14
O	8	0.500	95.0	1.33 10^{-3}	0.74	1.754	4.321	0.118	3.291	0.00
Ne	10	0.496	137.0	8.36 10^{-4}	0.59	2.074	4.642	0.081	3.577	0.00
Al	13	0.482	166.0	2.70	32.86	0.171	3.013	0.080	3.635	0.12
Si	14	0.498	173.0	2.33	31.06	0.201	2.872	0.149	3.255	0.14
Ar	18	0.451	188.0	1.66 10^{-3}	0.79	1.764	4.486	0.197	2.962	0.00
Fe	26	0.466	286.0	7.87	55.17	-0.001	3.153	0.147	2.963	0.12
Cu	29	0.456	322.0	8.96	58.27	-0.025	3.279	0.143	2.904	0.08
Ge	32	0.441	350.0	5.32	44.14	0.338	3.610	0.072	3.331	0.14
Kr	36	0.430	352.0	3.48 10^{-3}	1.11	1.716	5.075	0.074	3.405	0.00
Ag	47	0.436	470.0	10.50	61.64	0.066	3.107	0.246	2.690	0.14
Xe	54	0.411	482.0	5.49 10^{-3}	1.37	1.563	4.737	0.233	2.741	0.0
Ta	73	0.403	718.0	16.65	74.69	0.212	3.481	0.178	2.762	0.14
W	74	0.403	727.0	19.30	80.32	0.217	3.496	0.155	2.845	0.14
Au	79	0.401	790.0	19.32	80.22	0.202	3.698	0.098	3.110	0.14
Pb	82	0.396	823.0	11.35	61.07	0.378	3.807	0.094	3.161	0.14
U	92	0.387	890.0	18.95	77.99	0.226	3.372	0.197	2.817	0.14

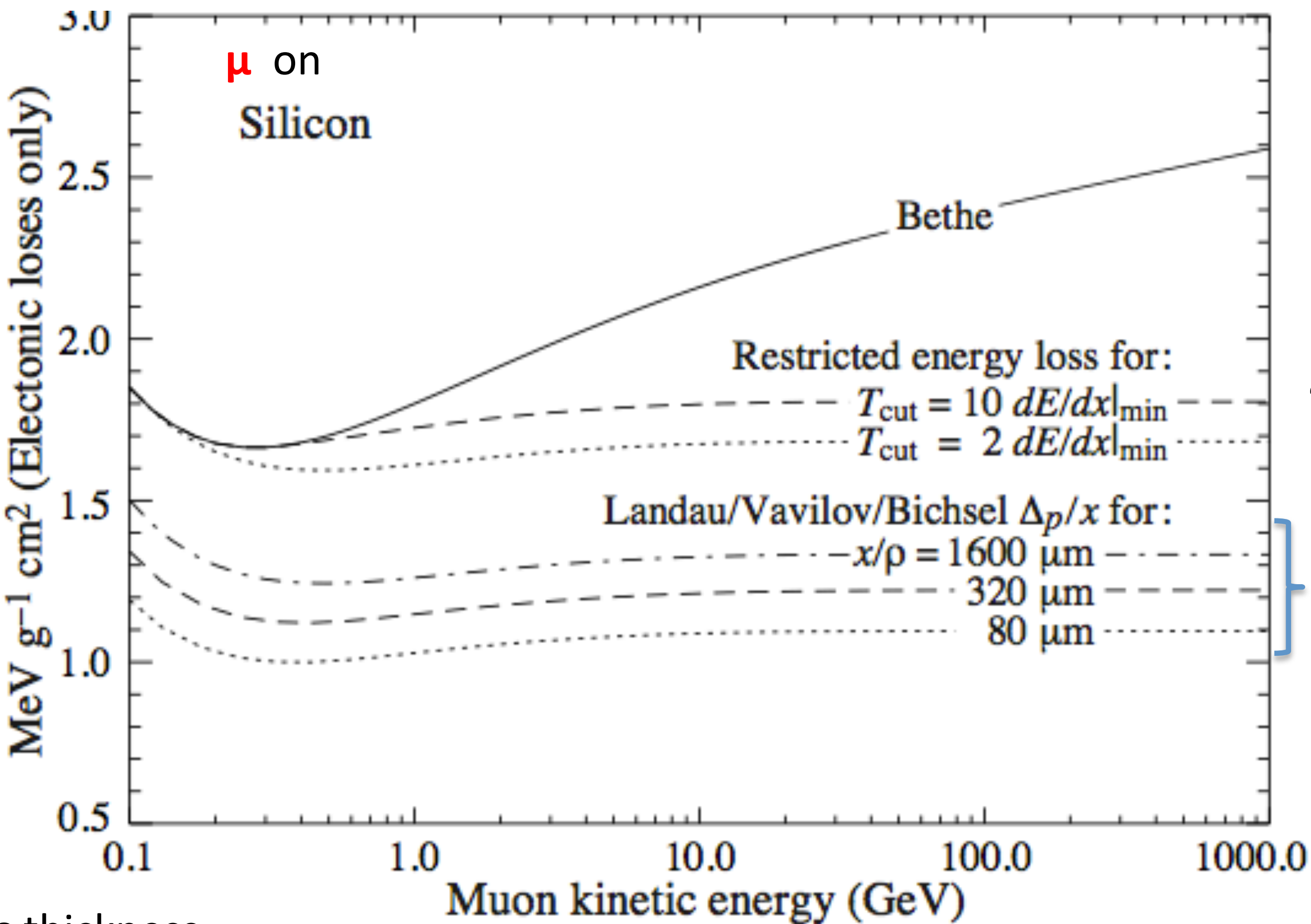
Data are from [Sternheimer, Berger and Seltzer (1984)]

Restricted energy loss

-dE/dx

or

Δ_p/x



'Fermi plateau'

Δ_p/x

x = **mass** thickness

Another important parameter is :

Δ_p = most probable energy loss (explained later)

-dE/dx Fluctuations → Energy straggling

- **Bethe-Bloch** formula describes **mean energy loss** per unit of lenght.

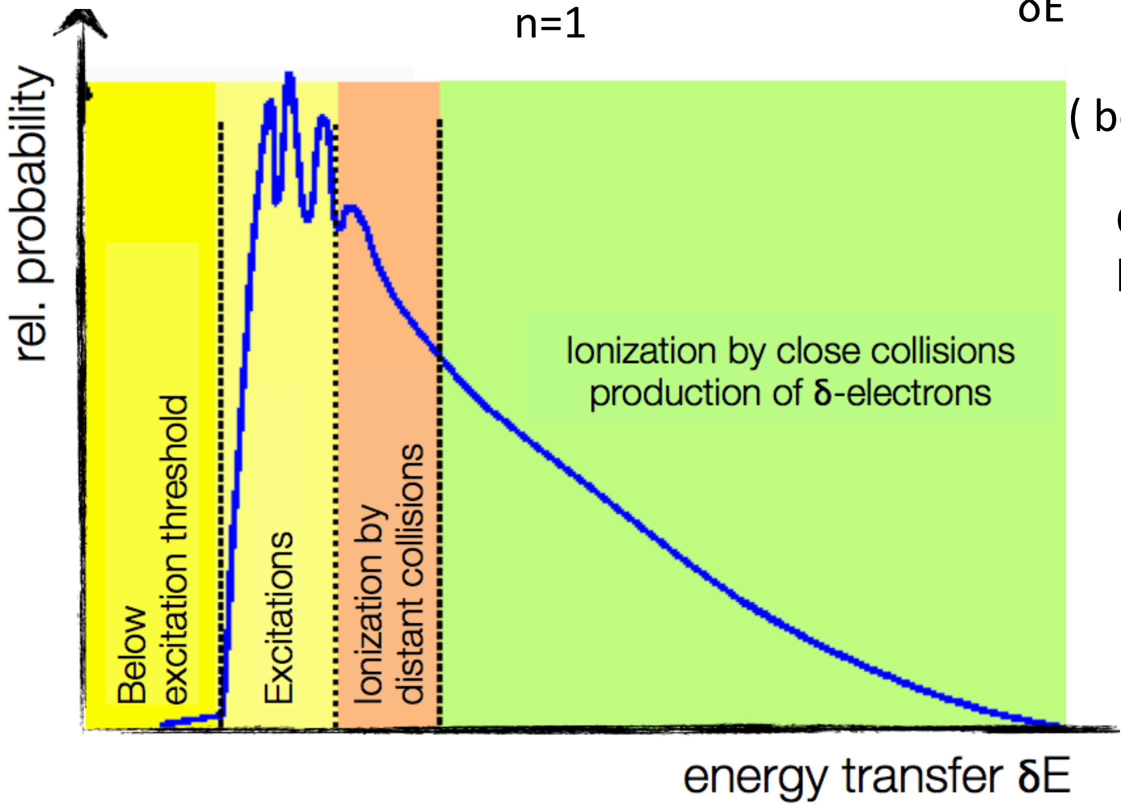
The actual energy loss **ΔE** in a material of thickness **x** is:

$$\Delta E = \sum_{n=1}^{N} \delta E_n (\beta)$$

- N number of collisions
- δE energy loss in a **single one collision**

δE stochastic fluctuations → **energy straggling**

(besides it depends on β of the particle)



Complex subject first studied by L. **Landau** and then by P.V. **Vavilov**

No general exact solutions, few approximate formulas help to estimate it. Introduce :

Significance parameter : K

$$K = \epsilon / T_{max}$$

mean energy loss in thickness ρdx = $153.4 \frac{z^2}{\beta^2} \frac{Z}{A} \rho \delta x \quad \text{keV},$

NB: ΔE depends on thickness x

ΔE (energy loss) distribution

- **Thin absorbers (K << 1):**
  - **Landau distribution**. Not analytic, useful approximation :

ΔE_{MP} = Most Probable value

ϵ See previous slide

$$L(\lambda)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\lambda+e^{-\lambda})\right) \qquad \lambda=\frac{\Delta E-\Delta E_{MP}}{\epsilon}$$

$$\Delta E_{MP}=\Delta E_{Bethe}+\epsilon\left(\beta^2+\ln\left(\frac{\epsilon}{T_{max}}\right)+0.194\right)\ MeV$$

  - **Improved (I)** generalized energy loss distribution : convolution of a Landau with a Gaussian (takes better into account **distant collisions**)

$$f(\Delta E,x)_I=\frac{1}{\sqrt{2\pi\sigma_I^2}}\int_{-\infty}^{+\infty} L(\Delta E-\Delta E',x)\exp\left(\frac{-\Delta E'^2}{2\sigma_I^2}\right)d(\Delta E')$$

$\sigma_I \sim (1/\beta^2)\ln\beta$

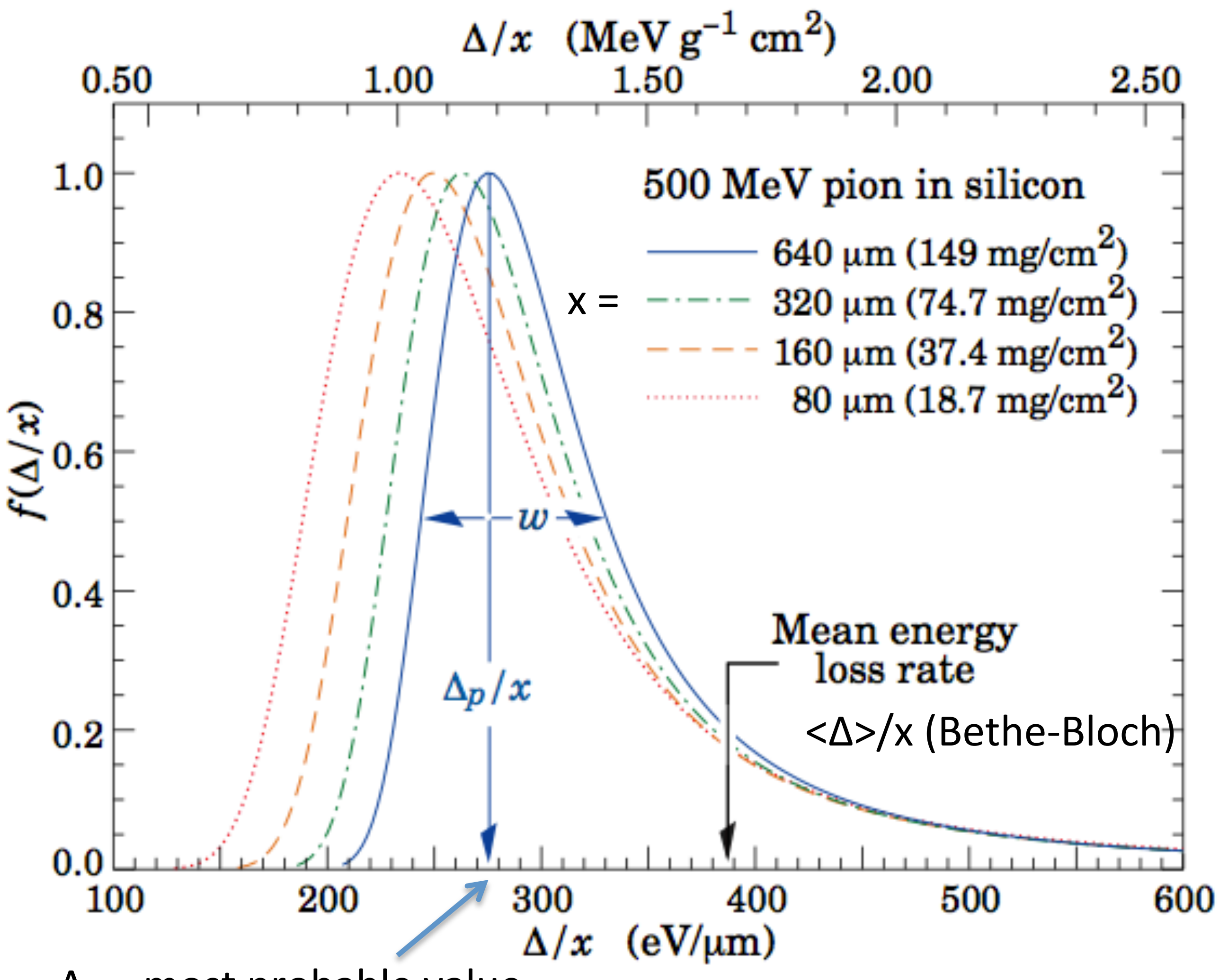
- **Thick absorbers (K >> 1):**

The distribution tends to a Gaussian

$$f(\Delta E,x)\simeq\frac{1}{\sqrt{2\pi T_{max}\epsilon\left(1-\frac{\beta^2}{2}\right)}}\exp\left(-\frac{(\Delta E-\Delta E_{Bethe})^2}{2T_{max}\epsilon\left(1-\frac{\beta^2}{2}\right)}\right)$$

x = thickness

Energy loss (Δ) distribution (straggling function)

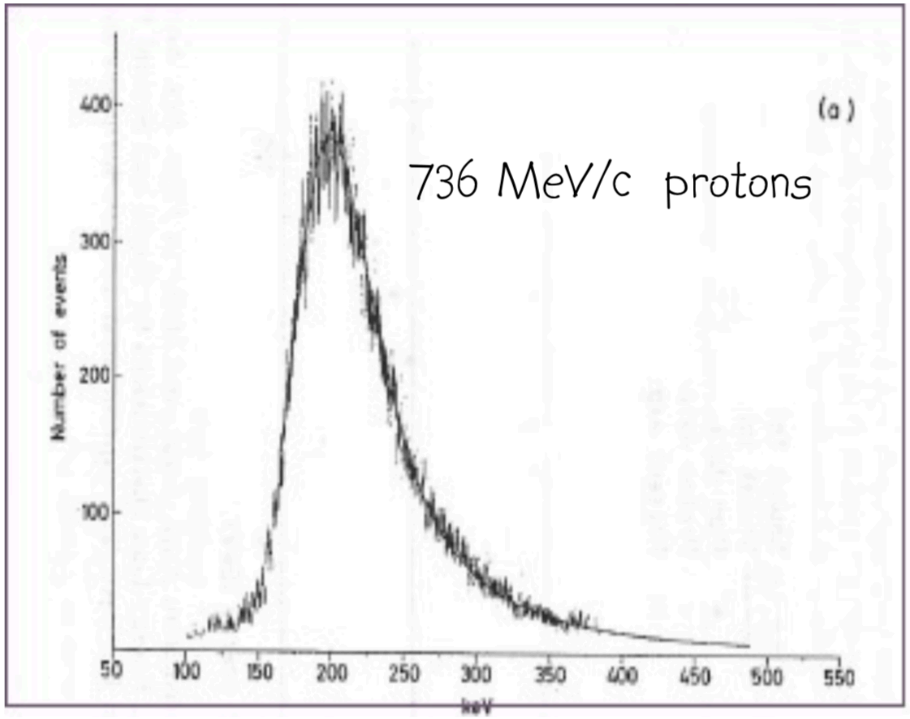


Δ = energy loss

x = thickness

Δ_p = most probable value

Important to describe the energy loss by a single particle



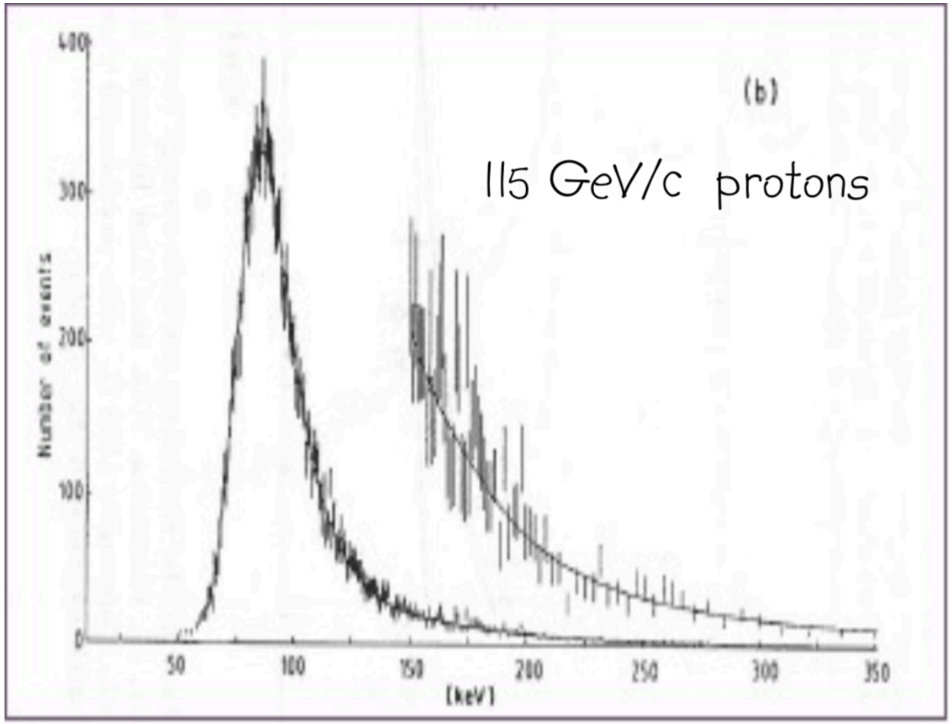


Fig. 2.10 Curves (a) and (b) (adapted and republished with permission from Hancock, S., James, F., Movchet, J., Rancoita, P.G. and Van Rossum, L., *Phys. Rev. A* **28**, 615 (1983); Copyright (1983) by the American Physical Society) show the energy loss spectra at 0.736 and 115 GeV/c of incoming particle momentum. Continuous curves are the complete fit to experimental data, i.e., the Landau straggling function folded over the Gaussian distribution taking into account distant collisions.

Stopping power of a compound medium

- For a compound of f elements:

$$-\frac{dE}{\rho\, dx} = \sum_1^f w_i \frac{dE}{\rho_i\, dx}$$

ρ_i = density of element i

$\frac{dE}{\rho_i\, dx}$ = stopping power of element i

w_i = mass fraction of element i

$w_i = (N_i\ A_i)/A_m$

N_i = number of atoms of element i
A_i = atomic weight of element i
A_m = molar mass of compound

$A_m = \sum N_i A_i$

- It is also possible to use effective quantities (empirical):

$$Z_{eff} = \sum N_i Z_i$$

$$A_{eff} = \sum N_i A_i$$

$$\ln I_{eff} = (\sum N_i Z_i \ln I_i)/ Z_{eff}$$

$$\delta_{eff} = (\sum N_i Z_i \delta_i)/ Z_{eff}$$

$$C_{eff} = \sum N_i C_i$$

Particle Range in matter : R

- Charged particles ionize, loose energy until their energy E_k is (almost) zero. The distance to this point is called the **range of the particle**: R

$$R\,(E_k) = \int_{E_k}^{\sim 0} \left(\frac{dx}{dE}\right) dE = \int_{E_k}^{\sim 0} \left(\frac{dE}{dx}\right)^{-1} dE$$

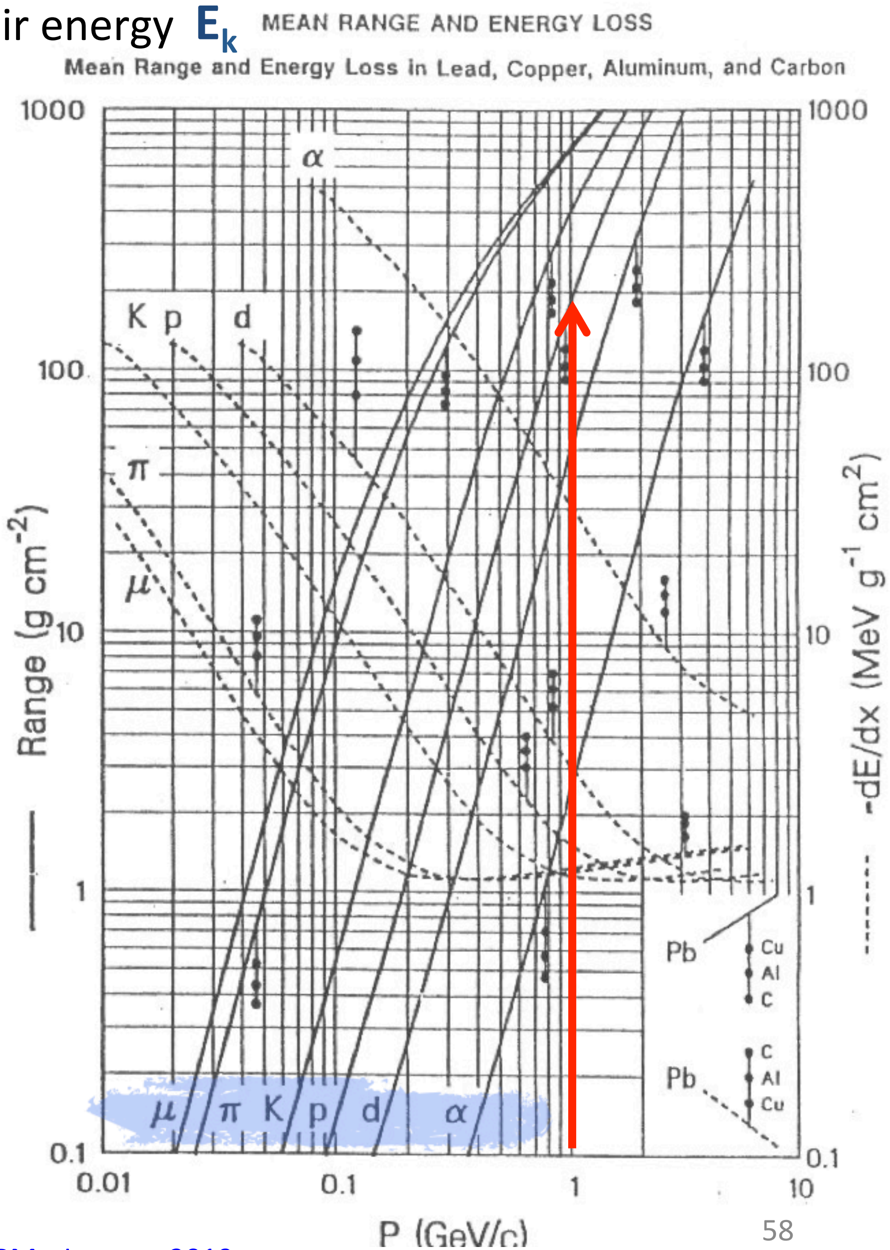
- This expression ignores the Coulomb **scattering** (producing a zig-zag trajectory of P_a)

- A **mean range < R >** is defined as the distance at which **half of the initial particles** have been stopped.

If E_k **> 1 MeV** , R ≈ <R>

Proton with **p = 1 GeV** on target lead (ρ = 11.34 g/cm³)

R = 200/11.34 ~ 20 cm



Particle Range in matter : R

- **R** may be used to evaluate the particle energy

  $R \propto E_k^{\,b}$ b ~ 1.75 for E_k < minimum ionisation

- **Scaling laws**
  - Particle 1 and 2, in same material

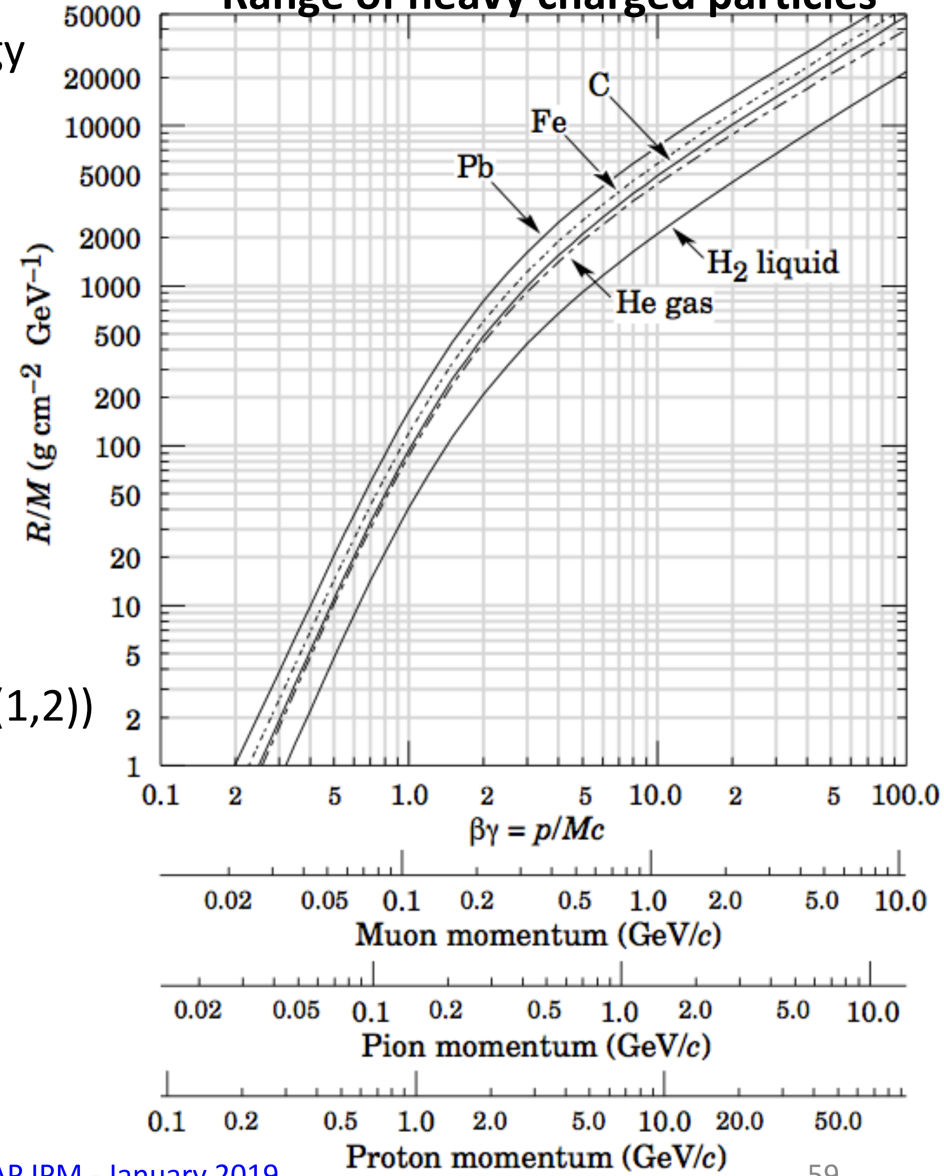
$$R_2(E_{k2}) \propto \frac{M_2\, z_1^2}{M_1\, z_2^2}\, R_1(E_{k1} * M_1/M_2)$$

  - Same particle in two different materials(1,2))

$$\frac{R_1}{R_2} = \frac{\rho_2}{\rho_1}\,\frac{\sqrt{A_1}}{\sqrt{A_2}}$$ R(cm)

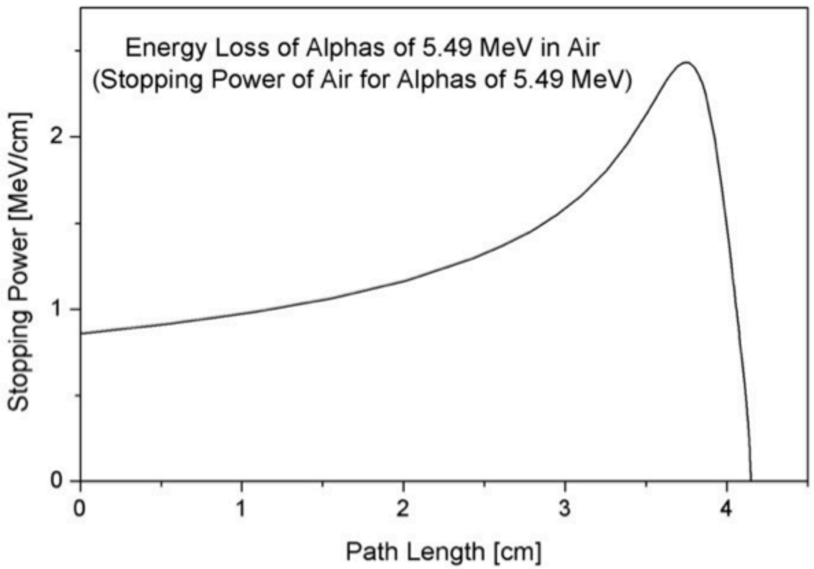
- Compounds: $R_{compound} = A_m \,/\, \sum (N_i\, A_i\, /R_i)$

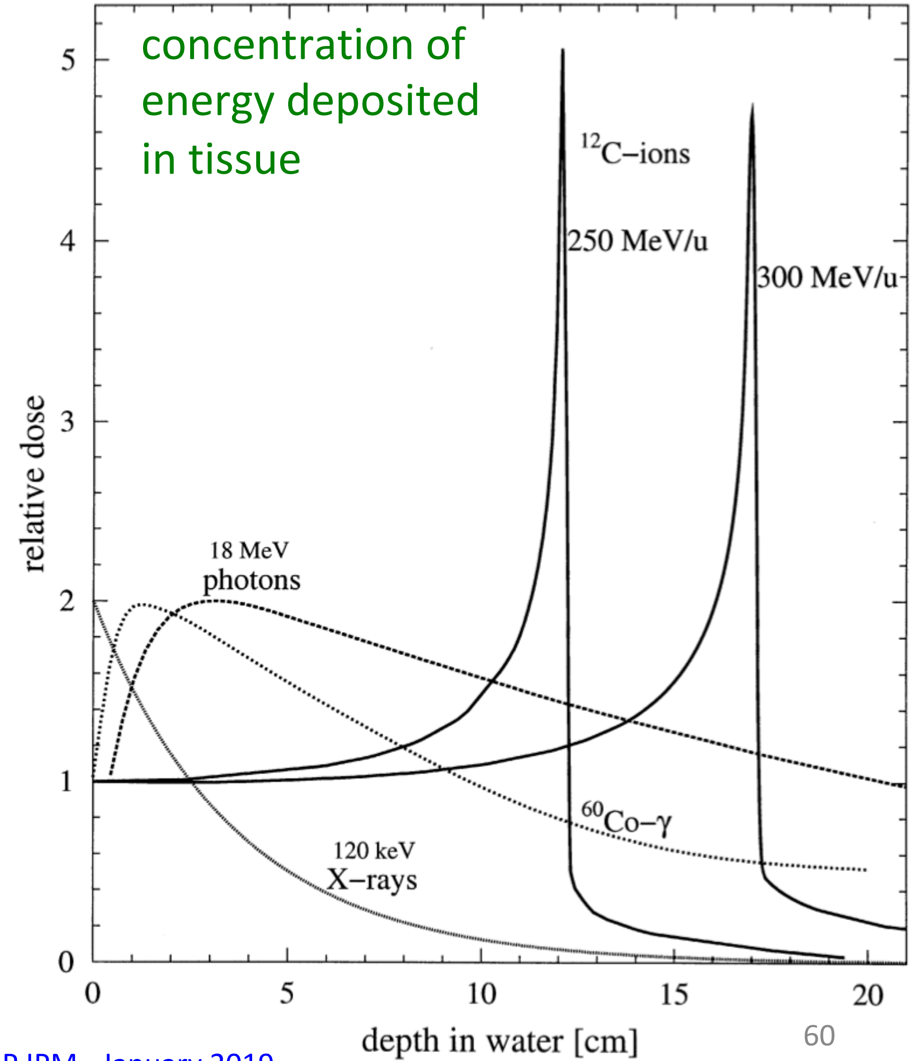
Range of heavy charged particles



Mean Particle Range

- If the medium is thick enough, a particle will progressively decelerate while increasing its stopping power ($\beta^{-5/3}$) until it reaches a maximum (called the **Bragg peak**).

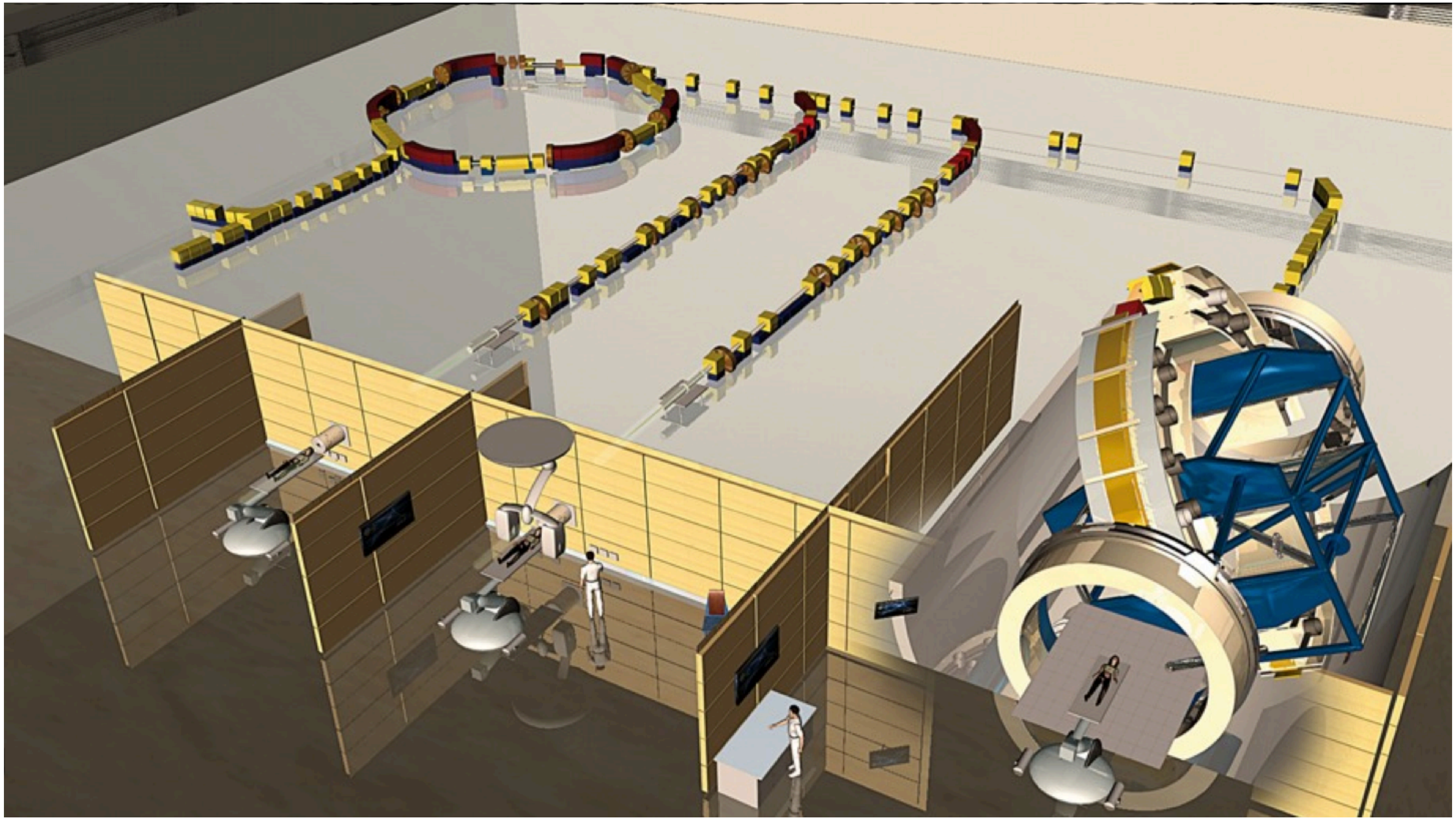




- Possibility to precisely deposit dose at well defined depth dependent on E_{beam} (Remember also dependence on $\mathbf{z^2}$)

Applications: Tumor therapy

Heidelberg Ion-Beam Therapy Center (HIT)



~ 50 centers around the world

Stopping power of $e^{\pm}$ by ionization and excitation in matter

For $e^{\pm}$ the **Bethe-Bloch formula** must be **modified** since:

1) the change in direction of the particle was neglected; for $e^{\pm}$ this approximation is not valid (scattering on particle with same mass)
2) Pauli Principle : the incoming and outgoing particles are the identical particles

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln \frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} + F(\tau) - \delta - 2\frac{C}{Z} \right]$$

For electrons: $F(\tau) = 1-\beta^2 + \frac{(\tau^2/8) - (2\tau+1)\ln 2}{(\tau+1)^2}$ $\tau = \frac{1}{\sqrt{1-\beta^2}} - 1 = E_k/(mc^2)$

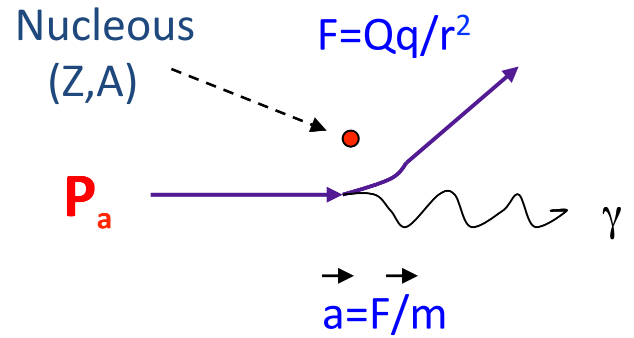
For positrons : $F(\tau) = 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^2}\right)$

$e^{\pm}$ loose more energy wrt heavier particles since they interact with particles of the same mass

- When a positron comes to a rest it annihilates : $e^+ + e^- \rightarrow \gamma\gamma$ of 511 keV each
- A positron may also undergo an annihilation in flight: with a cross section : $\sigma(Z,E) = \frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln(\gamma+\sqrt{\gamma^2-1}) - \frac{\gamma+3}{\sqrt{\gamma^2-1}} \right]$

2. Bremsstrahlung. Mean radiative energy loss.

- An accelerated (or decelerated) charged particle (P_a) emits electromagnetic radiation (γ)
- **Very fundamental process !**
- Here the process takes place in **the Coulomb field of the nucleus.** The amount of **screening** from the atomic electrons plays an important role
- Relevant in particular for $e^{\pm}$ due to their small mass



$$-\left(\frac{dE}{dx}\right)_{\text{brem}} = N \int_{\sim 0}^{\nu_0 = E_o/h} h\nu \frac{d\sigma}{d\nu} d\nu = N E_0 \phi(Z^2)$$

N = atoms/cm^3 (N= ρ N_A/A)
Z = atomic number

E_0 = Initial energy of particle P_a
$\nu_0 = E_0/h$
$h\nu$ = energy of emitted γ
$\frac{d\sigma}{d\nu}$ = Differential cross section of the bremsstralung process

If P_a = electron:

If $E_0 \gg m_e c^2$ et $E_0 \ll 137\, m_e c^2/Z^{1/3}$ $\phi(Z^2) = 4\alpha Z^2 r_e^2 \ln(2E_0/m_e c^2 - 1/3 - f(Z))$

If $E_0 \gg 137\, m_e c^2/Z^{1/3}$ $\phi(Z^2) = 4\alpha Z^2 r_e^2 \ln(183 Z^{-1/3} - 1/18 - f(Z))$

$\alpha = 1/137$

$r_e = \alpha/(m_e c^2)$

f(Z)= Coulomb correction

See W.R. Leo

2. Bremsstrahlung – Energy Spectrum

LPM =Landau–Pomeranchuk–Migdal cross section.

Normalized bremsstrahlung cross section vs y (= k / E_0) where $k = E_\gamma$

→ y = **fraction** of the electron energy (E_0) transferred to the radiated γ

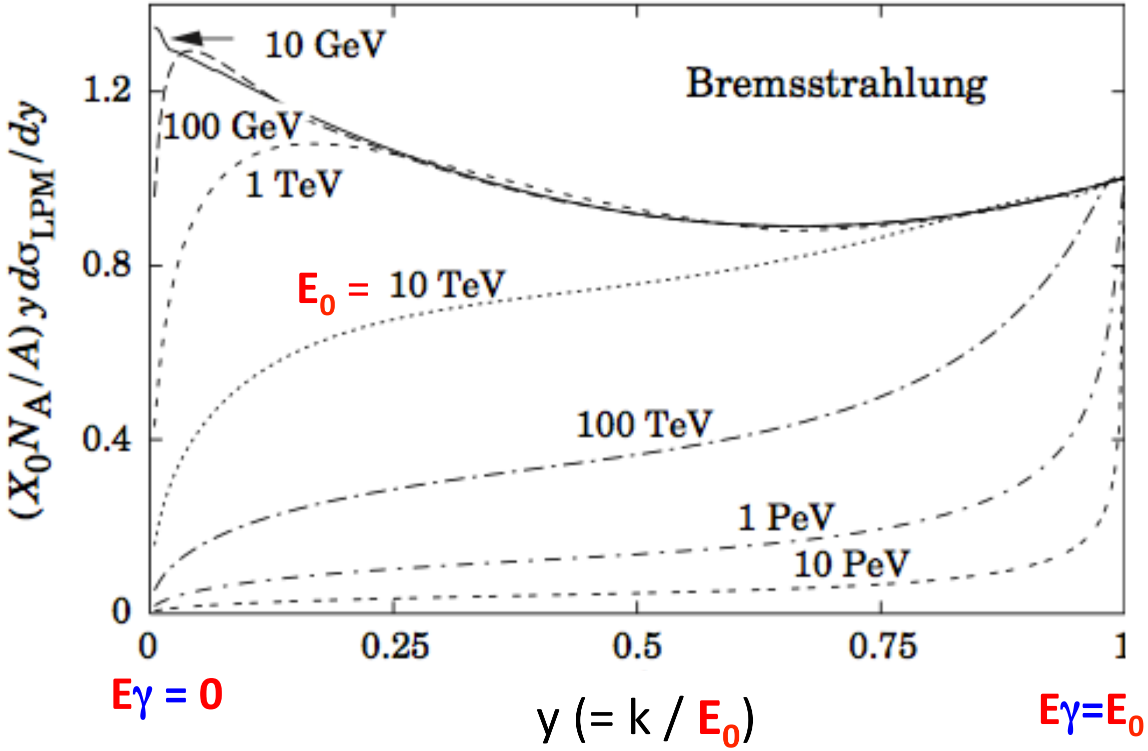
K dσ/dk = ν dσ/dν (for given E_0)

For high energy E_0 (small y):

$$\frac{d\sigma}{dk} = \frac{1}{k}\frac{A}{X_0 N_A}\left(\frac{4}{3} - \frac{4}{3}y + y^2\right)$$

Formula accurate except for y =1 and y = 0

see PDG for further details



LPM =Landau–Pomeranchuk–Migdal cross section.

Bremsstrahlung. Mean radiative energy loss

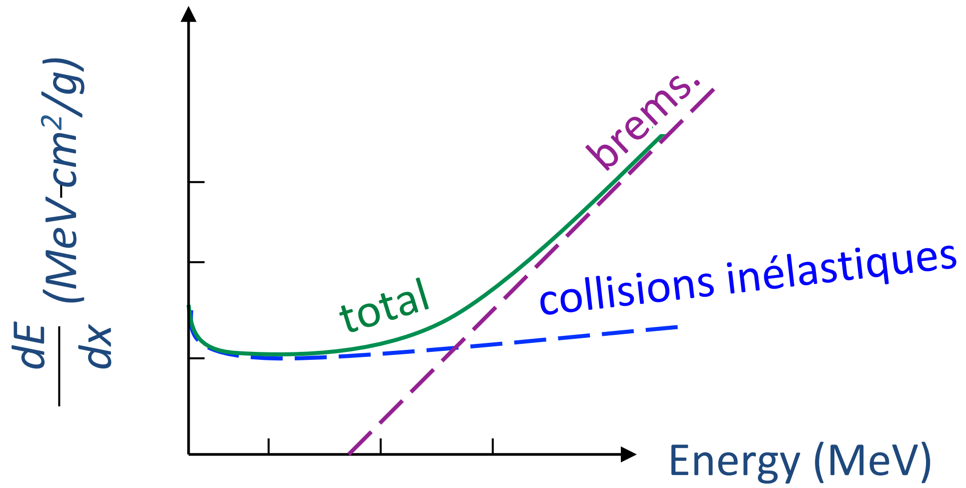
For a particle of charge **z** and mass **m**:

$$\frac{dE}{dx}_{\text{brem}}(z,m) = \left(\frac{m_e}{m}\right)^2 z^2 \frac{dE}{dx}_{\text{brem}}(e^-)$$

- Relevant in particular for $e^{\pm}$ due to their small mass

- Shown so far is the mean energy loss due interaction **in the field of the nucleus**
- Contribution also from radiation which arises in the fields of the **atomic electrons**.
- Cross section are given by the above formula but replacing $\mathbf{Z^2}$ with **Z**.
- The overall contribution can be approximated by replacing $\mathbf{Z^2}$ by **Z (Z+1)** in all the above formulas

Comparison -dE/dx Bremsstrahlung vs ionisation/excitation



- The **average energy loss** due to **ionisation/excitation** increases with the **log of the energy** and linearly with **Z** :

$$\left.\frac{dE}{dx}\right)_{\text{ion./excit.}} \propto Z/A, \quad 1/\beta^2 \ln E$$

- The **average energy loss** of due to **brem** increases **linearly** with **energy** and **linearly** with **E** and **Z²** :

$$\left.\frac{dE}{dx}\right)_{\text{brem}} \propto Z^2/A, \quad E, \; 1/m^2$$

Energie loss due to **brem** is a discrete process: results from the emission of ~ 1 γ ou 2 γ
--> fluctuations

Critical energy (E_c)

- The relevance of **bremsstralung** wrt **ionisation** depends on the **critical energy (E_c)** of the particle **P_a** in the material
- **The critical energy (E_c)** is the energy at which the **ionization stopping power** is **equal** to the mean **radiative energy loss**.

@ $E = E_c$

$$\left(\frac{dE}{dx}\right)_{brem.} = \left(\frac{dE}{dx}\right)_{ion}$$

@ $E > E_c$

$$\left(\frac{dE}{dx}\right)_{brem.} > \left(\frac{dE}{dx}\right)_{ion}$$

@ $E < E_c$

$$\left(\frac{dE}{dx}\right)_{brem.} < \left(\frac{dE}{dx}\right)_{ion}$$

For **$e^{\pm}$** in :

Pb Ec = 9.5 MeV
Cu Ec = 24.8 MeV
Fe Ec = 27.4 MeV
Al Ec = 51 Mev

For liquid and solids: $E_c \sim$ 610 MeV/(Z+1.24)

For gas $E_c \sim$ 710 MeV/(Z + 0.92)

For other particles **E_c** would scale according to the square of their masses with respect to the electron mass.

Radiation lenght X_0

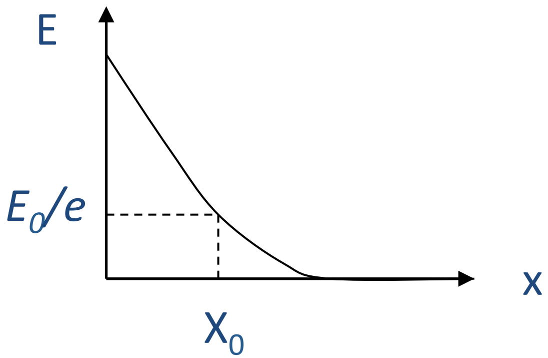
For E >> Ec $\left(\frac{dE}{dx}\right)_{tot} \approx \left(\frac{dE}{dx}\right)_{brem.}$

N = atoms/cm^3

$$-\frac{dE}{dx} = N E_0 \phi$$

$$\Rightarrow -\frac{dE}{E} = N \phi \, dx \Rightarrow \boxed{E = E_0 \, e^{-x/X_0}}$$

$X_0 \equiv 1/(N \phi)$



$X_0 \equiv 1/(N \phi)$ ≡ radiation lenght ≡
distance after which an high energy electron has lost 1/e of his energy by radiation

Mean radiated energy of an electron over a path x in the medium:

$$E_{brem.}(e^-) = E\left(1 - e^{-x/X_0}\right)$$

Radiation lenght X_0

$$X_0 \begin{cases} Pb = 0.56\ cm \\ Fe = 1.76\ cm \\ Air = 30050\ cm \end{cases}$$

$X'_0 \equiv X_0\,\rho$

$$X'_0 = \frac{716.4\ \mathrm{g\ cm^{-2}}\ A}{Z(Z+1)\ln(287/\sqrt{Z})}$$

Expressing the mean radiated energy in unit of X_0'

→ The probability of the process becomes less dependent on the material

Pour un composé de N éléments :

$$\frac{1}{X_0} = \sum_i w_i \frac{1}{X_{0i}}$$

w_i = fraction in mass of element i

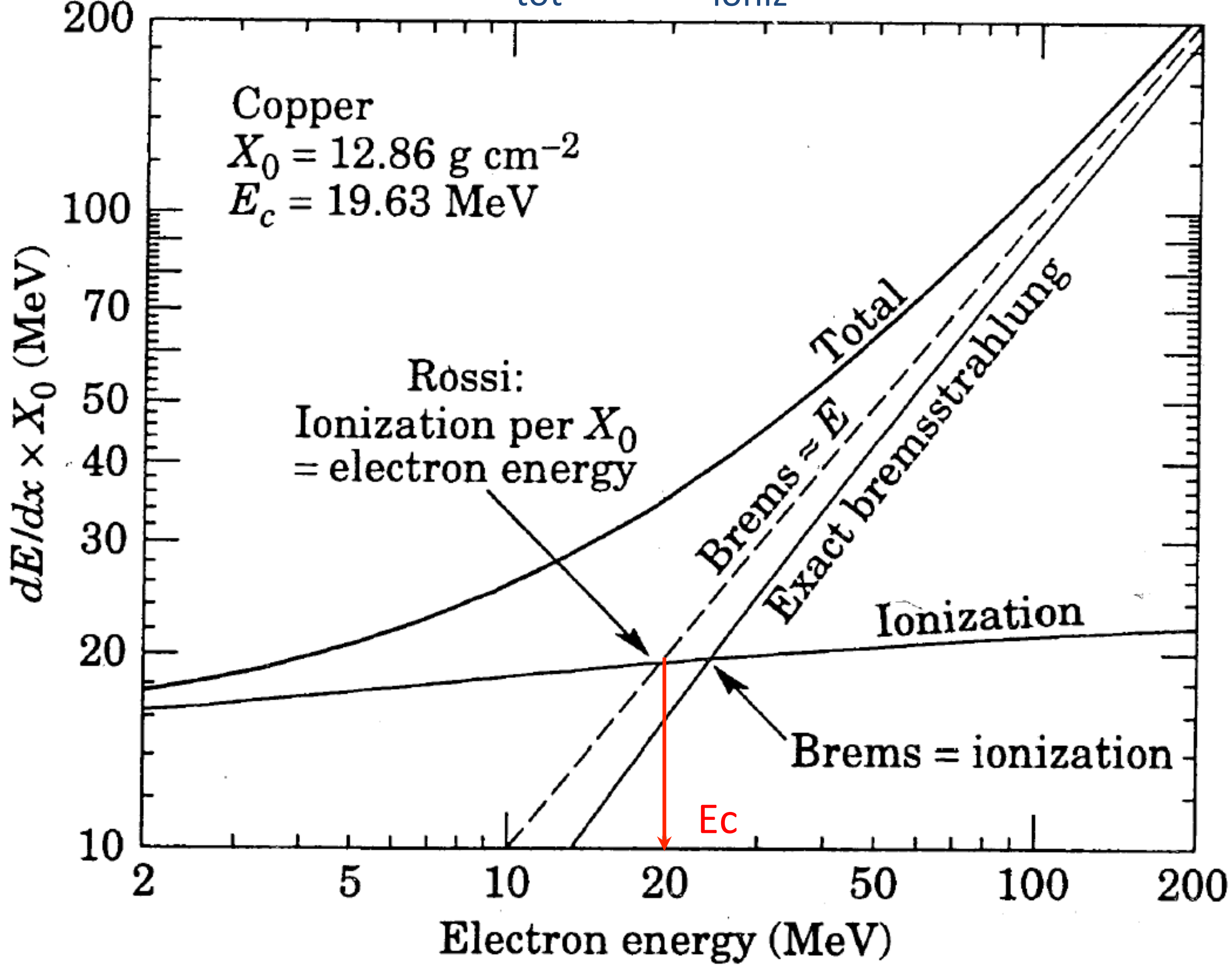
X_{0i} = radiation lenght of element i

For electrons

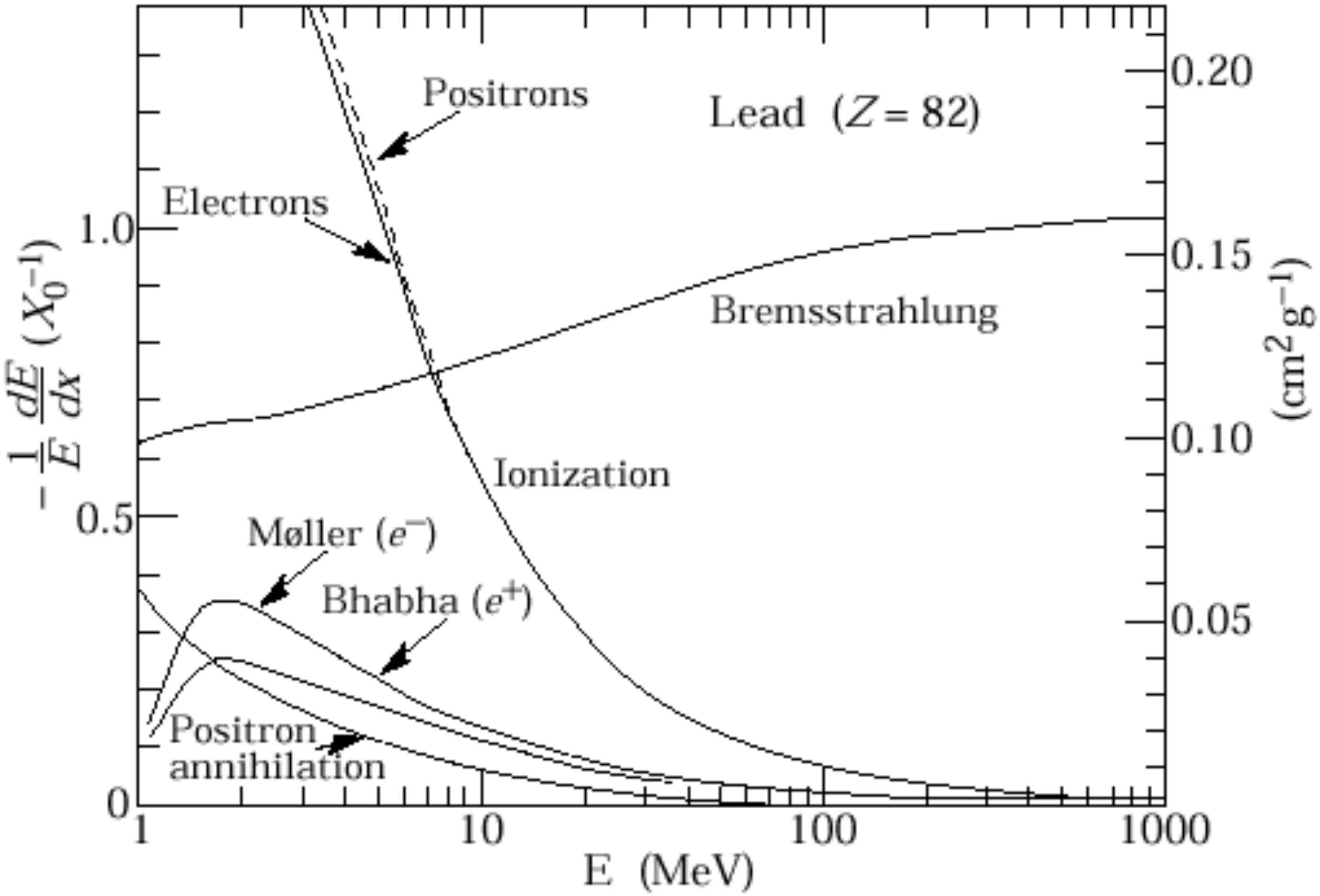
medium	*Z*	*A*	*X_0 (g/cm^2)*	*X_0 (cm)*	*E_C (MeV)*
hydrogen	1	1.01	63	700000	350
helium	2	4	94	530000	250
lithium	3	6.94	83	156	180
carbon	6	12.01	43	18.8	90
nitrogen	7	14.01	38	30500	85
oxygen	8	16	34	24000	75
aluminium	13	26.98	24	8.9	40
silicon	14	28.09	22	9.4	39
iron	26	55.85	13.9	1.76	20.7
copper	29	63.55	12.9	1.43	18.8
silver	47	109.9	9.3	0.89	11.9
tungsten	74	183.9	6.8	0.35	8
lead	82	207.2	6.4	0.56	7.4
air	7.3	14.4	37	30000	84
silica (SiO_2)	11.2	21.7	27	12	57
water	7.5	14.2	36	36	83

Electron interactions in copper : higher energies

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{ioniz} + \left(\frac{dE}{dx}\right)_{brem}$$



Interactions of electrons in lead: a more complete picture

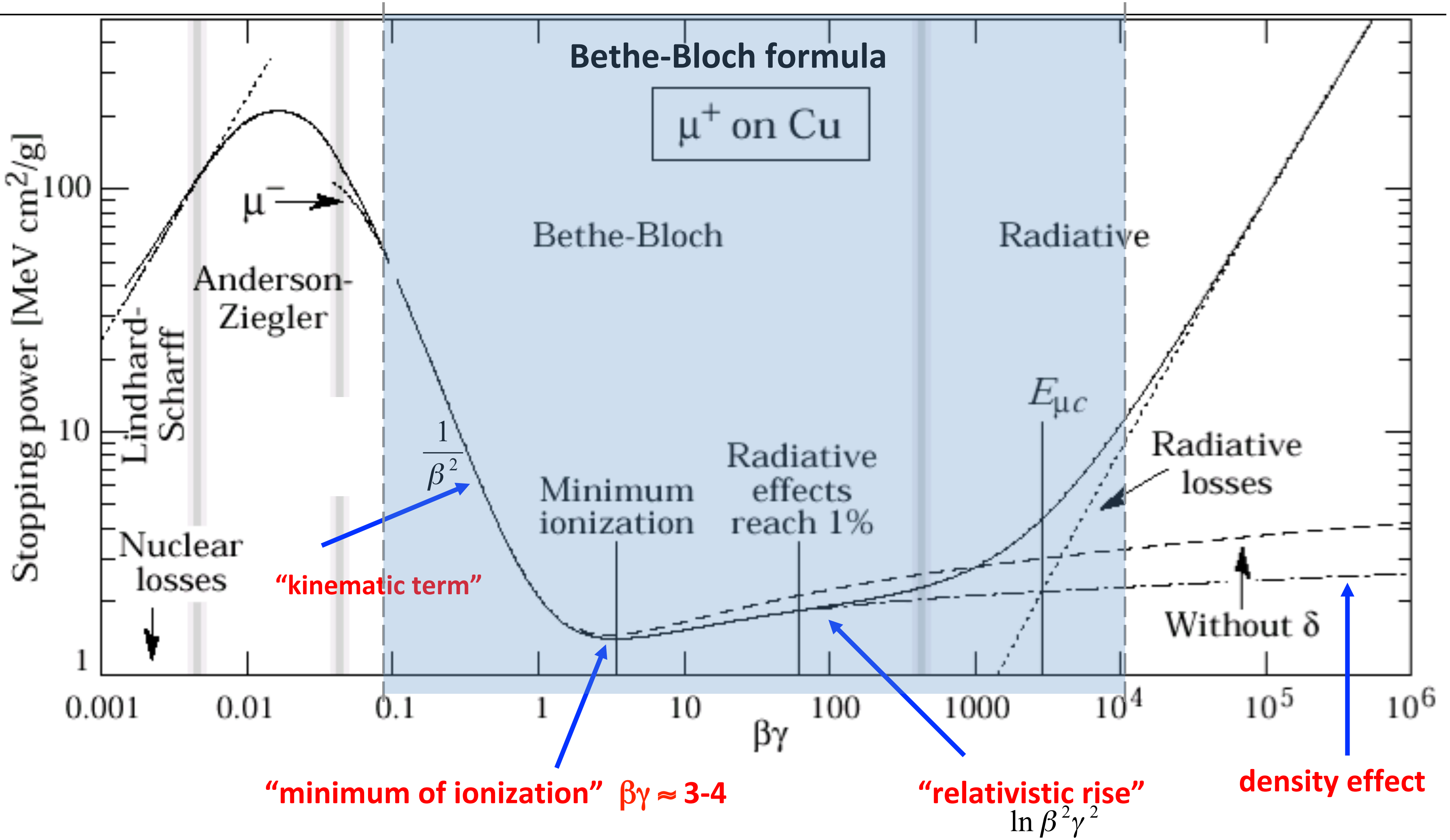


Moller scattering e- e- → e- e-

Bhabha scattering e+ e- → e+ e-

Positron annihilation e+ e- → 2γ

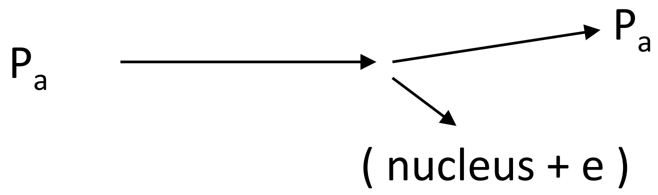
Total energy lost by a muon (μ) per unit length in copper



At very low energy the **Bethe-Bloch** formula is not valid since the speed of the interacting particle is ~ speed of electrons in the atoms. For $\beta\gamma < 0.05$ there are only phenomenological fitting formulae

3. Elastic scattering with nuclei

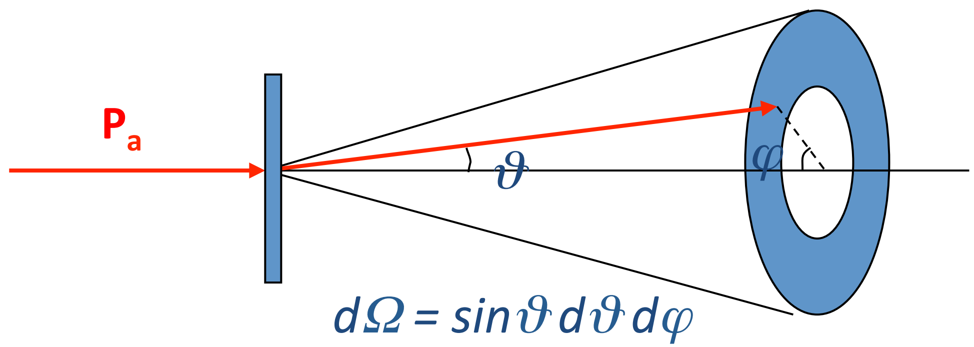
A **charged particle P_a** traversing a medium is deflected many times (mainly) by small-angles essentially due to **Coulomb scattering** in the electromagnetic field of **the nuclei**.



The **energy loss** (or transferred to the nuclei) is small ($m_{nucleus} >> m_{Pa}$) therefore **neglected**, The change of direction is important.

- **A single collision** is described by the Rutherford formula (ignores spin and screening effects)

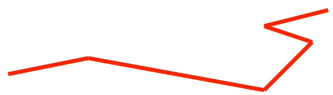
$$\frac{d\sigma}{d\Omega} = 4zZr_e^2\left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{\sin^4\theta/2}$$



- **Multiple scattering:** $N_{collisions} > 20$

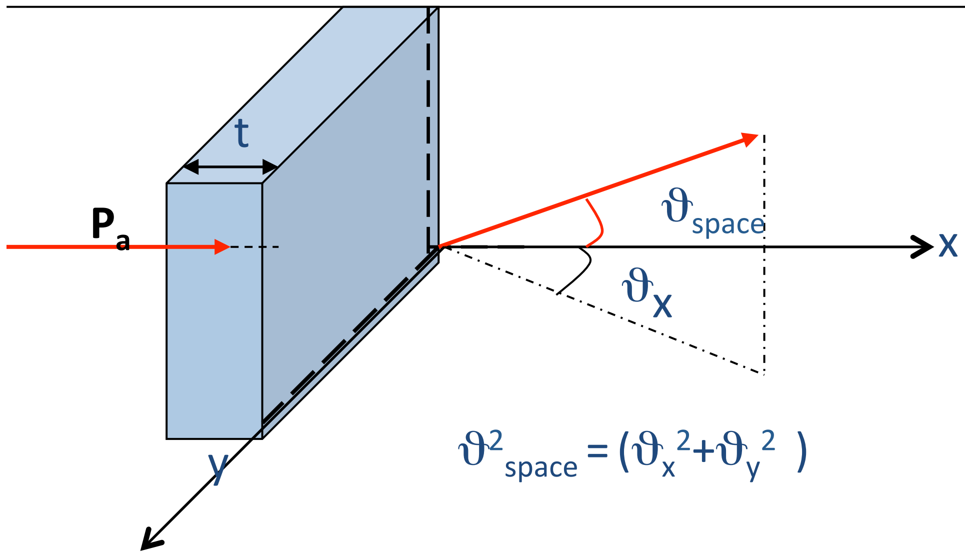
The particle follows a zig-zag trajectory

Deflection angles are described by the **Molière theory**



H. A. Bethe" "Molière's Theory of Multiple Scattering" Phys. Rev. **89, 1256 – Published March 1953**

3. Multiple scattering through small angles (< ~10^0)



For small scattering angles, the distribution of $\vartheta_x \approx$ Gaussian

$$\text{prob}(\vartheta_x)\, d\vartheta_x = \frac{1}{\sqrt{2\pi}\,\sigma_0} \exp(-\vartheta_x^2/(2\sigma_0^2))\, d\vartheta_x$$

(similar for ϑ_y and $\vartheta_{space}^2 = \vartheta_x^2+\vartheta_y^2$)

Where:

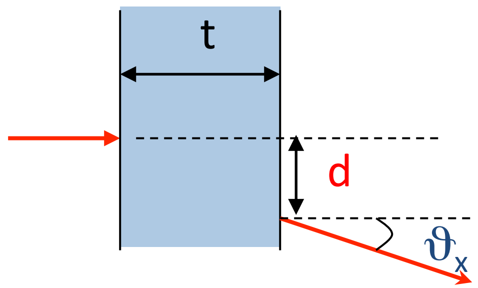
$$\sigma_0 = \frac{13.6 \text{ MeV}}{\beta p} |z| \sqrt{\frac{t}{X_0}} \left(1+0.038 \ln \frac{t}{X_0}\right)$$

t = medium thickness

ρ = matter density

X_0 = radiation lenght

β, p = speed/c and momentum of the incident particle



Particles emerging from the the medium are also laterally shifted :

$$d^{rms} = \frac{1}{\sqrt{3}}\, t\, \sigma_0$$

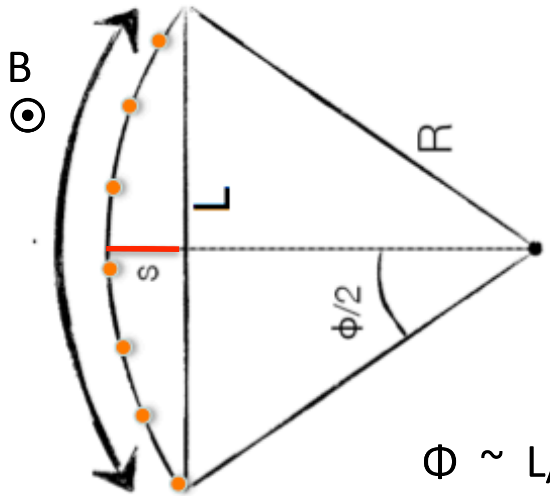
Momentum resolution

Multiple scattering impacts the measurement of the momentum

Assume B ⊥ v particle:

$Mv^2/R = q\,|\vec{v} \wedge \vec{B}|$ $\quad\quad$ **p** = B **R** $\quad\quad$ (q = 1)

The momentum is measured from R, which is obtained from **L** and **s**



$\Phi \sim L/R$

$$s = R - R\cos\frac{\phi}{2} \approx R\,\frac{\phi^2}{8}$$

$$s = R\frac{L^2}{8R^2} = \frac{L^2}{8R}$$

$$\boxed{R = \frac{L^2}{8s}}$$

The precision on the momentum will depend on the precision on the track reconstruction and also on the **multiple scattering that the particle undergoes**

Momentum resolution

$p = q\,B\,R \sim B\,L^2$

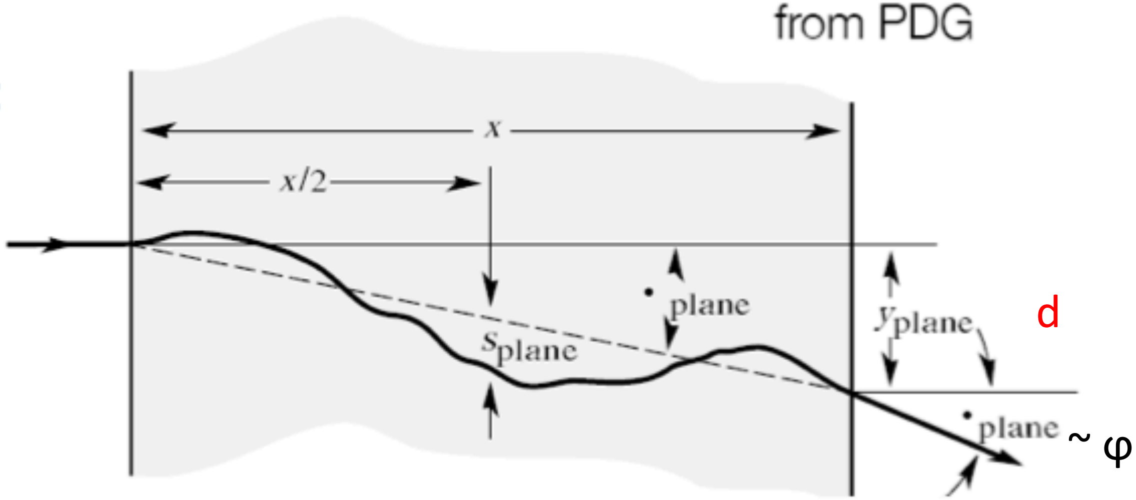
Multiple scattering introduces an apparent sagitta (0.5% in Argon gas)

from PDG

Multiple scattering contribution:

z=1

$$\sigma_\phi \approx \frac{14\ \mathrm{MeV}/c}{p}\sqrt{\frac{L}{X_0}}$$



p = q B **R** $\quad R = \frac{L}{\phi}$

At small momenta this limits resolution of momentum measurement ...

momentum independent

$$\frac{\sigma_\phi}{\phi} = \frac{14\ \mathrm{MeV}/c}{p}\sqrt{\frac{L}{X_0}} \cdot \frac{R}{L} = \frac{14\ \mathrm{MeV}/c}{p}\sqrt{\frac{1}{LX_0}} \cdot \frac{p}{eB} \sim \frac{1}{\sqrt{LX_0}B}$$

$$\left(\frac{\sigma_p}{p}\right)^2 = \boxed{\mathrm{const}\cdot\left(\frac{p}{BL^2}\right)^2} + \boxed{\mathrm{const}\cdot\left(\frac{1}{B\sqrt{LX_0}}\right)^2}$$

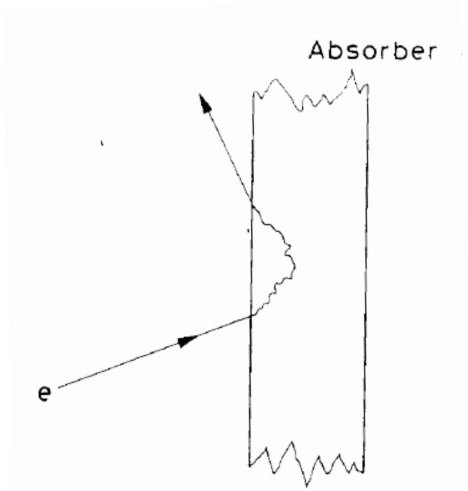
Due to multiple scattering

3. Back-scattering of electrons

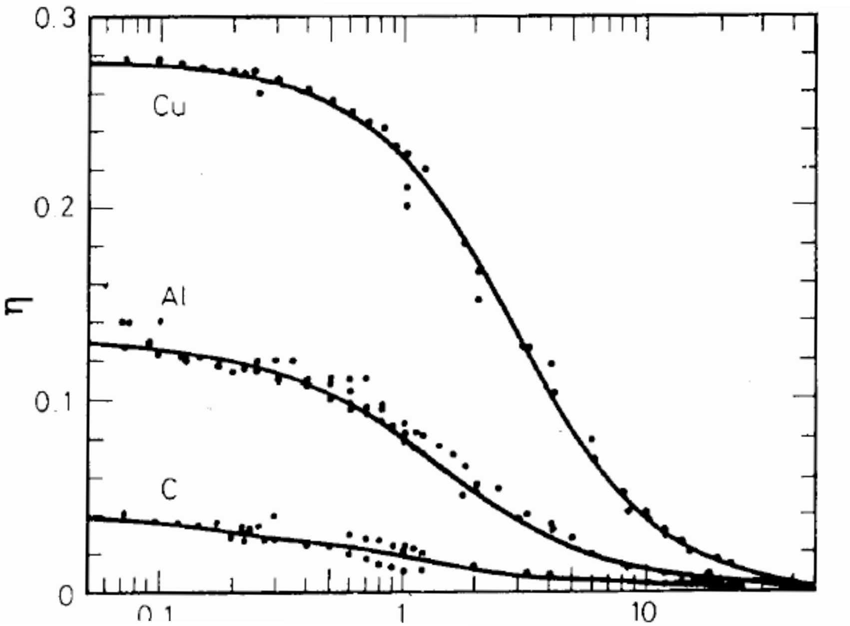
* increases with Z of the material

* is relevant for low energy electrons

$$\eta = \frac{\text{Number of backscattered electrons}}{\text{Number of incident electrons}}$$

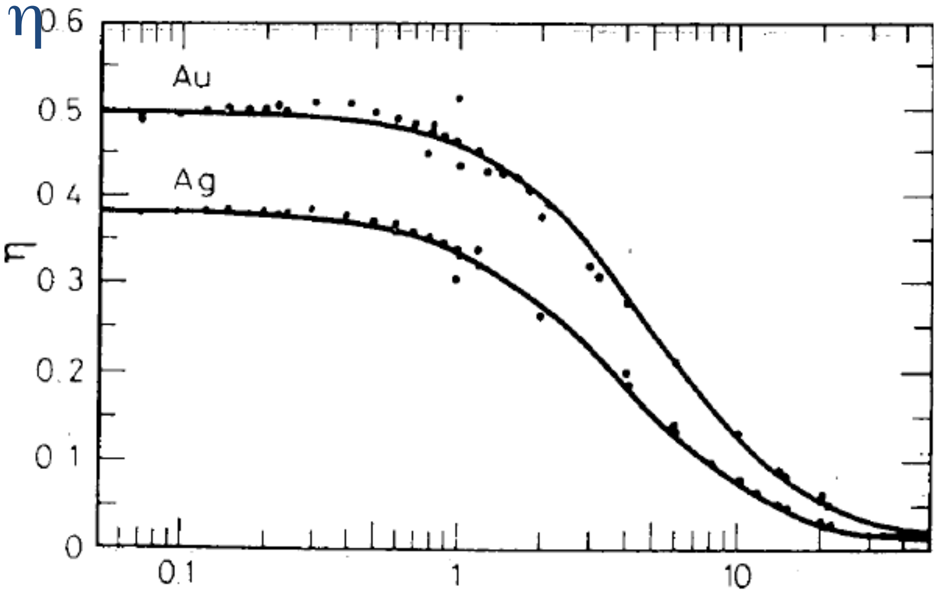


η



Kinetic energy (MeV)

η

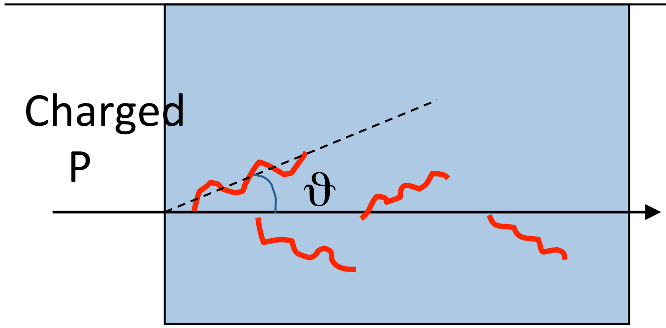


Energie cinétique(MeV)

Effect to take into account when building a detector for low energy electrons (< ~ 10 MeV)

78

4.Cherenkov light emission

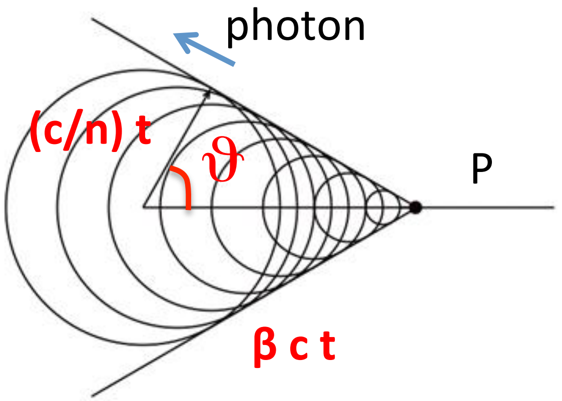


Radiation emitted when a charged particle crosses a medium at a speed > than the **phase velocity of light** in the medium

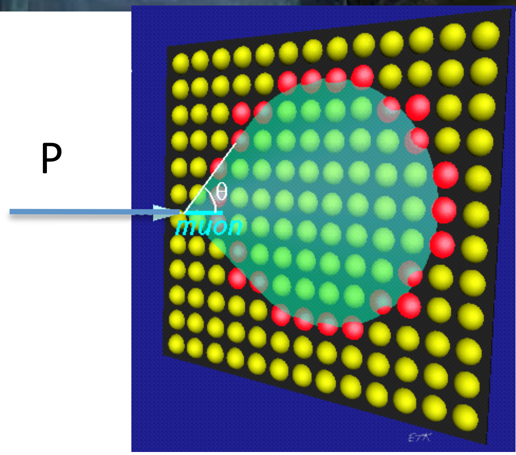
$$v_{particle} > c/n$$

n = refracting index

- The medium is **electrically polarized** by the particle's electric field (oscillating dipoles)
- When the particle travels fast this effect is left in the **wake** of the particle.
- The emitted energy radiates as a **coherent shockwave**



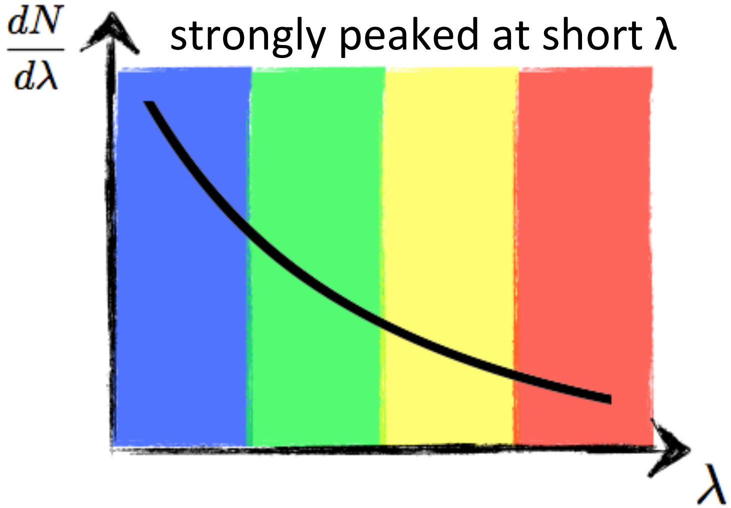
$$\cos\vartheta = \frac{1}{\beta n}$$



4.Cerenkov light emission

- Number of photons N emitted per unit path length and unit of wave length

$$\frac{dN}{dx\,d\lambda} = 2\pi\alpha \frac{1}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) z^2$$



- Number of photons per unit path length is:

$$\frac{dN}{dx} = 2\pi\alpha z^2 \int_{\beta n > 1} \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{d\lambda}{\lambda^2}$$

Assuming *n* ~ *const* over the wavelength region detected

n = refracting index

$$\frac{dN}{dx} = 2\pi\alpha \sin^2\theta \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) z^2$$

in λ range 350-500 nm (photomultiplier sensitivity range),

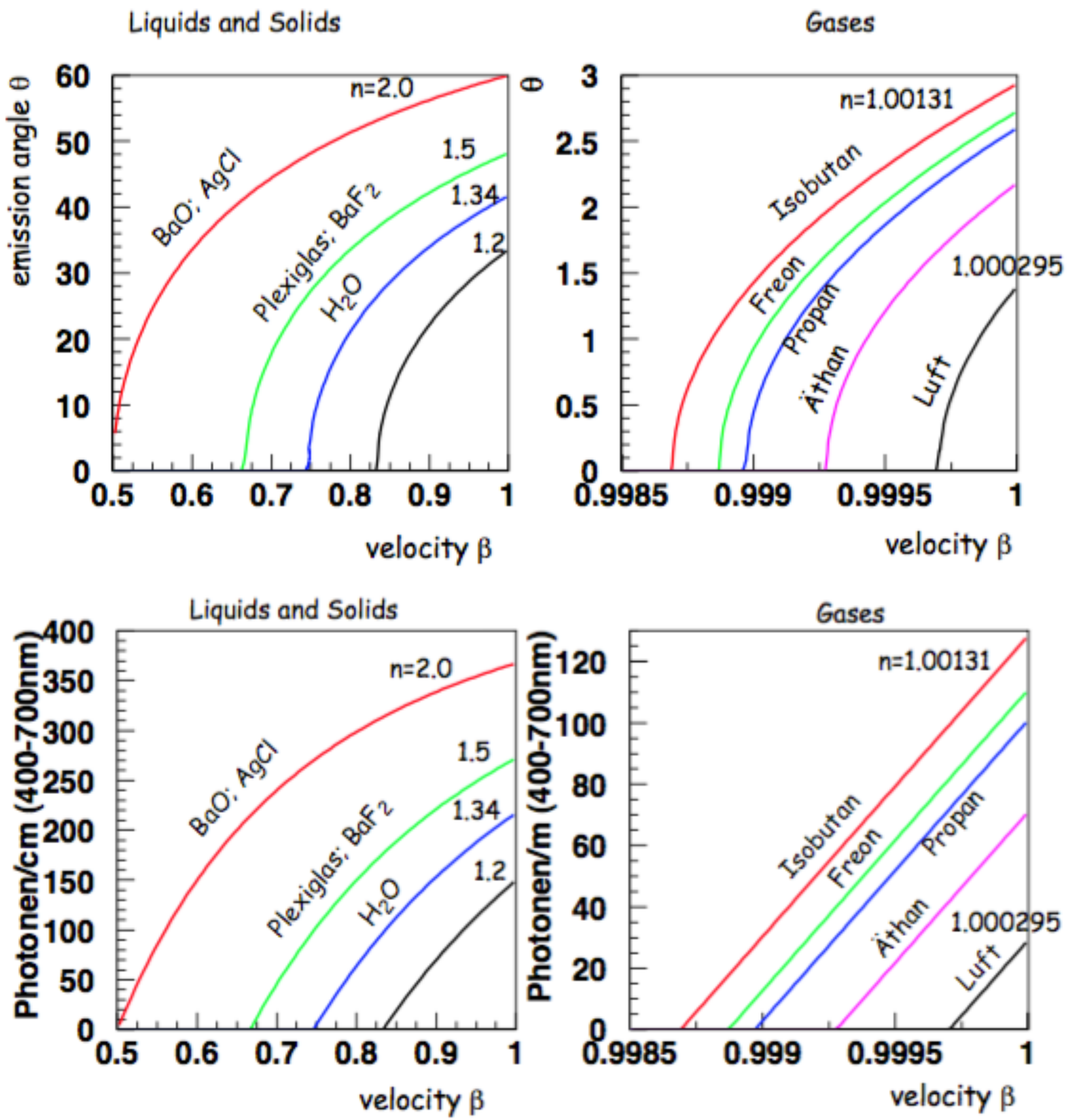
$$\frac{dN}{dx} = 390 \sin^2\theta \ \textit{photons/cm}$$

dE/dx due to Cherenkov radiation is small compared to ionization loss (< 1%) and much weaker than scintillating output. It can be neglected in energy loss of a particle, but is Important for particle detection

Cerenkov angle

$$\cos\vartheta = \frac{1}{\beta n}$$

Photon yield



4.Cerenkov light emission

$$\beta > 1/n$$

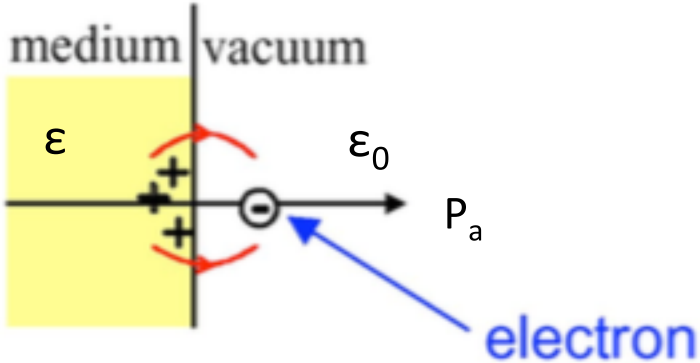
Parameters of Typical Radiator

Medium	n	β_{thr}	θ_{max} [β=1]	N_{ph} [eV^{-1} cm^{-1}]
Air	1.000283	0.9997	1.36	0.208
Isobutan	1.00127	0.9987	2.89	0.941
Water	1.33	0.752	41.2	160.8
Quartz	1.46	0.685	46.7	196.4

4.Transition radiation

- When a relativistic charged particle crosses a boundary between media of different **dielectric properties** radiation is emitted mostly in the **X- ray domain (5-15 KeV)**

The electric field generated by the particle is different on the two sides

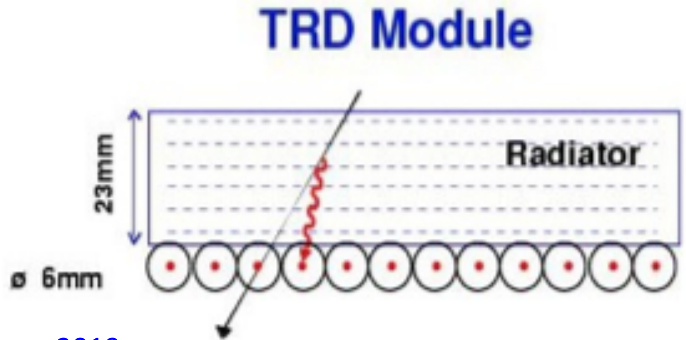


https://arxiv.org/pdf/1111.4188v1.pdf

(ω_p= plasma energy of medium

- The radiation is emitted in a cone at an angle **cos θ = 1/γ**
- Number of photons: $$N_\gamma(\hbar\omega > \hbar\omega_0) = \frac{\alpha z^2}{\pi}\left[\left(\ln\frac{\gamma\hbar\omega_p}{\hbar\omega_0} - 1\right)^2 + \frac{\pi^2}{12}\right]$$
- The probability of radiation per transition surface **is low ~ 1/2 α** (fine structure constant)

TR in AMS detector:
- polypropylene/polyethylene fibers
- Xe/CO_2 straw tubes



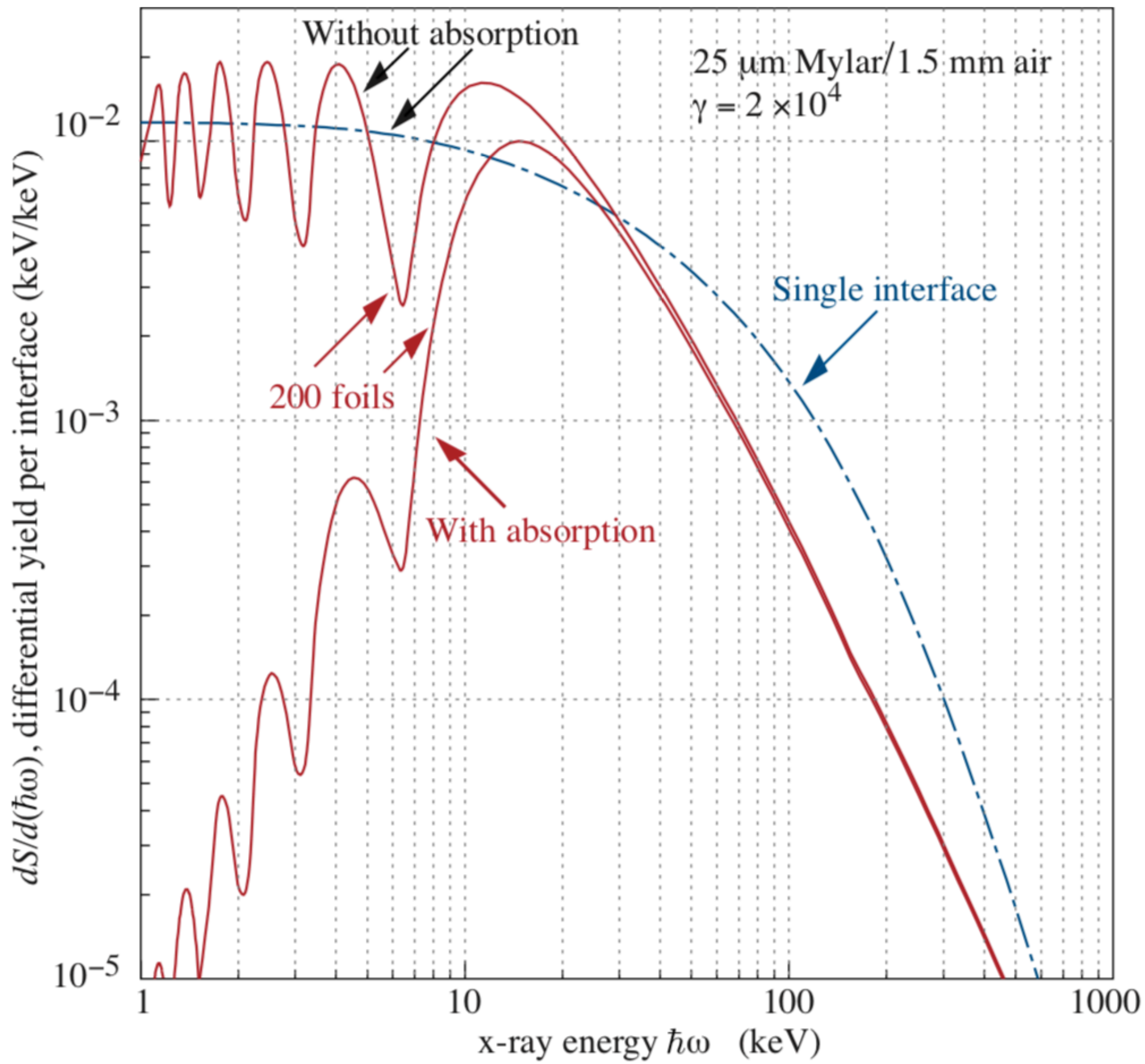


Figure 33.27: X-ray photon energy spectra for a radiator consisting of 200 25 μm thick foils of Mylar with 1.5 mm spacing in air (solid lines) and for a single surface (dashed line). Curves are shown with and without absorption. Adapted from Ref. 88.

4.Transition radiation

- The energy of radiated photons increases as a function of **γ of** particle

Energy radiated when a particle z crosses the boundary between vacuum et medium (ω_p= plasma energy)

$$I = \alpha z^2 \gamma \hbar \omega_p / 3$$

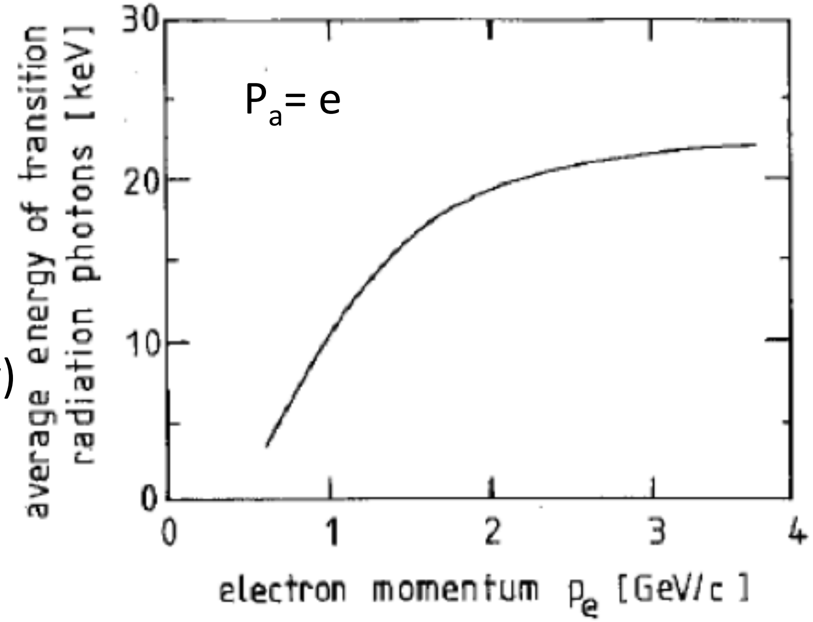
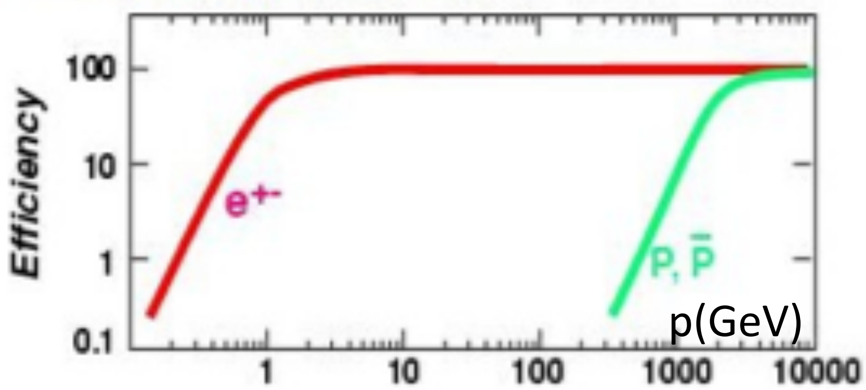
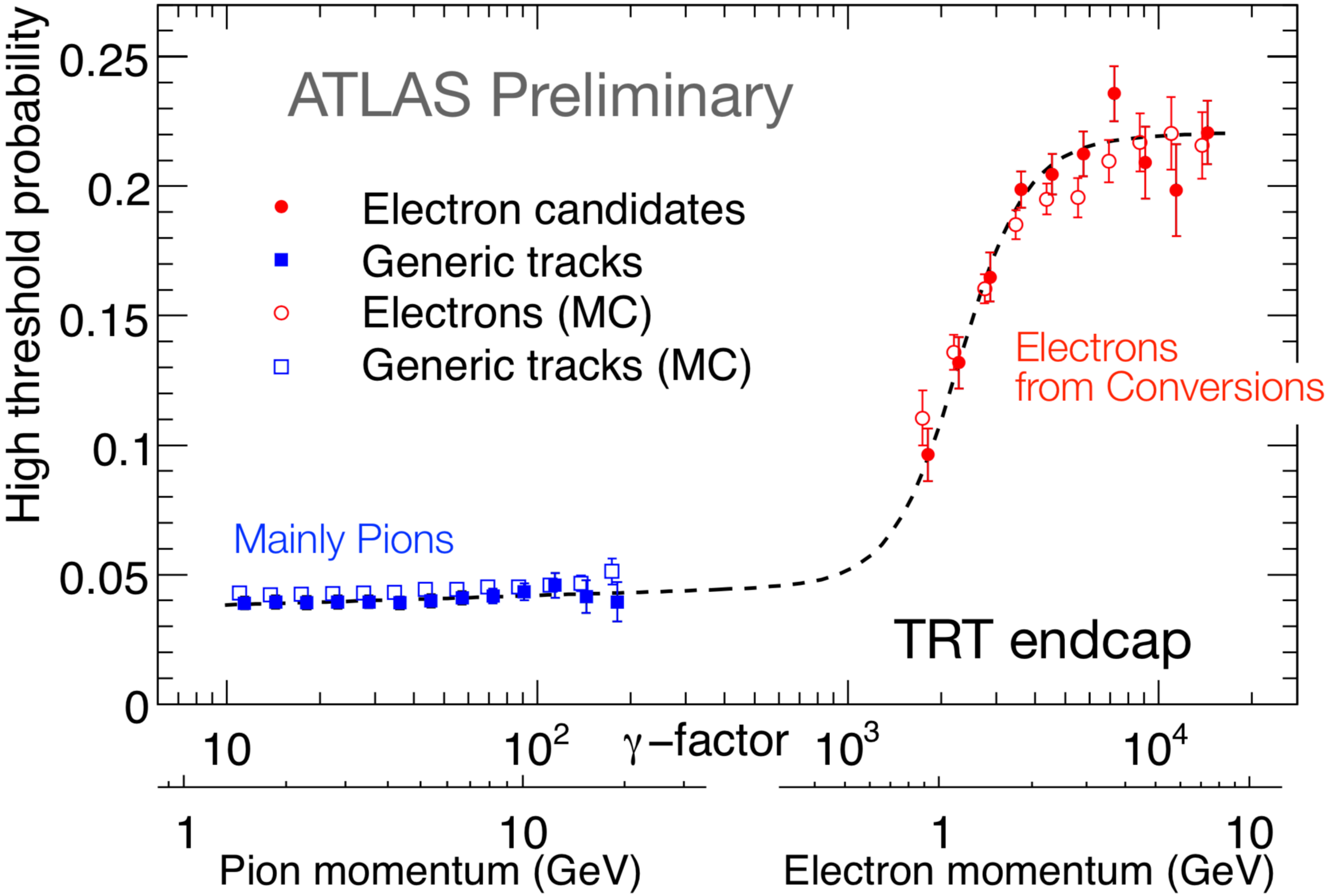


Fig. 6.21. Typical dependence of the average energy of transition radiation photons on the electron momentum for standard radiator arrangements [450].

Useful for particle identification

$e^{\pm}$ / hadron rejection > 10^3




ATLAS Preliminary
High threshold probability
0.25
0.2
0.15
0.1
0.05
0
Electron candidates
Generic tracks
Electrons (MC)
Generic tracks (MC)
Electrons
from Conversions
Mainly Pions
TRT endcap
10
10^2
γ–factor
10^3
10^4
1
10
Pion momentum (GeV)
1
10
Electron momentum (GeV)

END

Important for detection: deposited energy

- **Deposited energy** is what generates the signal in a particle detector
- The **energy loss** is never equal to the **deposited energy** as the radiated photons or the secondary particles may escape the medium
- Deposited energy is subjected to large stochastic fluctuations.
  (stopping power is the **mean** energy loss)
- If the medium is **thin** and the number of interactions is small, the deposited energy distribution is asymmetric : it is sometimes called a **Landau distribution**.
- If the medium is thick or the number of interactions is large, the deposited energy distribution tends to a Gaussian.
- There are no simple and exact analytical formulae to compute deposited energy.
- Nowadays, to estimate the energy deposited in a detector or more generally in a medium we use a Monte-Carlo program which simulates the propagation of the particle through matter : e.g. Geant4

Important for detection: creation of electron-ion pairs

When the measured signal is a current or a charge liberated through ionizing interactions, it is useful to compute the **mean number of created electron-ion pairs**

$$n = \frac{\Delta E_{deposited}}{W}$$

where : **W** is the required **mean energy to produce an e-ion pair**

W > I (mean excitation and ionization potential)

In many gas W ~ 30 eV.

In semiconductor detectors (Ge, Si), W is much lower : e.g. W=3.6 eV for Si and W=2.85 eV for Ge

Better statistics → better resolution