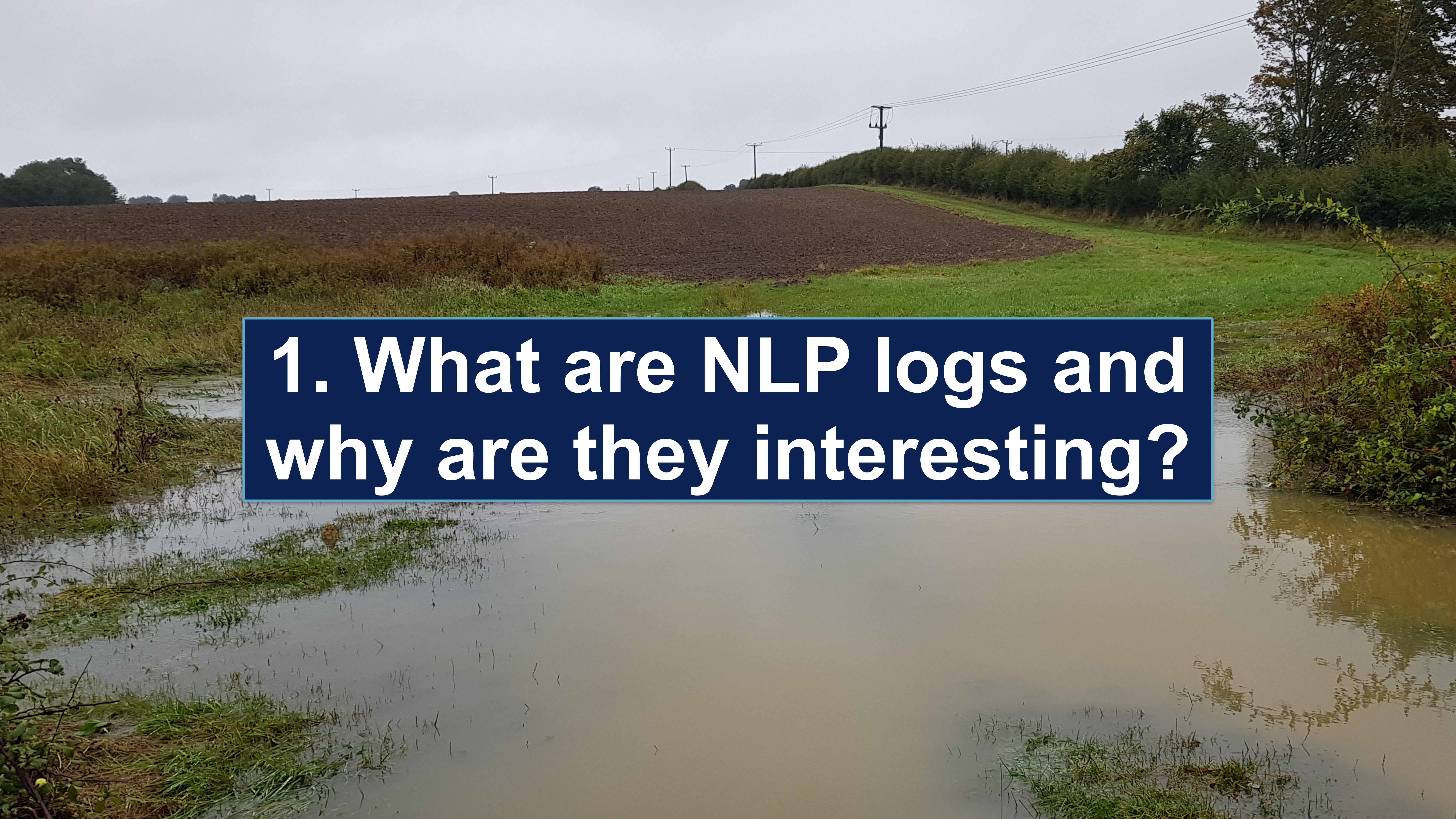
Power up your logarithm: the large impact of ‘subleading’ terms

Melissa van Beekveld

1905.08741 (with Wim Beenakker, Eric Laenen, Chris White)
1905.11771 (with Wim Beenakker, Eric Laenen, Anuradha Misra)
2012/2101.XXXX (with Eric Laenen, Jort Sinninghe Damste, Leonardo Vernazza)

1. What are NLP logs and why are they interesting?



Perturbation theory

A generic observable can be written as

$$\sigma = \sum_n c_n \alpha_s^n$$

The c_n are computed using Feynman diagrams

*Hopefully, only a **limited** number of orders is sufficient to describe the process…*

… which is only true if the c_n are small enough

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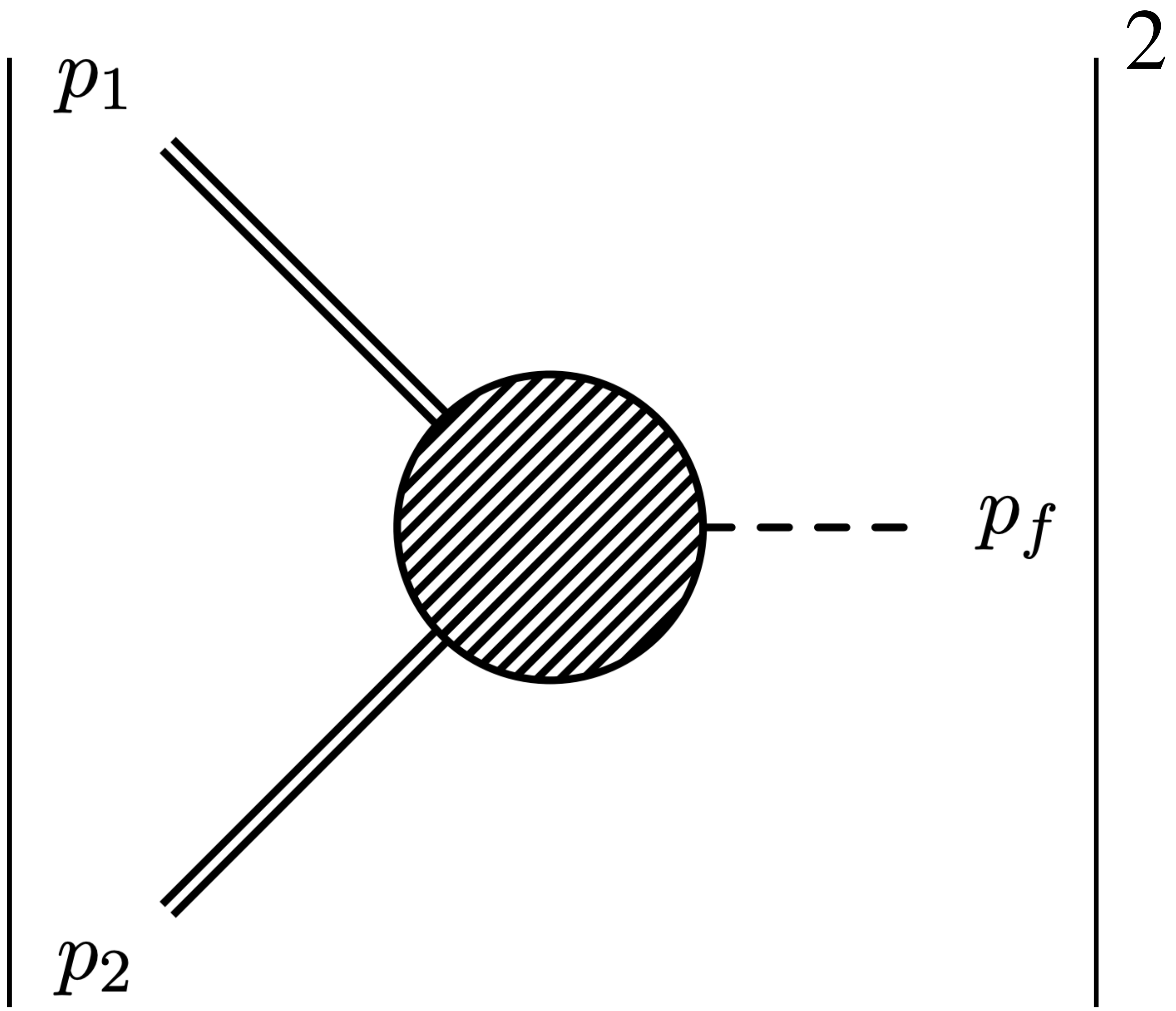
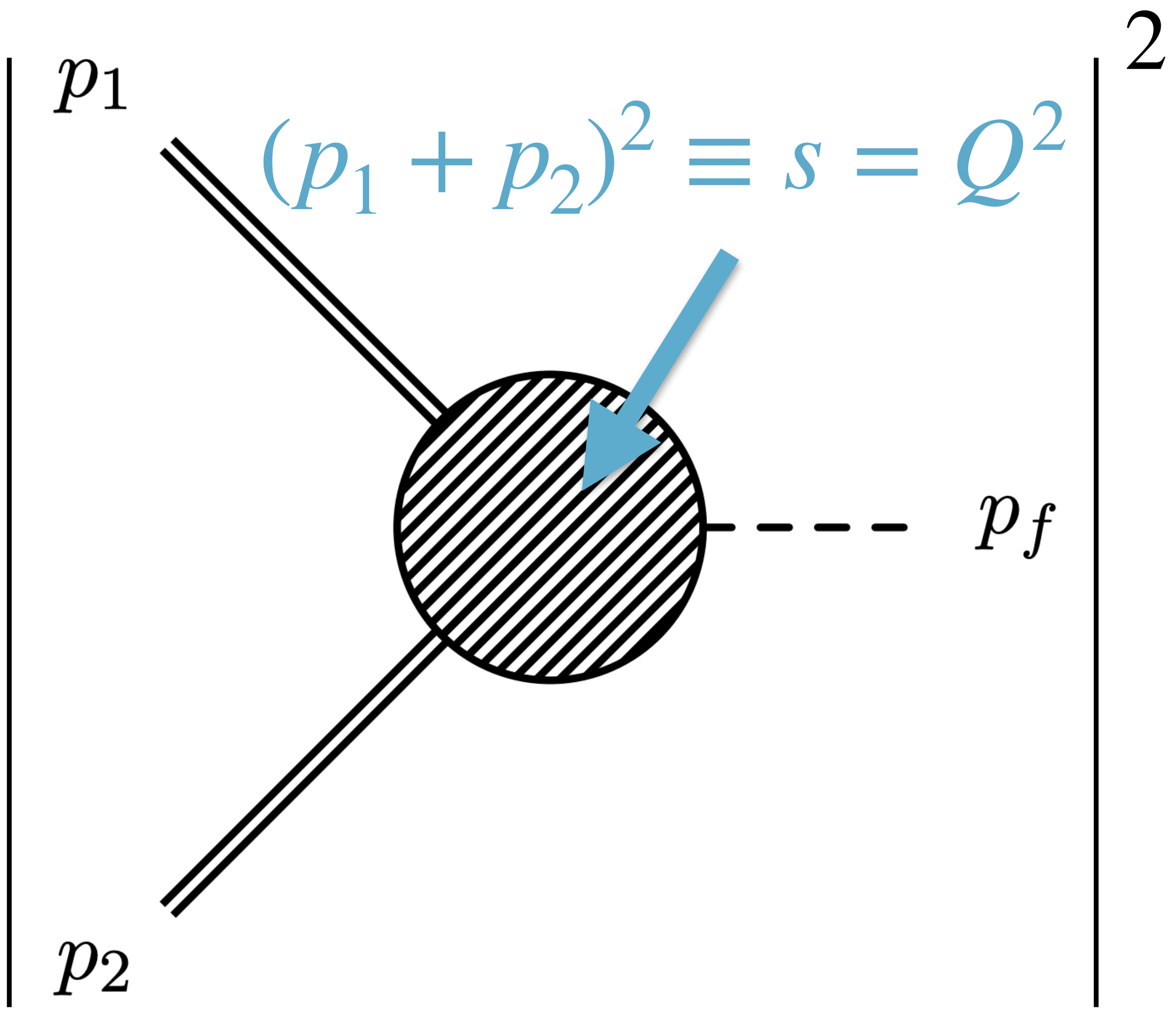
LO process



$\sim \sigma_0$

LO process



$\sim \sigma_0$

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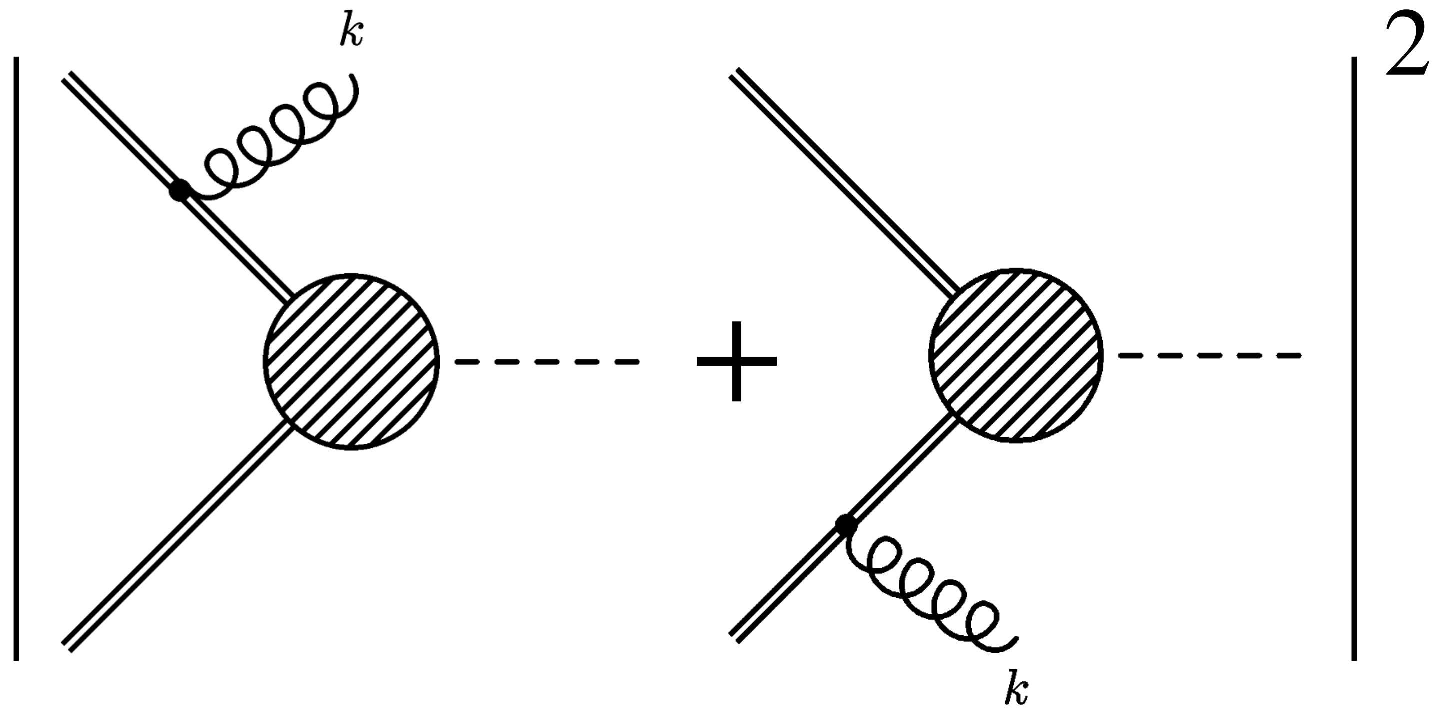
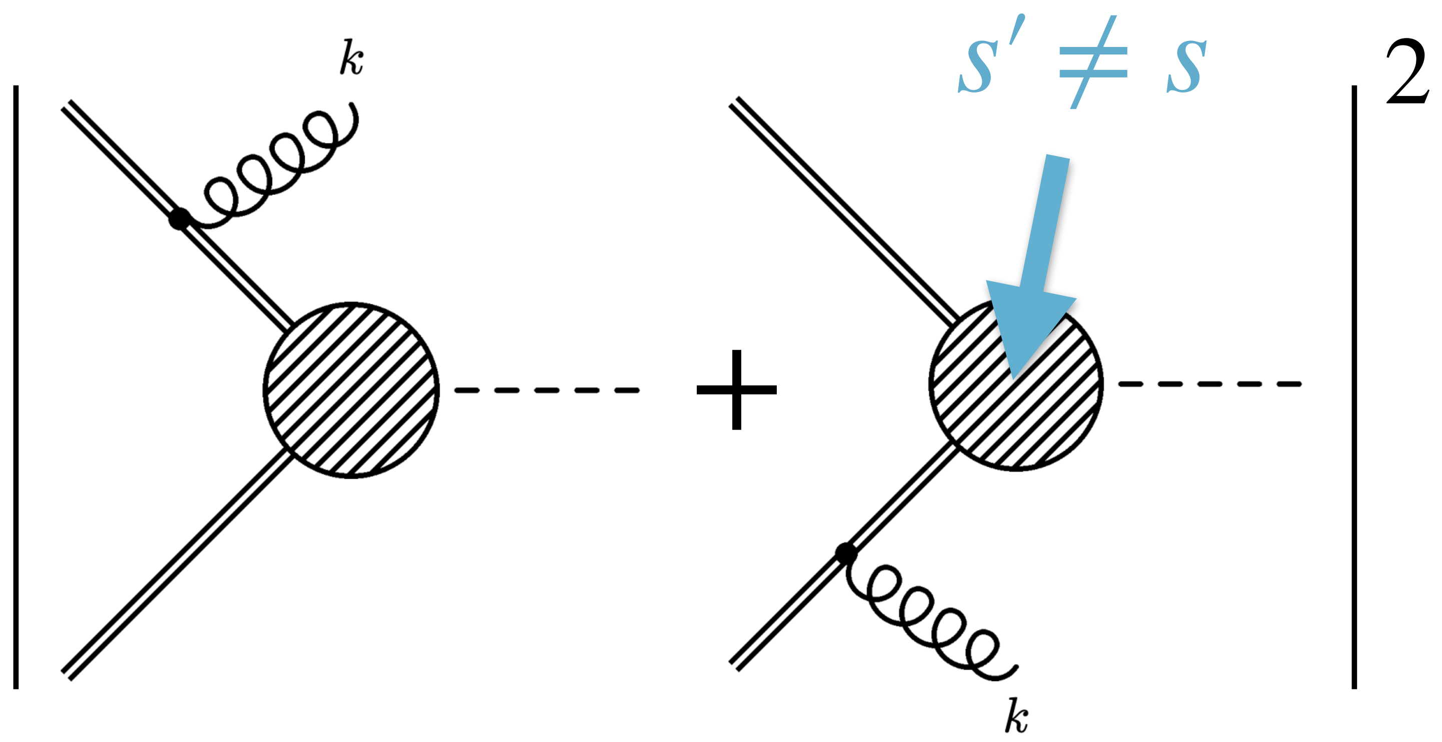
NLO process



Real emission of a gluon

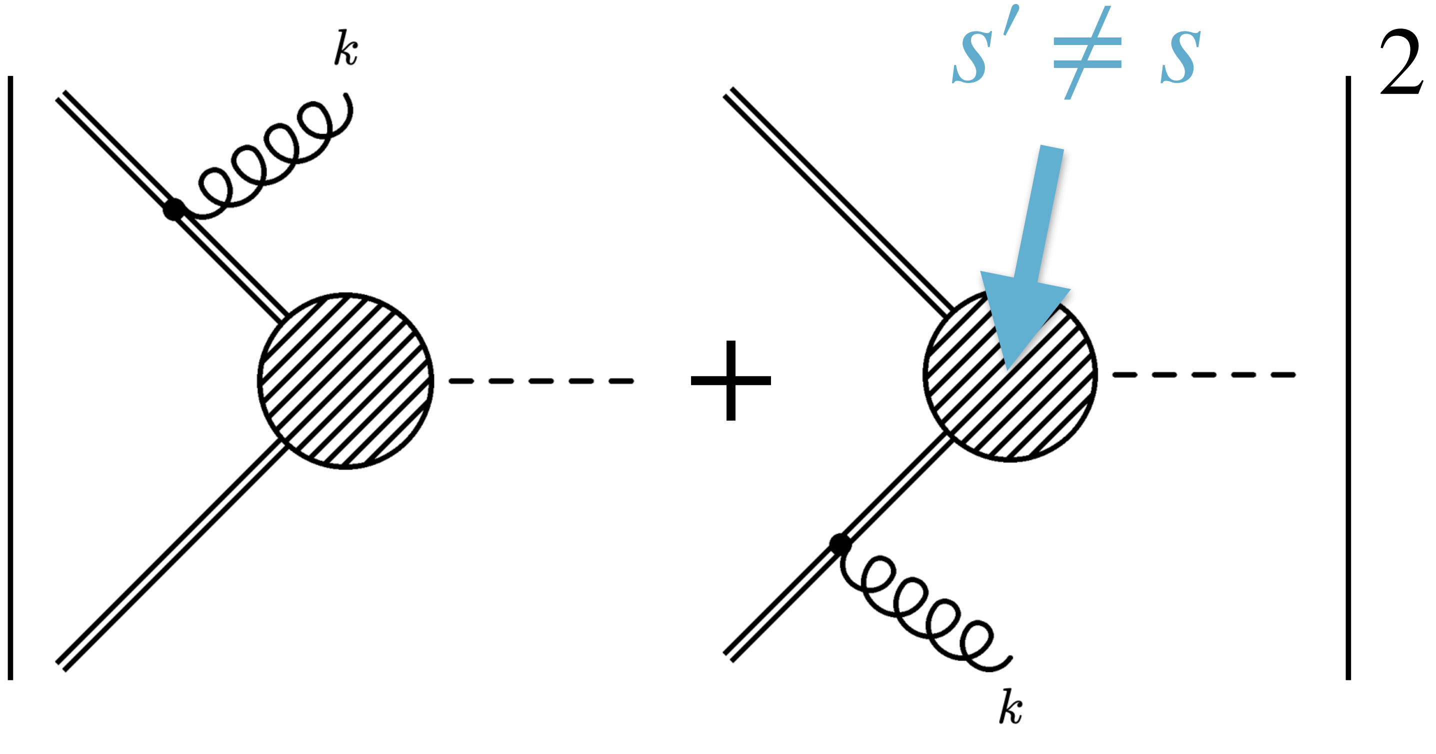
NLO process



Real emission of a gluon

$$s' = (p_1 + p_2 - k)^2 \equiv zs = Q^2$$

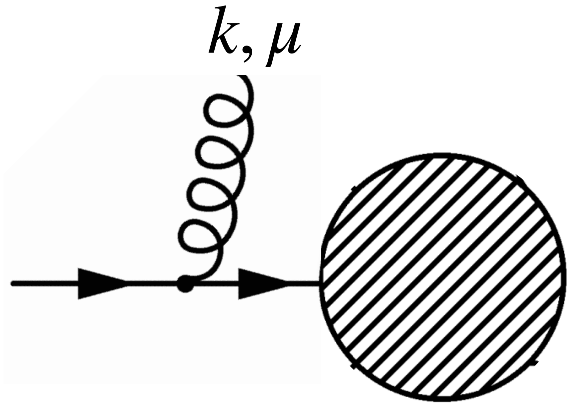
NLO process



Real emission of a gluon

$$s' = (p_1 + p_2 - k)^2 \equiv zs = Q^2$$

Emission of a soft gluon: the eikonal Feynman rule



$$= g_s \mathbf{T} \frac{p^\mu}{p \cdot k} u(p) \epsilon^*_\mu(k)$$

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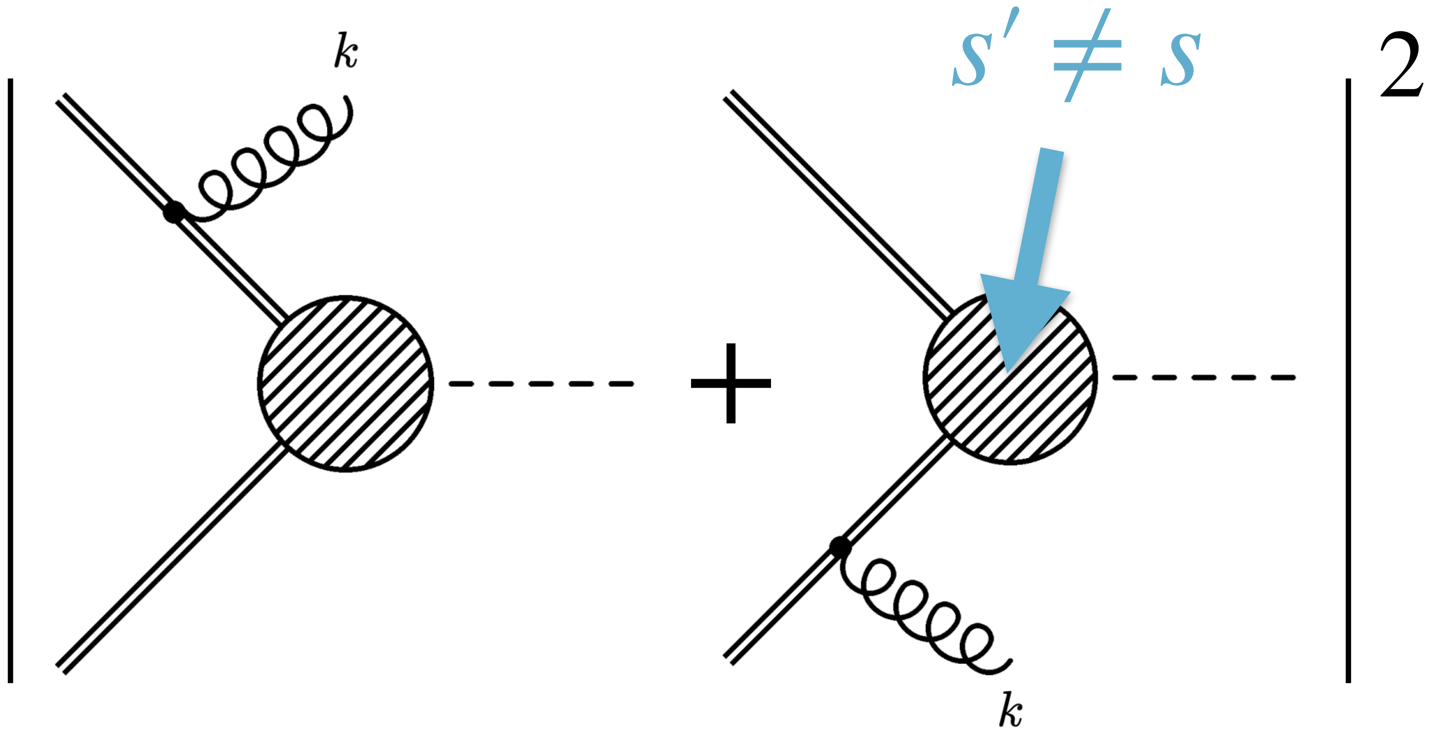
NLO process



Real emission of a gluon

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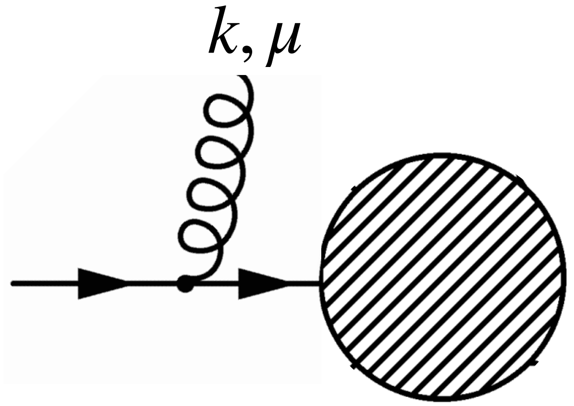
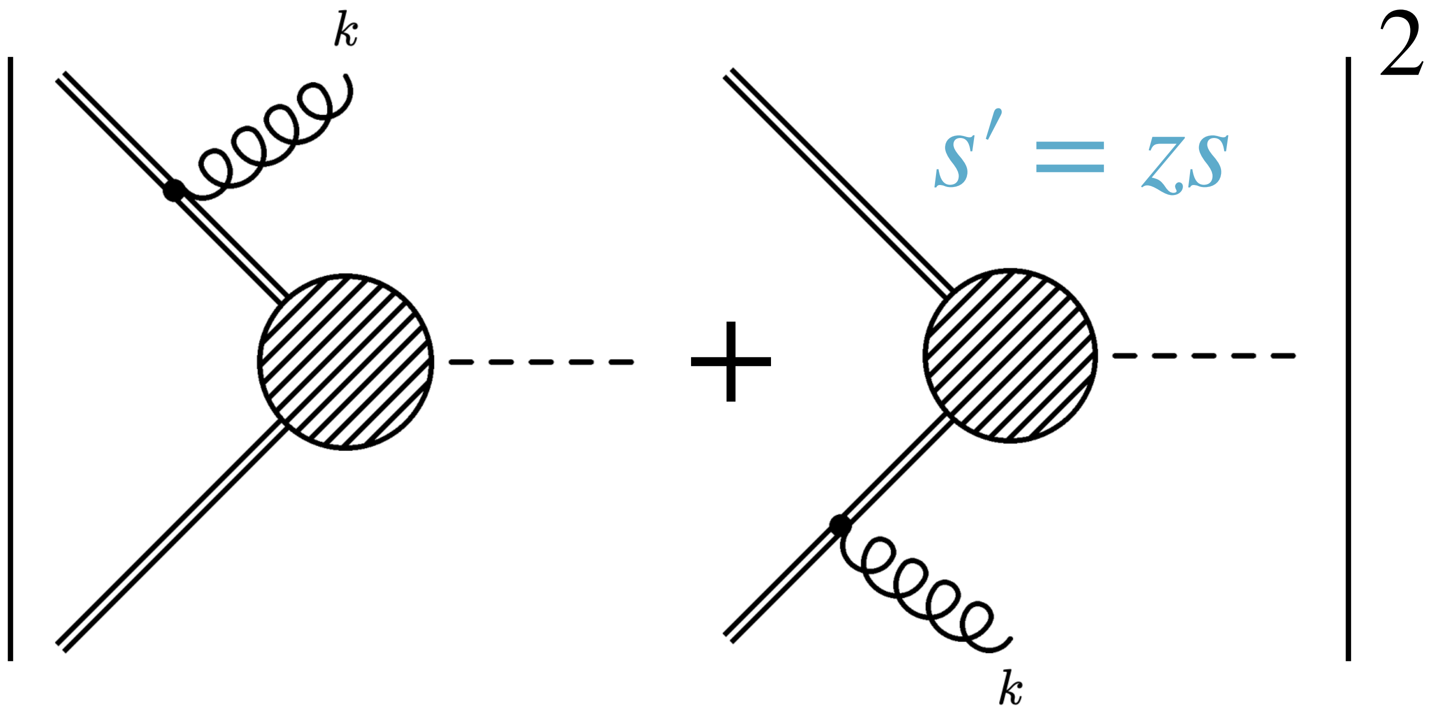
Emission of a soft gluon: the eikonal Feynman rule



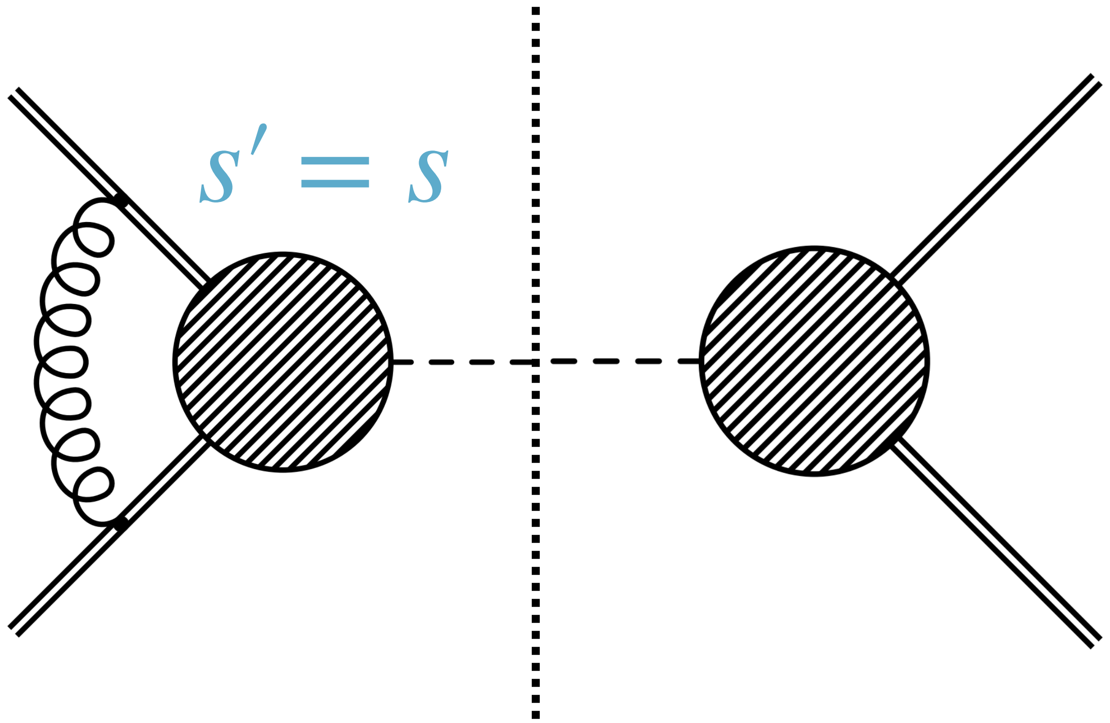
$$= g_s \mathbf{T} \frac{p^\mu}{p \cdot k} u(p) \epsilon^*_\mu(k)$$

Diverges for $k \to 0$ and $k // p$

NLO process



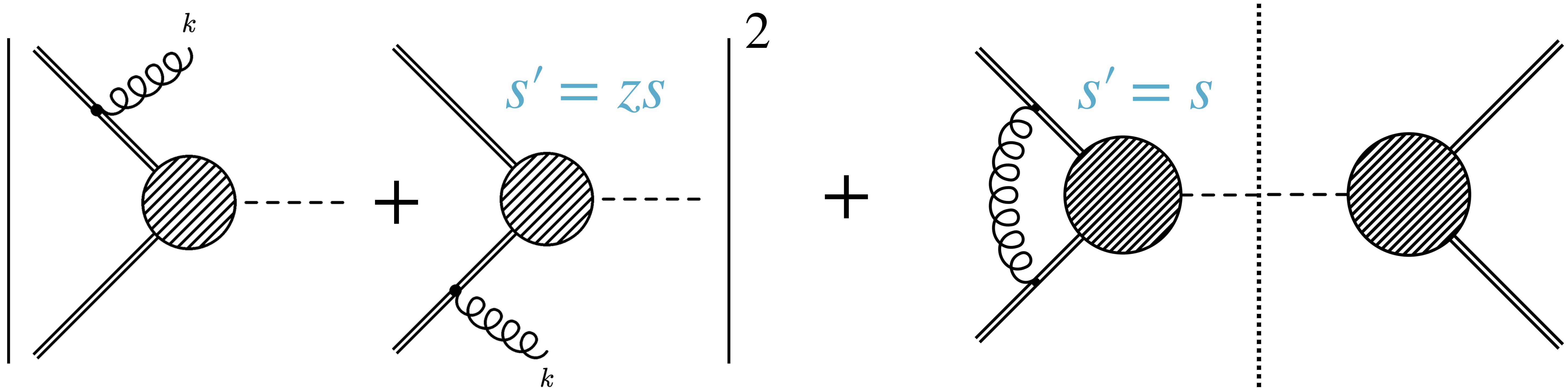
+



Real emission of a gluon

Virtual exchange of a gluon

Origin of large logarithms



$$\sim \frac{\mathrm{d}\sigma_1}{\mathrm{d}z} = \alpha_s \left(d_{11} \left(\frac{\ln(1-z)}{1-z} \right)_+ + d_1' \delta(1-z) + f_1 \right)$$

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Why is this a problem?

Perturbation theory:

$$\frac{d\sigma}{dz} = \sum_n c_n \alpha_s^n = \sigma_0 \delta(1-z) + \alpha_s \left(\sum_{m=0}^{m=1} d_{1m} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + d_1' \delta(1-z) + f_1 \right) + \ldots$$

*Hopefully, only a **limited** number of orders is sufficient to describe the process*

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for $z \to 1$ this is not small…

*Hopefully, only a **limited** number of orders is sufficient to describe the process*

… which is only true if the c_n are small enough

It gets worse…

There is no guarantee that the next order will get smaller!

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \sum_{n=0}^{\infty} \alpha_s^n \left[\sum_{m=0}^{2n-1} d_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + d_n' \delta(1-z) + f_n \right]$$

Can we trust the perturbative result in the domain $z \to 1$?

What if… *We could predict the form of d_{nm} for all n?*

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \sum_{n=0}^{\infty} \alpha_s^n \left[\sum_{m=0}^{2n-1} d_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + d'_n \delta(1-z) + f_n \right]$$

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What if… *We could predict the form of d_{nm} for all n?*

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \sum_{n=0}^{\infty} \alpha_s^n \left[\sum_{m=0}^{2n-1} d_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + d_n' \delta(1-z) + f_n \right]$$

And we would organise the perturbative series in a new way

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \sum_{n=1}^{\infty} \alpha_s^n d_{2n-1} \left(\frac{\ln^{2n-1}(1-z)}{1-z} \right)_+ + \sum_{n=1}^{\infty} \alpha_s^n d_{2n-2} \left(\frac{\ln^{2n-2}(1-z)}{1-z} \right)_+ + \ldots + \sum_{n=0}^{\infty} \alpha_s^n \left[f_n \right]$$

Resummation requires that:

1. You find a predictive pattern for the logarithms that works up to all orders

2. You can factorise these contributions from everything else that is going on in your process at higher orders

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1. You find a predictive pattern for the logarithms that works up to all orders

2. You can factorise these contributions from everything else that is going on in your process at higher orders

One of several methods may then be exploited to prove that the logarithms organise themselves in exponents ***thereby they are resummed***

Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	…
N^{n}LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	…

$$L^{2n} \sim \left(\frac{\ln^{2n-1}(1-z)}{1-z}\right)_+$$

$$\sigma_{\text{resum}} = \sigma_0 e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \ldots$$

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LO	1			
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$$L^{2n} \sim \left(\frac{\ln^{2n-1}(1-z)}{1-z}\right)_+$$

$$\sigma_{\text{resum}} = \sigma_0 \boxed{e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)}} e^{g^{(2)}(\alpha_s L)} \ldots$$

Leading-Log (LL)

Resummation: A new series

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	...
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$$L^{2n} \sim \left(\frac{\ln^{2n-1}(1-z)}{1-z}\right)_+$$

$$\sigma_{\text{resum}} = \sigma_0 e^{\frac{1}{\alpha_s} g^{(1)}(\alpha_s L)} e^{g^{(2)}(\alpha_s L)} \ldots$$

Next-to-Leading-Log (NLL)

Leading-power contributions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \left[\sum_{m=0}^{2n-1} d_{nm} \boxed{\left(\frac{\ln^m(1-z)}{1-z} \right)_+} + d_n' \delta(1-z) + f_n \right]$$

- Universal process-independent form
- Localized at threshold
- Linked to the soft and collinear divergences
- Resummation well understood

But there is more…

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \left[\sum_{m=0}^{2n-1} d_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + d_n' \delta(1-z) + \underbrace{d_{nm}'' \ln^m(1-z) + f_n'}_{f_n} \right]$$

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Next-to-leading-power (NLP)

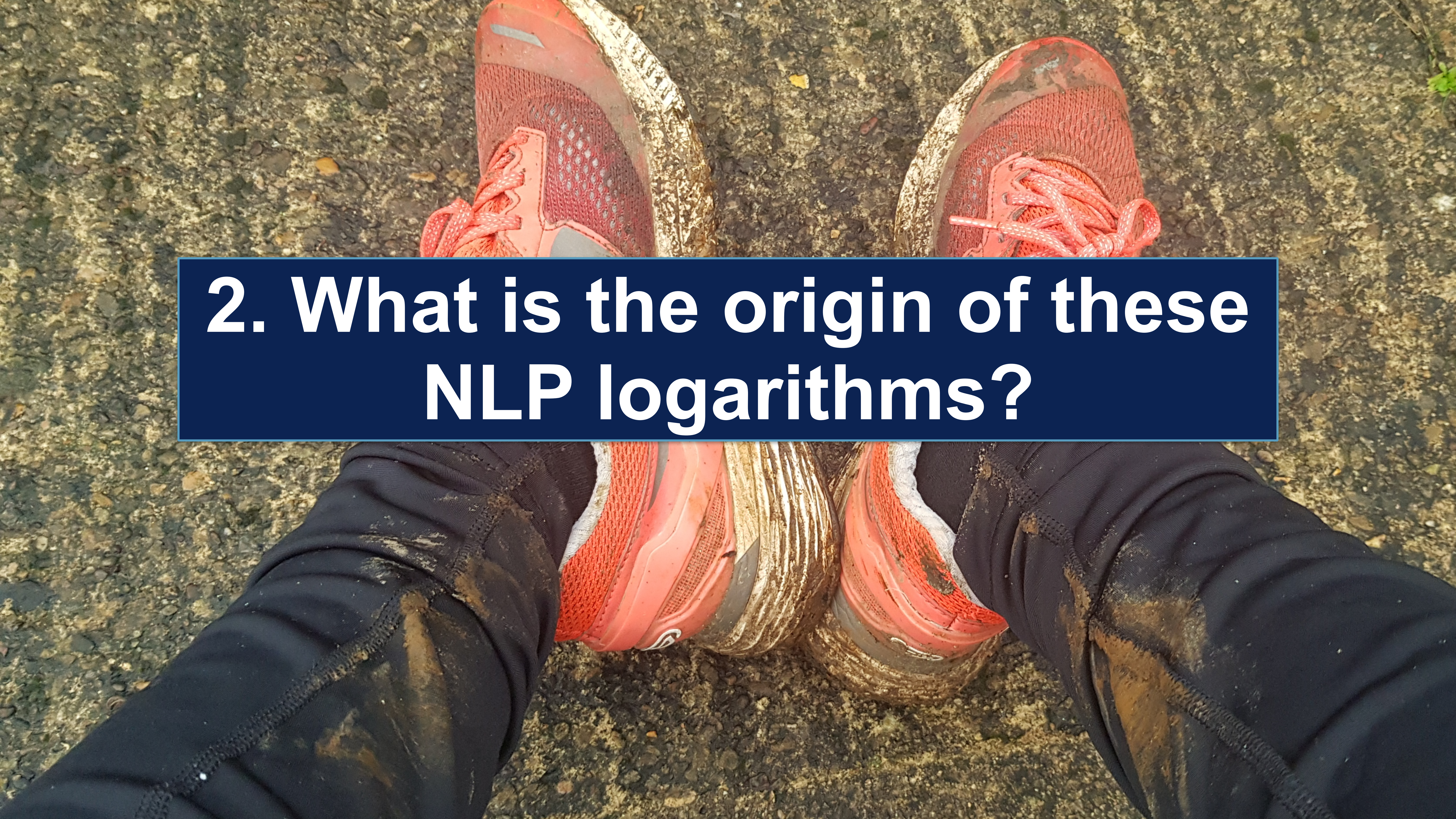
$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} \propto \sum_{n=0}^{\infty} \alpha_s^n \left[\sum_{m=0}^{2n-1} d_{nm} \left(\frac{\ln^m(1-z)}{1-z} \right)_+ + d'_n \delta(1-z) + d''_{nm} \ln^m(1-z) + f'_n \right]$$

No general resummation framework for these!

Understanding them is important because:

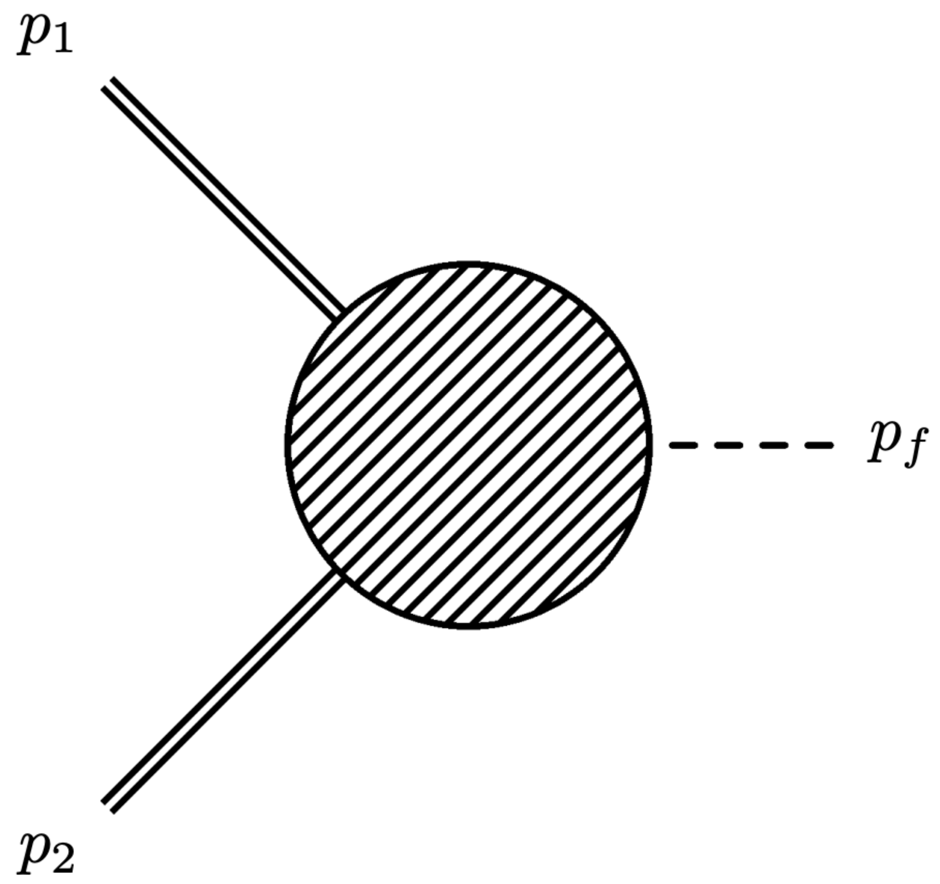
- Increasing experimental precision makes them relevant
- Check of higher-order corrections
- Help to reduce scale uncertainties

2. What is the origin of these NLP logarithms?



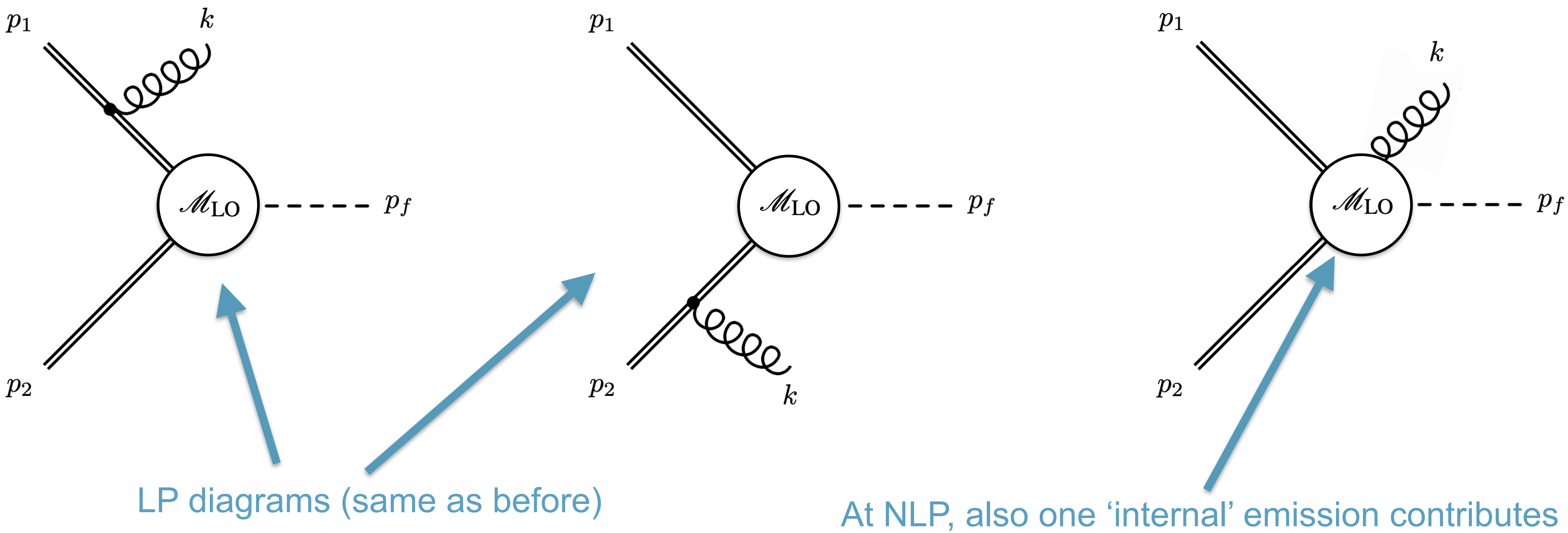
Universality of NLP logs

Let us first examine what happens when a *colourless* final state is produced



[1706.04018]

NLO Amplitude at NLP



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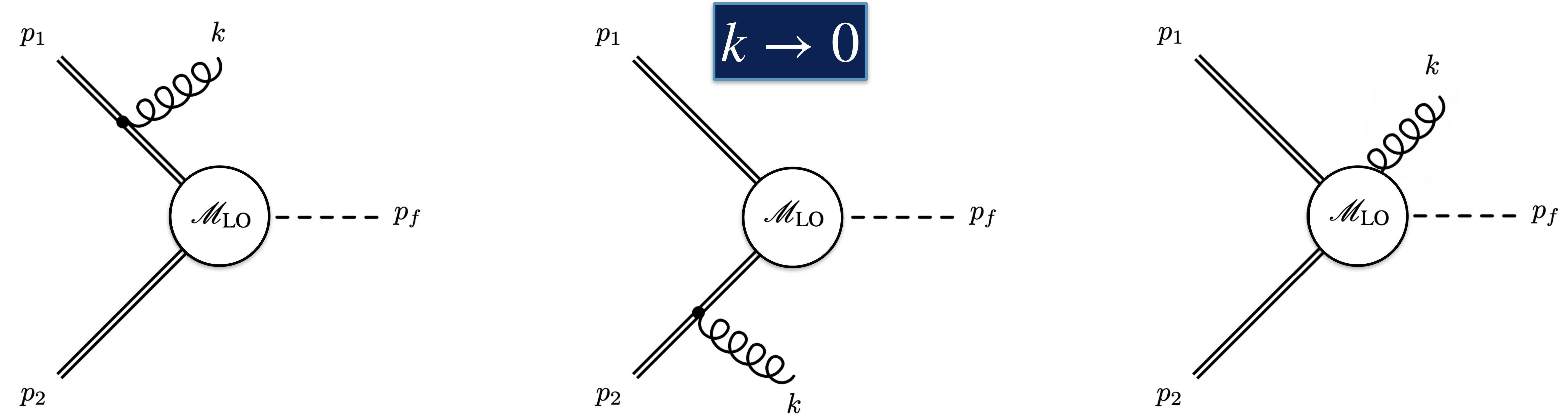
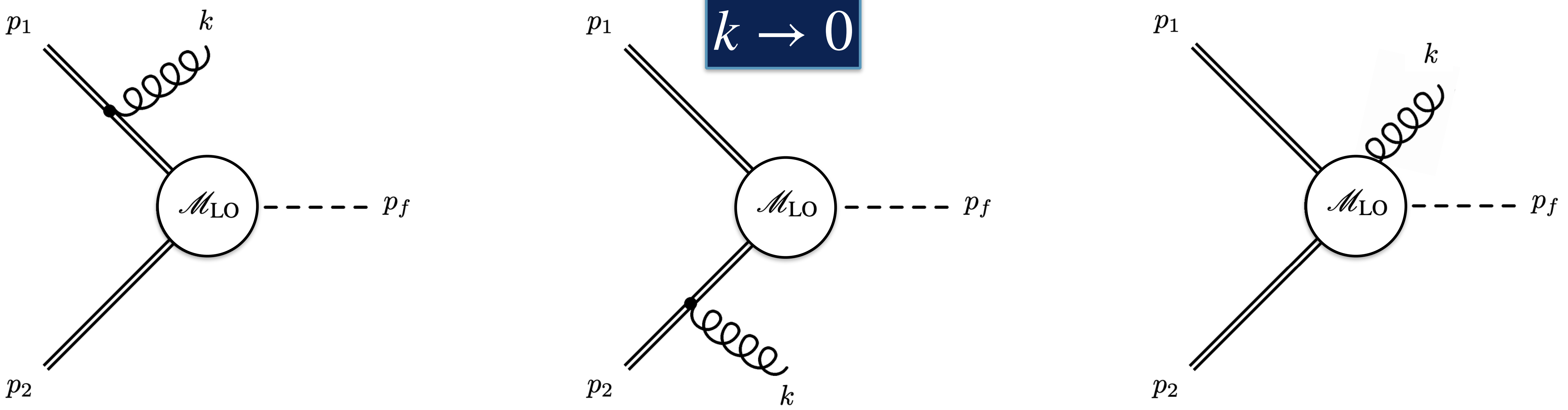


$$\mathscr{A}_{\text{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\text{LO}}\, \varepsilon_\sigma^*(k)$$

NLO Amplitude at NLP

$$\boxed{k \to 0}$$



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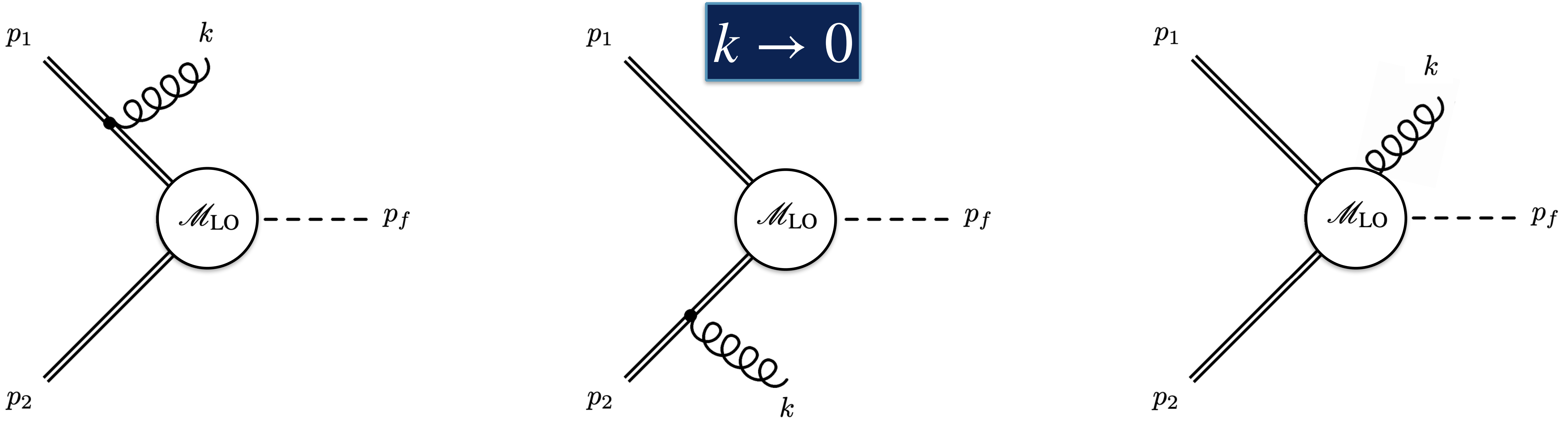
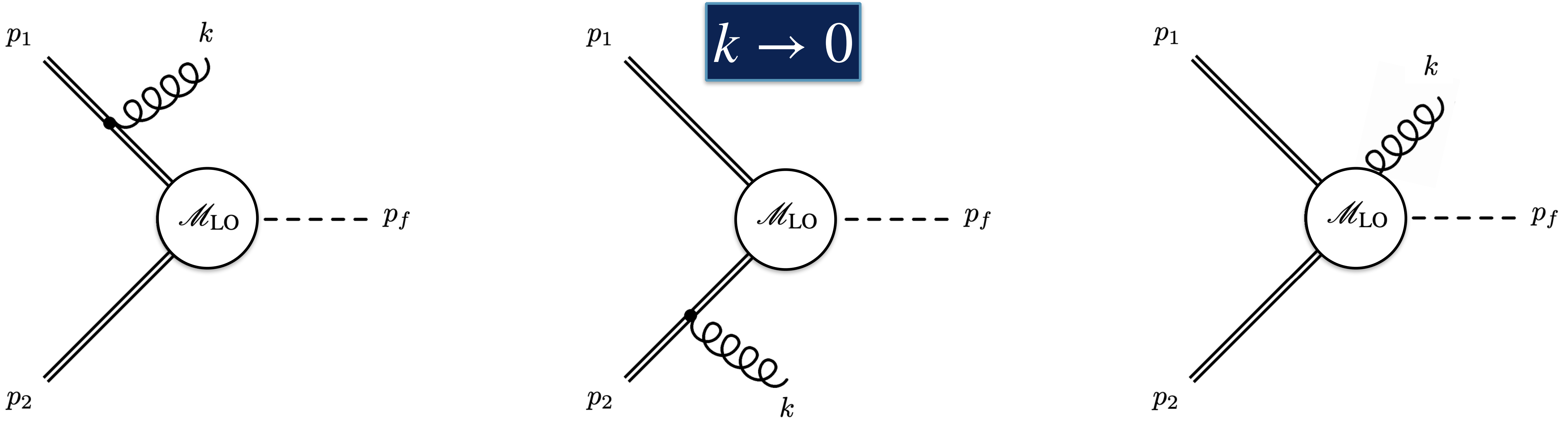


$$\mathscr{A}_{\mathrm{NLP}} = \sum_{i=1}^{n=2} \mathrm{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\mathrm{LO}}\, \varepsilon_\sigma^*(k)$$

NLO Amplitude at NLP



$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathcal{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

NLO Amplitude at NLP

Eikonal

$$\mathscr{A}_{\mathrm{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\mathrm{LO}}\, \varepsilon_\sigma^*(k)$$

$$\mathcal{O}\left(\frac{1}{k}\right)$$

NLO Amplitude at NLP

Scalar

$$\mathscr{A}_{\rm NLP} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\boxed{\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k}} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\rm LO}\, \varepsilon_\sigma^*(k)$$

$$\mathcal{O}\left(\frac{1}{k}\right) + \mathcal{O}(1)$$

NLO Amplitude at NLP

$$\Sigma^{\sigma\alpha} \begin{cases} \frac{i}{4}[\gamma^\sigma, \gamma^\alpha] \equiv S^{\sigma\alpha} \\ i(g^{\rho\sigma}g^{\alpha\nu} - g^{\sigma\nu}g^{\alpha\rho}) \equiv M^{\sigma\alpha,\rho\nu} \end{cases}$$

Spin

$$\mathscr{A}_{\mathrm{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \boxed{\frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha}} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\mathrm{LO}}\, \varepsilon_\sigma^*(k)$$

$$\mathscr{O}(1)$$

Needs to be inserted at the right place in the matrix element!

NLO Amplitude at NLP

Orbital

$$\mathscr{A}_{\text{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \boxed{\frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha}} \right) \otimes \mathscr{M}_{\text{LO}}\, \varepsilon_\sigma^*(k)$$

$\mathcal{O}(1)$

$$L_i^{\sigma\alpha} = -i\left(p_i^\sigma \frac{\partial}{\partial p_{i\alpha}} - p_i^\alpha \frac{\partial}{\partial p_{i\sigma}} \right)$$

NLO Amplitude at NLP

$$\mathscr{A}_{\mathrm{NLP}} = \sum_{i=1}^{n=2} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k}\Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\mathrm{LO}}\, \varepsilon_\sigma^*(k)$$

$$= \mathscr{A}_{\mathrm{scal}} + \mathscr{A}_{\mathrm{spin}} + \mathscr{A}_{\mathrm{orb}}$$

Towards the NLP cross section

$$\left|\mathscr{A}_{\mathrm{NLP}}\right|^2 = \sum_{\mathrm{colors}} \left|\mathscr{A}_{\mathrm{scal}}\right|^2 + 2\mathrm{Re}\left[\left(\mathscr{A}_{\mathrm{spin}} + \mathscr{A}_{\mathrm{orb}}\right)^{\dagger} \mathscr{A}_{\mathrm{scal}}\right]$$

$$\propto \mathscr{O}\left(\frac{1}{k^2}\right)$$

$$\propto \mathscr{O}\left(\frac{1}{k}\right)$$

Towards the NLP cross section

$$\begin{aligned} |\mathscr{A}_{\mathrm{NLP}}|^2 &= \sum_{\mathrm{colors}} |\mathscr{A}_{\mathrm{scal}}|^2 + 2\mathrm{Re}\left[\left(\mathscr{A}_{\mathrm{spin}} + \mathscr{A}_{\mathrm{orb}}\right)^\dagger \mathscr{A}_{\mathrm{scal}}\right] \\ &= K \frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} |\mathscr{M}_{\mathrm{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)|^2 \end{aligned}$$

Towards the NLP cross section

$$
\begin{aligned}
|\mathscr{A}_{\mathrm{NLP}}|^2 &= \sum_{\mathrm{colors}} |\mathscr{A}_{\mathrm{scal}}|^2 + 2\mathrm{Re}\left[(\mathscr{A}_{\mathrm{spin}} + \mathscr{A}_{\mathrm{orb}})^\dagger \mathscr{A}_{\mathrm{scal}}\right] \\
&= K \boxed{\frac{2p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k}} |\mathscr{M}_{\mathrm{LO}}(p_1 + \delta p_1, p_2 + \delta p_2)|^2
\end{aligned}
$$

Eikonal factor

Towards the NLP cross section

$$
\begin{aligned}
|\mathscr{A}_{\mathrm{NLP}}|^2 \;&=\; \sum_{\text{colors}} |\mathscr{A}_{\mathrm{scal}}|^2 + 2\mathrm{Re}\left[\left(\mathscr{A}_{\mathrm{spin}} + \mathscr{A}_{\mathrm{orb}}\right)^{\dagger}\mathscr{A}_{\mathrm{scal}}\right] \\
&=\; K\frac{2p_1\cdot p_2}{p_1\cdot k\, p_2\cdot k}\left|\mathscr{M}_{\mathrm{LO}}(p_1+\delta p_1, p_2+\delta p_2)\right|^2
\end{aligned}
$$

Shift in Born matrix element

$$
\delta p^{\alpha}_{i;j} \equiv -\frac{1}{2}\left(k^{\alpha} + \frac{p_j\cdot k}{p_i\cdot p_j}p_i^{\alpha} - \frac{p_i\cdot k}{p_i\cdot p_j}p_j^{\alpha}\right)
$$

Towards the NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,K_{\mathrm{NLP}}\,\sigma_{\mathrm{Born}}(Q)$$

[1706.04018]

Towards the NLP cross section

Integration over phase space: $$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\, \boxed{K_{\mathrm{NLP}}}\, \sigma_{\mathrm{Born}}(Q)$$

$$K_{\mathrm{NLP}} = \frac{\alpha_s}{\pi}\left(\frac{\bar{\mu}^2}{s}\right)^{\epsilon}\left[-\frac{2}{\epsilon}\left(\left(\frac{1}{1-z}\right)_+ - 1\right) + 4\left(\frac{\ln(1-z)}{1-z}\right)_+ - 4\ln(1-z) + \frac{1}{\varepsilon^2}\delta(1-z) + \ldots\right]$$

[1706.04018]

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Towards the NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,\boxed{K_{\mathrm{NLP}}}\,\sigma_{\mathrm{Born}}(Q)$$

$$K_{\mathrm{NLP}} = \frac{\alpha_s}{\pi}\left(\frac{\bar{\mu}^2}{s}\right)^{\epsilon}\left[-\frac{2}{\epsilon}\left(\left(\frac{1}{1-z}\right)_+ - 1\right) + \boxed{4}\left(\frac{\ln(1-z)}{1-z}\right)_+ - \boxed{4\ln(1-z)} + \frac{1}{\varepsilon^2}\delta(1-z) + \ldots\right]$$

NLP log with the same coefficient as the LP log!

[1706.04018]

Towards the NLP cross section

Integration over phase space:

$$\frac{\mathrm{d}\sigma_{\mathrm{NLP}}}{\mathrm{d}Q} = K\,K_{\mathrm{NLP}}\,\sigma_{\mathrm{Born}}(Q)$$

$$K_{\mathrm{NLP}} = \frac{\alpha_s}{\pi}\left(\frac{\bar{\mu}^2}{s}\right)^{\epsilon}\left[-\frac{2}{\epsilon}\left(\left(\frac{1}{1-z}\right)_+ - 1\right) + 4\left(\frac{\ln(1-z)}{1-z}\right)_+ - 4\ln(1-z) + \frac{1}{\varepsilon^2}\delta(1-z) + \ldots\right]$$

$$= zK_{\mathrm{LP}}$$

[1706.04018]

Let's extend these results

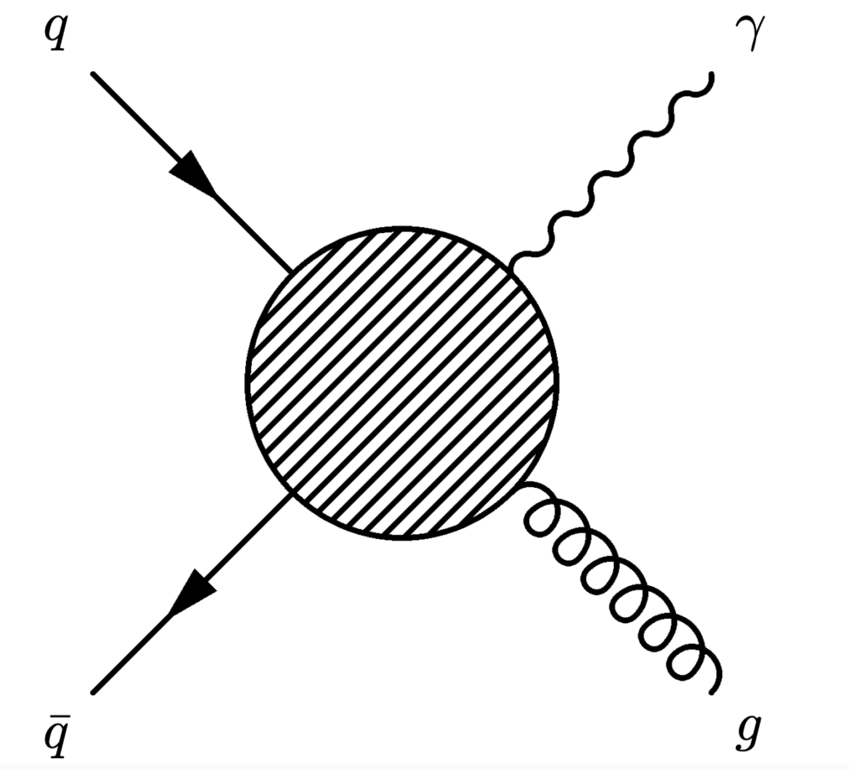
- What happens with coloured particles in the final state?

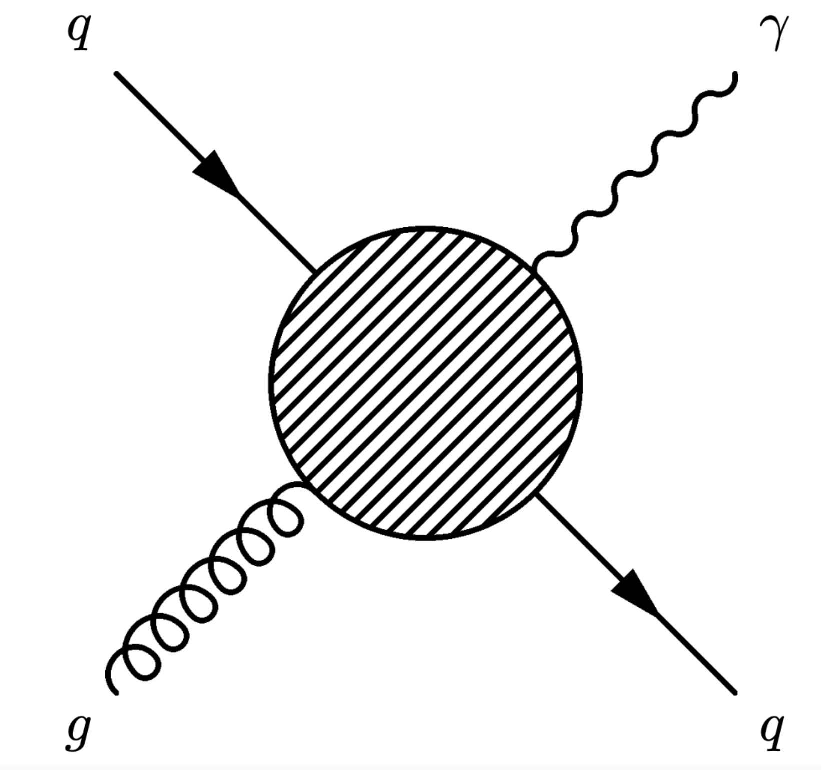
- What role do soft quarks play?

[1905.08741]

Prompt photon production

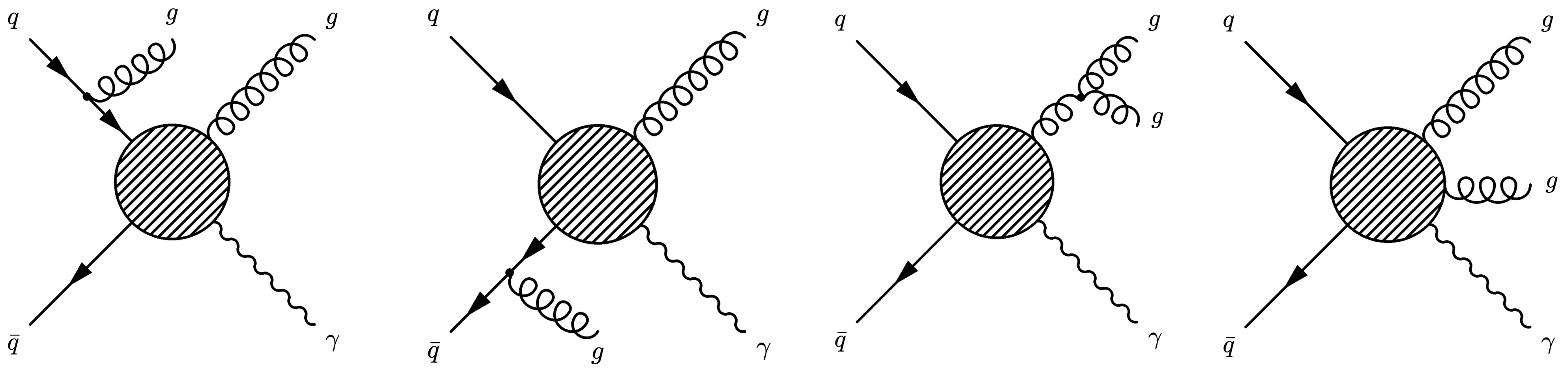
$$pp \rightarrow \gamma + X$$





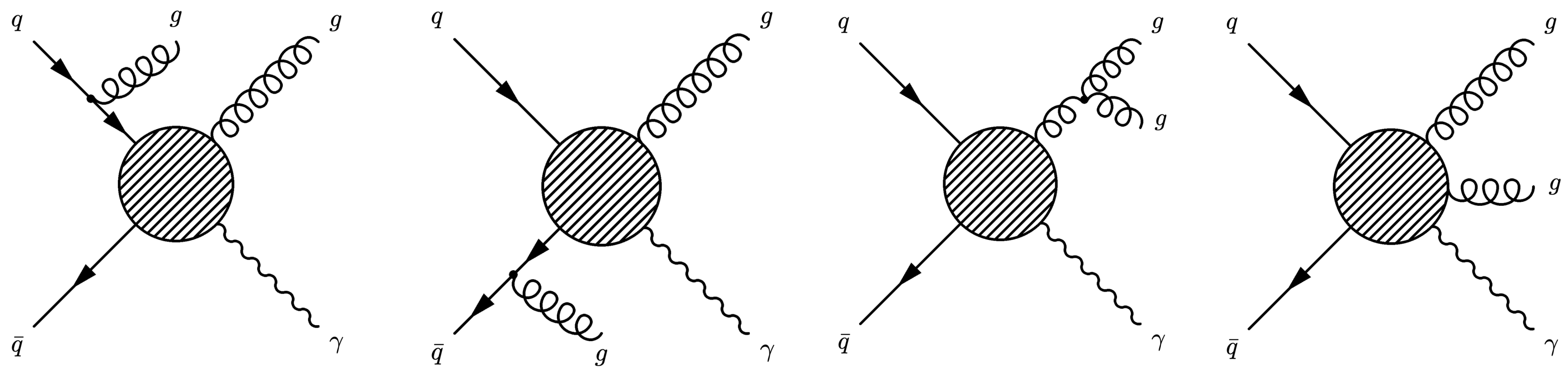
Simplest channel: $q\bar{q}$

$$q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$$



Similar NLP amplitude emerges!

$$\mathscr{A}_{\mathrm{NLP}} = \sum_{i=1}^{n=3} \mathbf{T}_i \left(\frac{2p_i^\sigma \pm k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\mathrm{LO}}\, \varepsilon_\sigma^*(k)$$



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Difference:
sign change for final state radiation

Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

$$\begin{aligned}
|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 \quad = \quad & \frac{C_F}{C_A}\Bigg[C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \\
& +\frac{1}{2}C_A \frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;R}, p_R-\delta p_{R;1})\right|^2 \\
& +\frac{1}{2}C_A \frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2 \\
& -\frac{1}{2}C_A \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2\Bigg]
\end{aligned}$$

Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

Eikonal factors

$$
\begin{aligned}
|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 \;=\; \frac{C_F}{C_A}\Bigg[& C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \\
& + \frac{1}{2}C_A \frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)} \left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;R}, p_R-\delta p_{R;1})\right|^2 \\
& + \frac{1}{2}C_A \frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)} \left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2 \\
& - \frac{1}{2}C_A \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \Bigg]
\end{aligned}
$$

Interferences are created!

Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

Shifts in Born amplitude

$$
\begin{aligned}
|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 \quad = \quad & \frac{C_F}{C_A}\Bigg[C_F\frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \\
& +\frac{1}{2}C_A\frac{2p_1\cdot p_R}{(p_1\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;R}, p_R-\delta p_{R;1})\right|^2 \\
& +\frac{1}{2}C_A\frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2 \\
& -\frac{1}{2}C_A\frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)}\left|\mathscr{M}^{q\bar{q}}_{\mathrm{LO}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2\Bigg]
\end{aligned}
$$

Towards the NLP cross section

Process: $q(p_1)\bar{q}(p_2) \to \gamma(p_\gamma)g(k)g(p_R)$

Shifts in Born amplitude

$$|\mathscr{A}_{\mathrm{NLP},q\bar{q}}|^2 = \frac{C_F}{C_A}\Bigg[C_F \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \left|\mathscr{M}_{\mathrm{LO}}^{q\bar{q}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2$$

After integration over phase space all LL terms up to NLP are obtained. Missing LP NLL terms are recovered by adding the $g \to gg(q\bar{q})$ splittings.

$$+\frac{1}{2}C_A \frac{2p_2\cdot p_R}{(p_2\cdot k)(p_R\cdot k)} \left|\mathscr{M}_{\mathrm{LO}}^{q\bar{q}}(p_2+\delta p_{2;R}, p_R-\delta p_{R;2})\right|^2$$

$$-\frac{1}{2}C_A \frac{2p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \left|\mathscr{M}_{\mathrm{LO}}^{q\bar{q}}(p_1+\delta p_{1;2}, p_2+\delta p_{2;1})\right|^2 \Bigg]$$

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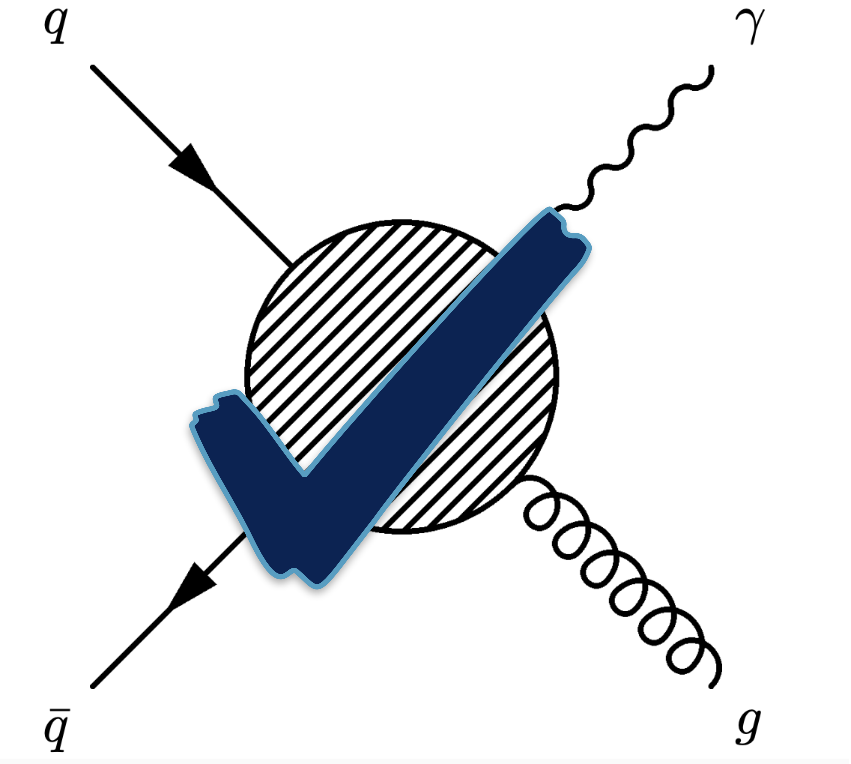
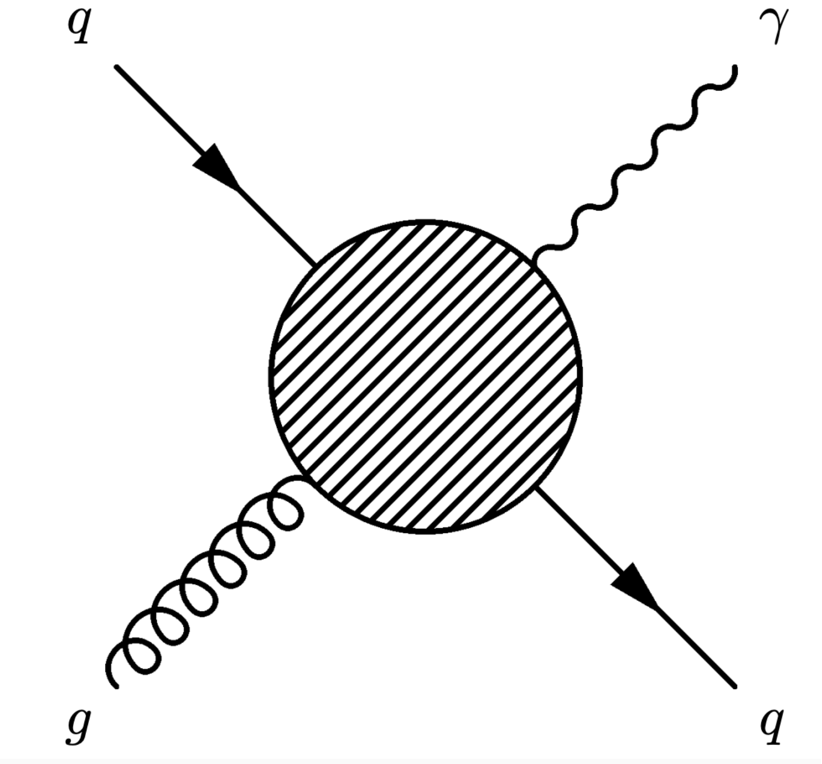
Prompt photon production

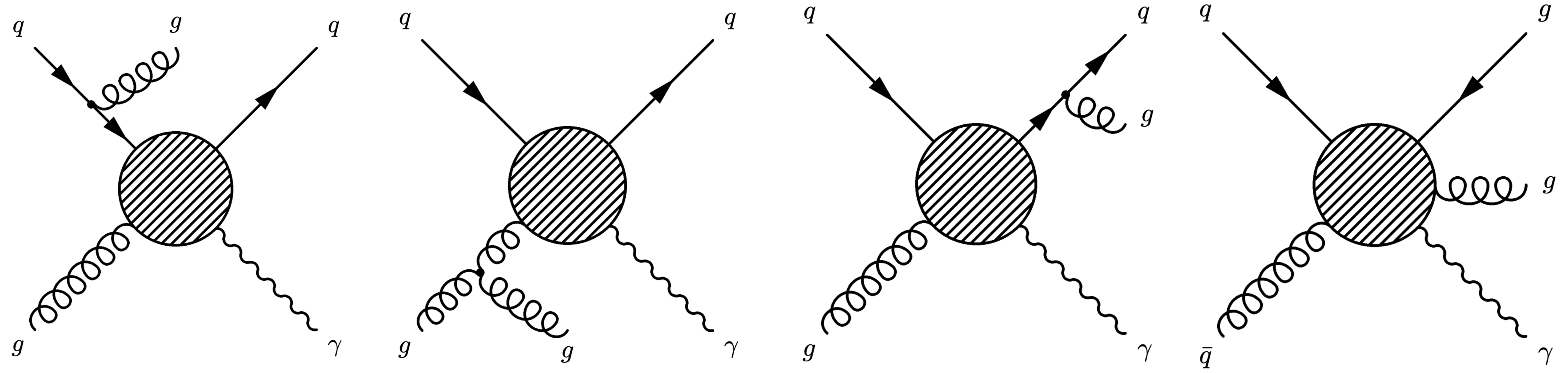
$$pp \to \gamma + X$$





qg channel

$g(p_1)q(p_2) \to \gamma(p_\gamma)g(k)q(p_R)$



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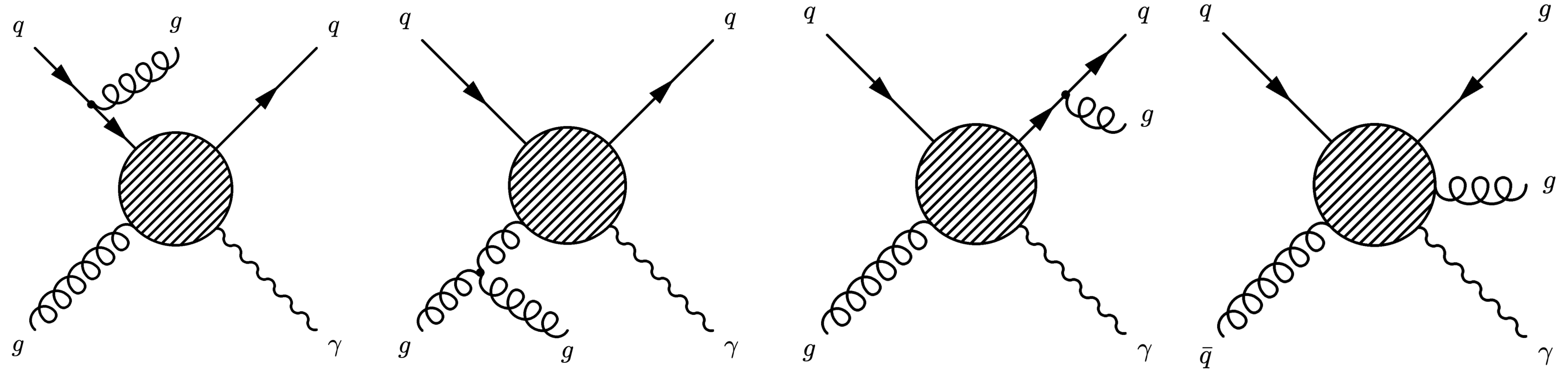
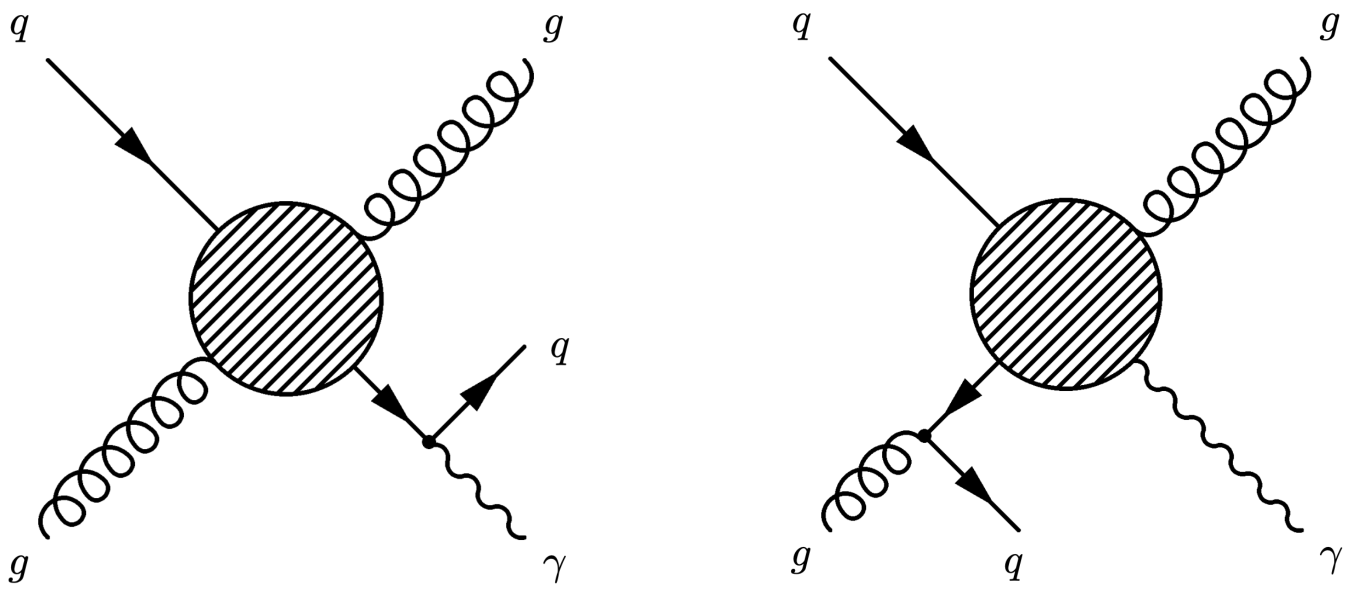
$$g(p_1)q(p_2) \to \gamma(p_\gamma)g(k)q(p_R)$$



but also:



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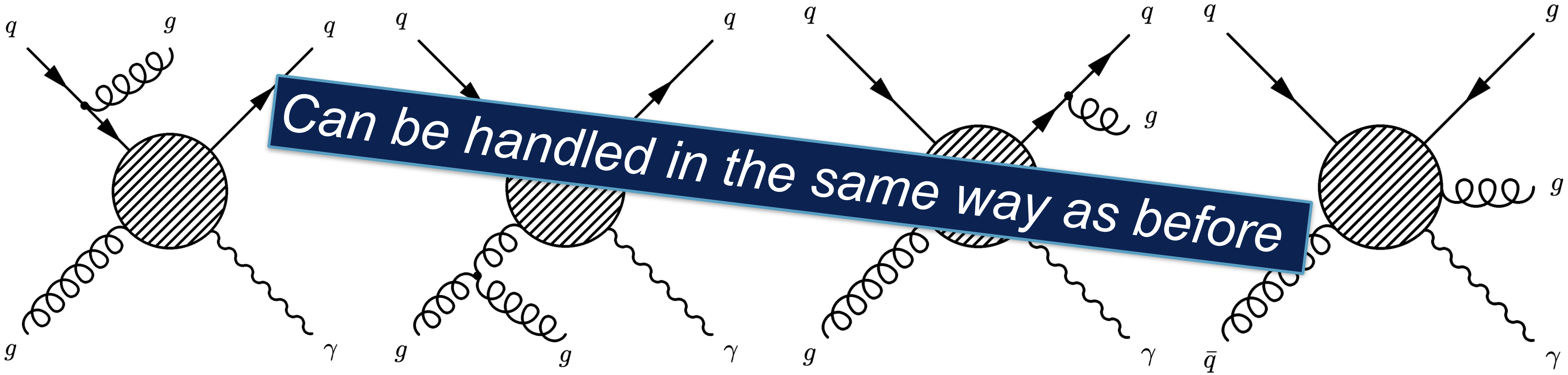

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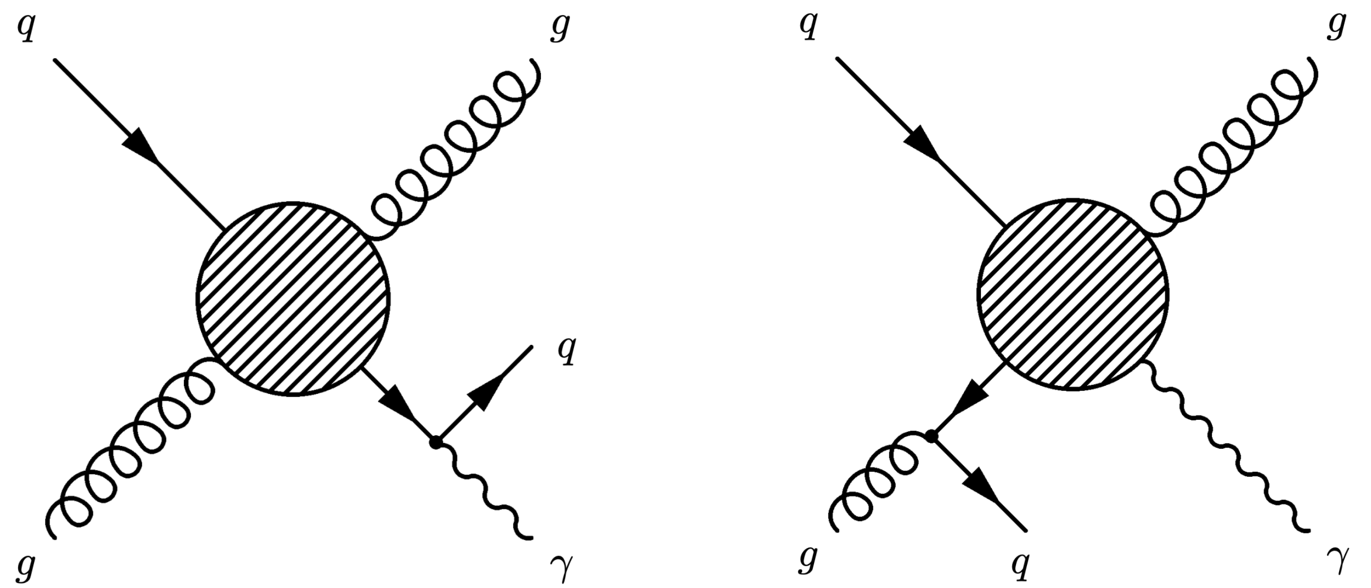
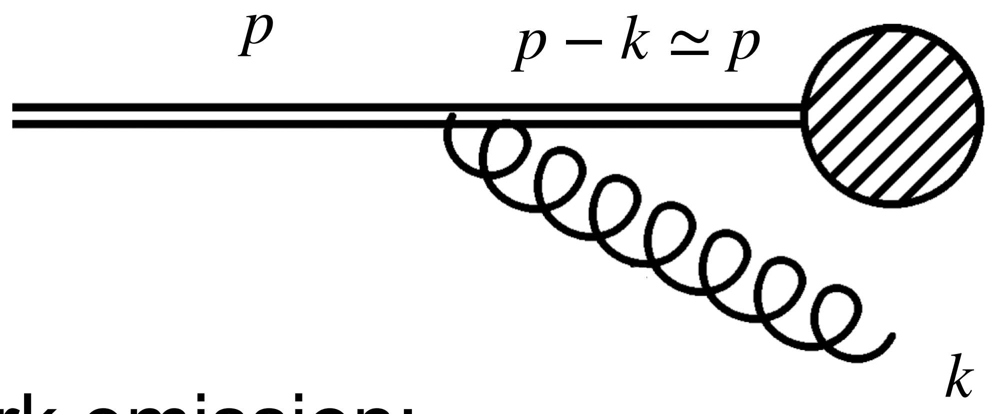


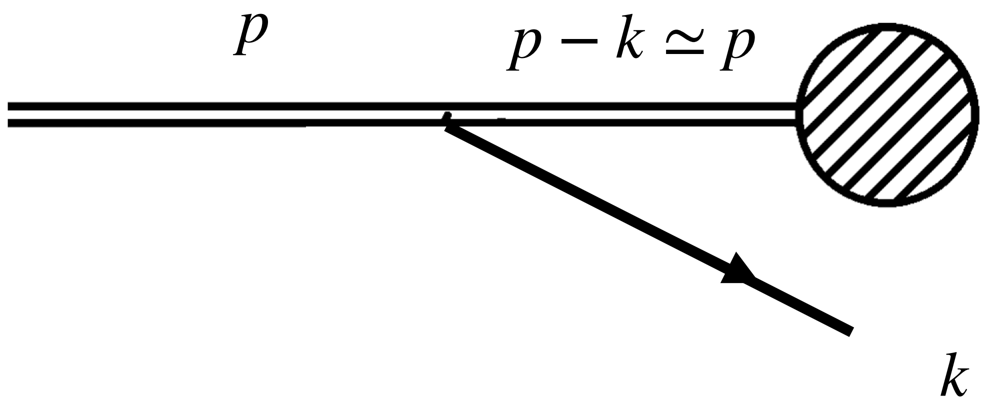
Why only talk about gluon emission?

Soft gluon emission:



$$\propto \mathcal{O}\left(\frac{1}{k}\right)$$

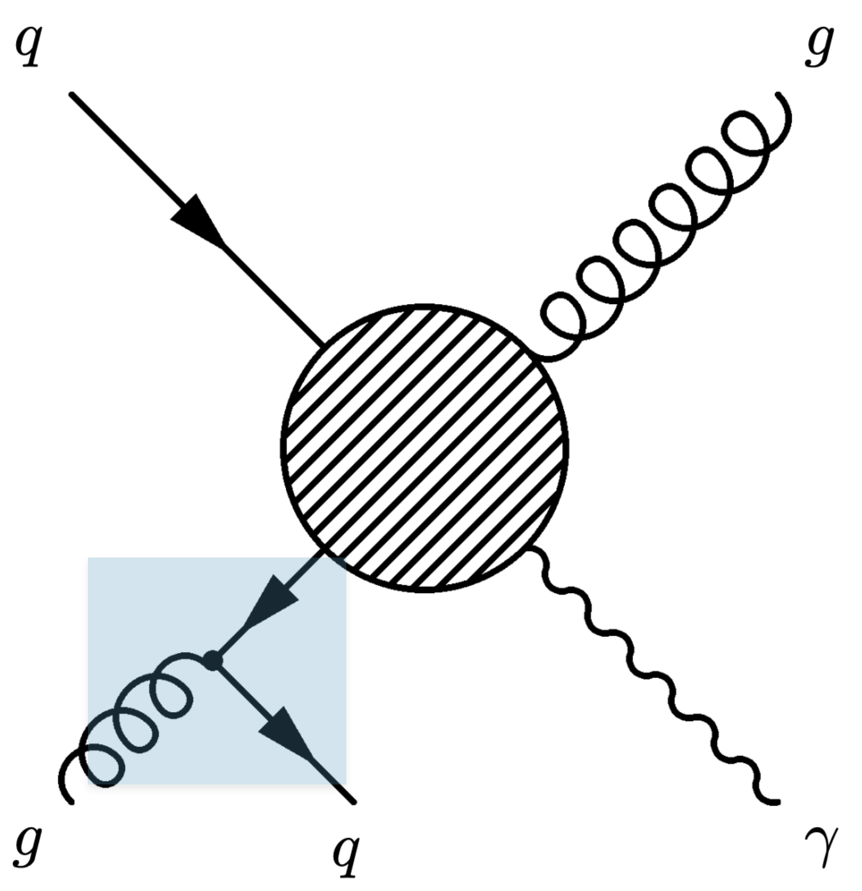
Soft quark emission:



$$\propto \mathcal{O}\left(\frac{1}{\sqrt{k}}\right)$$

How to handle the soft quark contributions?

What happens?



When q becomes soft, this creates a contribution to the NLP logs

Note:
The hard process has now changed from $qg \to q\gamma$ *to* $q\bar{q} \to g\gamma$

Similar for final state splittings…

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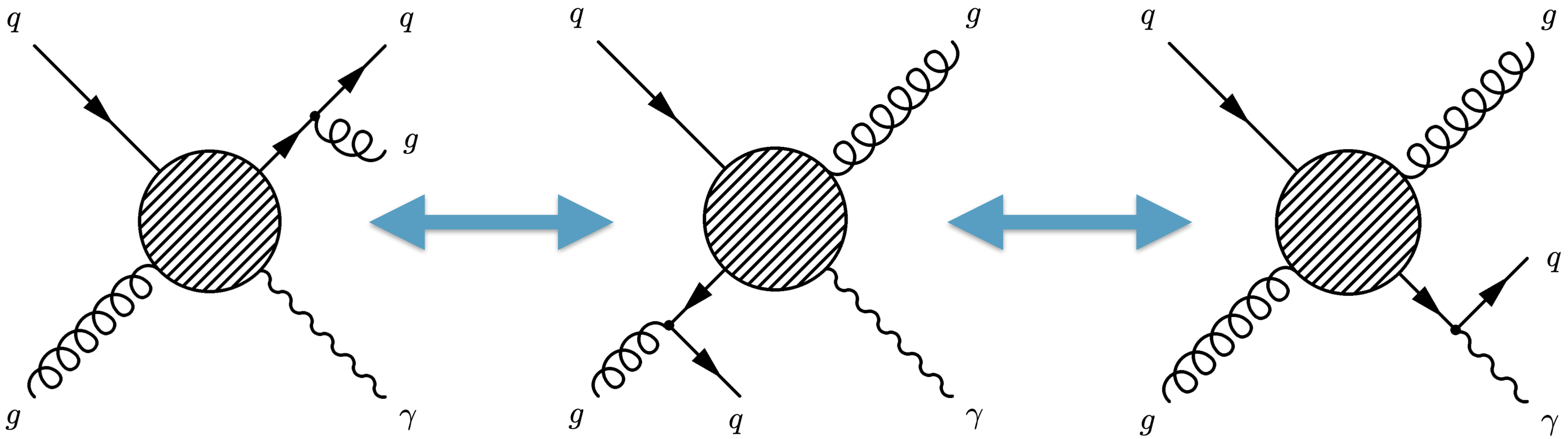
But they also interfere!



Full NLP NLO amplitude

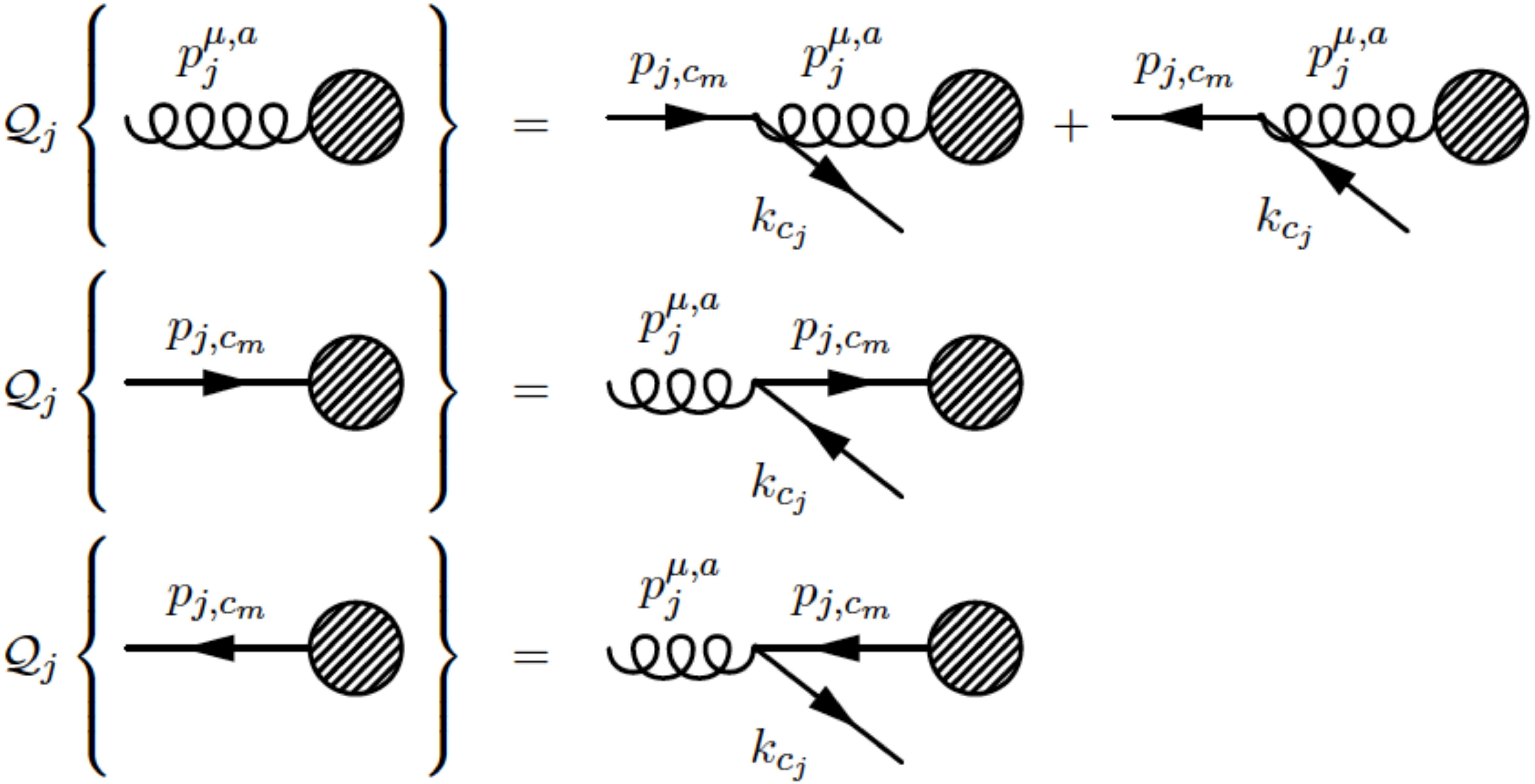
$$\mathscr{A}_{\text{NLP}} = \sum_{i=1}^{n} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\text{LO}}$$

Soft gluon contribution

$$+ \sum_{i=1}^{m} \mathbf{T}_i \frac{1}{2p_i \cdot k} \mathscr{Q}_i \otimes \mathscr{M}_{i,\text{LO}}$$

Soft quark contribution

Quark emission operator



LL terms at LP and NLP

- By combining the soft quark and gluon amplitude, all LP + NLP LL terms are obtained
- All 7 NLO prompt photon channels [Gordon, Vogelsang, 1993] can be correctly described up to LL NLP
- Also works for DIS and e+e- to jets

Soft gluon contribution

$$\mathscr{A}_{\mathrm{NLP}} = \sum_{i=1}^{n} \mathbf{T}_i \left(\frac{2p_i^\sigma - k^\sigma}{2p_i \cdot k} - \frac{ik^\alpha}{p_i \cdot k} \Sigma_i^{\sigma\alpha} - \frac{ik^\alpha}{p_i \cdot k} L_i^{\sigma\alpha} \right) \otimes \mathscr{M}_{\mathrm{LO}}$$

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- By combining the soft quark and gluon amplitude, all LP + NLP LL terms are obtained

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Soft quarks and gluons generate all NLP LL contributions at NLO

Open questions:
1. How does this extend to higher orders?
2. What happens at NLP NLL, in particular with final state non-soft contributions?

3. NLP LL resummation for colour-singlet processes

LP resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

Partonic cross section at LP:

$$\sigma = \frac{1}{2s}\left[\int \mathrm{d}\Phi_{\mathrm{LP}} \, |\mathscr{M}|^2_{\mathrm{LP}} + \ldots\right]$$

LP resummation for colour-singlet processes

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LP matrix element for DY and Higgs at LL is governed by *soft emissions* only, which can be factorized from the hard scattering

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★ Eikonal diagrams exponentiate before phase space integration (Gatheral ’83, Frenkel and Taylor ’84)

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★ Phase space for n soft-gluon radiations factorises (e.g. Forte and Ridolfi, 2002)

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Therefore, the *eikonal* cross section at LP has an exponentiated form!

$$\sigma \propto \sigma_{\mathrm{hard}}(z, s) \times \sigma^{\mathrm{eik}}(z) \quad \text{with} \quad \sigma^{\mathrm{eik}} \propto \exp\left[S_{\mathrm{LP}}(z)\right]$$

LP resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

$$\sigma^{\text{eik}} \propto \exp\left[S_{\text{LP}}(z)\right] \quad \text{with} \quad \sigma \propto \sigma_{\text{hard}}(z, s) \times \sigma^{\text{eik}}(z)$$

To separate kinematics of soft function from the hard part: go to Mellin space

$$\int_0^1 \mathrm{d}z\, f(z)\, z^{N-1}$$

Threshold limit $z \to 1$ 'selected' for $N \to \infty$

LP resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

$$\sigma^{\mathrm{res,LP}} = \sigma_{\mathrm{hard}} \exp\left[\int_0^1 \mathrm{d}z^{N-1} S_{\mathrm{LP}}(z)\right]$$

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$$= \sigma_{\mathrm{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1} \int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\mathrm{LP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1} \frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

LP resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

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Soft-collinear contributions (splitting functions)

LP resummation for colour-singlet processes

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

$$\sigma^{\mathrm{res,LP}} = \sigma_{\mathrm{hard}} \exp\left[\int_0^1 \mathrm{d}z^{N-1} S_{\mathrm{LP}}(z)\right]$$

$$= \sigma_{\mathrm{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1} \int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\mathrm{LP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1} \frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

wide-angle contributions

NLP resummation for colour-singlet processes

[1905.13710]

Consider $p_1 + p_2 \to Q + k_1 + \ldots + k_n$

Partonic cross section at NLP:

$$\sigma = \frac{1}{2s}\left[\int d\Phi_{\mathrm{LP}} |\mathscr{M}|^2_{\mathrm{LP}} + \int d\Phi_{\mathrm{LP}} |\mathscr{M}|^2_{\mathrm{NLP}} + \int d\Phi_{\mathrm{NLP}} |\mathscr{M}|^2_{\mathrm{LP}} + \ldots\right]$$

NLP resummation for colour-singlet processes

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NLL only!

NLP resummation for colour-singlet processes

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Partonic cross section at NLP:

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LL

This contains only next-to-soft corrections at LL,
non-soft NLP enhancements are NLP NLL (and beyond)

[1410.6406, 1807.09246]

NLP resummation for colour-singlet processes

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This contains only next-to-soft corrections at LL,
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Factorised ('external') next-to-soft-gluon emissions exponentiate [0811.2067,1010.1860]

NLP resummation for colour-singlet processes

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Non-factorisable (‘internal’) emissions are linked by a shift in kinematics: $\int d\Phi_{\mathrm{LP}}\,|\mathscr{M}|^2_{\mathrm{LP+NLP}} = zK_{\mathrm{LP}}\,\sigma_{\mathrm{LO}}(Q^2)$

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NLP resummation for colour-singlet processes

[1905.13710]

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Partonic cross section at NLP:

$$\sigma = \frac{1}{2s}\left[\int d\Phi_{\rm LP}\,|\mathscr{M}|^2_{\rm LP} + \int d\Phi_{\rm LP}\,\boxed{|\mathscr{M}|^2_{\rm NLP}} + \cancel{\int d\Phi_{\rm NLP}\,|\mathscr{M}|^2_{\rm LP}} + \ldots\right]$$

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Non-factorisable (‘internal’) emissions are linked by a shift in kinematics: $\int d\Phi_{\rm LP}\,|\mathscr{M}|^2_{\rm LP+NLP} = zK_{\rm LP}\,\sigma_{\rm LO}(Q^2)$

$$\boxed{\sigma^{\rm res,NLP} = \sigma_{\rm hard}\exp\left[\int_0^1 dz^{N-1} z\, S_{\rm LP}(z)\right]}$$

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NLP resummation for colour-singlet processes

[1905.13710]

$$\sigma^{\mathrm{res,NLP}} = \sigma_{\mathrm{hard}} \exp\left[\int_0^1 \mathrm{d}z^{N-1} z\, S_{\mathrm{LP}}(z)\right]$$

$$= \sigma_{\mathrm{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1} \int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\mathrm{NLP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1} \frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

NLP resummation for colour-singlet processes

[1905.13710]

$$\sigma^{\mathrm{res,NLP}} = \sigma_{\mathrm{hard}} \exp\left[\int_0^1 \mathrm{d}z^{N-1} z\, S_{\mathrm{LP}}(z)\right]$$

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$$P_{ii}^{\mathrm{NLP}} = \frac{\alpha_s}{2\pi} C_i \left[\left(\frac{1}{1-z}\right)_+ - 1 + \cdots\right] + \mathcal{O}(\alpha_s^2)$$

Key is that the LL LP and NLP contributions come from a pole in ϵ
that needs to be absorbed in parton distribution functions
→the NLP expansion of the splitting function generates this information

NLP resummation for colour-singlet processes

[1905.13710]

$$\sigma^{\text{res,NLP LL}} = \sigma_{\text{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1}\int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\text{NLP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1}\frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

Note that this only works at NLP LL for ‘LP-induced’ colour-singlet processes:

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NLP resummation for colour-singlet processes

[1905.13710]

$$\sigma^{\text{res,NLP LL}} = \sigma_{\text{hard}} \exp\left[2\int_0^1 \mathrm{d}z^{N-1} \int_{\mu_F^2}^{Q^2(1-z)^2} \frac{\mathrm{d}q^2}{q^2} P_{ii}^{\text{NLP}}(z, \alpha_s(q^2)) + \int_0^1 \mathrm{d}z^{N-1} \frac{1}{1-z} D(\alpha_s((1-z)^2 Q^2))\right]$$

Note that this only works at NLP LL for 'LP-induced' colour-singlet processes:

★ Beyond LL the phase space needs to be modified (leading to $Q^2(1-z)^2 \to Q^2(1-z)^2/z$)

★ What is the contribution from non-soft collinear emissions?

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NLP resummation for colour-singlet processes

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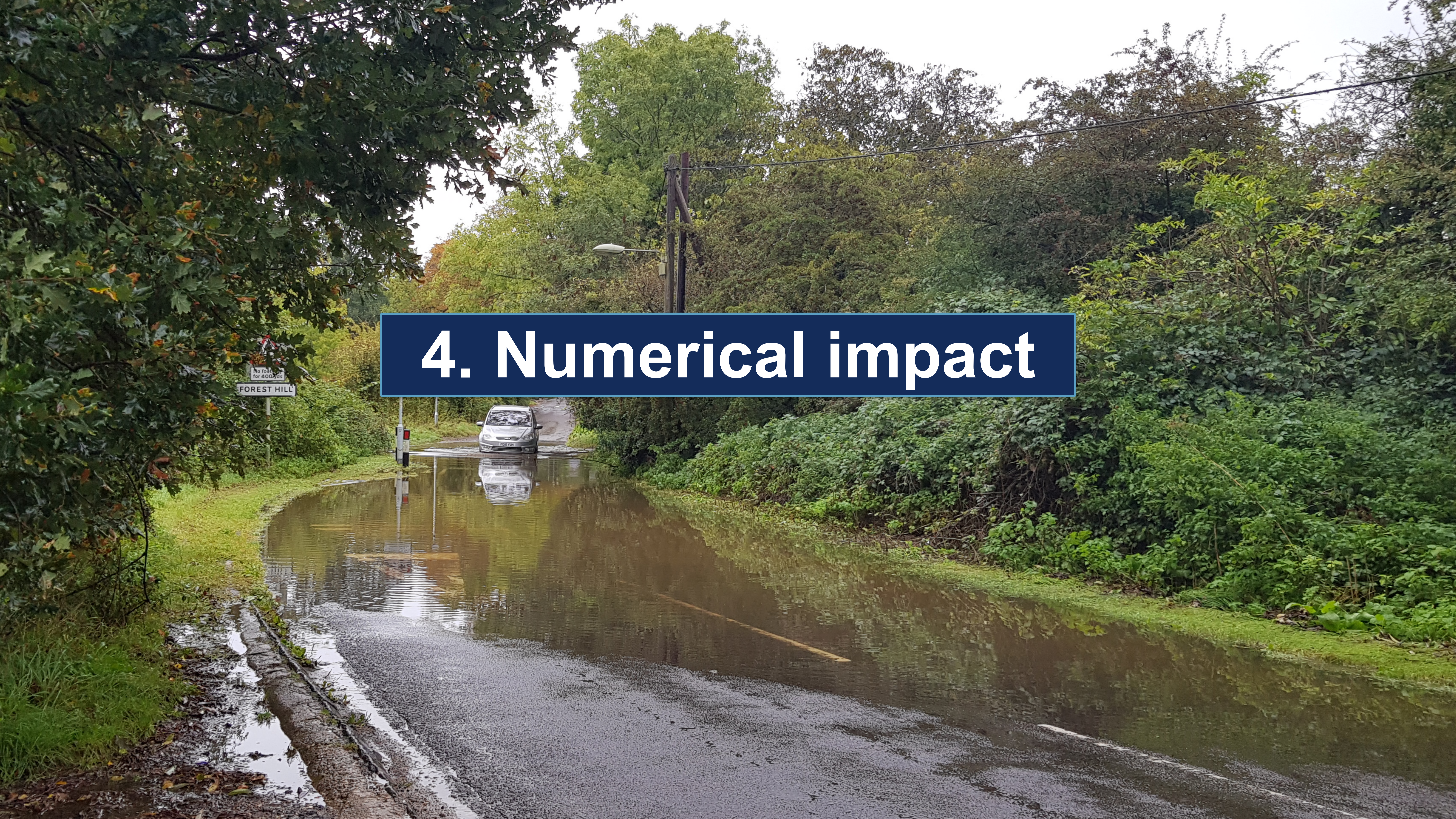
★ We saw that the kinematic shift for prompt photon is not factorisable

4. Numerical impact



Consider single Higgs and DY

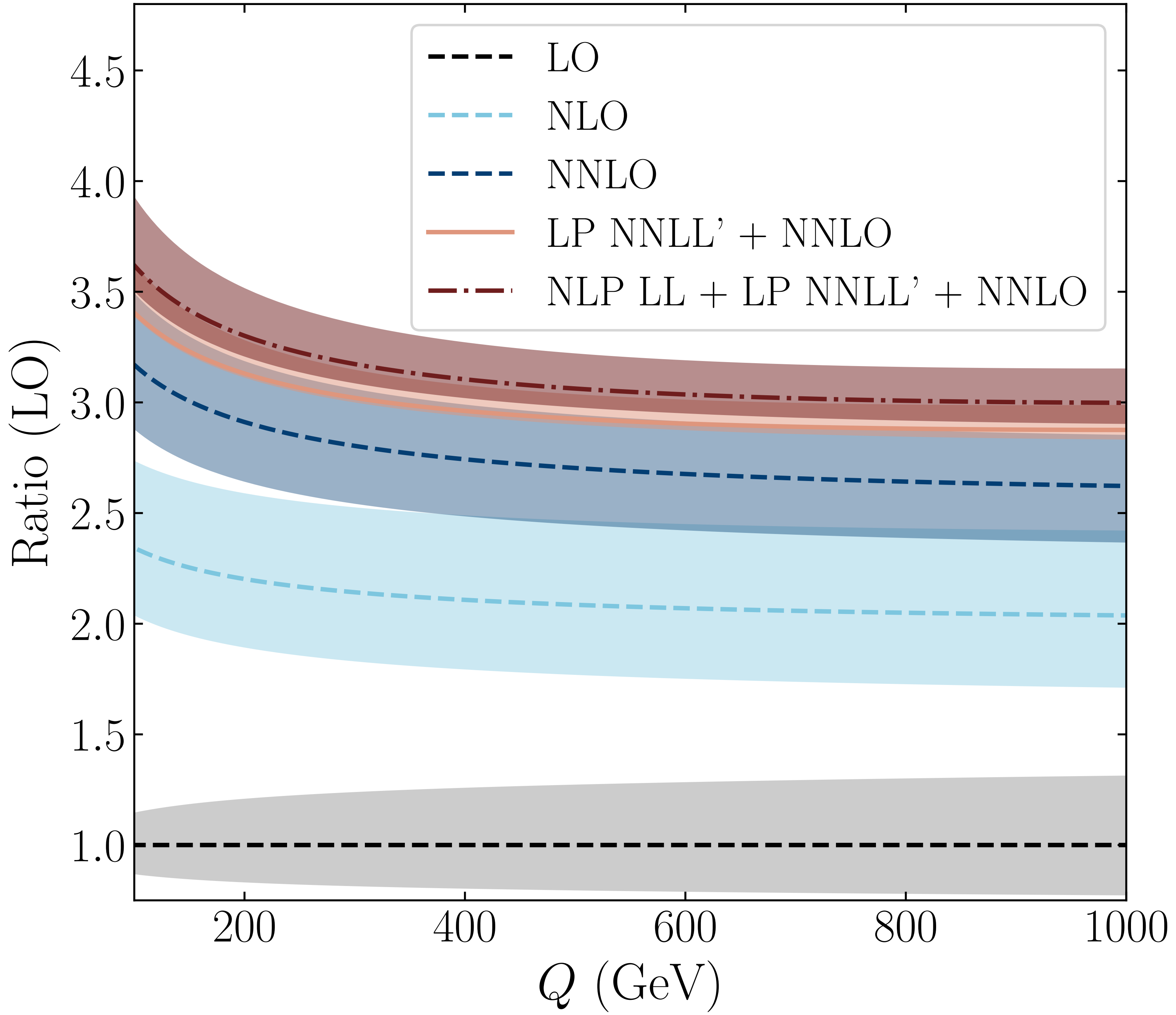
We take both processes at NNLL + NLP LL resummed and match to NNLO

Use PDF4LHC NNLO PDF set (so not resummed ones…)

Set $\mu_R = \mu_F$

Verified our set-up with the results from existing codes

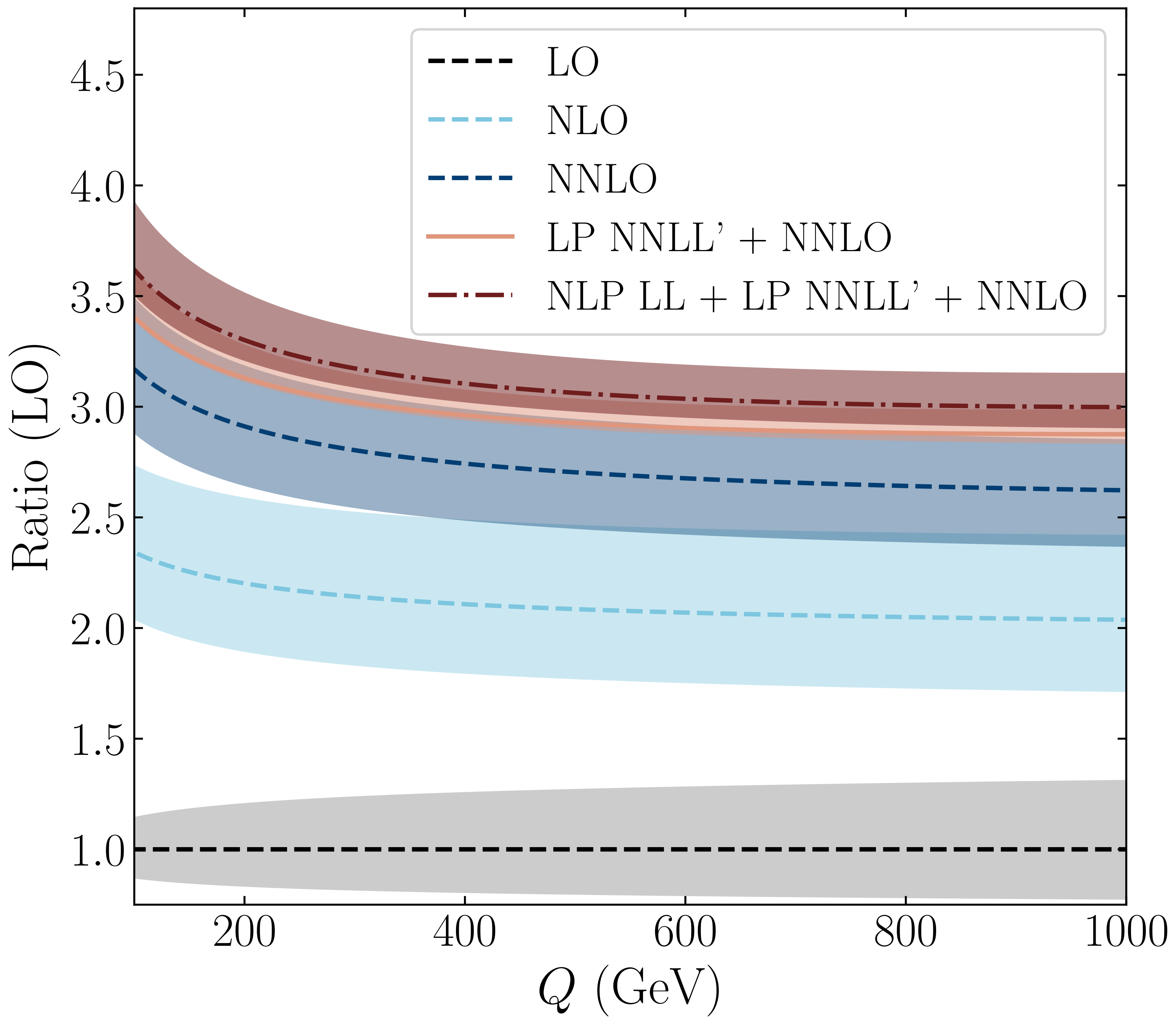
Consider single Higgs and DY

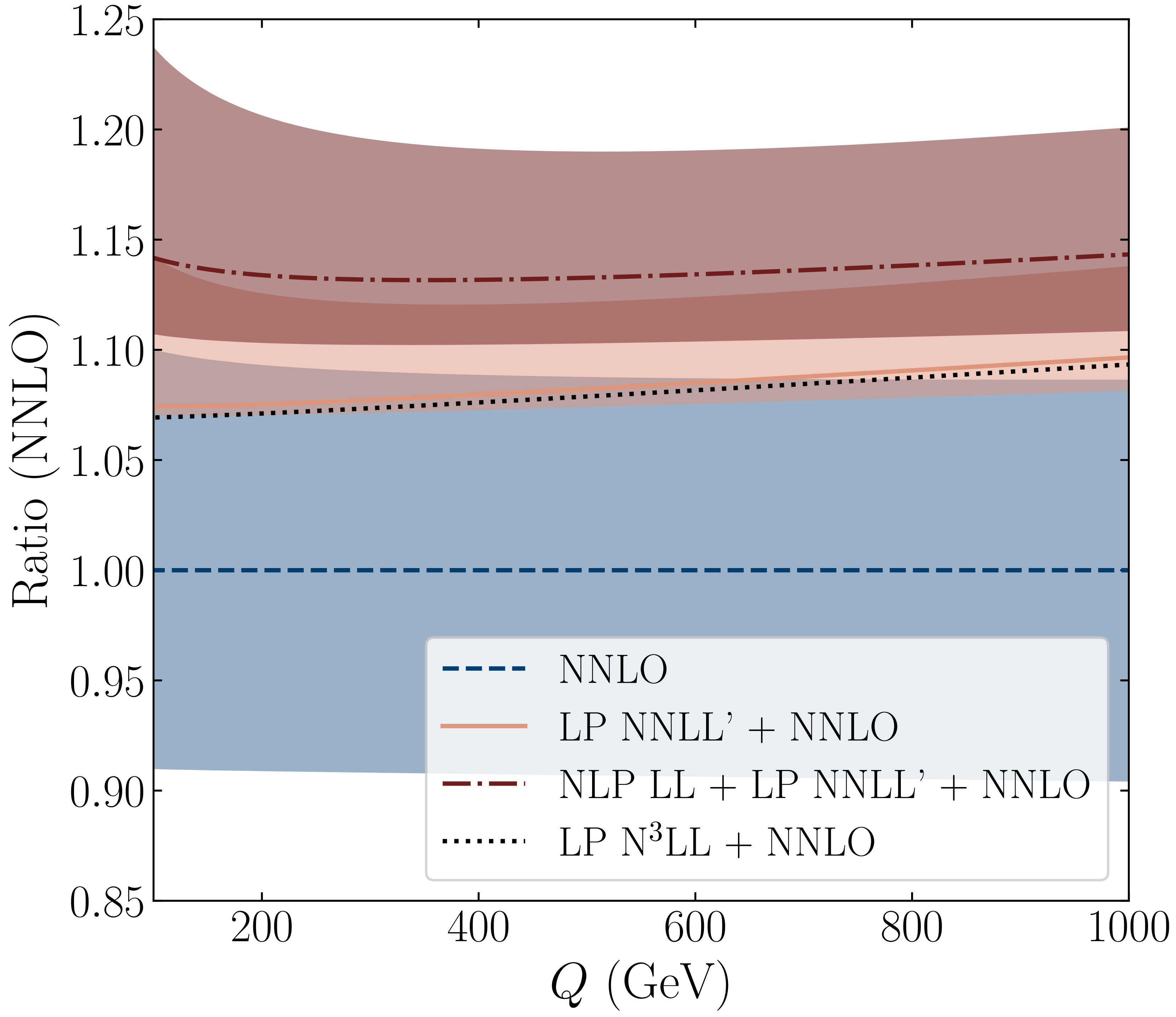


We vary $Q = m_h$

It seems that the NLP contribution is indeed subleading…

Consider single Higgs and DY





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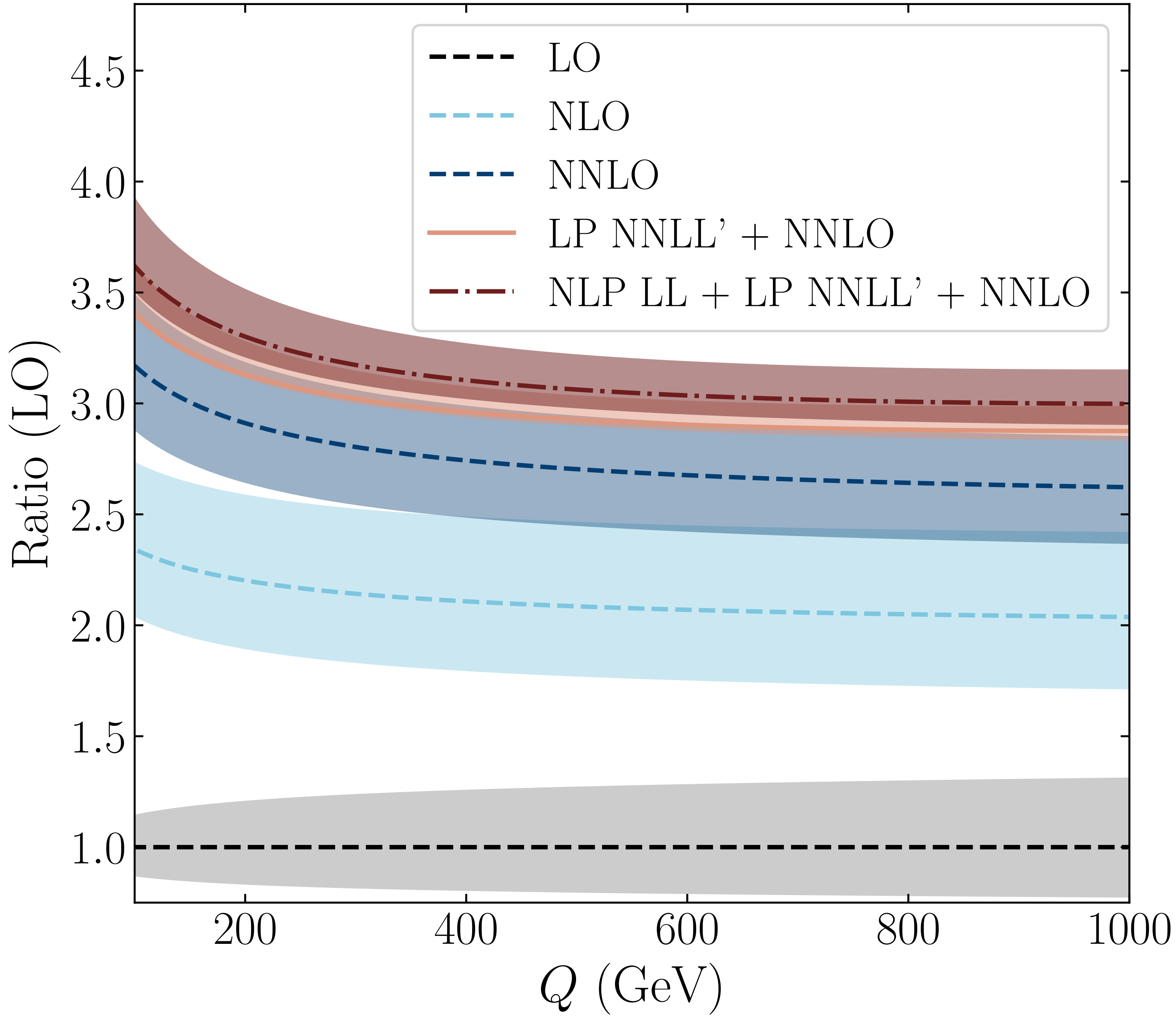
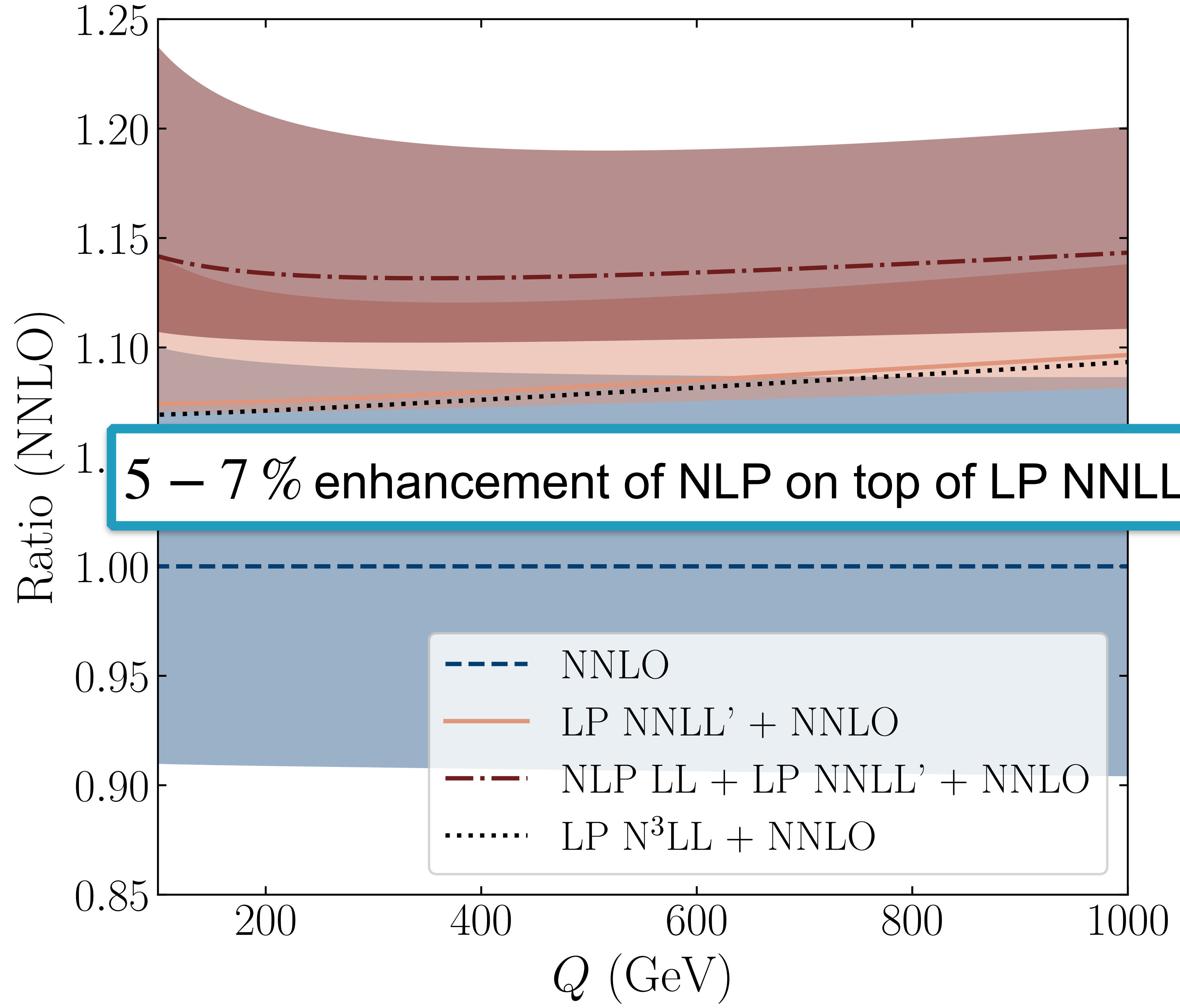
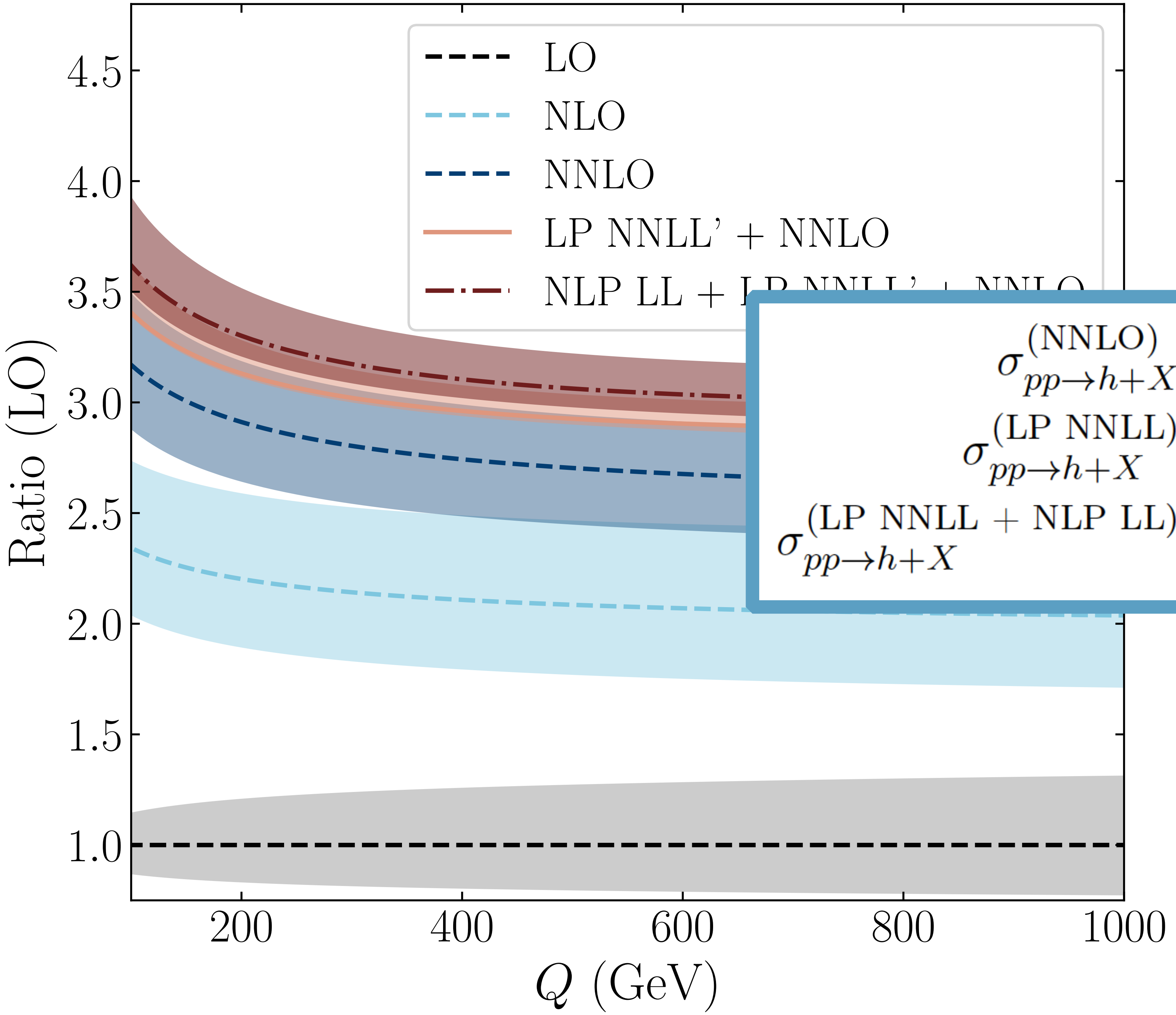
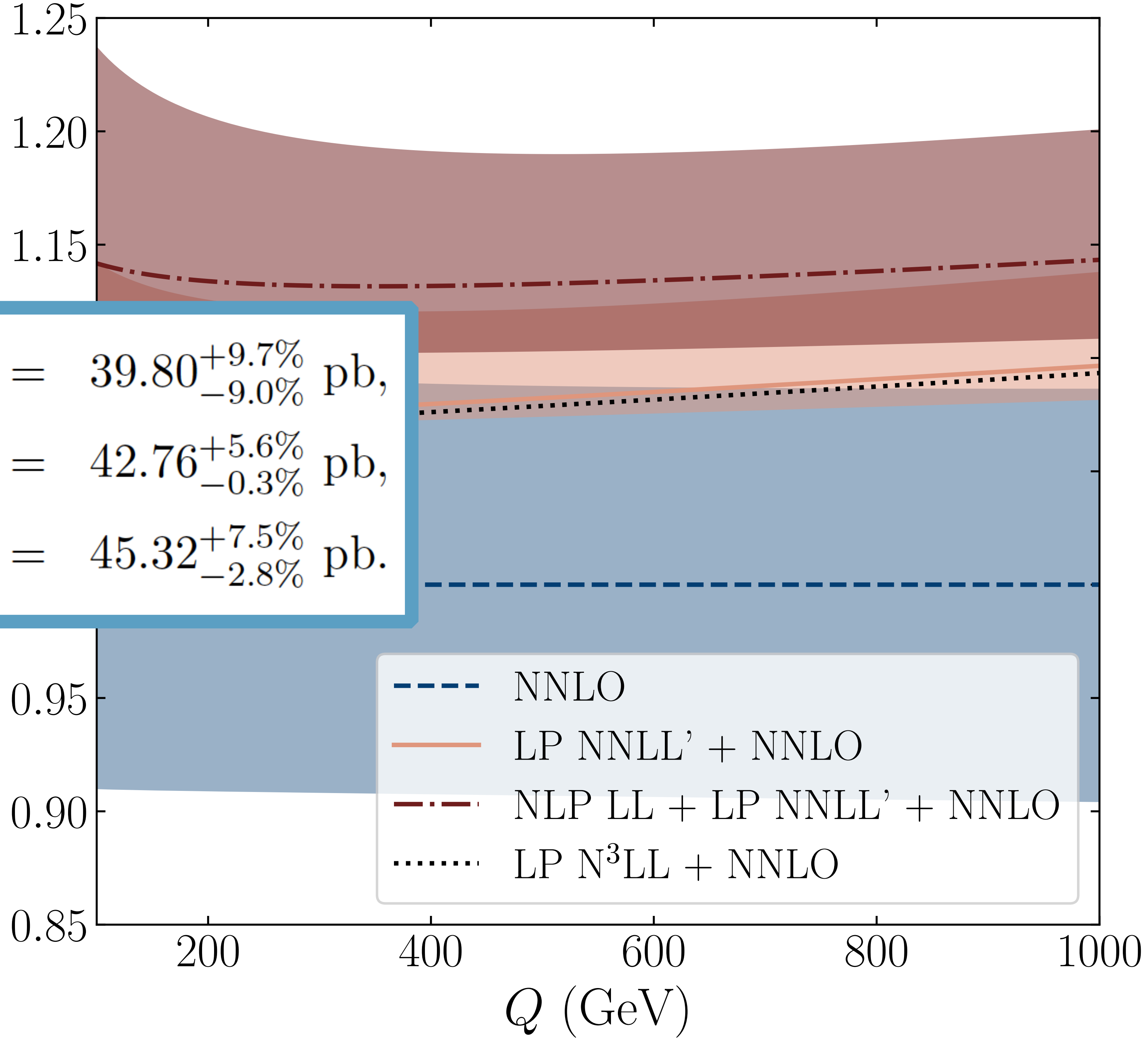




$5-7\,\%$ enhancement of NLP on top of LP NNLL

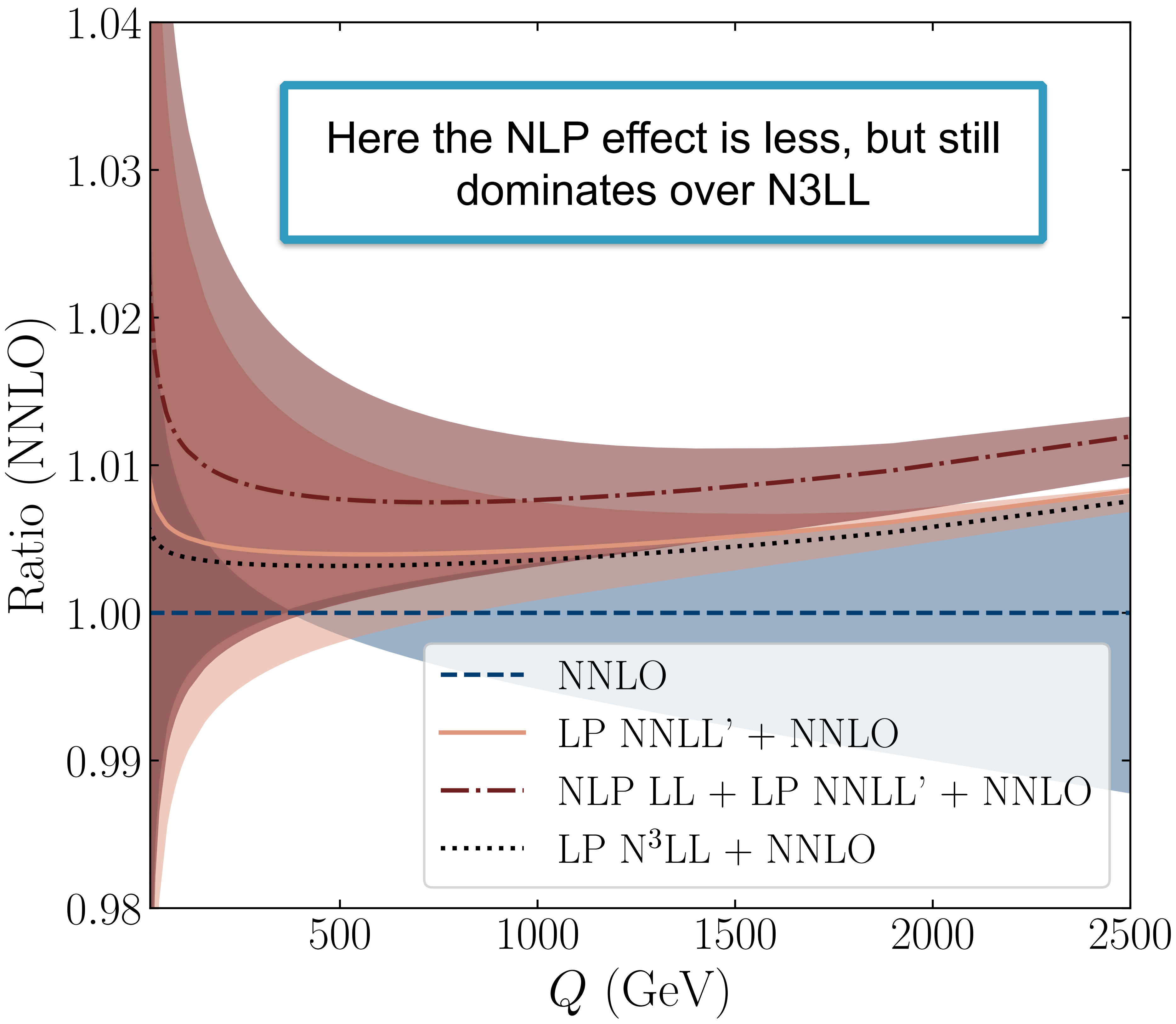
Consider single Higgs and DY





$$\sigma^{(\mathrm{NNLO})}_{pp\to h+X}(m_\mathrm{h}) = 39.80^{+9.7\%}_{-9.0\%}\ \mathrm{pb},$$
$$\sigma^{(\mathrm{LP\ NNLL})}_{pp\to h+X}(m_\mathrm{h}) = 42.76^{+5.6\%}_{-0.3\%}\ \mathrm{pb},$$
$$\sigma^{(\mathrm{LP\ NNLL\ +\ NLP\ LL})}_{pp\to h+X}(m_\mathrm{h}) = 45.32^{+7.5\%}_{-2.8\%}\ \mathrm{pb}.$$

Consider single Higgs and DY



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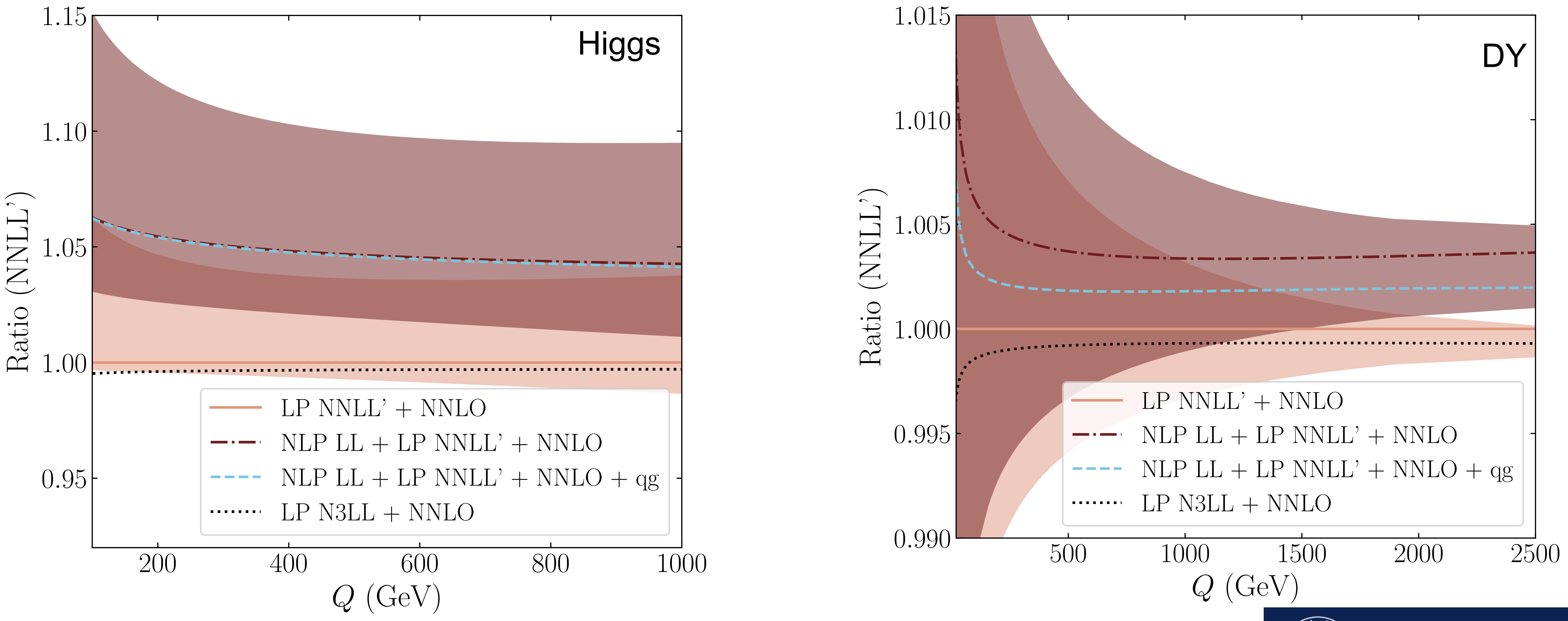

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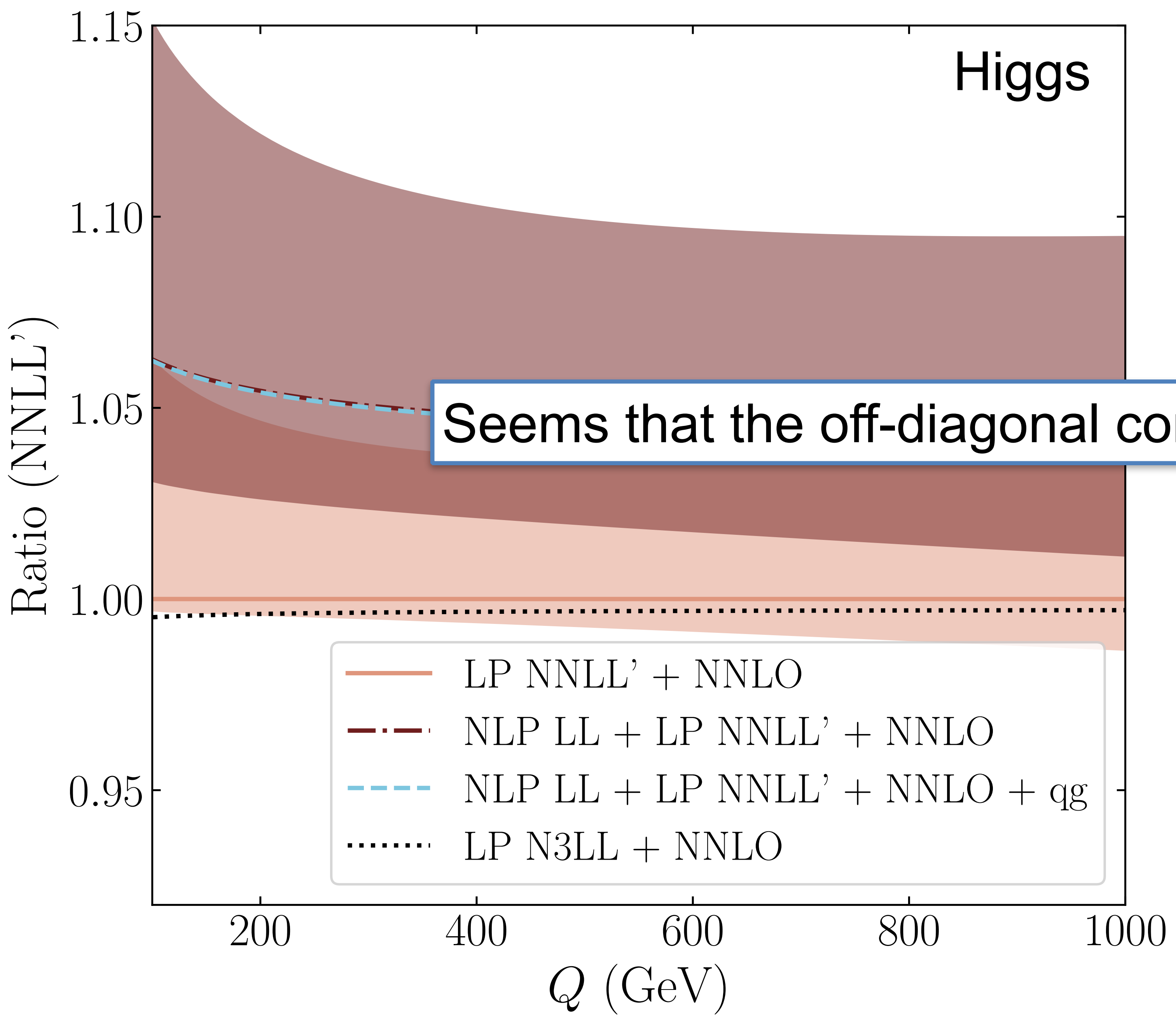

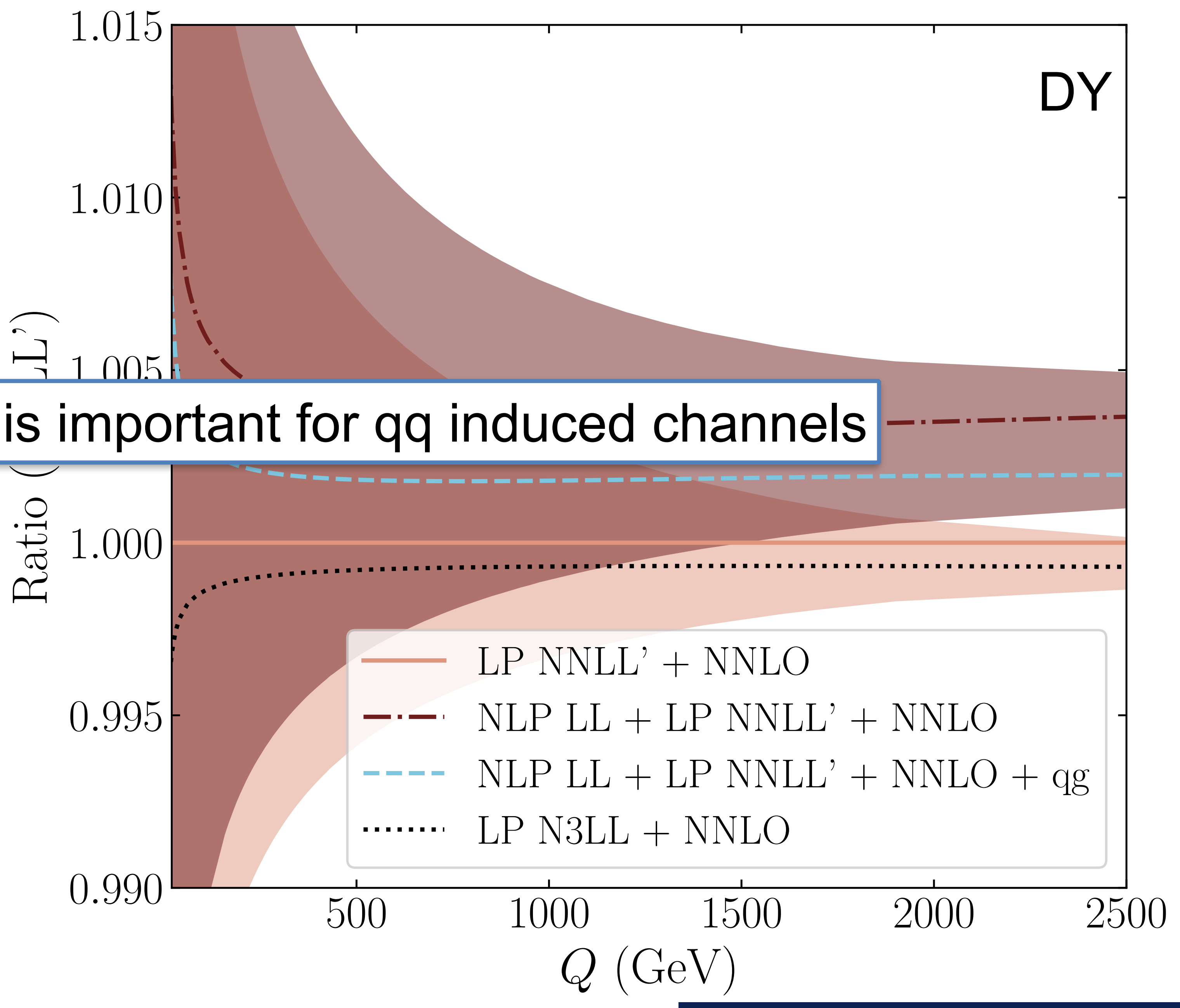

Seems that the off-diagonal contribution is important for qq induced channels

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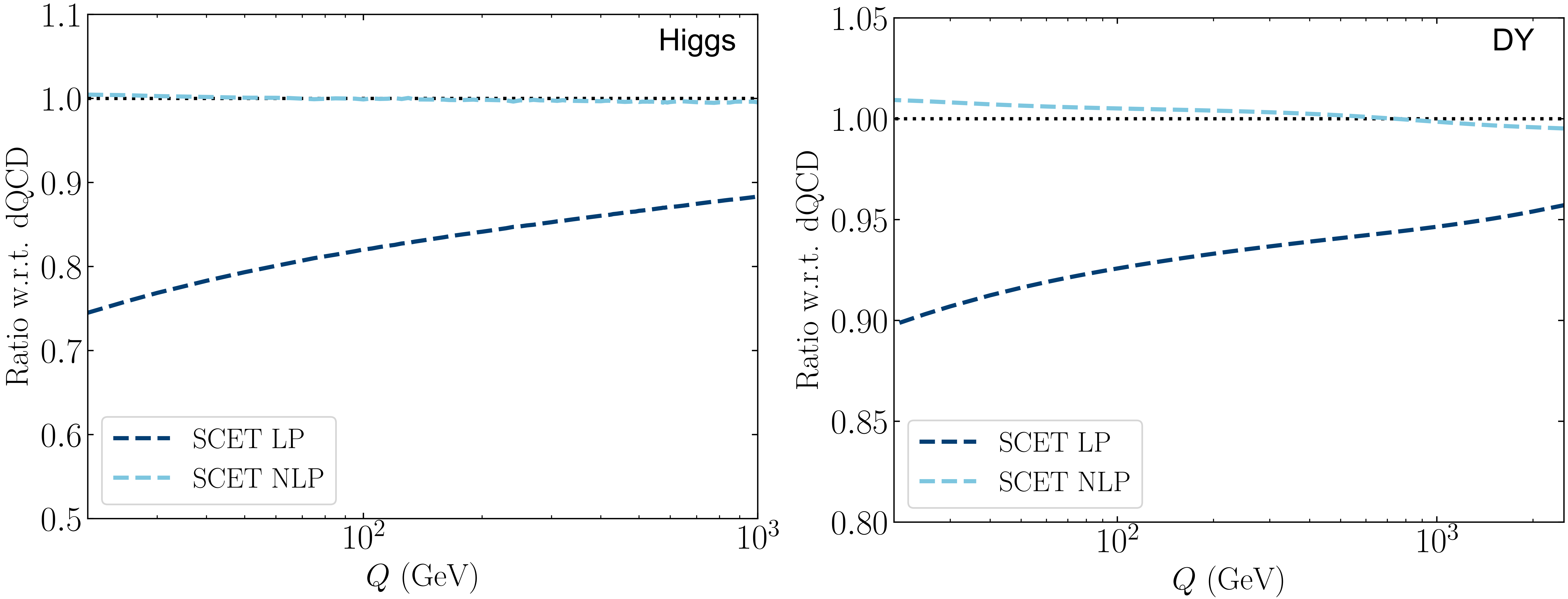
SCET vs dQCD at NLP



Conclusions

– NLP amplitude for soft gluons is universal and creates a shift to the Born matrix element

– This leads to NLP LL resummation for colour-singlet processes

– Numerical contribution of LL NLP terms varies for different processes, but in general it is not ‘negligible’

– Understanding soft quark emissions is of importance, especially for $q\bar{q}$-initiated processes

Relevant for NLP LL for colour-singlet: *Can we resum soft quarks?*

Relevant for NLP LL in general: *How to deal with ‘wide-angle’ NLP emissions?*

Relevant for NLP NLL: *What are ‘next-to-collinear/non-soft’ contributions?*

What about prompt photon?

Here we do not know the NLP resummation, but can we use what we have learned from the DY and Higgs cases to estimate the class of NLP contributions that arise due to next-to-soft collinear momentum configurations?

Option 1: use diagonal splitting functions at NLP
Option 2b: use the DGLAP equations with off-diagonal dependence up to LL NLP
Option 2c: use the DGLAP equations with off-diagonal dependence without approximating

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But remember: no interference effects are taken into account in this way!

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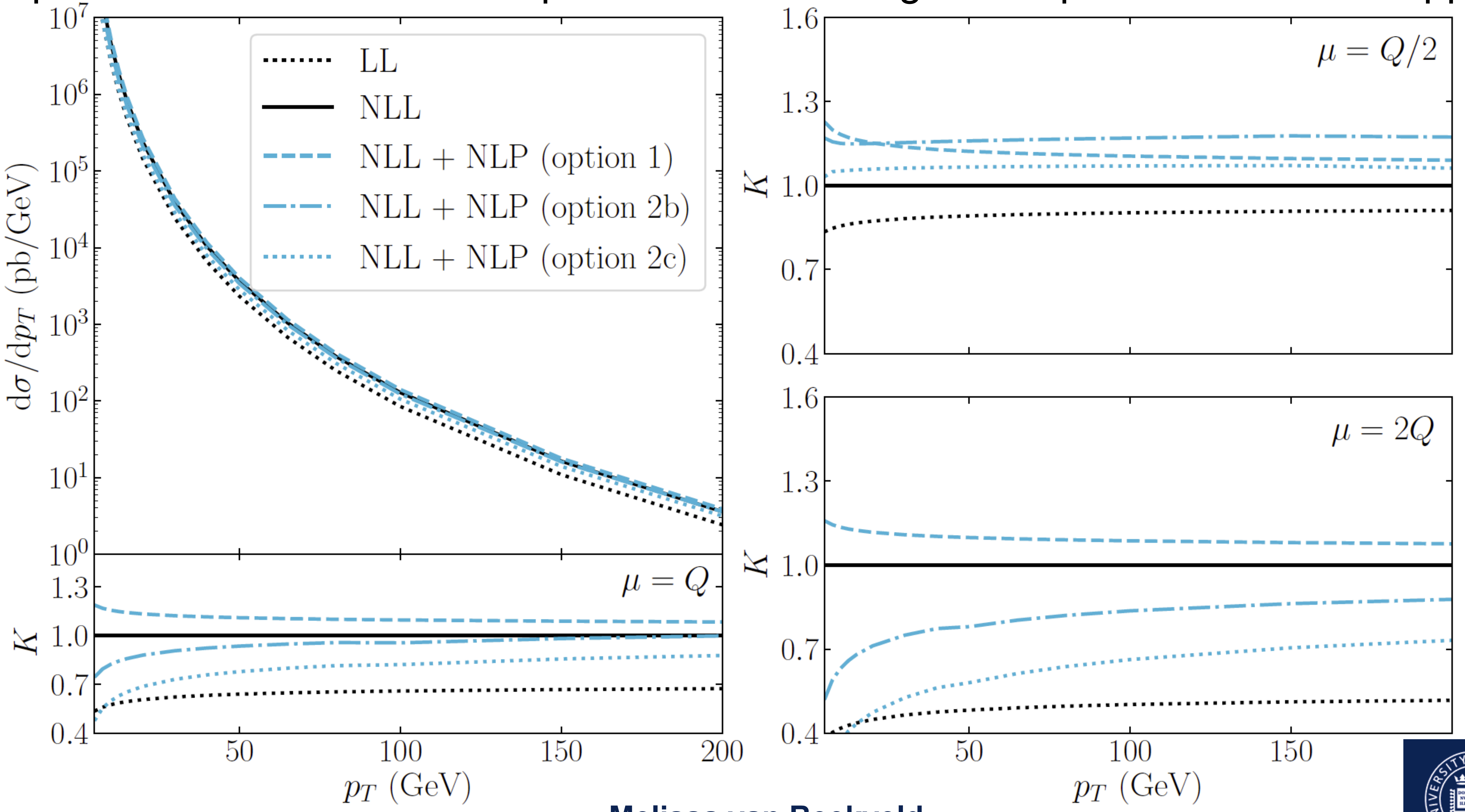

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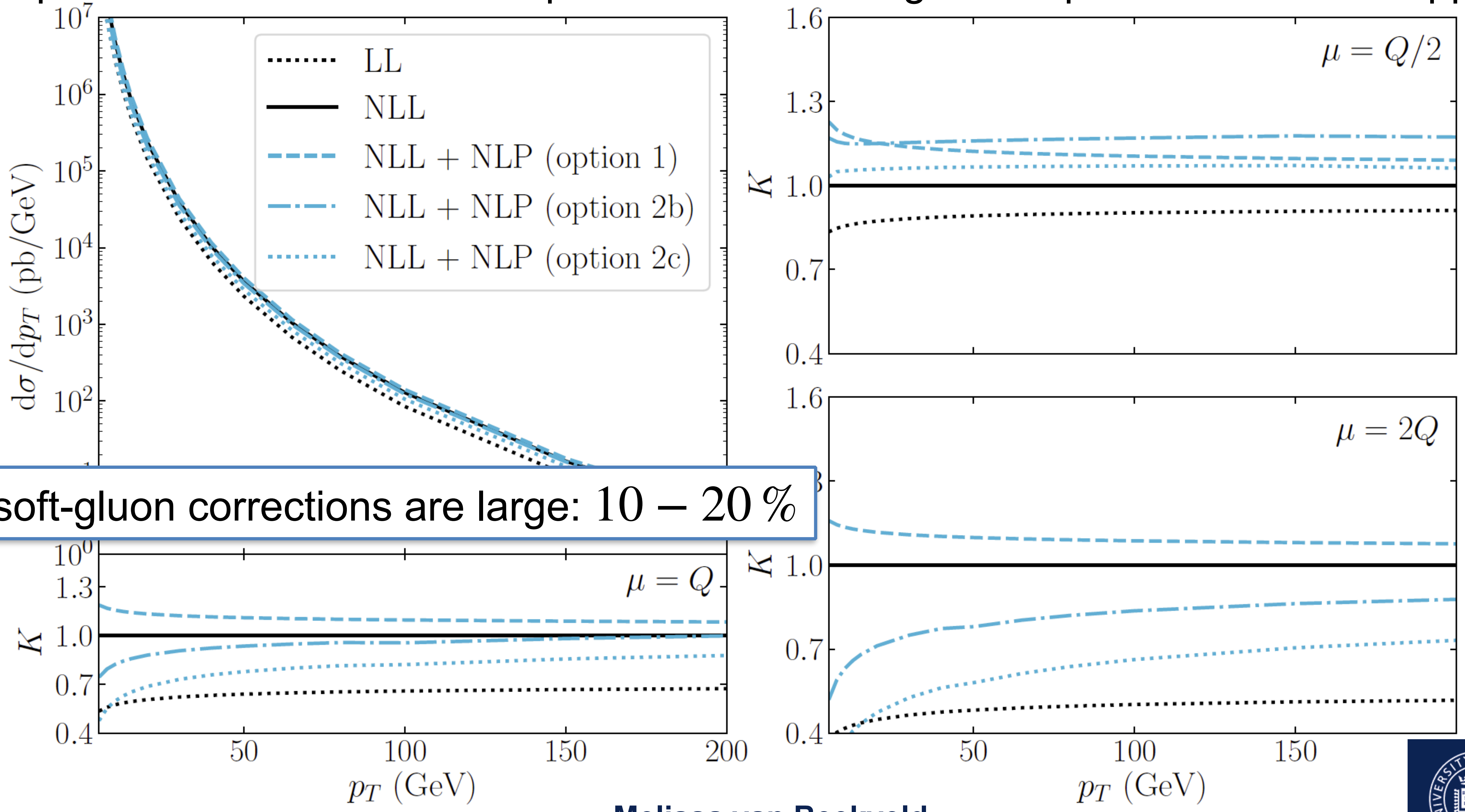

NLP LL soft-gluon corrections are large: $10 - 20\ \%$

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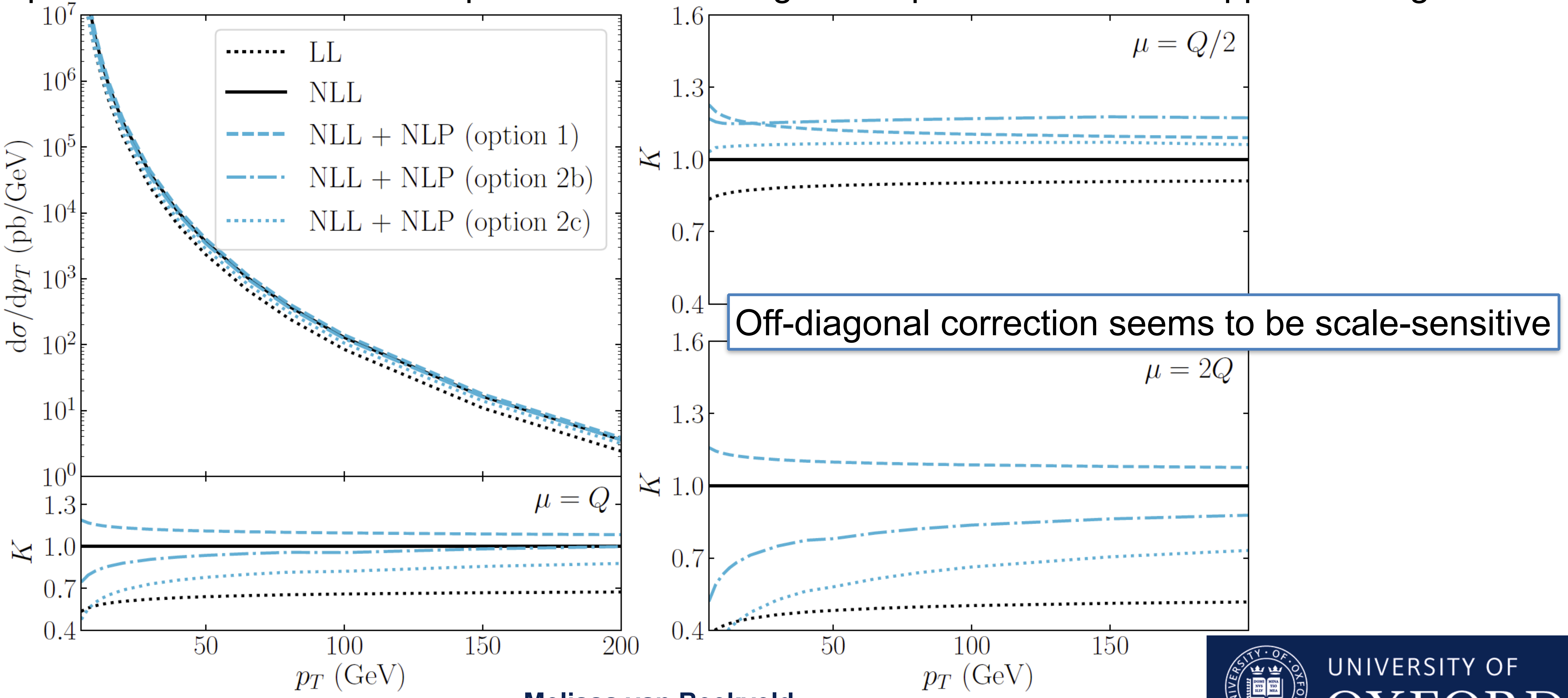

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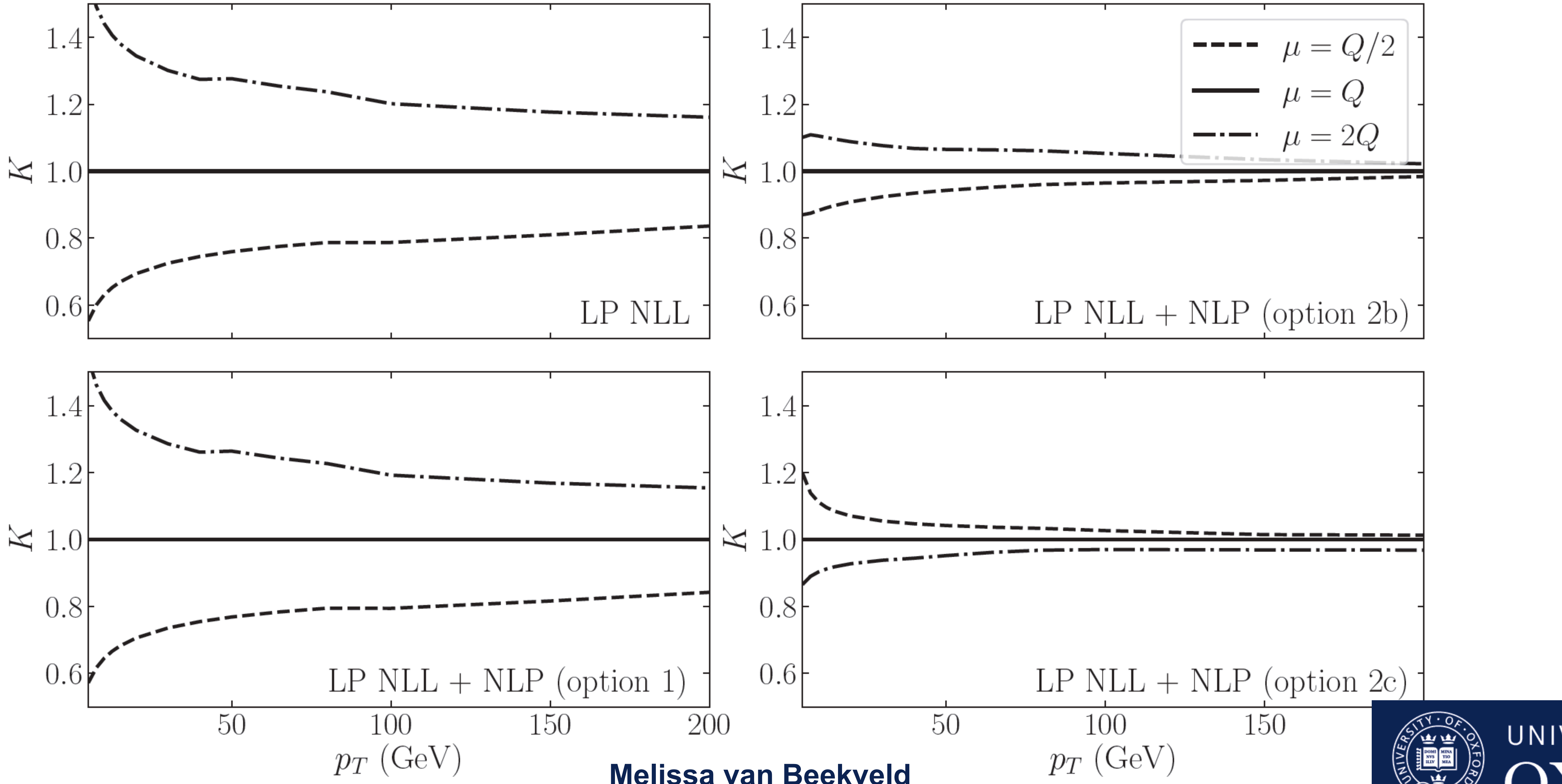

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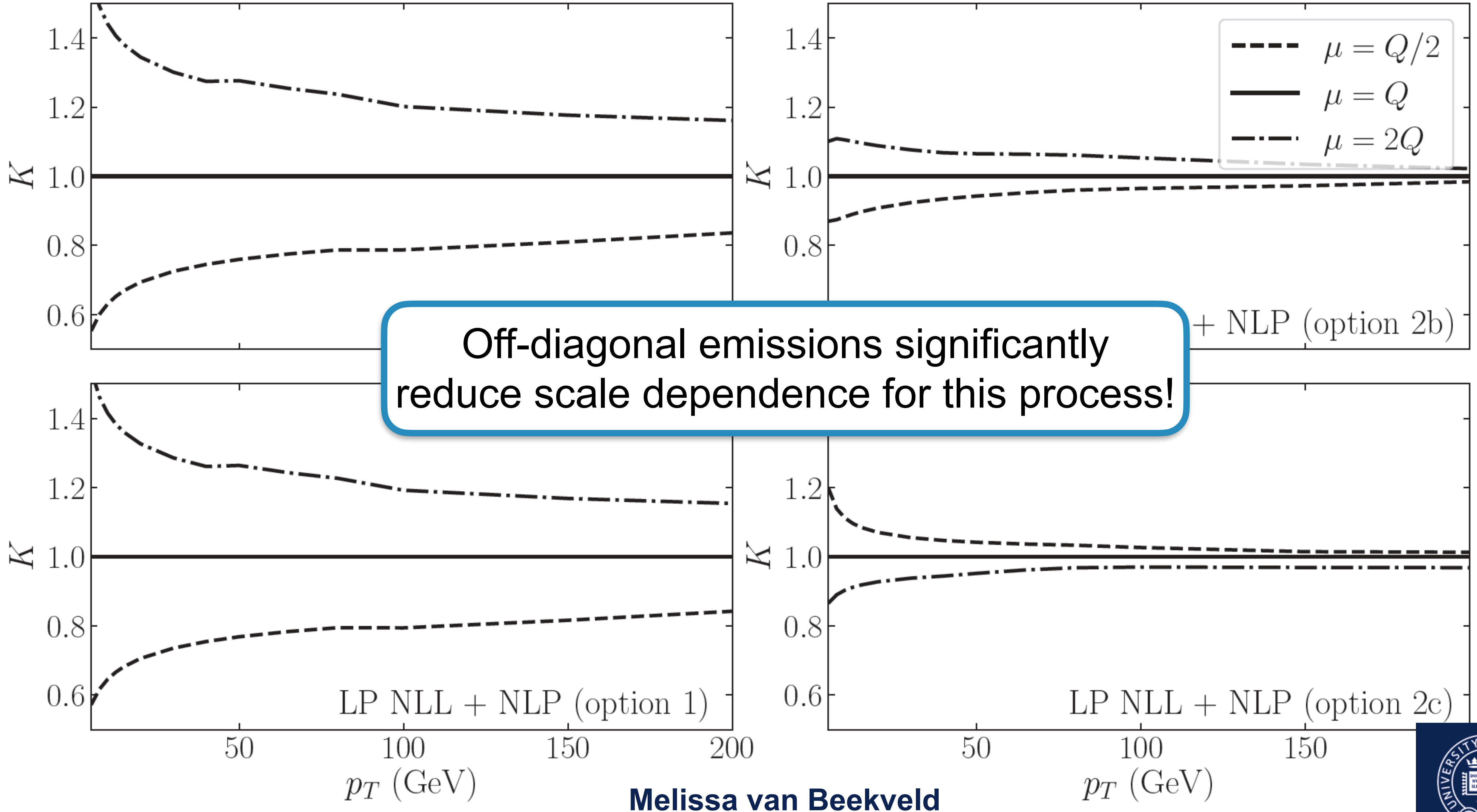

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