Projectile Motion: Surveying High-School Students understanding after Using column vector.

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Abstract

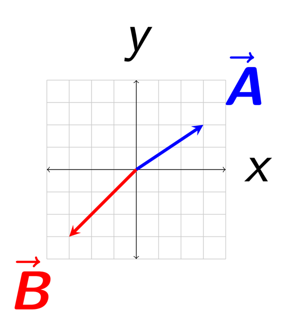
The objective of this study was to improve student's understanding and problem-solving skill for high-school students in the topic of projectile motion. This work shows a new but conventional and important representation. We used the column vector to teach grade-10 students at a school in Bangkok in academic year 2020. After finishing the topic of projectile motion, the test was given to 48 students. The post-test results revealed that the former group performed better in solving problem. In addition, students who learned with column vector realized more that the situation in the problem was two dimensions.

Introduction

- Projectile motion is a the motion in the case of constant acceleration on any 2-dimensional plan that any beginner high-school, major in science, students has to study in the first year of high-school physics subject.
- The traditional way of teaching on this topic's problem solving is considering the horizontal and vertical components of kinematics vector quanties just like displacement, instantaneous velocity and acceleration separately and try to solve a problem from one componnent to the other component respectively by using equations of motion in case of constant acceleration.
- By the way, we can use another approach to solving the projectile motion problems by using vector algebra directly instead without components separation. There are many representation in vector, matrix representation in column vector is our choice that we use to teach on this paper. Students can simultaneously consider x-component and y-component of kinematic quantities by using just a column vector for one quantities. The students can improve their problem-soving skill and using vector algebra in physics on the way that vector quantity in physics can be worked.

Column Vector Representation

Vectors are mathematical quantities defined in the mathematics of linear algebra according to the Linear Algebra text books. It can be transformed from vector diagram in cartesian coordinates and writen in matrix-form of column vector as shown.



$$\vec{A} = \begin{bmatrix} +3 \\ +2 \end{bmatrix} \qquad \vec{B} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

Figure: Column vector representation of vector diagram

We can write kinematical vector-quantities such as displacement, velocity etc. in form of column vector representation without considering each scalar component one by one separately. This form of vector is more useful for problem-solving in higher dimensions more than 1-dimension mechanics.

Projectile Problem-Solving

Procedure

1. Draw vector diagram ralate to a problem.

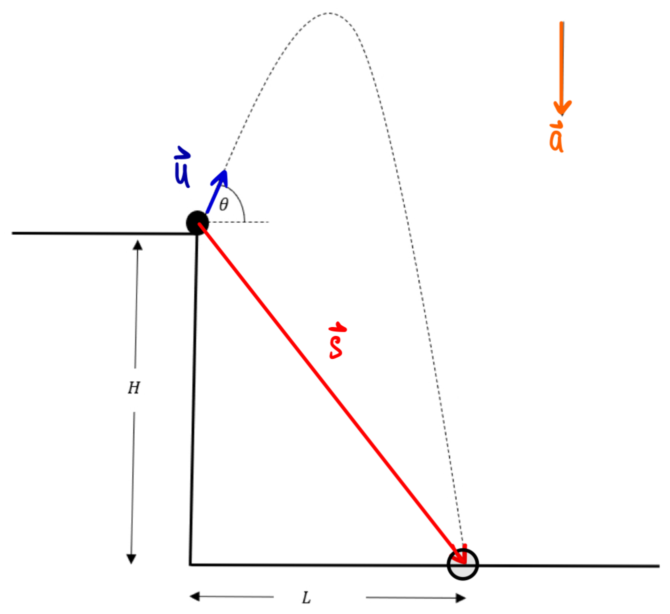


Figure: Vector diagram of a projectile problem

2. Transform all vector quantities in vector diagram into column vector form.

Quantities	Column Vector Form
Displacement	$\vec{S} = \begin{bmatrix} +L \\ -H \end{bmatrix}$
Initial velocity	$\vec{u} = \begin{bmatrix} +u\cos(\theta) \\ +u\sin(\theta) \end{bmatrix}$
Final velocity	$\vec{v} = \begin{bmatrix} +v_x \\ -v_y \end{bmatrix}$
Acceleration	$\vec{a} = \begin{bmatrix} 0 \\ -g \end{bmatrix}$
Time	t(scalar quantity)

3. Substitute all column vector into the equations of motion for constant of acceleration. For example,

$$\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\begin{bmatrix} +L \\ -H \end{bmatrix} = \begin{bmatrix} +u\cos(\theta) \\ +u\sin(\theta) \end{bmatrix} t + \frac{1}{2}\begin{bmatrix} 0 \\ -g \end{bmatrix} t^2 \qquad (1)$$

4. Using addition and scalar mutiplication properties to rearrange eq.(1). We will find that all component will be revealed in each element of column vector.

$$\begin{bmatrix} +L \\ -H \end{bmatrix} = \begin{bmatrix} u\cos(\theta)t \\ u\sin(\theta)t - \frac{1}{2}gt^2 \end{bmatrix} \qquad (2)$$

5. From eq.(2), we can solve the system of equations for a solution of a problem.

Research Methodology

- We used the column vector to teach grade-10 students at a school in Bangkok on the topic of projectile motion. By using pre-post test design and analysis, the test was given to 48 students that was splitted into 2 groups, control and treatment group. We used traditional way for teaching control group and used column vector approach for teaching another one.
- Pretest and Posttest consist of 10 items of multiple 5-choices about projectile motion calculation in different case of problems. Maximum score of the test is 10 points and minimum score is 0.
- T-test statistics, normalized gain and effective size by using Cohen's d had been used for analysed the data to distinguish the different between pretest-posttest and between outcome of different approach of teaching too.
- Students were interviewed after they had finished their posttest for additional information.

Results

T-Test Independent Mean Comparison

Group Statistics

	group	N	Mean	Std. Deviation	Std. Error Mean
pre-test	treatment	24	2.0417	.75060	.15322
	control	24	2.4167	.65386	.13347
post-test	treatment	24	8.6250	1.13492	.23166
	control	24	4.2083	.93153	.19015
normailized_gain	treatment	24	.8270	.14438	.02947
	control	24	.2322	.13370	.02729

Figure: Descriptive group statistics

	t	df	Sig.(2-tailes)	Cohen's d
Pre-test	-1.845	45.151	0.72	-.533
Post-test	14.737	44.316	<.001	4.254

Table: Mean Comparison of pre-post test between group by using T-Test, and Cohen's d effective size.

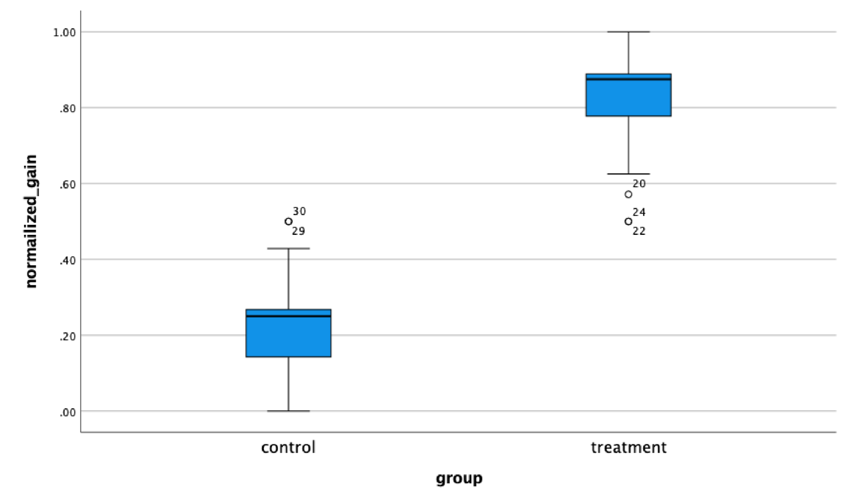


Figure: Bar chart comparison of average normalized gain

Conclusions

- There was not a significant difference in the scores of pretest of treatment group. It's mean that before teaching, all student had the same less problem-solving skill on this projectile motion topic.
- There was a significant difference in the scores of posttest of treatment group (M=8.63,SD=1.13) and control group (M=4.21,SD=0.93). condition; t(45)=14.74,p<.001. The normalized gain value of treatment group was equal to 0.83, on the high-gain level, where the other group was equal to 0.23 that was on the low-gain level. It could be interpreted that our new approach had much more effective than a traditional way.
- The Cohen's d effective size of posttest between two groups was 4.25. It might be interpreted that it had a difference outcome between two approach of teaching on this topic. By using this statistics, teaching on this topic by using column vector had slightly more effective than a traditional approach.
- In additional interview, we found that students in treatment group realized more that the situation in the problem was two dimensional consideration simultaneously.

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