Cumulants of the chiral order parameter in a nonequilibrium Bjorken expansion with a QCD critical point

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Abstract. To understand experimentally obtained net-proton number cumulants in the search for the QCD critical point, we study a dynamical model based on an effective quark-meson Lagrangian with chiral symmetry. We investigate the evolution of the expanding medium created in a heavy-ion collision using a spatially homogeneous fluid and a time-dependent order parameter, the sigma field evolved by a Langevin equation. We extract cumulants of the sigma field along a parametrized freeze-out curve and match the obtained freeze-out points to corresponding beam energies. These cumulants can be related to cumulants of the net-proton number through the sigma-proton coupling to provide a qualitative comparison to experimental data from STAR's beam energy scan program. We demonstrate that the presence of the spinodal or mixed phase region around the first-order chiral phase transition allows for a wide interval of cumulants at the lowest beam energies.

1. Introduction

Quark–gluon plasma (QGP) is a new state of matter that was present at extremely high temperatures and densities in the early universe. Contrary to a hadronic medium (protons, neutrons, pions, etc.), it is characterized by deconfinement and chiral symmetry restoration.

The study of heavy ion collisions aims to explore the phase structure of nuclear matter as a function of baryochemical potential μ_B and temperature T [1,2]. At $\mu_B = 0$, a smooth crossover has been found while for large μ_B, a discontinuous first-order phase transition is widely expected which ends at a second-order critical point (CP) [3,4].

Event-by-event fluctuations of conserved quantities such as net-baryon, net-charge, and net-strangeness are sensitive to the correlation length and connected proxy to thermodynamic susceptibilities calculated in lattice QCD [5,6] or effective model calculations. The fluctuations of conserved quantities are defined in the form of cumulants. To compare with the experimental data, ratios of baryon number susceptibilities are used to eliminate the dependence on volume and temperature of the system which are notoriously difficult to access. Higher order cumulants of conserved quantities depend directly on higher powers of the correlation length [7], and the correlation length of the nonequilibrium system depends on expansion time and is limited by the system size. It has previously been shown that the correlation length increases to about $2-3$ fm near the CP [8]. Experimental programs such as the beam-energy scan program of the Relativistic Heavy Ion Collider (RHIC) try to measure how e.g. net-proton skewness and kurtosis deviate from baseline calculations with e.g. UrQMD or a hadron resonance gas model to locate the QCD CP [9].

2. The NχFD Bjorken model

The spontaneous chiral symmetry breaking in vacuum as well as the restoration or chiral symmetry at large T or μ_B is described by the widely used quark-meson model whose Lagrangian reads

$$\mathcal{L} = \overline{q}\left(i\gamma^\mu\partial_\mu - g\sigma\right)q + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma)\ . \tag{1}$$

In QCD, the quark mass term explicitly breaks chiral symmetry which is introduced as a term $H\sigma$ in the chiral potential as follows,

$$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}\left(\sigma^2 - {f_\pi}^2\right)^2 - H\sigma\ . \tag{2}$$

Here, q represents the light quark doublet $q=(u,d)$. The grand potential given in the mean-field approximation becomes

$$\Omega(T,\mu) = U(\sigma) + \Omega_{q\bar{q}}\ . \tag{3}$$

In this equation, the quark and antiquark contribution is given by

$$\Omega_{\bar{q}q} = -2N_cN_fT\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left\{\ln\left[1+\exp\left(\frac{E-\mu}{T}\right)\right] + \ln\left[1+\exp\left(\frac{E+\mu}{T}\right)\right]\right\}\ . \tag{4}$$

This describes the negative pressure of a fermionic gas of quarks and antiquarks with quasiparticle energies $E=\sqrt{p^2+M^2}$, and effective mass of (constituent) quarks $M(\sigma)=g\sigma$ with the coupling g depending on the contributed quark mass in a vacuum. The parameters of this model are chosen as $f_\pi = 93$ MeV, $m_\pi = 138$ MeV and $H = f_\pi m_\pi^2$.

Note here that one can think of further extensions of this Lagrangian, most notably by including the Polyakov loop to account for a deconfinement transition as in [10]. Here, we focus on the criticality of the chiral phase transition which is expected to determine the singular behavior near the QCD critical point and which is captured with the quark-meson model. Although the model is admittedly crude, the addition of the Polyakov loop would shift the pseudocritical temperature at $\mu_{\mathrm{B}}=0$ away from the value obtained in lattice QCD calculations [11]. For a realistic model and equation of state including both quark and hadronic degrees of freedom, a large number of fields is needed which requires a fine-tuning of many parameters [12]. This would render any nonequilibrium simulation based on such a model unnecessary complicated and highly sensitive to the numerics.

From the Lagrangian equation (1), the nonequilibrium chiral fluid dynamics model (NχFD) is constructed by propagating the chiral order parameter σ with the Langevin equation of motion [13–15],

$$\ddot{\sigma} + \left(\frac{D}{\tau} + \eta\right)\dot{\sigma} + \frac{\delta\Omega}{\delta\sigma} = \xi\ . \tag{5}$$

Hereby, the dots refer to derivatives with respect to proper time τ, $D=1$ in the Hubble term for an expansion in one direction of the beam axis as assumed by the Bjorken model.

The damping coefficient η has been calculated as

$$\eta = \frac{12g^2}{\pi}\left[1-2n_{\mathrm{F}}\left(\frac{m_\sigma}{2}\right)\right]\frac{1}{m_\sigma^2}\left(\frac{m_\sigma^2}{4} - M^2\right)^{3/2}\ . \tag{6}$$

The white and Gaussian noise $\xi\,(t,x)$ in the above equation satisfies the fluctuation-dissipation theorem

$$\langle\xi(t)\xi(t')\rangle_\xi = \delta(t-t')\frac{m_\sigma\eta}{V}\coth\left(\frac{m_\sigma}{2T}\right) \ . \tag{7}$$

The mass of the field σ is equivalent to the curvature of the thermodynamic potential in equilibrium depending on temperature and chemical potential,

$$m_\sigma^2 = \left.\frac{\partial^2\Omega}{\partial\sigma^2}\right|_{\sigma=\langle\sigma\rangle} \ . \tag{8}$$

Under the assumption of the Bjorken model that the rapidity distribution of the charged particles is boost invariant, we obtain the hydrodynamic equations for energy density e and baryon number density n as [16]

$$\frac{\partial e}{\partial\tau} = -\frac{e+p}{\tau} + \left[\frac{\delta\Omega_{\bar{q}q}}{\delta\sigma} + \left(\frac{D}{\tau}+\eta\right)\frac{\partial\sigma}{\partial\tau}\right]\frac{\partial\sigma}{\partial\tau}, \tag{9}$$

$$\frac{dn}{d\tau} = -n/\tau. \tag{10}$$

We characterize fluctuations of the sigma field by cumulants C_n of the probability distribution defined as:

$$C_1 = \langle\sigma\rangle, \tag{11}$$

$$C_2 = \langle(\delta\sigma)^2\rangle, \tag{12}$$

$$C_3 = \langle(\delta\sigma)^3\rangle, \tag{13}$$

$$C_4 = \langle(\delta\sigma)^4\rangle - 3\langle(\delta\sigma)^2\rangle^2. \tag{14}$$

These are related to mean (M), variance (σ^2), skewness (S) and kurtosis (κ) via

$$M = C_1\ , \quad \sigma^2 = C_2\ , \quad S\sigma = \frac{C_3}{C_2}\ , \quad \kappa\sigma^2 = \frac{C_4}{C_2}\ . \tag{15}$$

We investigate the higher order cumulant ratios of the sigma field along a freeze-out curve which has been obtained from thermal model fits to experimental data [17] over a wide range of beam energies. A polynomial fit yields a parametrization,

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4\ , \tag{16}$$

where $a = 0.166$ GeV, $b = 0.139$ GeV^{-1} and $c = 0.053$ GeV^{-3}. We furthermore match the point where our Bjorken evolution hits this freeze-out curve with the corresponding beam energy via:

$$\mu_B(\sqrt{s}) = \frac{d}{1+e\sqrt{s}}\ , \tag{17}$$

where $d = 1.308$ GeV and $e = 0.273$ GeV^{-1} [18].

3. Fluctuations of the order parameter

We initialize the field and fluid in equilibrium at several starting points in the chirally restored phase, see the beginning of the trajectories in the phase diagram of the quark-meson model, figure 1. These points are conveniently characterized by a choice of initial values (T_0, μ_0). The

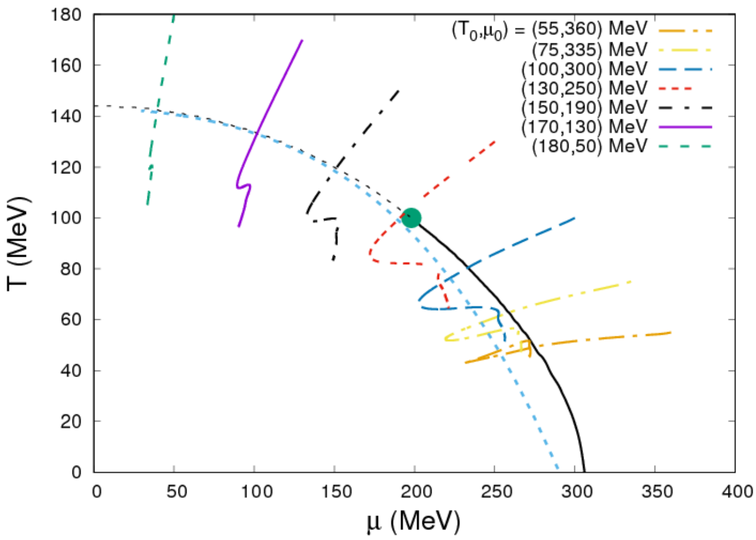

Figure 1. The phase structure of chiral phase transition in different initial conditions (T_0, μ_0) alongside trajectory of fireball.

coupled system of field and fluid is then evolved according to equations (5) and (10) which gives an evolution of T and μ that can be seen in the same figure.

The point where the trajectories hit the dashed freeze-out line is used to determine the event-by-event fluctuations. We hereby use a number of $N = 10^5$ events which are randomized through the stochastic noise term ξ that yields a non-deterministic evolution. The bending of the trajectories has been discussed in previous publications [16]. It is particularly worth noting here that, as a consequence of this behavior, the trajectories passing through the first-order phase transition cross the freeze-out curve twice. For these cases, we calculate the cumulants at both crossing points to give a range of possible values.

Figure 2 shows the so obtained σ^2, $S\sigma$ and $\kappa\sigma^2$ versus beam energy $\sqrt{s}$. We note that for all these cases, the consideration of the spinodal region and double-crossing of the freeze-out curve leads to an increased range of possible cumulant values for the lowest beam energies. The variance, which is more or less constant, decreases by a factor of 2 after passing through the second crossing, see figure 2(a). A similar effect is observed for the skewness, though somewhat less dramatic, see figure 2(b). The most dramatic impact is seen in the kurtosis. Although it increases monotonically within error bars, we see a clear sign change at beam energy $\sqrt{s} \sim 4$ GeV, see figure 2(c). As argued in [7] strong negative values of the kurtosis of the order parameter are understood as a direct consequence of a first-order phase transition.

As introduced previously, we ultimately aim to study fluctuations in the net-proton number N_p rather than the order parameter which is not experimentally accessible. However, assuming a σ-proton coupling, we can assume a relation between the fluctuations in N_p and the sigma field as

$$\delta N = \delta N^0 + V\, g\, \delta\boldsymbol{\sigma}\, d \int_{\boldsymbol{p}} \frac{\partial n_p}{\partial m} \,, \tag{18}$$

where g denotes the coupling constant, d the number of degrees of freedom, $f_{\boldsymbol{p}}$ the proton-number distribution function and m the dynamically generated mass [7].

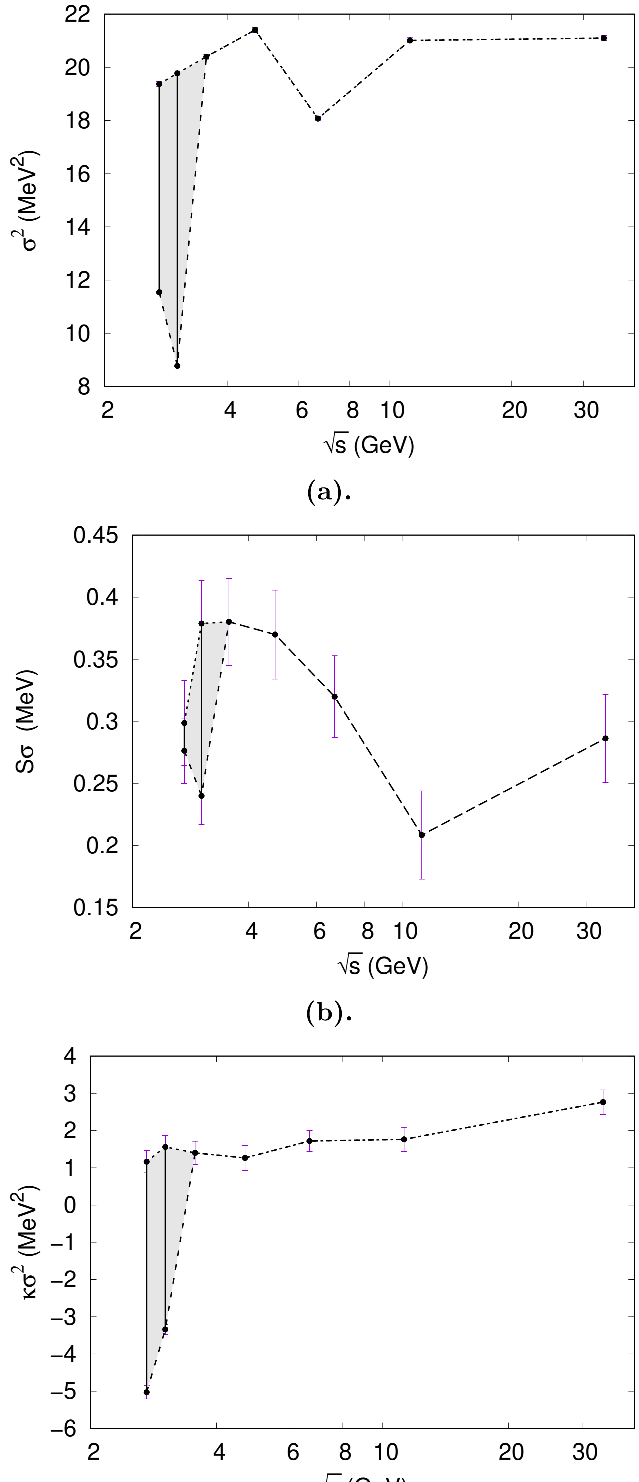

Figure 2. σ^2 (a) , Sσ (b) and $\kappa\sigma^2$ (c) of the sigma field as a function of beam energy $\sqrt{s}$.

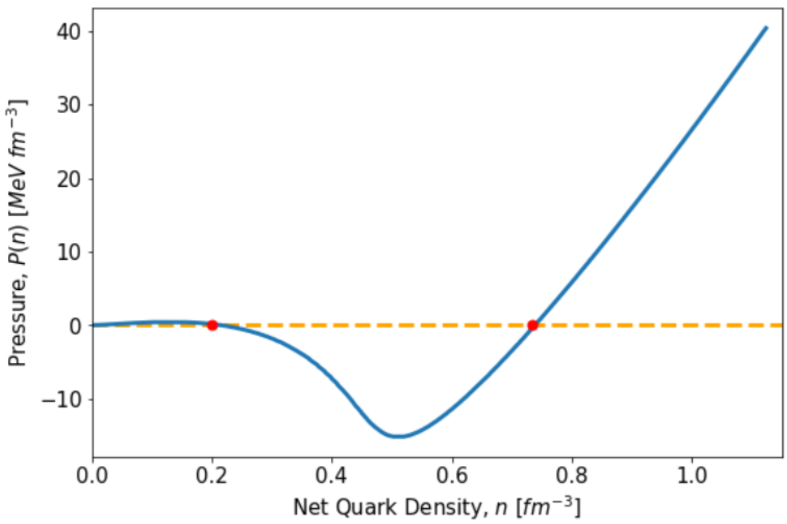

Figure 3. The equation of state at T = 0. The dashed orange line signifies the zero pressure threshold, which makes it easy to see the negative pressure portion. The red dots mark the negative pressure region in the x-axis.

For correct treatment of these particle number fluctuations, the effect of volume fluctuations needs to be considered which plays a major role especially for trajectories crossing the first-order phase transition. Work in this direction is currently in progress.

4. Equation of state at low T

The thermodynamics of strong interactions at low temperature and high density is important for heavy-ion collisions at low beam energies but also for astrophysics, specifically in understanding the inner structure of compact stars [19, 20]. Of particular importance for any hydrodynamic simulation is the equation of state (EoS), especially at ultra-low temperature ($T \approx 0$) [21] and in the presence of spinodal instabilities. As shown in [22], chiral models often suffer from the problem of negative pressures at low T which allows for the unphysical phenomenon of stable quark droplets in a vacuum and ultimately leads to a severe overestimation of fluctuation observables like e.g. net-baryon number moments.

The EoS is obtained by first determining the dynamical values of $\langle\sigma\rangle$ using the gap equation $\partial\Omega/\partial\sigma = 0$. After that, we determine the net-quark density by a derivative of the pressure P as $n = \frac{\partial P}{\partial \mu}$ and parametrize both $P = -\Omega$ and n through $\langle\sigma\rangle$.

An early result of this research work reveals several features of the obtained equation of state. Figure 3 shows that the pressure is only slightly above zero at $0.0 < n < 0.2$ fm^{-3}. Then, we have a negative pressure for $0.20 < n < 0.74$ fm^{-3}, before the pressure increases in the positive region for subsequently increasing density.

The negative pressure region poses the aforementioned problems to the nonequilibrium model. Therefore, more work needs to be done to find a reliable workaround. Currently, we are investigating comparisons with other models that include further hadronic degrees of freedom like the sigma-omega model.

5. Summary and conclusions

We have studied cumulant ratios of the sigma field for different beam energies within the nonequilibrium chiral Bjorken model based on the quark-meson Lagrangian. It must be stressed that although our theoretical framework is just a simple model it is nevertheless able to describe the complex nature of dynamical phenomena near a chiral CP. We found a large spread of possible values for variance, skewness, and kurtosis at the first-order phase transition. Especially the kurtosis with strongly negative values at low $\sqrt{s}$ can hint at the presence of a first-order phase transition. For comparison with experimental data, we need to evaluate the fluctuations in the net-proton number after a correct treatment of volume fluctuations and the possible need for an extension of the Lagrangian to cure the problem of negative pressure at lowest temperatures.

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