An investigation of Aharonov-Bohm effect towards the potential use for the gravitational wave detection

C Deesamer[1], N Wanwieng[2], P Chainakun[1,3,*] and A Watcharangkool[4]

[1]School of Physics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima 30000, Thailand
[2]Department of Physics and Materials Science, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand
[3]Centre of Excellence in High Energy Physics and Astrophysics, Suranaree University of Technology, Nakhon Ratchasima 30000, Thailand
[4]National Astronomical Research Institute of Thailand, Chiang Mai 50200, Thailand

*E-mail: `pchainakun@g.sut.ac.th`

Abstract. We investigate an alternative way to detect the gravitational wave using the concept of Aharonov-Bohm experiment in curved space-time. Our system consists of an electron beam which is split into two beams passing opposite sides of the solenoid and producing interference patterns. The change in interference patterns can be observed if the system is perturbed by the gravitational wave, and can be used to trace back to the nature of the gravitational wave. This system is described by the cylindrical coordinate in Minkowski space-time where we set the incoming wave propagating in the z-direction, perpendicular to the solenoid's cross-section. We found that the perturbation on the cross-section area due to gravitational strength is not strong enough to significantly change the phase shift. Contrarily, by changing the magnetic field generated by the current inside the solenoid, the results suggest that the significant phase shift could potentially be detected if the gravitational wave is allowed to propagate in the direction that is perpendicular to z-direction.

1. Introduction

The gravitational wave has been robustly detected for the first time in the stellar-mass binary black hole merging, GW150914, by the Laser Interferometer Gravitational-Wave Observatory (LIGO) in 2015 [1]. The measurement method of LIGO is based on the laser interferometry technique to measure the strain of the displacement of two perpendicular arms connecting to reflection mirrors, relating to the phase difference. Here, we investigate the gravitational wave detection in an alternative way using the Aharonov-Bohm experiment which provides the magnetic vector potential that can produce a specific interference pattern of the electron beams [2] once perturbed by the gravitational wave. Due to its nature, the wave passes through the earth with strength (h) weaker than 10^{-20} [3] and currently be detected with the minimum strain $\sim 10^{-21}$ that, when being treated as a weak perturbation from linearized theory [4], can slightly affect the space-time continuum [5]. Hence, the change in the interference pattern would be insignificant and difficult to be detected if the perturbation is beyond the first order. By setting the magnetic field in the Aharonov-Bohm experiment to be uniform, the change of magnetic

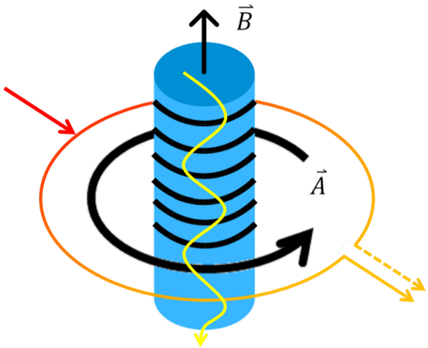

Figure 1. The schematic of Aharonov-Bohm experiment with gravitational wave. See text for more details.

flux and corresponding phase shift can be calculated from the change in the cross-section area of the solenoid. We then test the coupling between the gravitational wave and electromagnetic field from the variation of the current density using Maxwell's equation in curved space-time [6]. The metric tensor carrying the information of the weak perturbation contains the variation term that leads us to evaluate the potential use of this experimental set-up to detect the gravitational wave.

2. Experimental model

In the Aharonov-Bohm experiment, the incoming electron beam traveling in the x-y plane can be split into two beams passing opposite sides of the solenoid (figure 1). The solenoid is assumed to have infinite length where the magnetic field is zero outside the solenoid. Therefore, the magnetic vector potential $\mathbf{A}$ is only proportional to the magnetic field $\mathbf{B}$ inside the solenoid circuit, hence $\boldsymbol{\nabla} \times \mathbf{A} = \mathbf{B}$. The magnetic flux inside the solenoid circuit then can be written as

$$\Phi = \int \mathbf{B} \cdot d\mathbf{S} = \oint \mathbf{A} \cdot d\mathbf{l} \,, \tag{1}$$

where $\mathbf{S}$ is the cross-section area of the solenoid and $\mathbf{l}$ is the path around the solenoid. Without gravitational wave, the phase shift produced by the Aharonov-Bohm effect is suggested to be [2]

$$\Delta\phi = \frac{q}{\hbar} \int \mathbf{B} \cdot d\mathbf{S} = \frac{q}{\hbar} \oint \mathbf{A} \cdot d\mathbf{l} = \frac{q}{\hbar} \Phi \,. \tag{2}$$

Since the gravitational wave perturbation influences the ripples in the geometry of space-time, we employ a uniform magnetic field and observe the change in the path length and the cross-section area due to the perturbation by the gravitational wave. This leads to the change in the current density that, finally, allows us to quantify the change in phase shift when the gravitational wave is present. In this case, we do not concern the interaction between each particle, i.e., weak or strong force.

We employ the metric tensor g consisting of the Minkowski metric η in local flat space-time and a weak perturbation h for the wave propagating in the vacuum in z-direction using transverse-traceless gauge [4], having the polarization h_+ and $h_\times$ on the x-y plane [7]. Then, the length L from point P to Q can be described by

$$L = \int_P^Q \sqrt{g_{\mu\nu} dx^\mu dx^\nu} = \int_P^Q \sqrt{(\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu} \,. \tag{3}$$

In cartesian coordinate, the metric tensor for the wave propagating in z-direction is

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cos\left[\omega(t - z/c)\right] . \tag{4}$$

In cylindrical coordinate, using tensor transformation, the gauge is still transverse-traceless. The metric tensor then becomes

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & H_+ & rH_\times & 0 \\ 0 & rH_\times & -r^2 H_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} , \tag{5}$$

where

$$\begin{aligned} H_+(\phi, t) &= M \sin(A + 2\phi) \cos\left[\omega(t - z/c)\right] , \\ H_\times(\phi, t) &= M \cos(A + 2\phi) \cos\left[\omega(t - z/c)\right] , \\ M &= \sqrt{h_+^2 + h_\times^2} . \end{aligned}$$

Furthermore, we define the initial length in x- and y-axis as L_{x0} and L_{y0}, respectively. When the effect of perturbation due to the gravitational wave is included (figure 2), we calculate the cross-section area at a particular value of z and t.

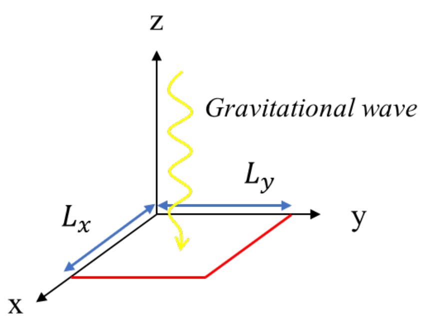

Figure 2. The schematic of the change in the cross-section area in Aharonov-Bohm experiment due to the gravitational wave perturbation.

The variation of current density j is evaluated via the derivative of metric tensor in case of a uniform magnetic field. According to Maxwell's equations in curved space-time [6], we can write

$$j^\nu = \frac{1}{\sqrt{-\det(g)}} \partial_\mu [\sqrt{-\det(g)} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}] \tag{6}$$

$$= \frac{1}{r} \partial_\mu [r g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}] , \tag{7}$$

where $F_{\rho\sigma}$ is the electromagnetic strength tensor in cylindrical coordinate. For the infinite length solenoid, we do not consider the point charge and leave the current alone, so the electric field is negligible due to the uniform magnetic field configuration.

3. Result and discussion

To calculate the change in the cross-section area shown in figure 2, we multiply the proper length from two axes given in the cartesian coordinate (x,y)

$$L_x = L_{x0}\{1 + h_+ \cos[\omega(t - z/c)]\}^{\frac{1}{2}} , \tag{8}$$

$$L_y = L_{y0}\{1 - h_+ \cos[\omega(t - z/c)]\}^{\frac{1}{2}} , \tag{9}$$

to estimate the cross-section area, from equations (8) and (9),

$$S = L_x L_y = L_{x0} L_{y0} + O(h^2) . \tag{10}$$

The result shows that the difference between perturbed and unperturbed cross-section is in the order of h^2, which is negligible in the linearized theory. However, more precise calculations can be done by taking into account the cross polarization in the surface integral, but the results are not significantly different from our estimation here.

Furthermore, in cylindrical coordinate, we set the solenoid's radius r to be constant. Then, Maxwell's equation for fields which are constant in time with the gravitation effect can be expressed in terms of

$$j_h^r = \frac{1}{r^2}\partial_\phi\big[(1 - H_\times^2)B_z\big] - \partial_z\big[(1 - H_+)B_\phi\big] + \frac{1}{r}\partial_z(H_\times B_r) , \tag{11}$$

$$j_h^\phi = -\frac{1}{r}\partial_r\Big[\frac{1}{r}(1 - H_\times^2)B_z\Big] + \frac{1}{r^2}\partial_z\big[(1 + H_+)B_r\big] + \frac{1}{r}\partial_z(H_\times B_\phi) , \tag{12}$$

$$j_h^z = \frac{1}{r}\partial_r\big[(1 - H_+)rB_\phi\big] + \frac{1}{r}\partial_r(H_\times B_r) - \frac{1}{r^2}\partial_\phi\big[(1 + H_+)B_r\big] + \frac{1}{r}\partial_\phi(H_\times B_\phi) , \tag{13}$$

while the variation of current density in each dimension without non-gravitational effect is

$$j^{r*} = -\frac{1}{r^2}\partial_\phi(H_\times^2 B_z) - \partial_z(H_+ B_\phi) + \frac{1}{r}\partial_z(H_\times B_r) , \tag{14}$$

$$j^{\phi*} = \frac{1}{r}\partial_r(\frac{1}{r}H_\times^2 B_z) + \frac{1}{r^2}\partial_z(H_+ B_r) + \frac{1}{r}\partial_z(H_\times B_\phi) , \tag{15}$$

$$j^{z*} = -\frac{1}{r}\partial_r\big(H_+ rB_\phi\big) + \frac{1}{r}\partial_r(H_\times B_r) - \frac{1}{r^2}\partial_\phi(H_+ B_r) + \frac{1}{r}\partial_\phi(H_\times B_\phi) . \tag{16}$$

Note that we discard the term that shows the second order of H^2 in $g^{\mu\rho}g^{\nu\sigma}$. As a result, all terms of each current density equation show no presence of B_z, hence the variation due to the perturbation in this way is approximately be zero.

On the other hand, by changing the orientation of the solenoid so that the wave propagation is tangent to the cross-section area, i.e. fixes x-axis and rotates 90°degree from the original z-direction, the perturbation is occurred only in one direction of the solenoid's cross-section as shown in figure 3. The perturbed area then becomes

$$S = \int\int \sqrt{g_{xx}g_{zz}}\, dxdz = \int\int \sqrt{1 + O(h)}\, dxdz \;\approx\; L_{x_0}L_{z_0}[1 + O(h)]^{\frac{1}{2}} . \tag{17}$$

Consequently, some of the metric tensor multiplication terms can be in the order of h which, in turn, allow the variation of the current density. Under this configuration, the significant phase shift should potentially be detected, which will be investigated further in the future.

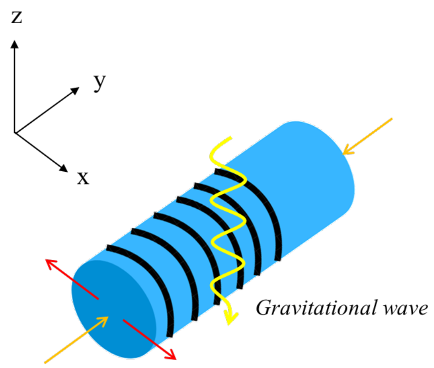

Figure 3. The schematic of the solenoid, rotating 90° from the original z-direction.

4. Conclusion

Our calculations show that the change in phase shift induced by the perturbation on the cross-section area due to the gravitational strength is not significant, especially if the gravitational wave is travelling along the z-direction. This system, instead, could be used to detect gravitational wave travelling in the tangent direction to the solenoid cross-section area.

Acknoledgement

This research was supported by the Program Management Unit for Human Resources & Institution Development, Research and Innovation [grant number B05F630108]

Reference

[1] Abbott B P *et al.* 2016 *Phys. Rev. Lett.* **116**(6) 061102
[2] Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
[3] Blair D 1991 *The Detection of Gravitational Waves* (Cambridge : University Press)
[4] Carroll S M 2019 *Spacetime and Geometry* (Cambridge : University Press)
[5] Einstein A 1916 *Ann. Phys.* **354** 769
[6] Hall G S 1984 *Gen. Relativ. Gravit.* **16** 495
[7] Hawking S W and Israel W 1987 *Three Hundred Years of Gravitation* (Cambridge : University Press)