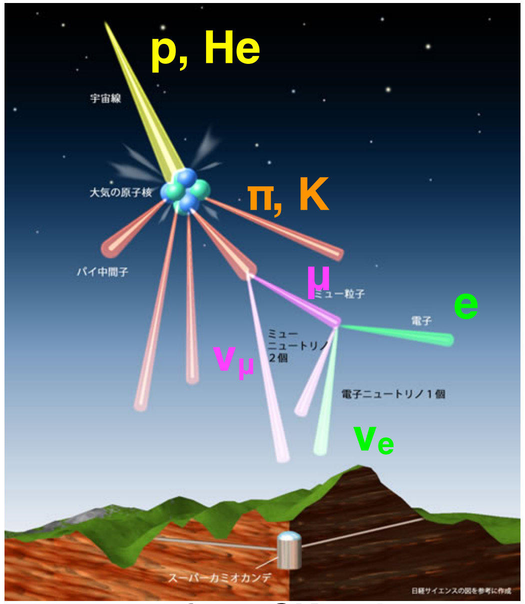
atmospheric neutrino simulation with beam data

Kazufumi Sato (Nagoya University)
10 Dec. 2020 @ NA61 low E workshop

- study to incorporate **accelerator hadron measurements** into **atmospheric neutrino simulation (Honda flux)**

atmospheric neutrino (atm. ν)



from SK web page

primary cosmic-ray particles (p,He) hit to air atoms

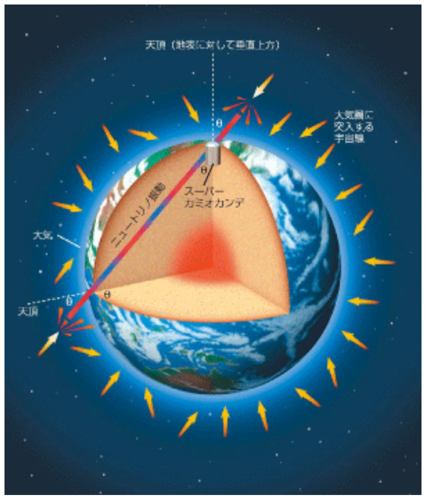
→ develop hadronic shower

→ cascade : $\pi \rightarrow \boldsymbol{\nu_\mu}$, $\mu \rightarrow \boldsymbol{\nu_\mu, \nu_e}$

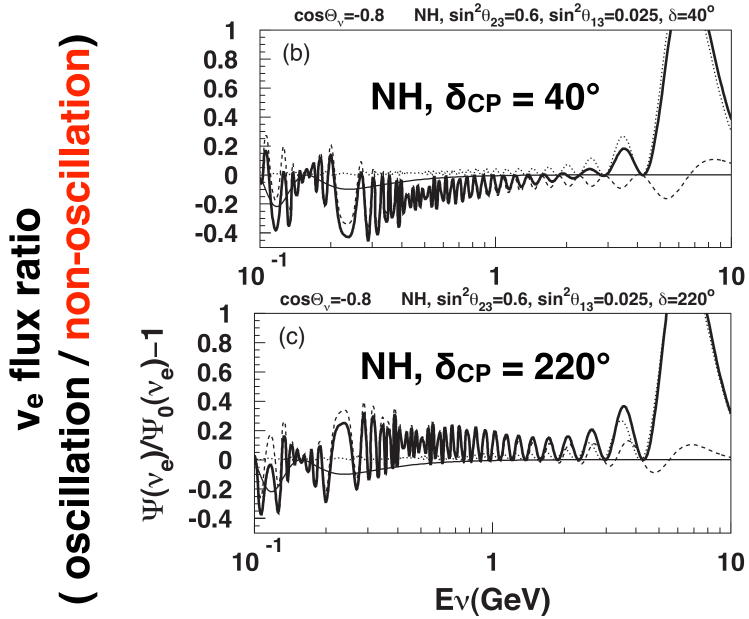
atm. ν's

- E_ν : O(10) MeV — O(10) TeV
- flight length L : 10 — O(10^4) km

→ **wide L/E**


from SK web page

from HK design report



- atm. ν flux possibly depends on δ_{CP} in E<1 GeV region

- for oscillation study, we have to know "**non-oscillated**" flux
  → **simulation!**

Honda flux code (ATMNC)

ATMNC: **ATM**ospheric **M**uon **N**eutrino **C**alculation code

developed by M. Honda (U of Tokyo, ICRR)

[PRD 83, 123001(2011) and references in it]

- full MC simulation for air shower

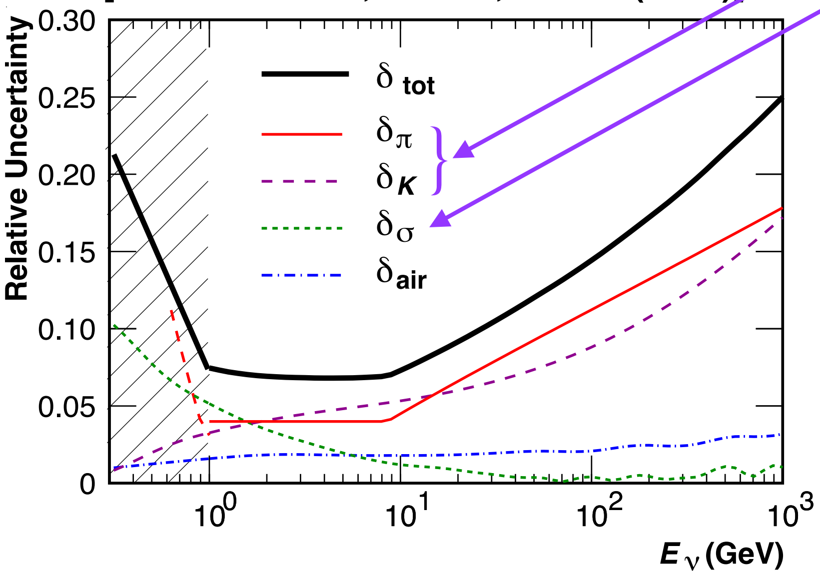
→ provides ν_μ, $\bar{\nu}_\mu$, ν_e, $\bar{\nu}_e$ **flux** at a given detector position

- **3D** simulation
  - air density model **NRLMSISE-00**
  - geomagnetic model **IGRF**
  - precise primary particle flux based on **AMS02** data
- has been used in **SK atm. ν analysis**

  → for analysis in HK, we want to improve its accuracy

uncertainty of atm. ν flux

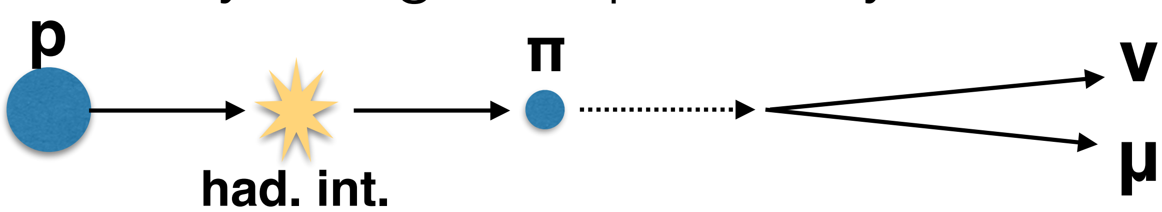
uncertainty of ATMNC flux
(except primary flux err ~5%)
[M. Honda et. al, PRD75, 043006(2007)]



hadron production
hadronic cross-section

- ***hadronic interactions in air shower***
  → ***dominant!***
    - Hadronic Model
        - JAM (E<31GeV)
        - dpmJet3 (otherwise)

- tuned by using atm. μ data by Honda-san



limitation of tuning

- low E_ν (<1GeV): E deposit of μ
- high E_ν (>10 GeV): K contribution

uncertainty of atm. ν flux

uncertainty of ATMNC flux
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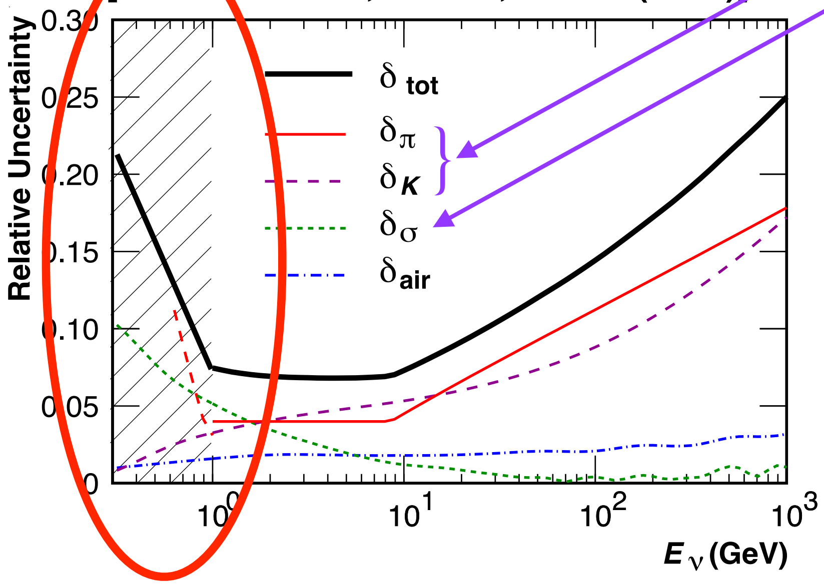
[M. Honda et. al, PRD75, 043006(2007)]



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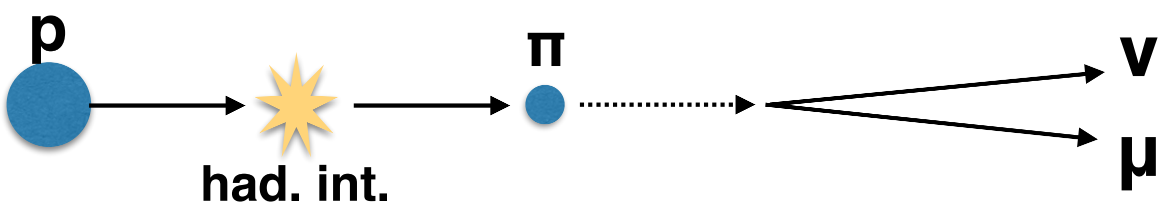

limitation of tuning

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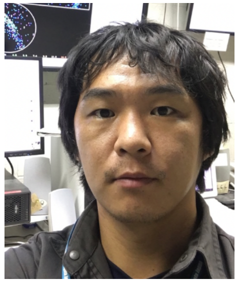
activity of Nagoya ISEE CR group

K. Sato (me) **H. Menjo** **Y. Itow**



studying to incorporate **hadron production data measured in beam experiments** into ATMNC

- several beam measurements are conducted/planned (mainly for long-baseline ν experiment)

  *HARP, BNL E910, **NA61/SHINE**, EMPHATIC …*

→ **reflect these measurement results into ATMNC**

Maybe the measurement data is insufficient but…

- can **reduce uncertainty** by combining the muon study
- can reveal **which phase space is important for atm. ν** production, and **feed back to the beam experiment**
- common treatment of sys. error between **T2K-SK**

method to incorporate beam data into ATMNC

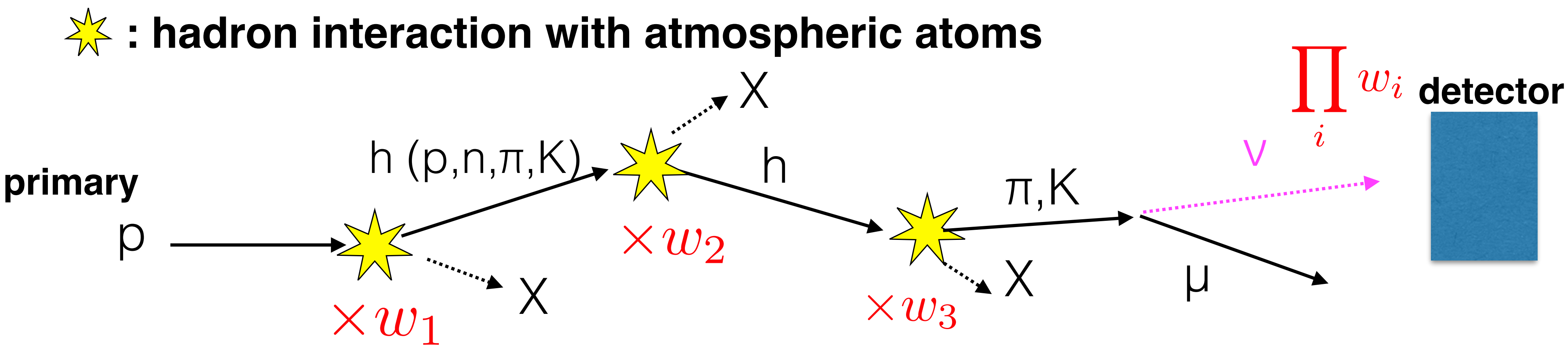
What we want to do:

- correct difference of $\dfrac{d\sigma}{dpd\Omega}$ between data and ATMNC

→ apply the *weight*

$$w = \frac{\left(\frac{d\sigma}{dpd\Omega}\right)_{data}}{\left(\frac{d\sigma}{dpd\Omega}\right)_{MC}}$$

for each hadron interaction in ATMNC simulation



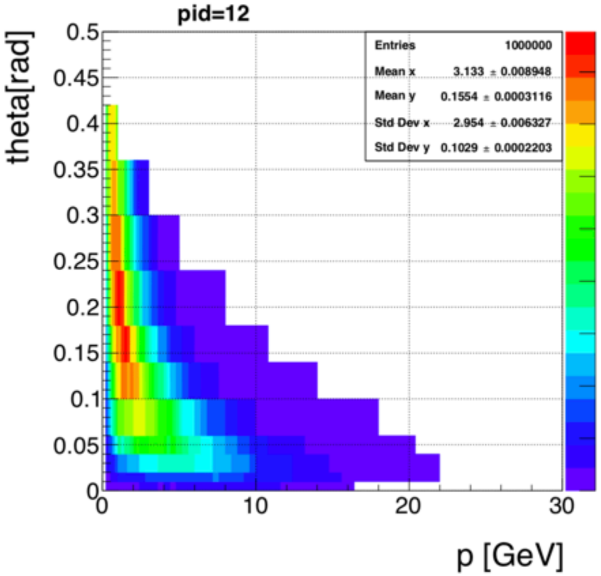
method to incorporate beam data into ATMNC

e.g.) p (31GeV) + Air → π⁺+ X

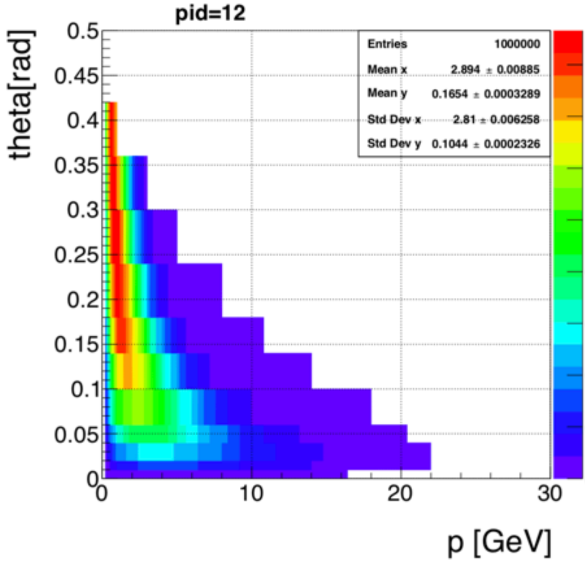
$$w = \frac{\left(\frac{d\sigma}{dpd\Omega}\right)_{data}}{\left(\frac{d\sigma}{dpd\Omega}\right)_{MC}}$$

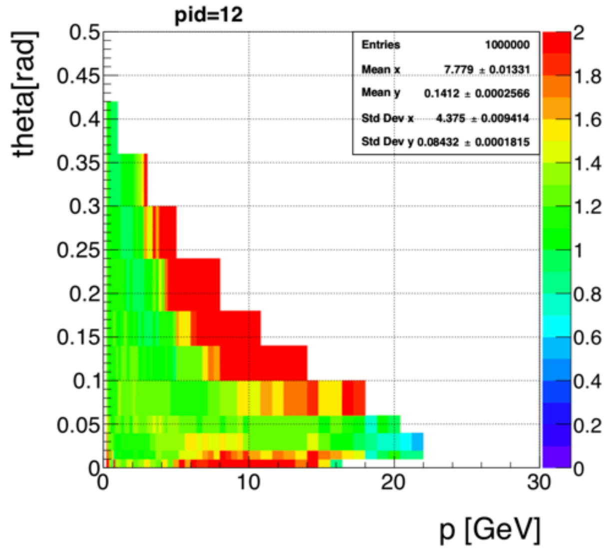
differential cross-seciton of outgoing π⁺

NA61/SHINE

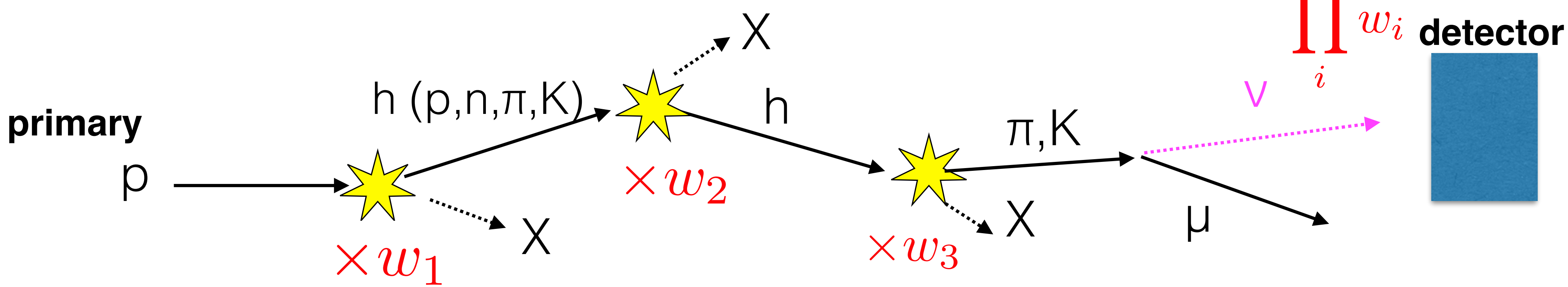


ATMNC





: hadron interaction with atmospheric atoms



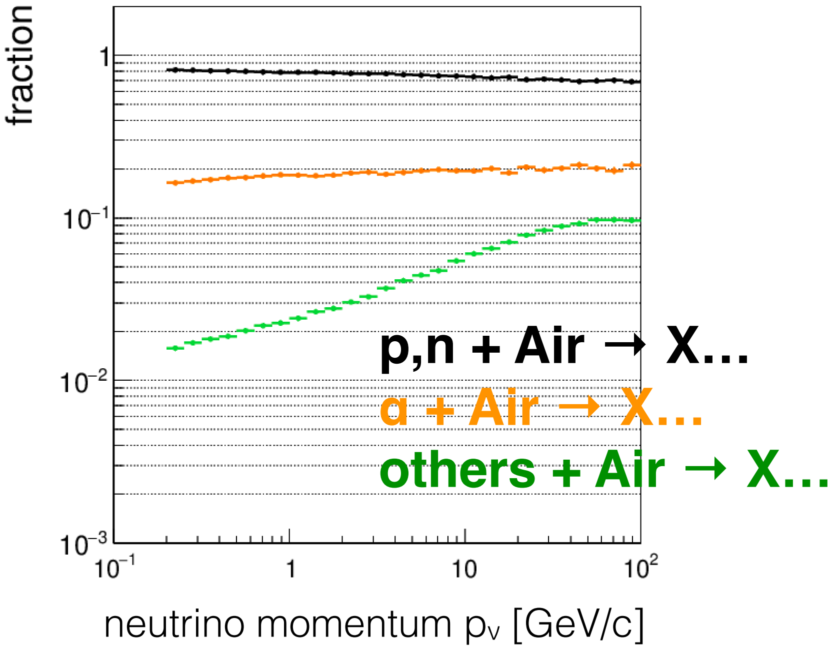
Beam particle, outgoing particle

What kind of hadron interaction is involved in **ν production?**

breakdown of hadron production

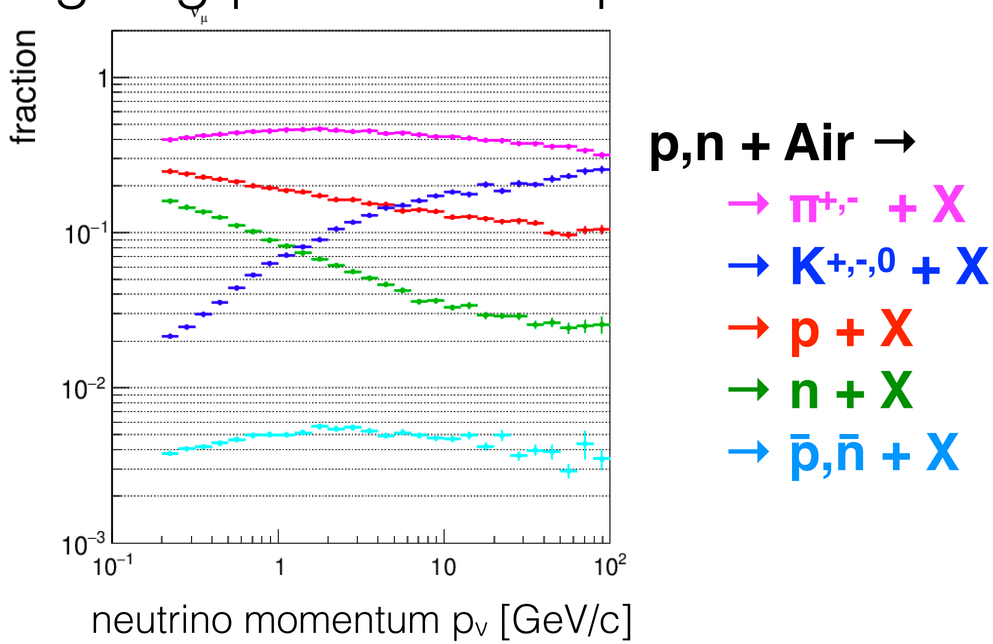
fraction to # of all hadron productions involved in atm. ν production

incident particle



80% are "**p** + Air → X"

outgoing particle from p+Air

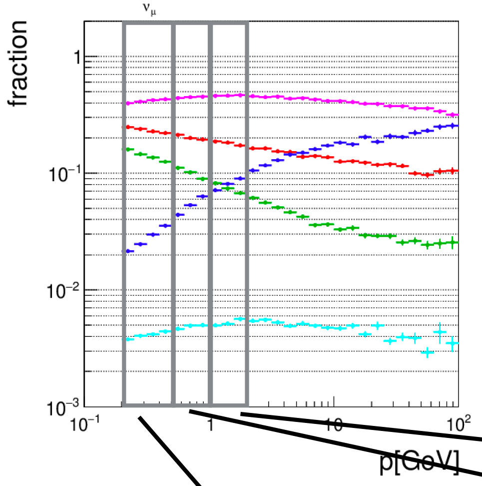


- "**p + Air → $\pi^{\pm}$ + X**" is dominant
- in < 1 GeV, **p**,**n** contribute.
- in > 5 GeV, **K** contributes

To explore sub-GeV ν, we need beam data of

p + A → $\pi^{\pm}$ + X and **p + A → p + X**

beam momentum



momentum of protons involved in ν production

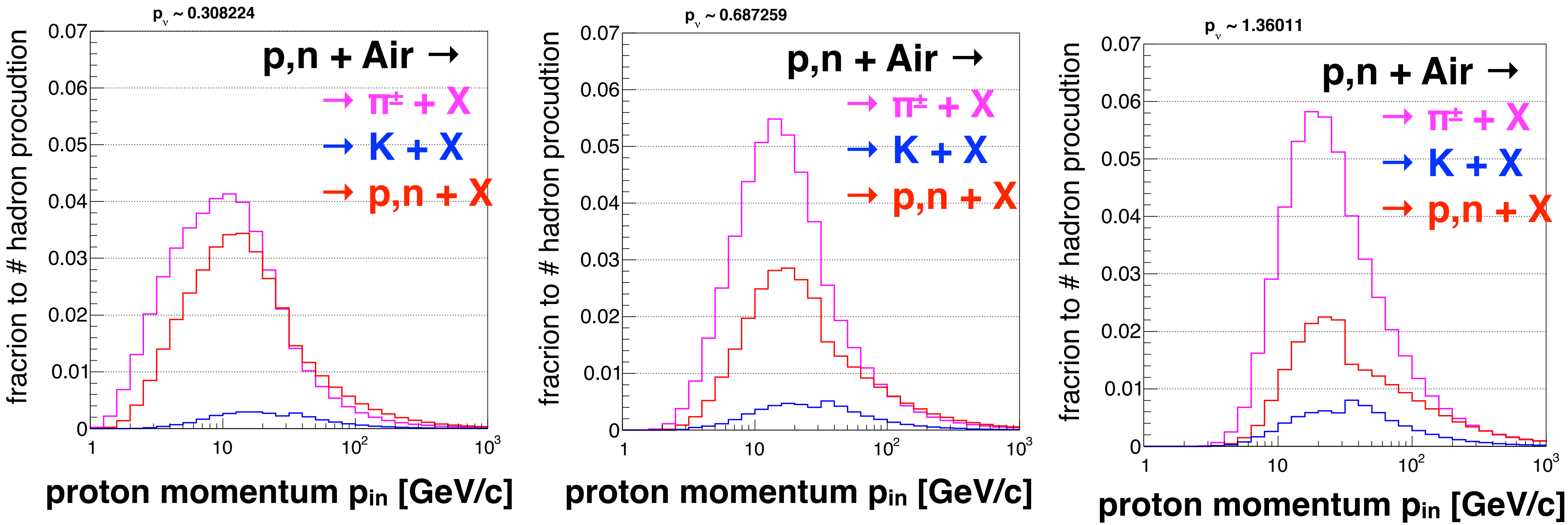
- for sub-GeV ν, **peaked at 10 GeV/c**

→ We need low energy beam data!

<pν>~ 0.31 GeV/c

<pν>~ 0.69 GeV/c

<pν>~ 1.36 GeV/c



available recent beam data p <= 31 GeV/c

p_{beam} [GeV/c]	3	5	6.4	8	12	12.3	17.5	31
p+Be	HARP $\pi^\pm$	HARP $\pi^\pm$	E910 $\pi^\pm$	HARP $\pi^\pm$	HARP $\pi^\pm$	E910 $\pi^\pm$	E910 $\pi^\pm$	
p+C	HARP $\pi^\pm$	HARP $\pi^\pm$		HARP $\pi^\pm$	HARP $\pi^\pm$			NA61 **$\pi^\pm$,K$^\pm$,p**
p+Al	HARP $\pi^\pm$	HARP $\pi^\pm$		HARP $\pi^\pm$	HARP $\pi^\pm$			

HARP : **3,5,8,12 GeV, p+ (Be,C,Al,Cu) → π^{+-} + X**

(Forward) Phys.Rev.C80, 035208 (2009)

(Large Angle) Eur. Phys. J. C 53, 177–204 (2008)

(Large Angle) Eur. Phys. J. C 54, 37–60 (2008)

BNL E910 : **6.4, 12.3, 17.5 GeV, p+ (Be,Cu,Au) → π^{+-} + X**

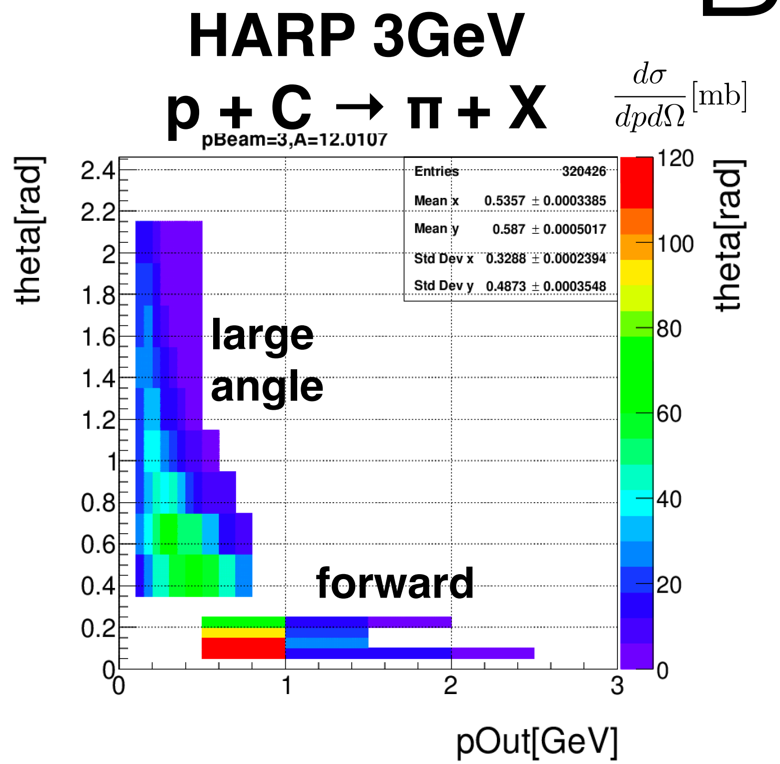
(Forward) Phys. Rev. C 77, 015209 (2008)

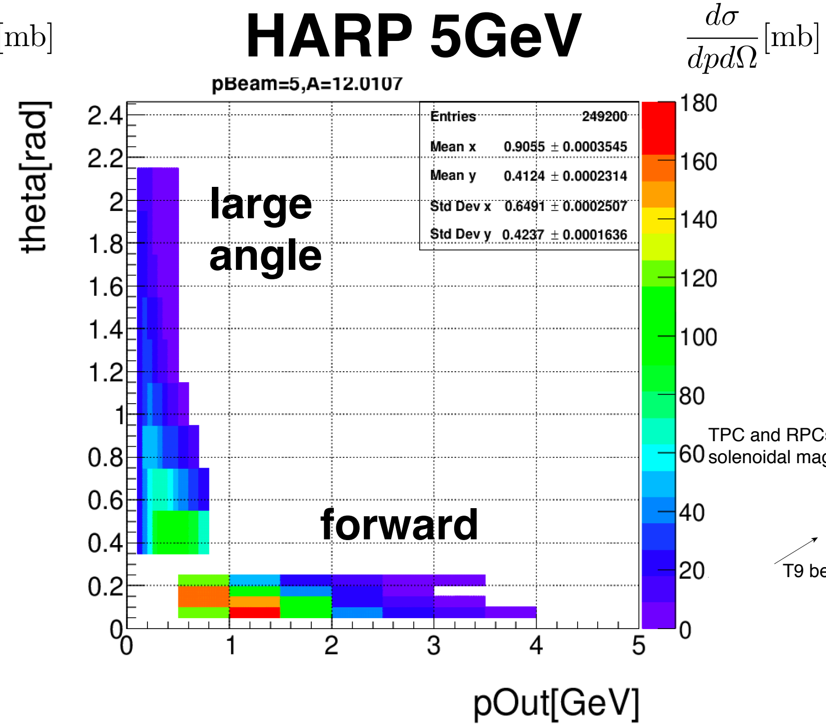
(Large Angle) Phys. Rev. C 65, 024904 (2002)

NA61/SHINE : **31 GeV, p + C → π, K, p + X**

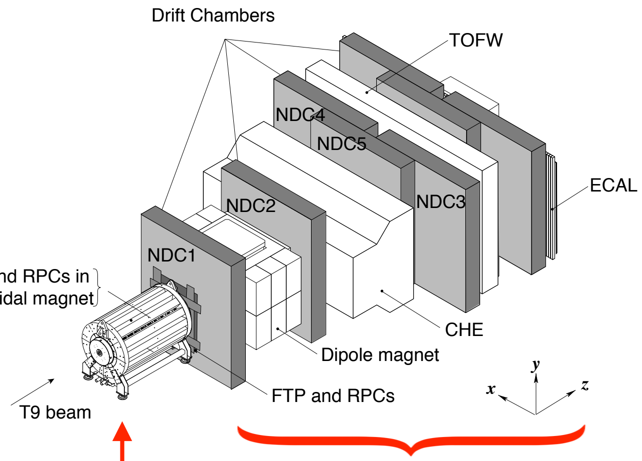
Eur. Phys. J. C 76, 84 (2016)

Beam data



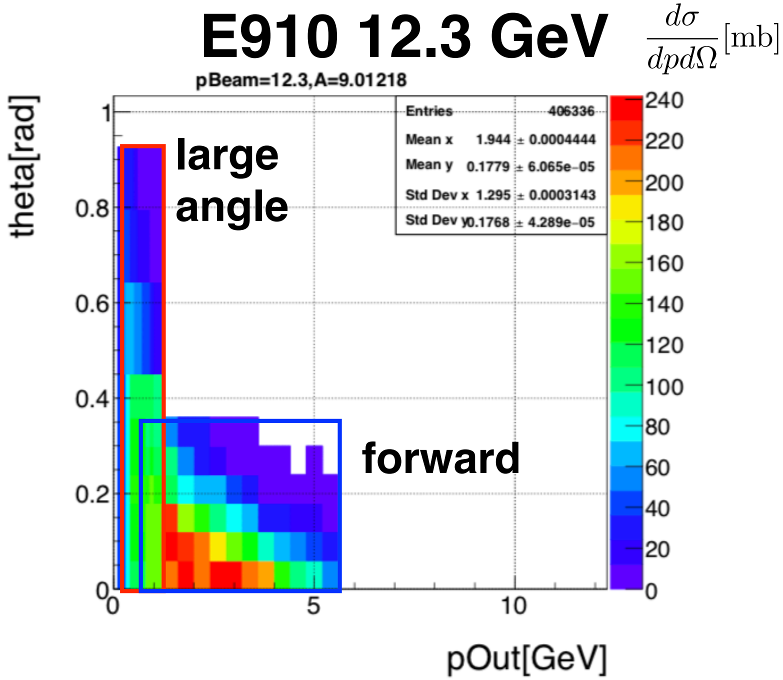


HARP detector



for forward
Acc : 0 ~ 0.25 rad

large-angle
Acc: 0.35 ~ 2.5 rad



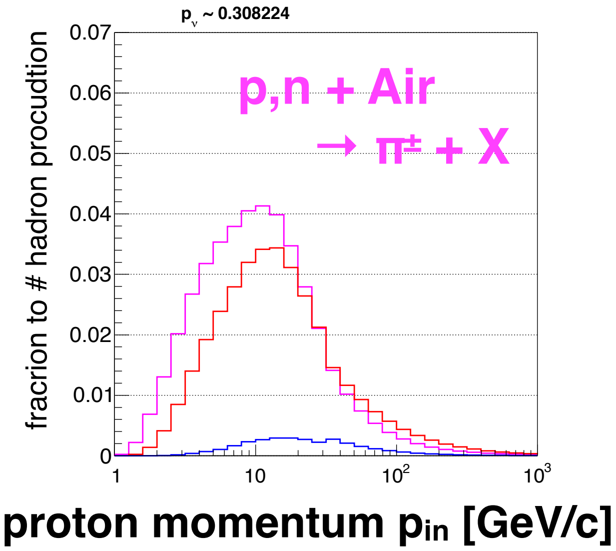
HARP coverage in p-θ plane:

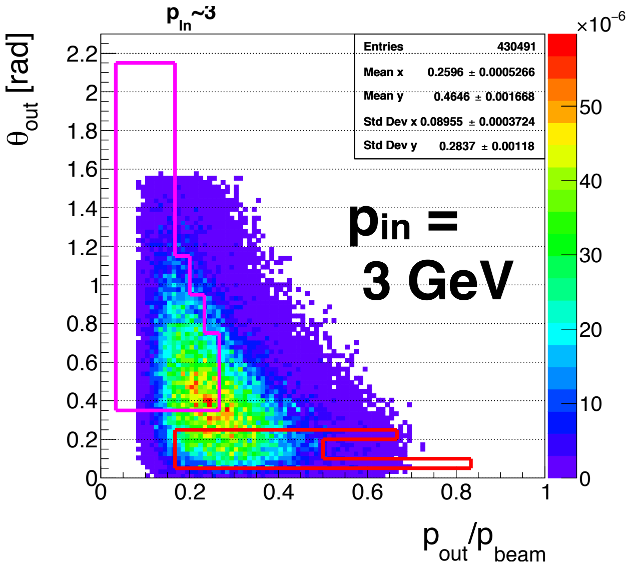
- forward: p > 0.5 GeV/c, θ < 0.25 rad
  - binning is rough
- large-angle: p < 0.8 GeV/c, θ > 0.35 rad

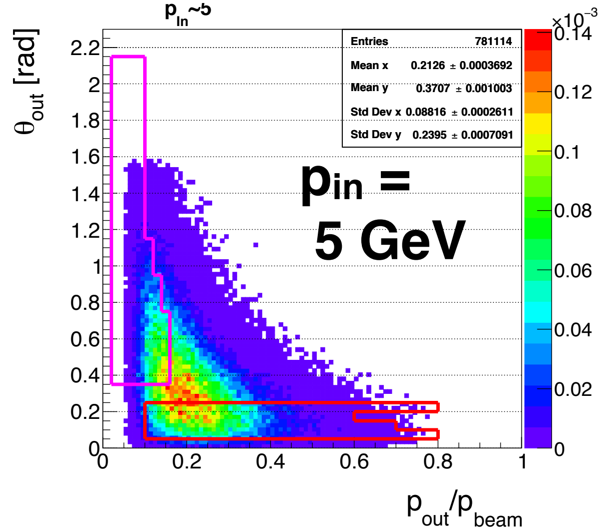
π phase space (for **0.3 GeV ν_μ**)

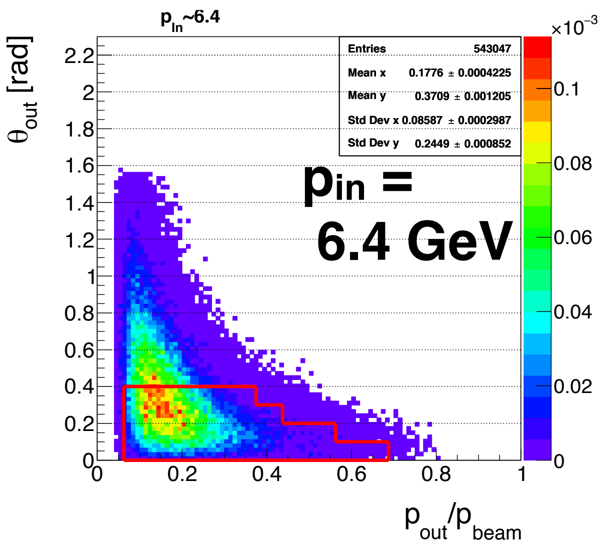
<p_ν>~ 0.31 GeV/c

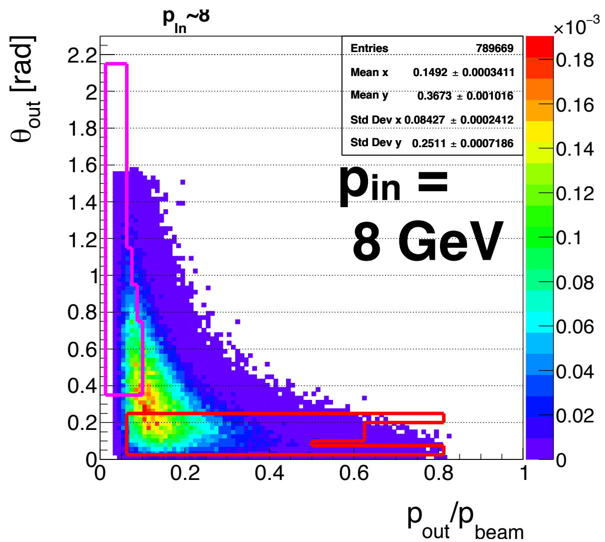
π's phase space of p+A→π±+X in air shower

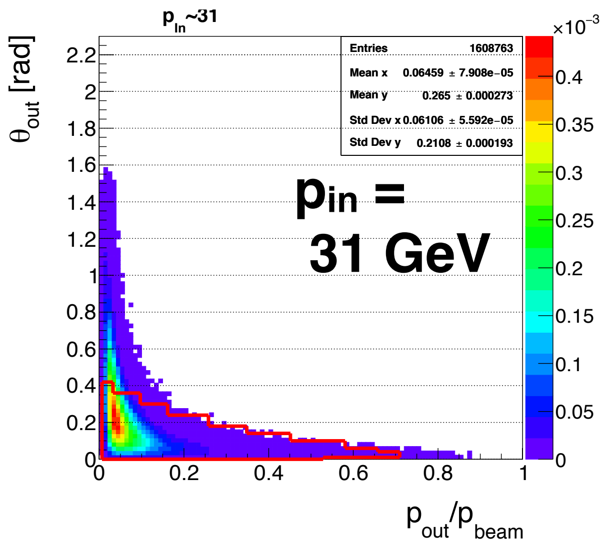






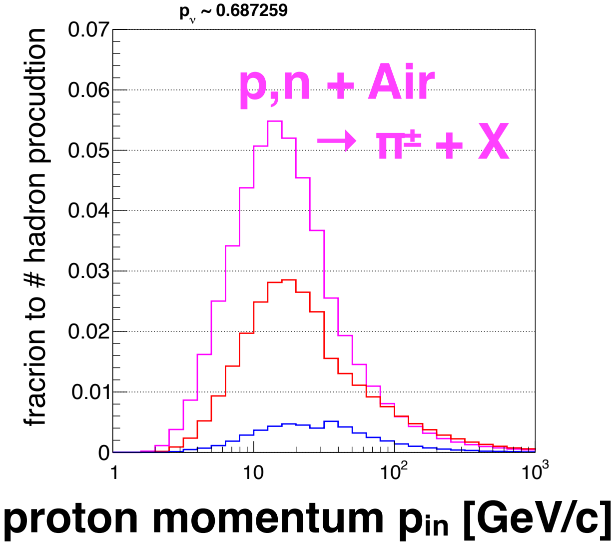




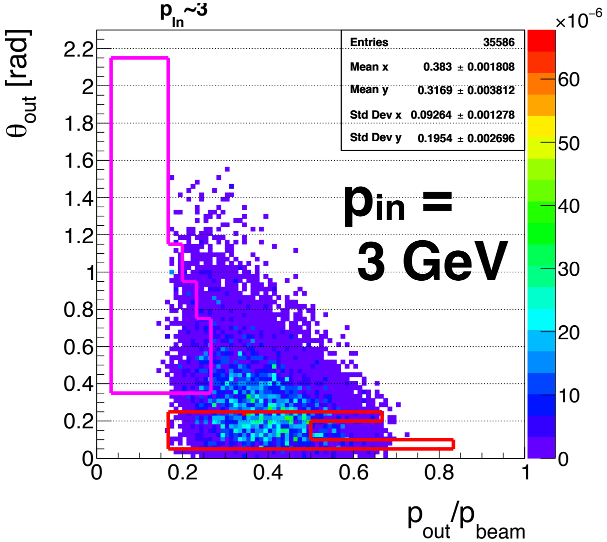


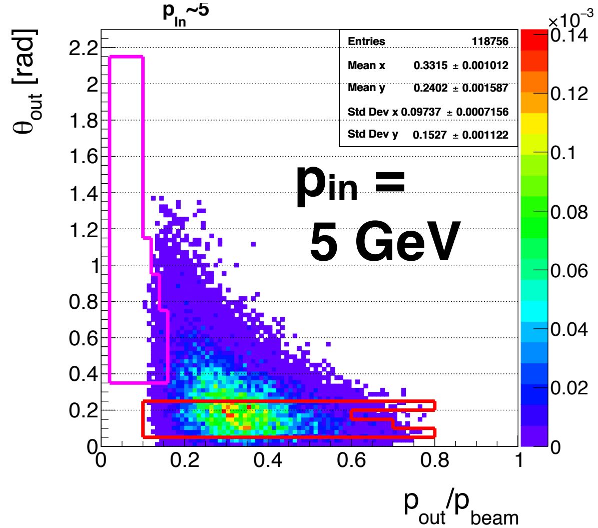
π phase space (for **0.69 GeV** ν_μ)

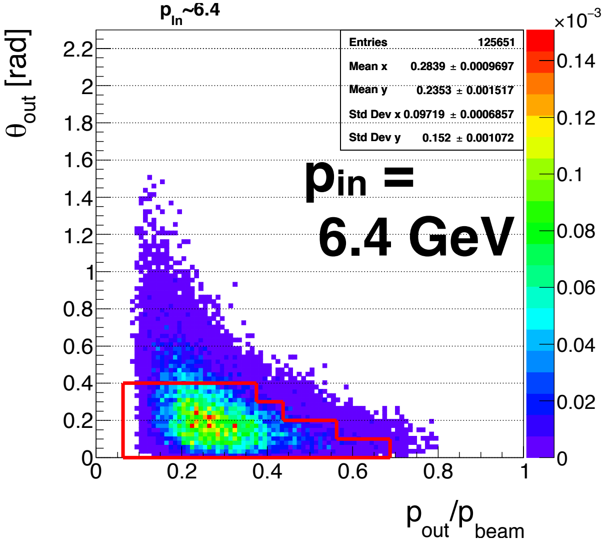
$<p_\nu>\sim$ 0.69 GeV/c

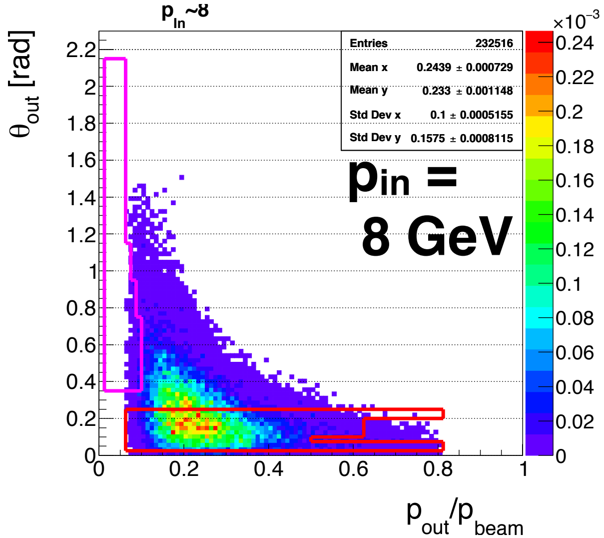


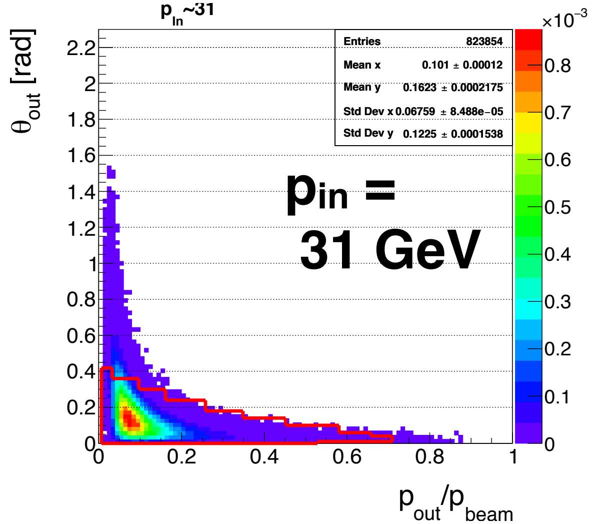
π's phase space of p+A→$\pi^\pm$+X in air shower





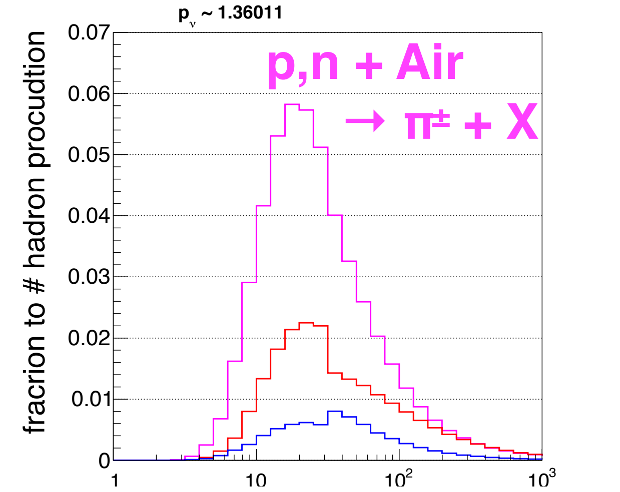






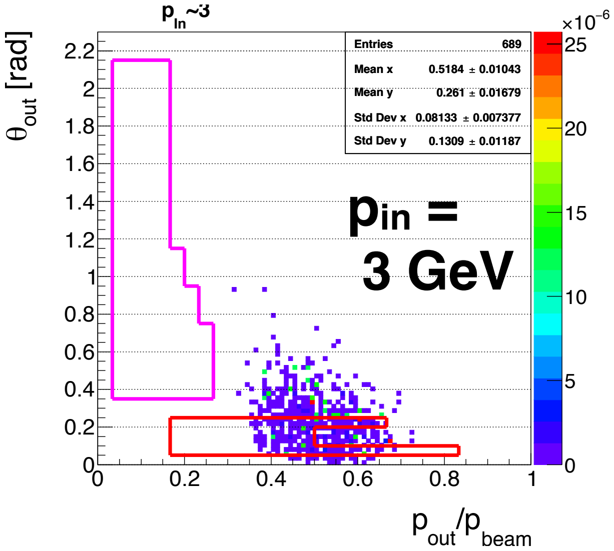
π phase space (for **1.4 GeV** ν_μ)

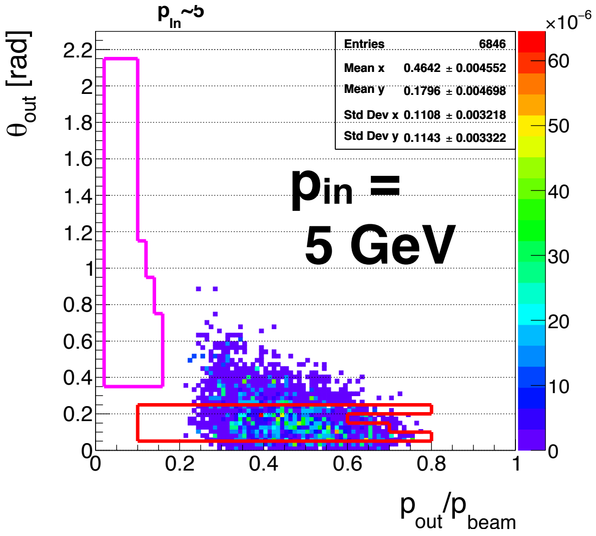
$\langle p_\nu \rangle \sim$ 1.4 GeV/c

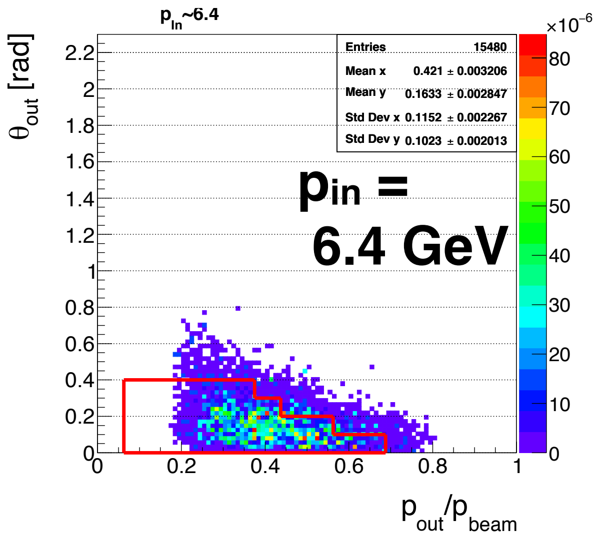


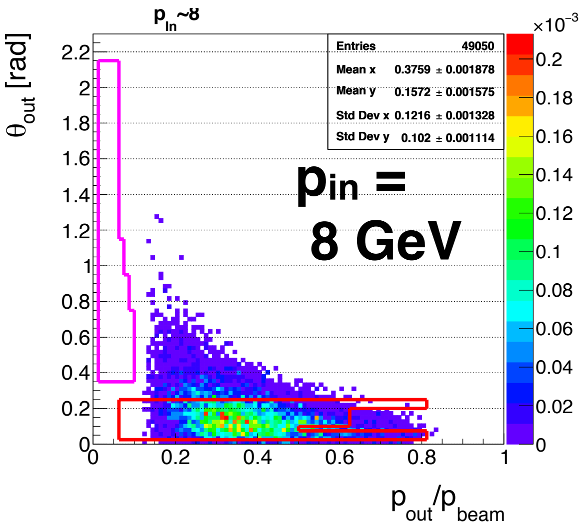
proton momentum p_{in} [GeV/c]

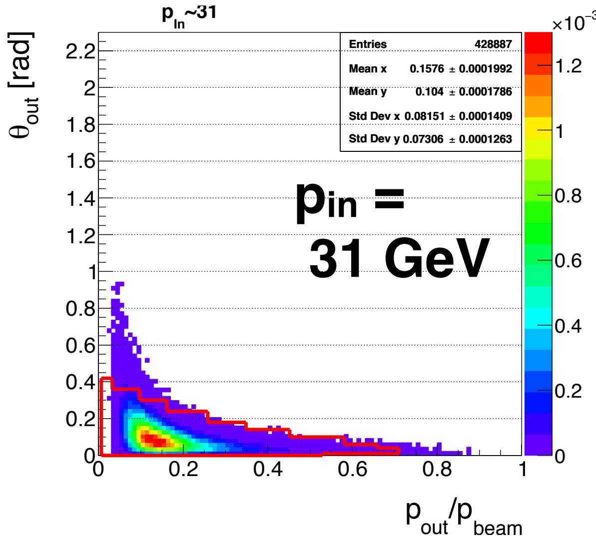
π's phase space of p+A→π±+X in air shower





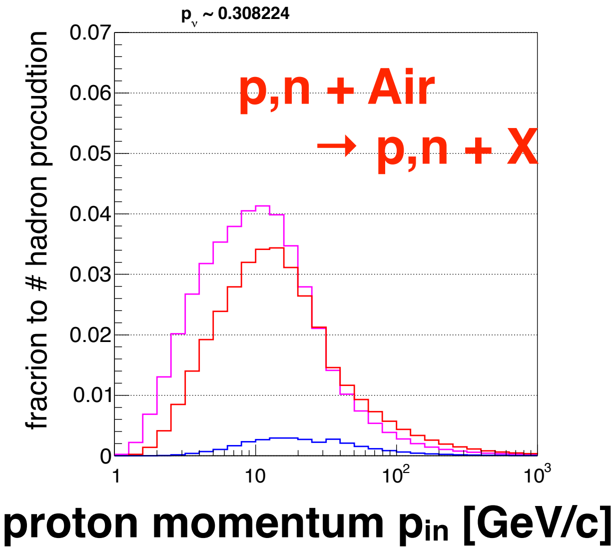




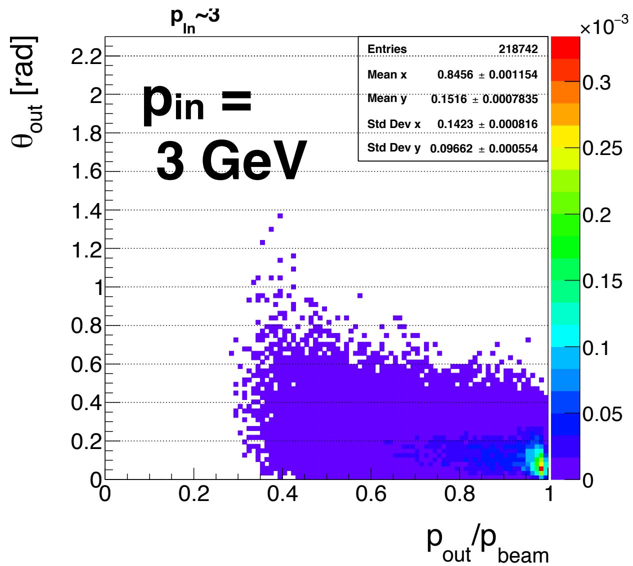


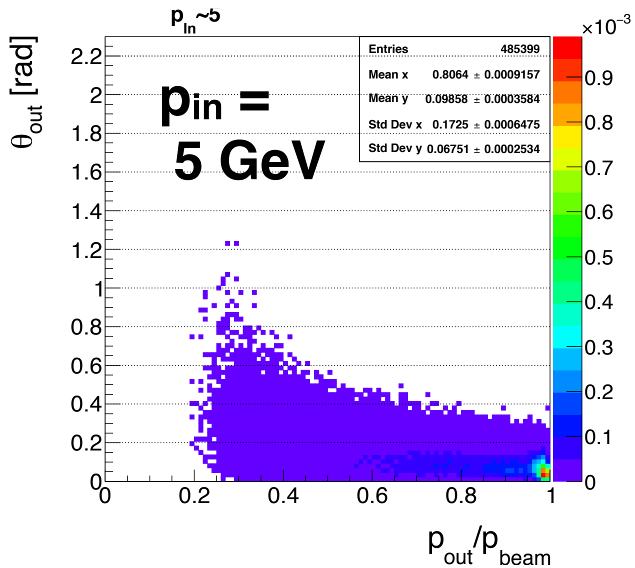
p phase space (for **0.3 GeV ν_μ**)

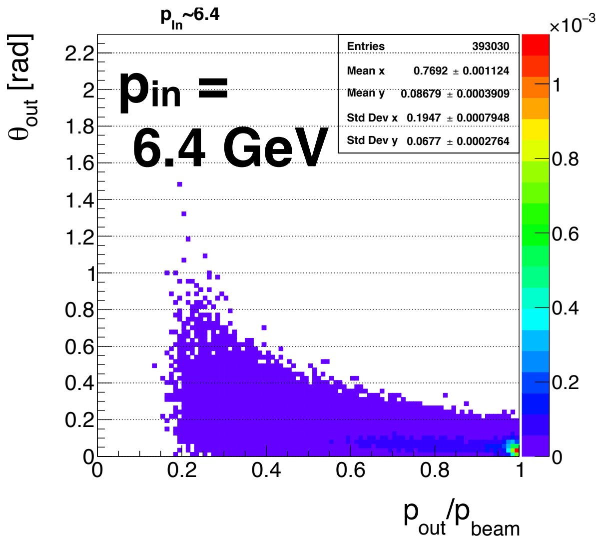
$\langle p_\nu \rangle$~ 0.31 GeV/c

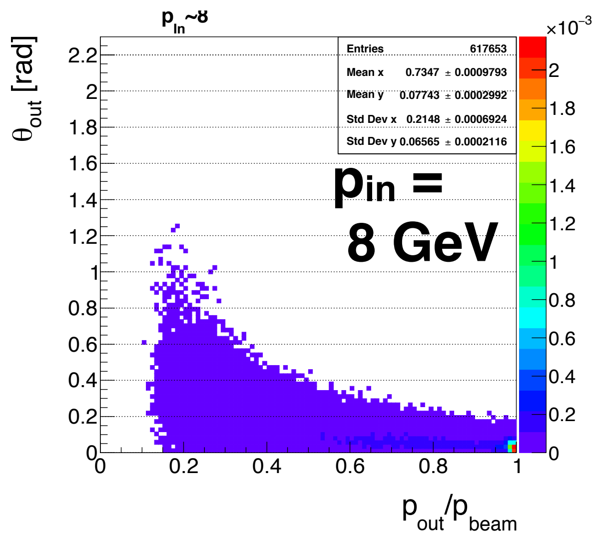


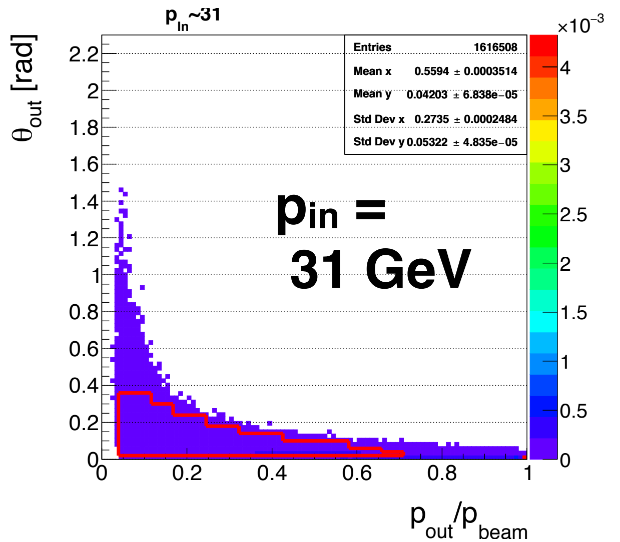
phase space of p+A→**p,n**+X in air shower









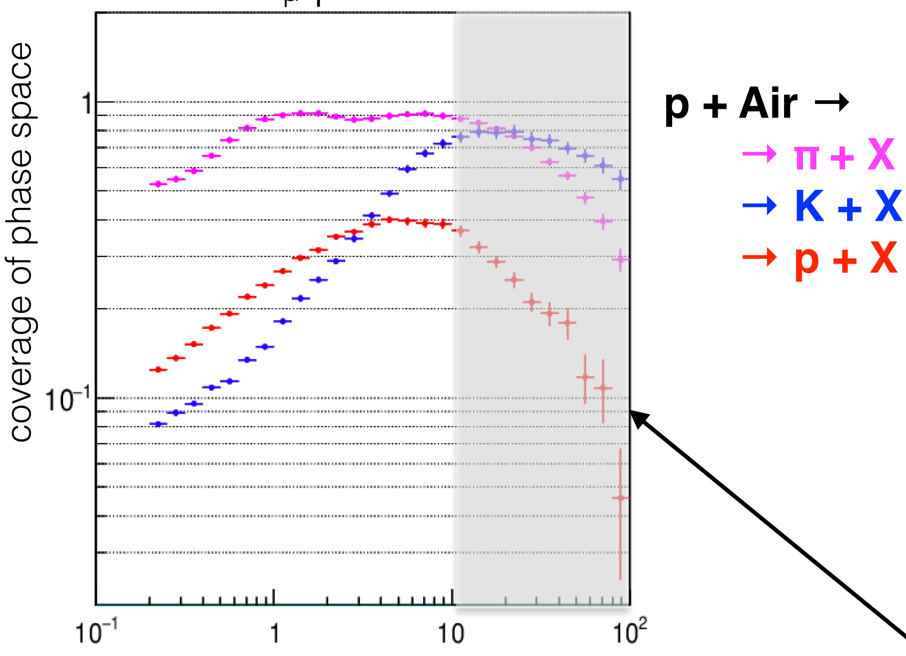


*Both HARP and BNL data are π only → **We need proton production data!**

coverage

How much the phase space is covered by beam data

for ν_μ production



neutrino momentum p_ν [GeV/c]

coverage of p+A→π+X

$$= \frac{\text{(\# of p+A→π+X covered by data)}}{\text{(\# of all p+A→π+X)}}$$

- For p+A→π+X,
90% coverage for >1GeV ν
⇄ ~**50%** for 0.3 GeV ν

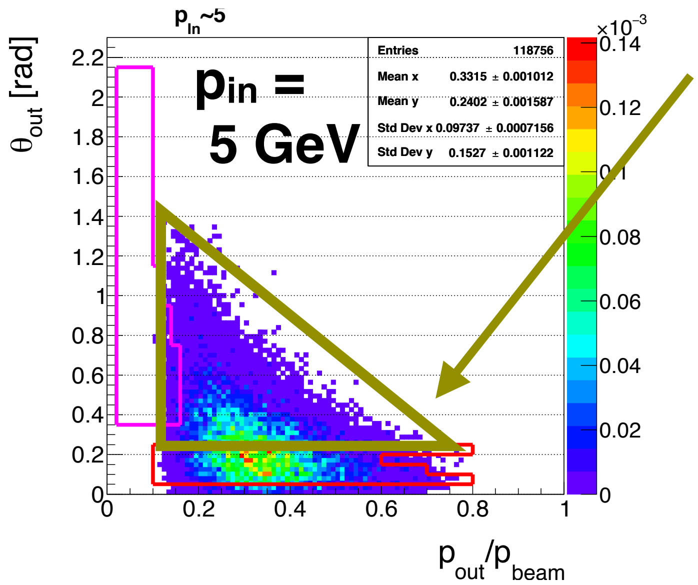
small coverage in low E!

for incident protons w/ high energy

- 31 GeV/c < p_{proton} < 1 TeV/c :
use NA61 31-GeV data with assuming perfect X_F scaling

- p_{proton} > 1 TeV :
ignored (coverage is calculated as 0)

parameterization?



want to cover this region by parameterization!

- *several candidates of parameterization*

Sanford & Wang (1967), Badhwar (1977), Mokhov (1998), Mariani (2011), BMPT(2001)

ref:
- S.R. Blattnig et al., PRD, 62, 094030 (2000)
- M. Bonesini et al., Eur. Phys. J. C 20 (2001)
- C. Mariani et al., PRD, 84, 114021 (2011)

BMPT (2001) (M. Bonesini et al., Eur. Phys. J. C 20 (2001))

$$E\frac{d^3\sigma}{dp^3} = A(1-x_R)^{\alpha}(1+Bx_R)x_R^{-\beta}\times$$

$$[1+\frac{a}{x_R^{\gamma}}p_T+\frac{a^2}{2x_R^{\delta}}p_T^2]e^{-a/x_R^{\gamma}p_T}$$

$$x_R \equiv \frac{E_{CM}}{\max\{E_{CM}\}}$$

: radial scaling
assume that x_R is independent from E_{beam}
[F. E. Taylor et al., PRD 14, 1217 (1976)]

*** see *backup* for definitions of other parameterizations**

fitting (forward and large-angle data)

test a simultaneous fit of *forward* and *large-angle* data

- use HARP p+C data • for each beam momentum

$$\chi^2 \equiv \sum_j^{\text{beam}} \sum_i \left(\frac{Y_i - N_j \bar{f}_i}{\delta Y_i} \right)^2$$

$Y_i \pm \delta Y_i$: measured $\frac{d\sigma}{dpd\Omega}$

$\bar{f}_i$: parameterization

N_j : normalization factor
fixed to 1 for forward data
free for large-angle data

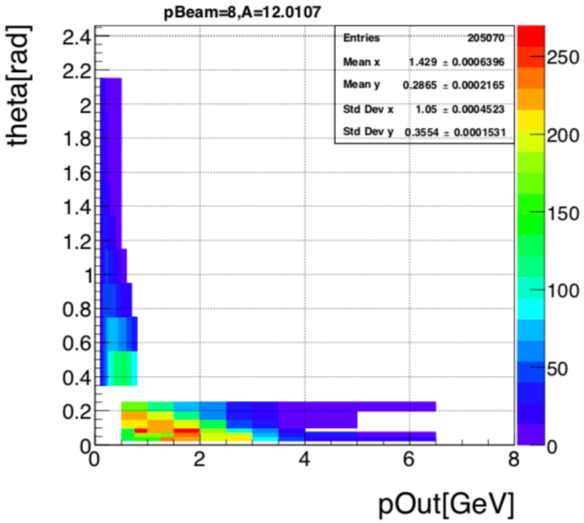
chi2 / NDF of BMPT fit

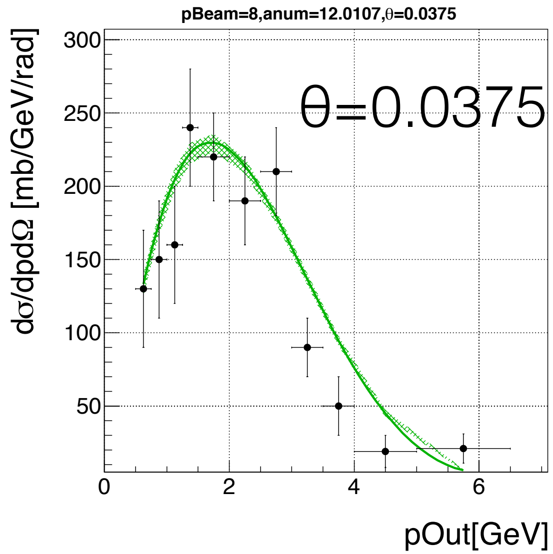
beam [GeV]	3	5	8	12
chi2 / NDF	125 / 80	151 / 97	338 / 129	304 / 135
N_1/N_0	2.05	1.7	1.6	1.6

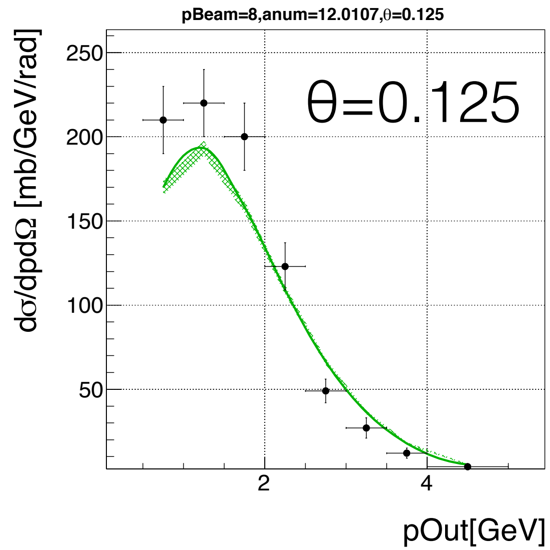
- we need **1.6~2** normalization factors
absolute value is inconsistent between forward and large-angle data

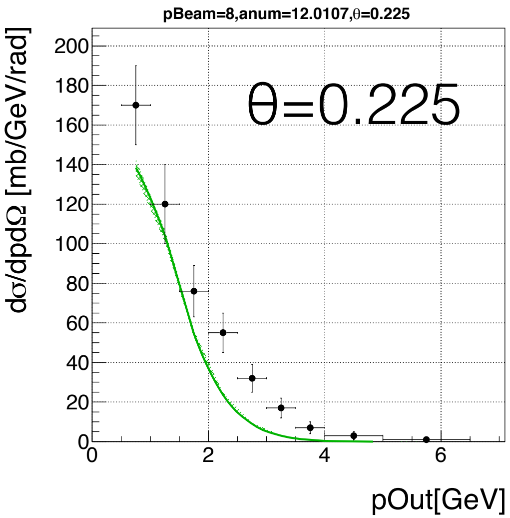
e.g.) BMPT fit result @ HARP-8GeV $p+C \rightarrow \pi^{+}+X$

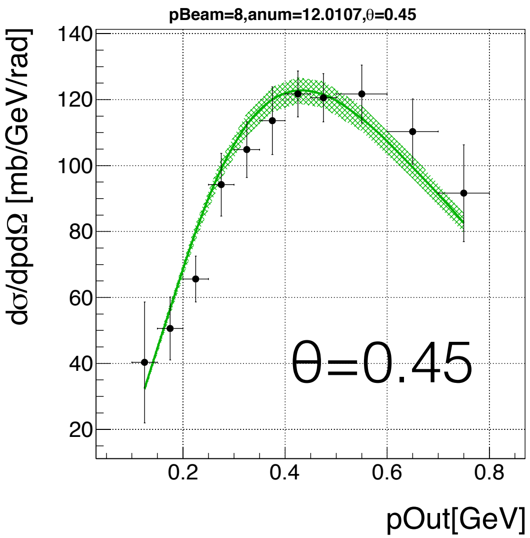
measured data

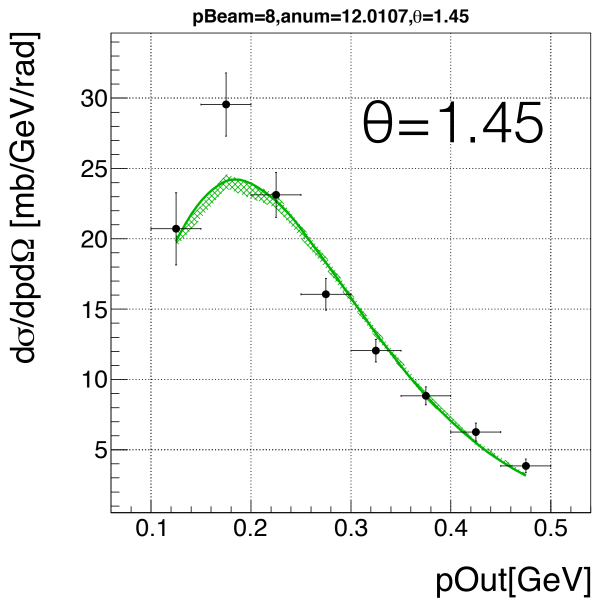












fitting (forward data only)

- use data whose beam momenta are close to each other

HARP p+(Be,C,Al) **BNLE910 p+Be** **NA61 p+C**

chi2/NDF of p+A → π^+ + X

beam P[GeV]	3,5	5, 6.4 ,8	8,12, 12.3	12,12.3,17.5	17.5, 31
BMPT	114.268 / 91	490.398 / 265	820.649 / 422	658.817 / 327	972.313 / 480

chi2/NDF of p+A → π^- + X

beam P[GeV]	3,5	5, 6.4 ,8	8,12, 12.3	12,12.3,17.5	17.5, 31
BMPT	40.8346 / 82	296.486 / 248	580.453 / 403	528.442 / 317	**1470. / 513**

p+A → K^+ + X

beam P[GeV]	31
BMPT	81.5 / 88

p+A → K^- + X

beam P[GeV]	31
BMPT	122 / 80

p+A → **p** + X

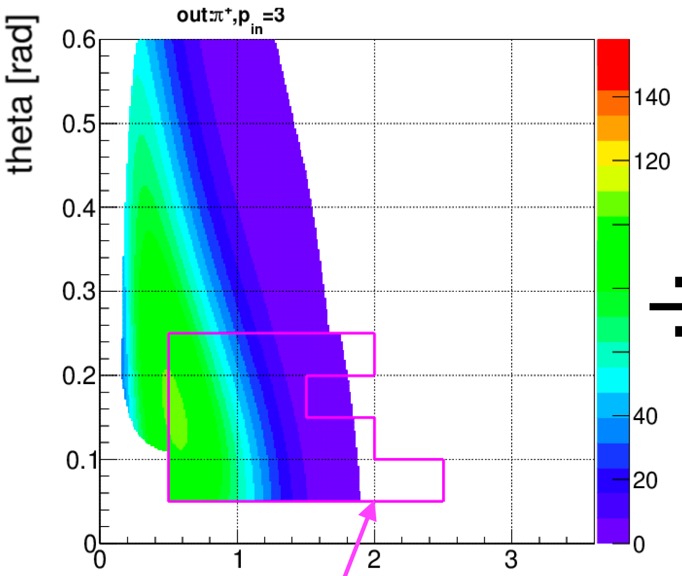
beam P[GeV]	31
BMPT	193 / 177

weight tables

- make weight tables for 27 momenta of incident particles
  - every 0.5 GeV/c for $3 \leqq p_{in} \leqq 10$ GeV/c
  - + p_{in} = 11, 12, 14, 16 17.5, 20, 25, 31.6, 50, 100, 300 GeV/c

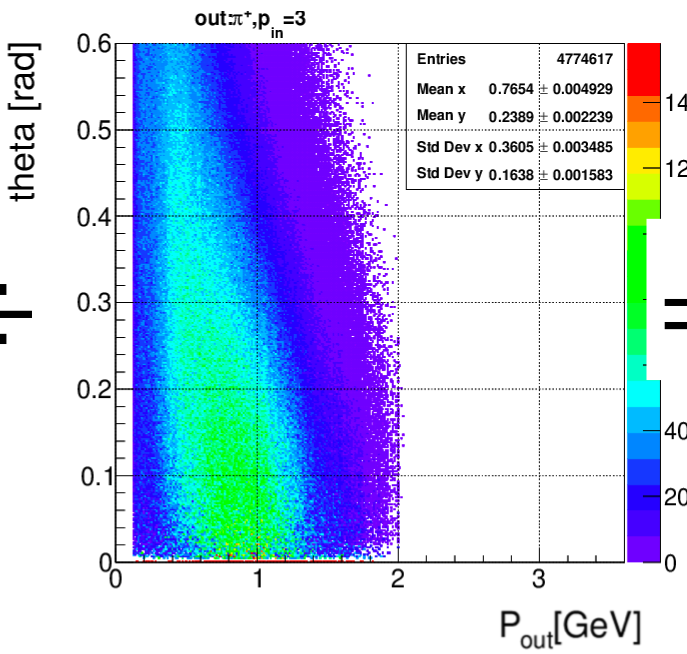
e.g.) p=3GeV $\frac{d\sigma}{dp d\Omega}$ @ 3GeV

HARP 3-5 GeV fit result



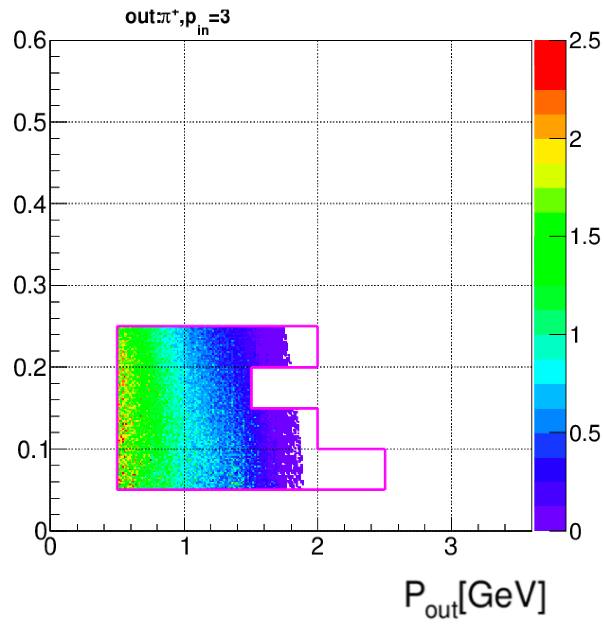
÷

ATMNC



=

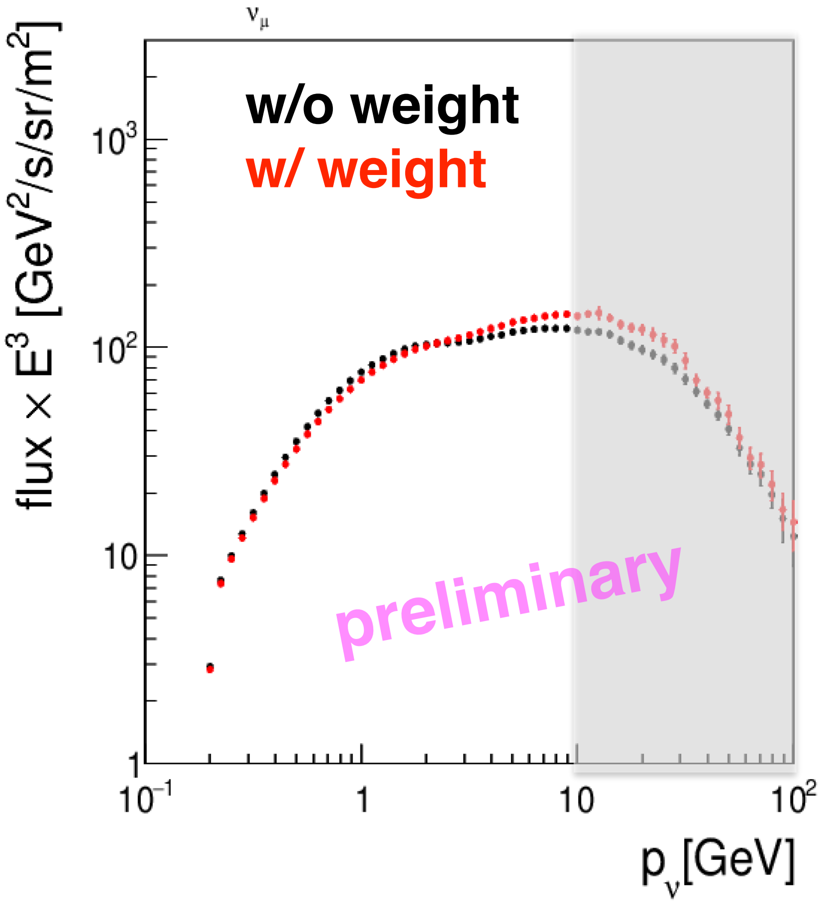
weight table
(ratio: data-fit / ATMNC)

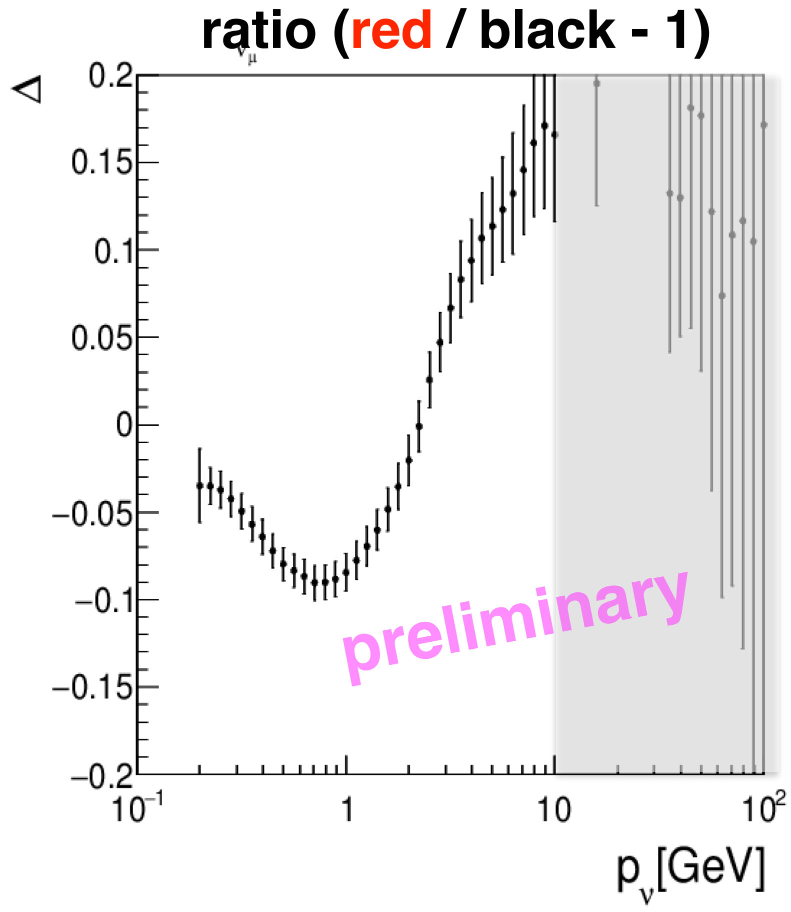


covered region
by HARP 3 GeV/c forward data

modification from original

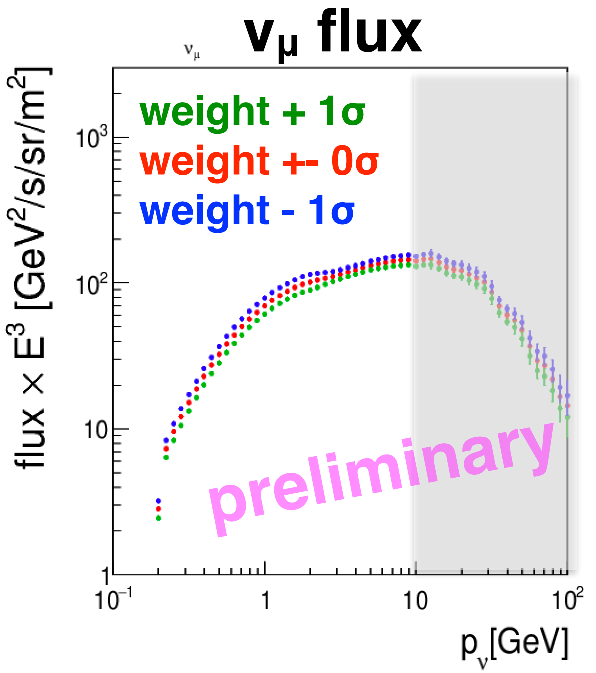
calculate flux by using only regions which covered by weight tables

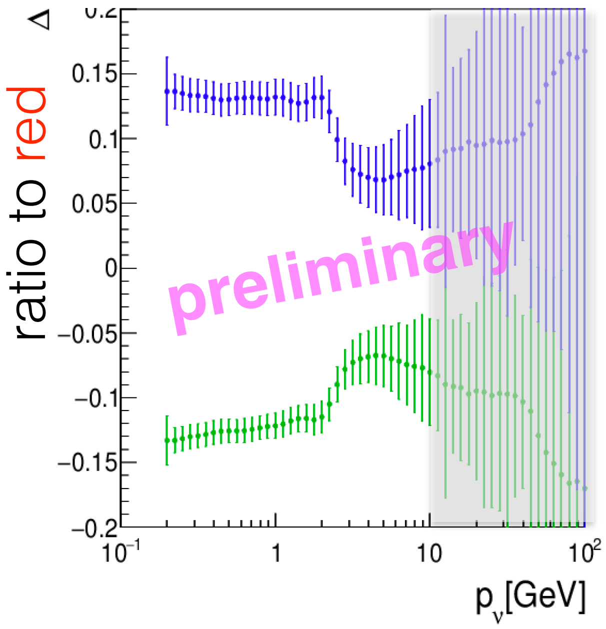




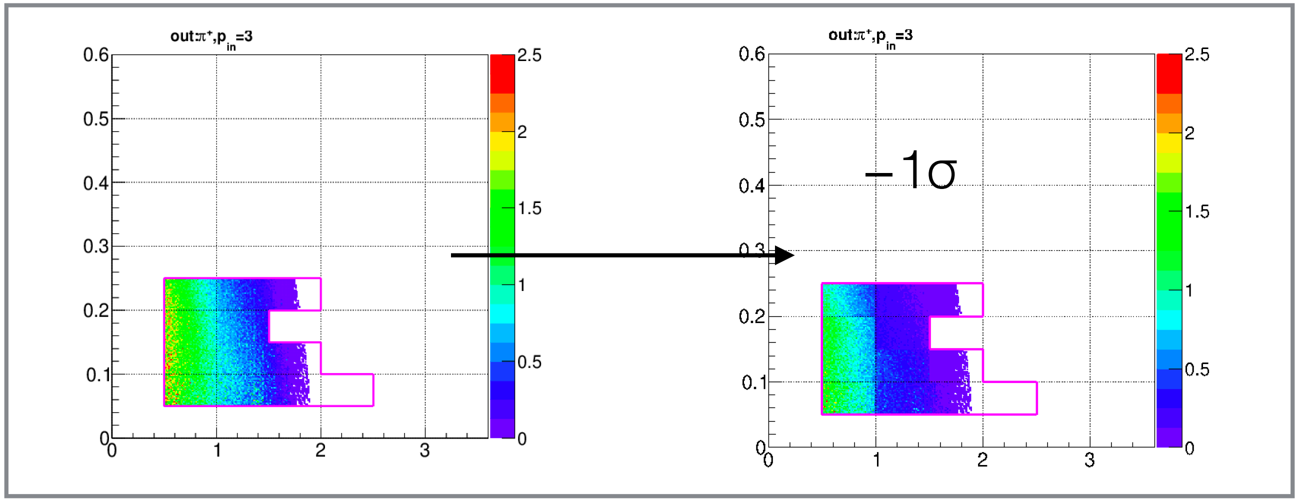
the flux change is -10%–15% = similar to Honda flux uncertainty

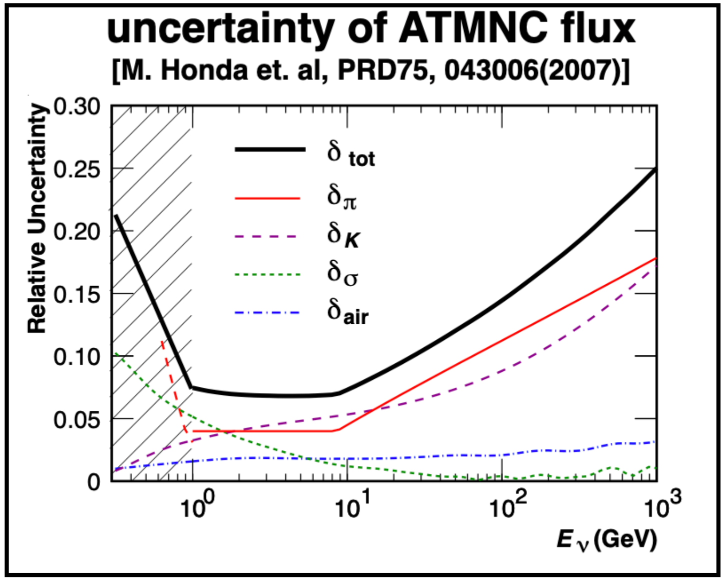
impact of beam data error





modify the *weight* by 1σ beam data





14% at sub-GeV region

→ similar to current ATMNC uncertainty

Summary

To reduce uncertainty of **Honda flux simulation** in <1 GeV region…

- we need …
  - **p + A → $\pi^{+,-}$ + X** & **p + A → p + X**
  - proton beam momentum : **~10 GeV/c**

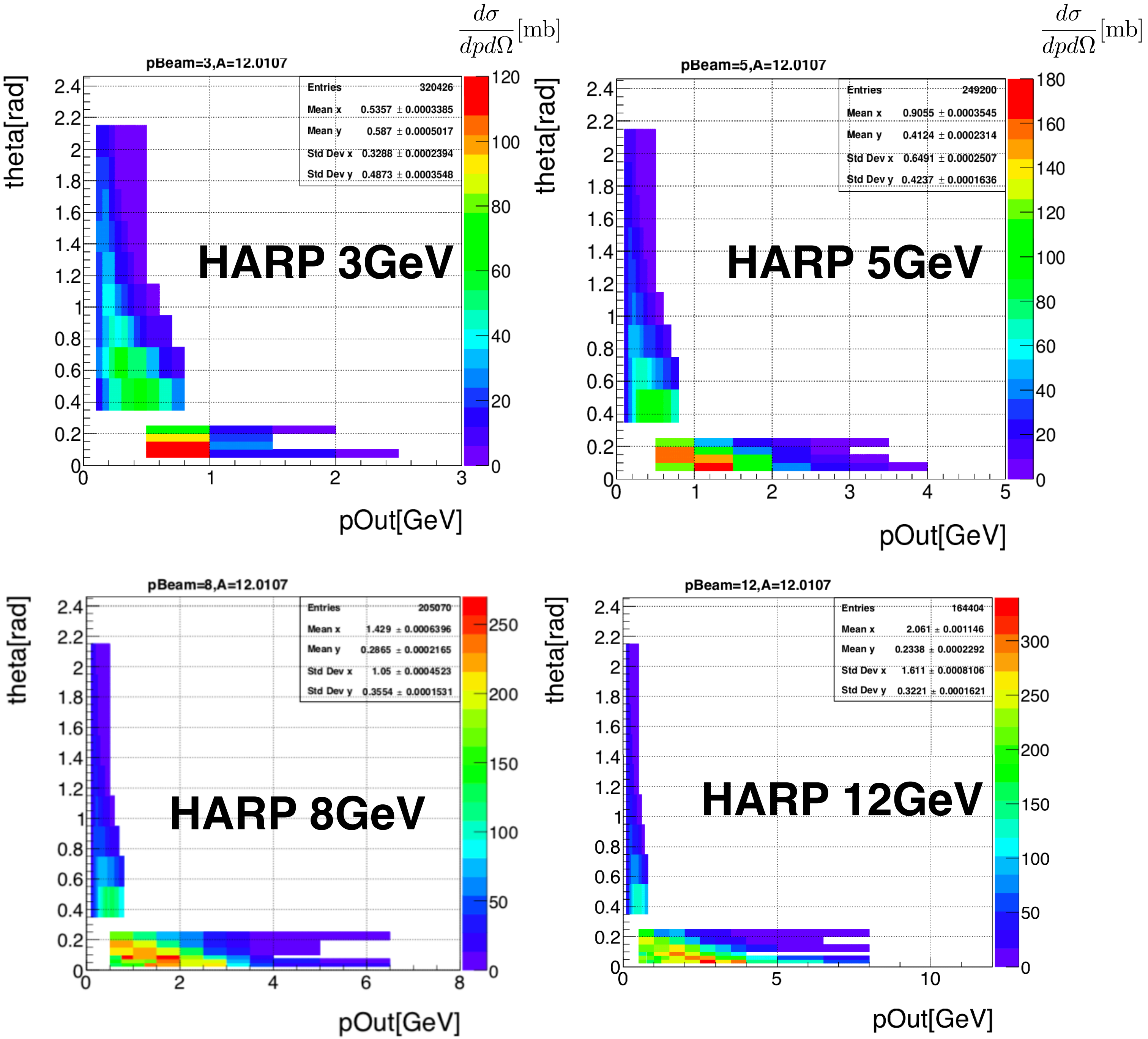
- phase space coverage of HARP and BNL-E910 data
  - only π data are available -> we want **low E proton production data**
  - coverage of phase space : **~ 50% of π production** for 0.3 GeV/c neutrino

- Parameterization to extrapolate the data
  - test several parameterizations
    - failed to fit the forward and large angle data simultaneously
    → precise measurements with low-E beam will help to find proper parameterization

- test weighting method (preliminary, using forward data only)
  - **1σ beam data error** fluctuates ν flux by **~14%**

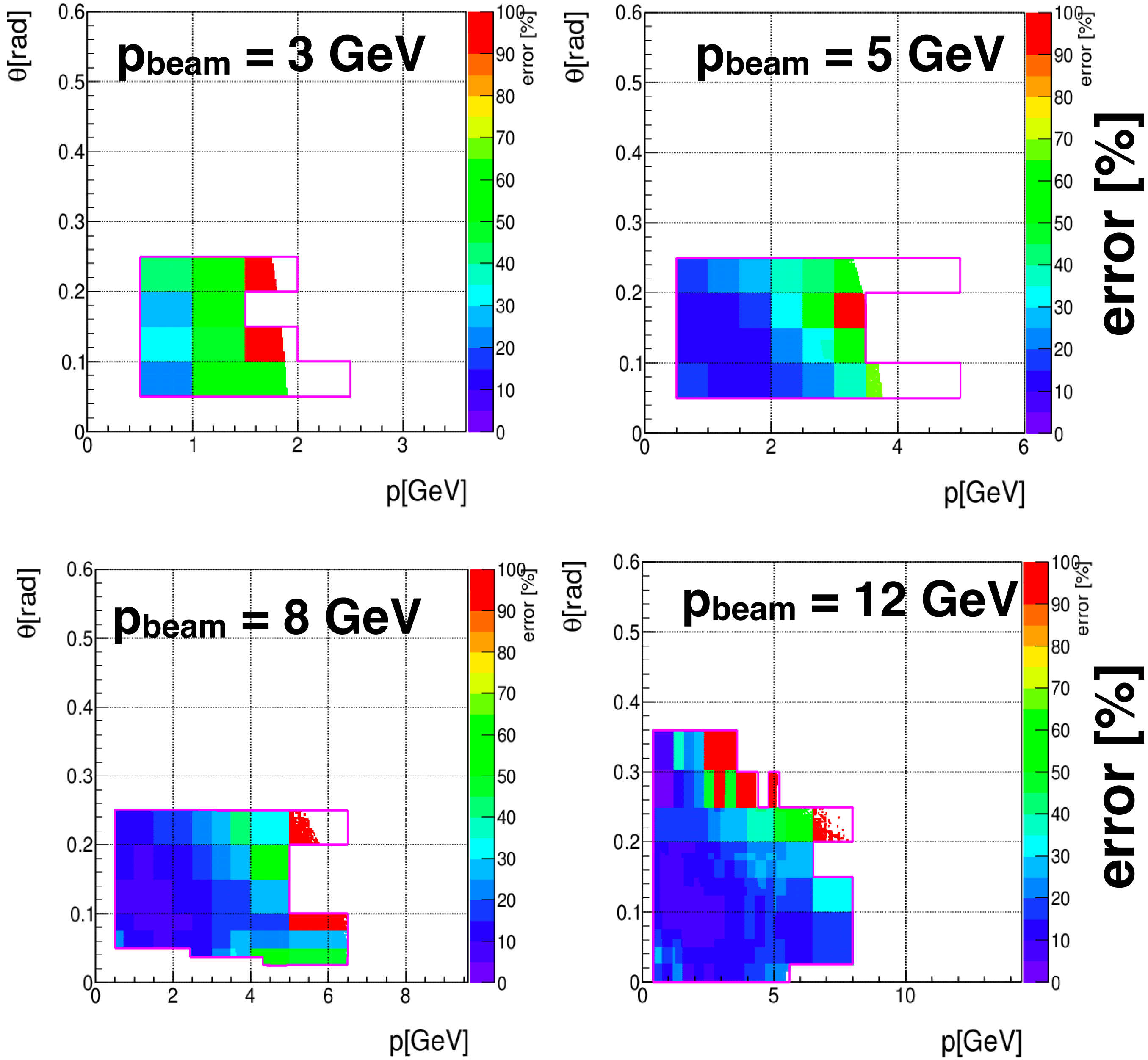
backup

HARP's data


pBeam=3,A=12.0107
Entries 320426
Mean x 0.5357 ± 0.0003385
Mean y 0.587 ± 0.0005017
Std Dev x 0.3288 ± 0.0002394
Std Dev y 0.4873 ± 0.0003548
HARP 3GeV
theta[rad]
pOut[GeV]
$\frac{d\sigma}{dpd\Omega}$[mb]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
0
1
2
3
0
20
40
60
80
100
120
pBeam=5,A=12.0107
Entries 249200
Mean x 0.9055 ± 0.0003545
Mean y 0.4124 ± 0.0002314
Std Dev x 0.6491 ± 0.0002507
Std Dev y 0.4237 ± 0.0001636
HARP 5GeV
theta[rad]
pOut[GeV]
$\frac{d\sigma}{dpd\Omega}$[mb]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
0
1
2
3
4
5
0
20
40
60
80
100
120
140
160
180
pBeam=8,A=12.0107
Entries 205070
Mean x 1.429 ± 0.0006396
Mean y 0.2865 ± 0.0002165
Std Dev x 1.05 ± 0.0004523
Std Dev y 0.3554 ± 0.0001531
HARP 8GeV
theta[rad]
pOut[GeV]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
0
2
4
6
8
0
50
100
150
200
250
pBeam=12,A=12.0107
Entries 164404
Mean x 2.061 ± 0.001146
Mean y 0.2338 ± 0.0002292
Std Dev x 1.611 ± 0.0008106
Std Dev y 0.3221 ± 0.0001621
HARP 12GeV
theta[rad]
pOut[GeV]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
0
5
10
0
50
100
150
200
250
300

uncertainty of HARP data (forward)

p+(C, Be, Al)→ π + X



parameterization functions

ref:

- S.R. Blattnig et al., PRD, 62, 094030 (2000)
- M. Bonesini et al., Eur. Phys. J. C 20 (2001)
- C. Mariani et al., PRD, 84, 114021 (2011)

Sanford & Wang (1967)

$$\frac{d^2\sigma^\pi}{dp d\Omega}(p,\theta) \quad = \quad c_1 p^{c_2}\left(1-\frac{p}{p_{\mathrm{beam}}}\right)\exp\left[-c_3\frac{p^{c_4}}{p_{\mathrm{beam}}^{c_5}} - c_6\theta\left(p - c_7 p_{\mathrm{beam}}\cos^{c_8}\theta\right)\right]$$

Badhwar (1977)

$$E\frac{d^3\sigma}{d^3p} = \frac{A}{(1+4m_p^2/s)^r}(1-\tilde{x})^q \exp\left[\frac{-Bp_T}{1+4m_p^2/s}\right] \qquad \tilde{x} \equiv \left[x_F^2 + \frac{4}{s}\left(p_T^2+m^2\right)\right]^{1/2} \quad q = \frac{C_1 + C_2 p_T + C_3 p_T^2}{\sqrt{1+4m_p^2/s}}$$

Mokhov (1998)

$$E\frac{d^3\sigma}{d^3p} = A\left(1 - \frac{p}{p_{\max}}\right)^B \exp\left(-\frac{p}{C\sqrt{s}}\right)V_1(p_T)V_2(p_T)$$

$$V_1(p_T) = (1-D)\exp(-Ep_T^2) + D\exp(-Fp_T^2) \quad \text{for } p_T \le 0.933 \text{ GeV}$$

$$= \frac{0.2625}{(p_T^2+0.87)^4} \quad \text{for } p_T > 0.933 \text{ GeV}$$

$$V_2(p_T) = 0.7363\exp(0.875p_T) \quad \text{for } p_T \le 0.35 \text{ GeV}$$

$$= 1 \quad \text{for } p_T > 0.35 \text{ GeV.}$$

Mariani (2011), (Mariani et al, Phys. Rev. D 84, 114021)

$$\frac{d^2\sigma}{dpd\Omega} = \frac{p_K^2}{E_K}\left(E_K\frac{d^3\sigma}{dp_K^3}\right) = \left(\frac{p_K^2}{E_K}\right)c_1 \times$$
$$\times \exp\left[c_3\,|x_F|^{c_4} - c_7\,|p_T \times x_F|^{c_6} - c_2 p_T - c_5 p_T^2\right]$$

BMPT (2001) (M. Bonesini et al., Eur. Phys. J. C 20 (2001))

$$E\frac{d^3\sigma}{dp^3} = A(1-x_R)^\alpha(1+Bx_R)x_R^{-\beta}\times$$
$$[1+\frac{a}{x_R^\gamma}p_T + \frac{a^2}{2x_R^\delta}p_T^2]e^{-a/x_R^\gamma p_T}$$

$$x_R \equiv \frac{E_{CM}}{\max\{E_{CM}\}}$$

: radial scaling assume that x_R is independent from E_{beam} **[F. E. Taylor et al., PRD 14, 1217 (1976)]**

consistency : forward ⇄ large-angle

simultaneous fit of HARP p+C forward and large-angle data

chi2 / NDF of BMPT fit

beam [GeV]	3	5	8	12
chi2 / NDF	125 / 80	151 / 97	338 / 129	304 / 135
N_1/N_0	2.05	1.7	1.6	1.6

chi2 / NDF of Mariani fit

beam [GeV]	3	5	8	12
chi2 / NDF	failed	132 / 98	318 / 129	304 / 135
N_1/N_0	—	1.7	1.7	1.7

chi2 / NDF of Mokhov fit

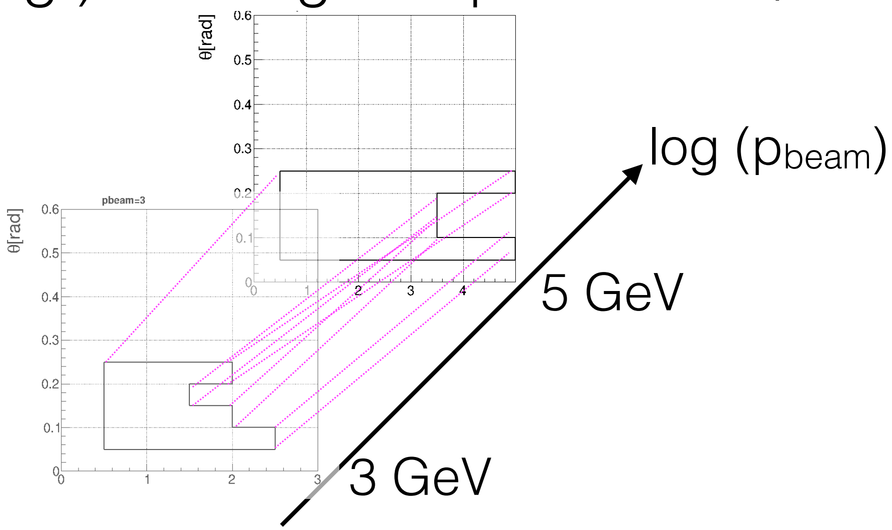
beam [GeV]	3	5	8	12
chi2 / NDF	153 / 84.	178 / 101	363 / 133	335 / 139
N_1/N_0	2.5	1.6	1.7	1.7

- we need 1.6~2 normalization factors
  absolute value is inconsistent between forward and large-angle data

phase space coverage of beam data

to estimate the phase space coverage of beam data,
interpolate phase spaces between neighboring beam momentums.

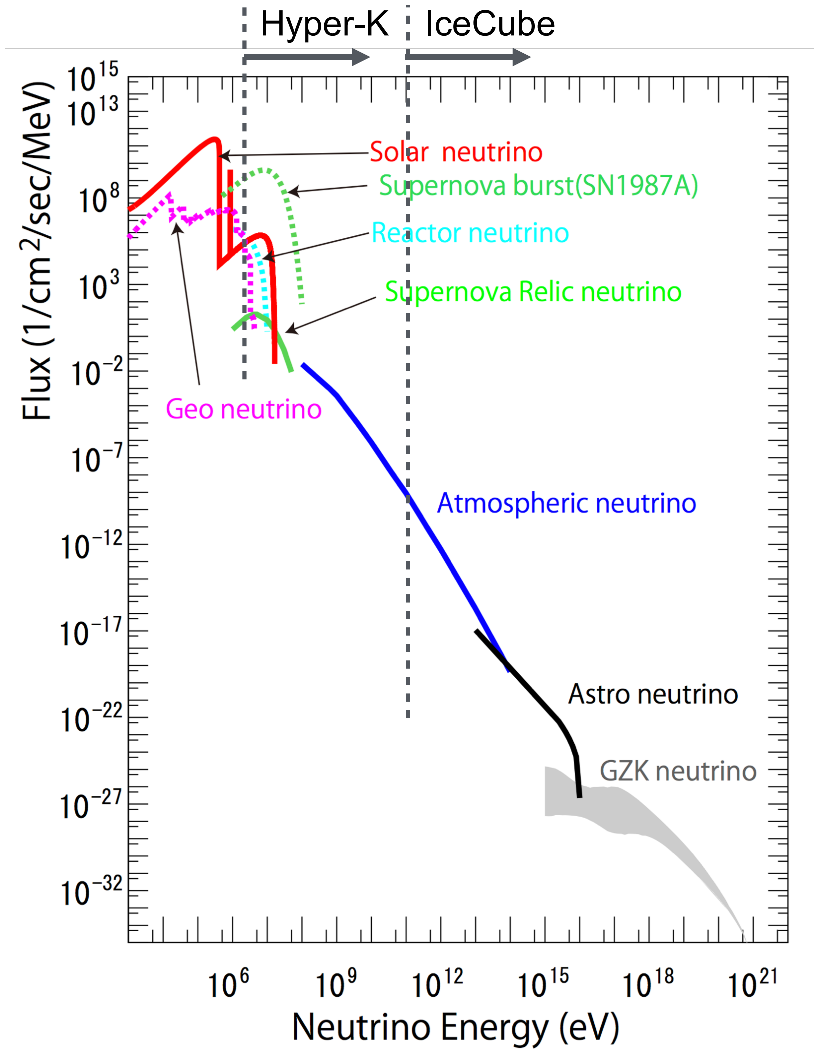
e.g.) coverage for p_{in} = 4 GeV/c



linearly interpolate phase spaces of HARP-3GeV and HARP-5GeV data

- for p+A→π+X
  - interpolate *3-5, 5-6.4, 6.4-8, 8-12, 12-17.5, 17.5-31GeV/c*

neutrino flux

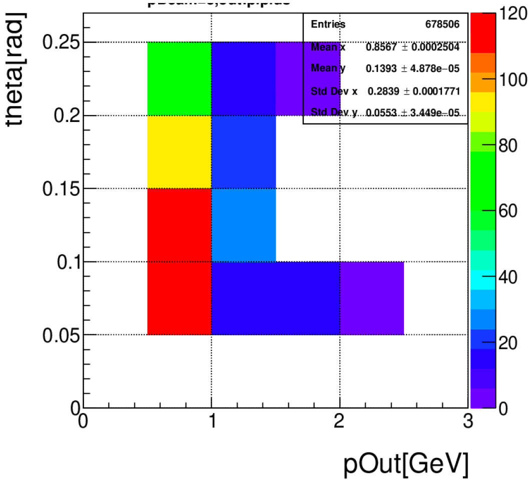


from Yano-san's slide
@ lowBG simposium 2019 in Sendai, Japan

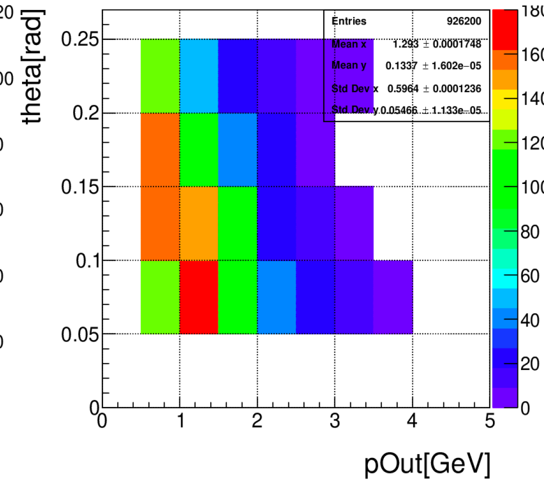
used beam data

HARP p+C → π^+ + X

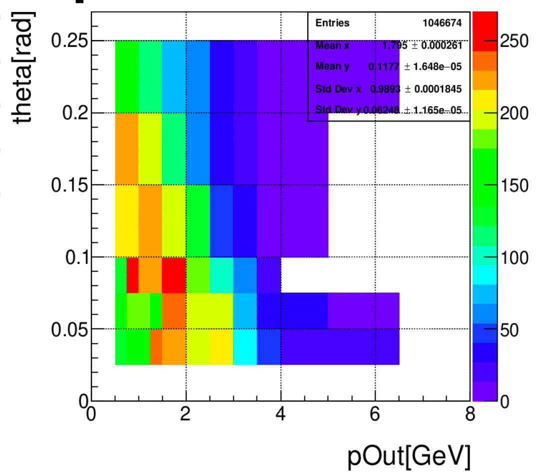
p_{beam}=3 GeV/c



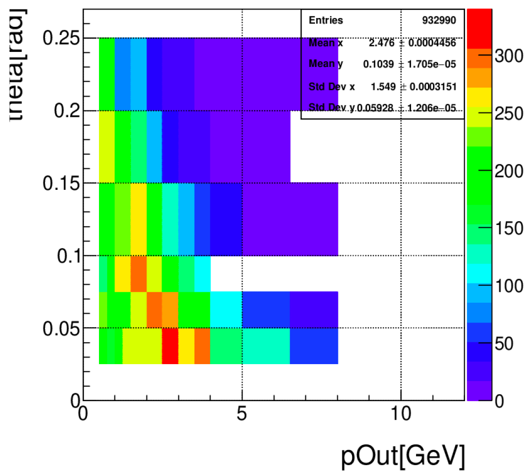
p_{beam}=5 GeV/c



p_{beam}=8 GeV/c

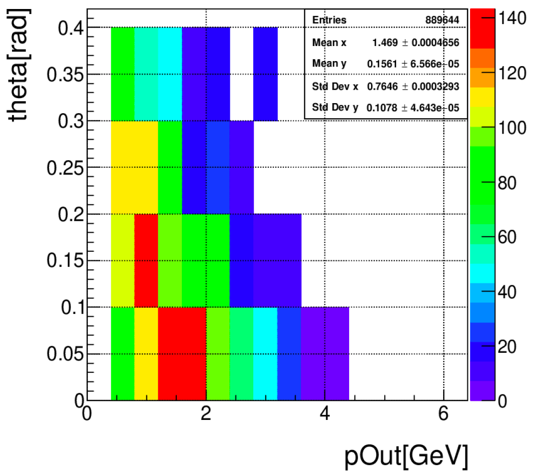


p_{beam}=12 GeV/c

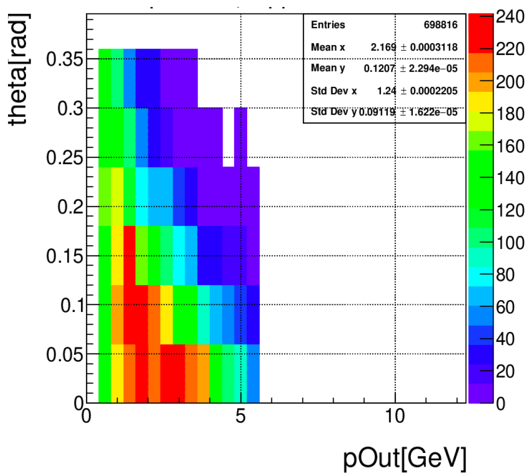


BNL p+Be → π^+ + X

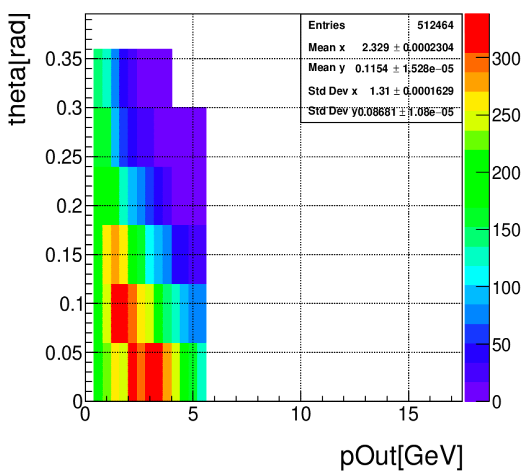
p_{beam}=6.4 GeV/c



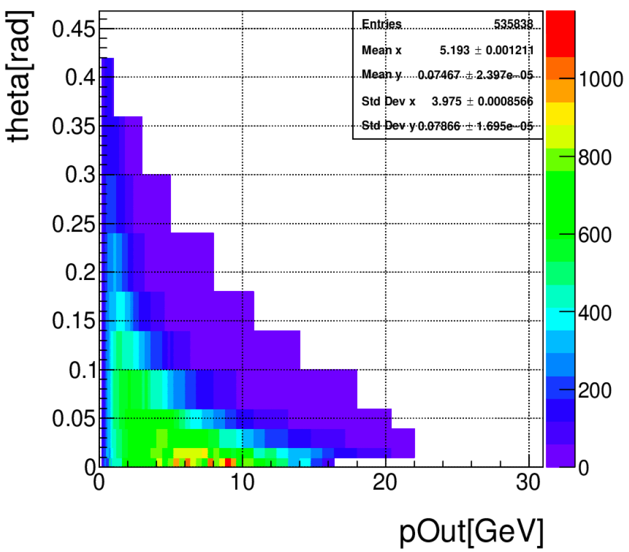
p_{beam}=12.3 GeV/c



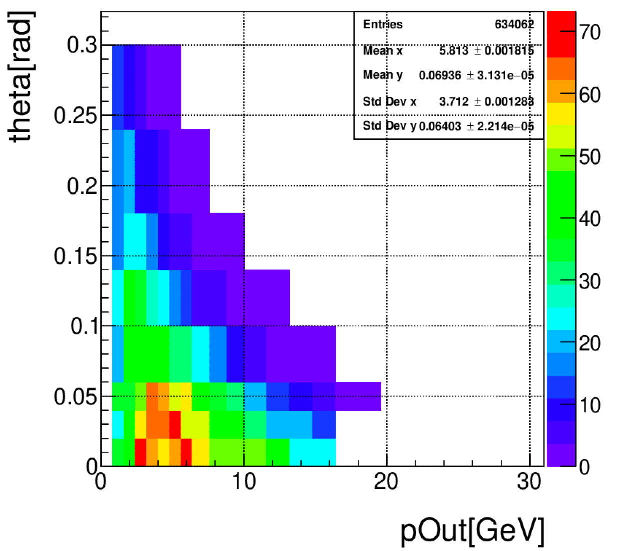
p_{beam}=17.5 GeV/c



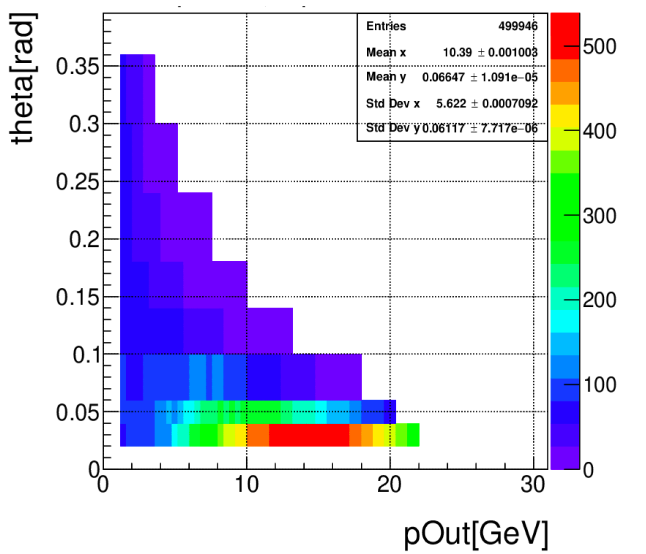
NA61 p+C → π⁺ + X



NA61 p+C → K⁺ + X

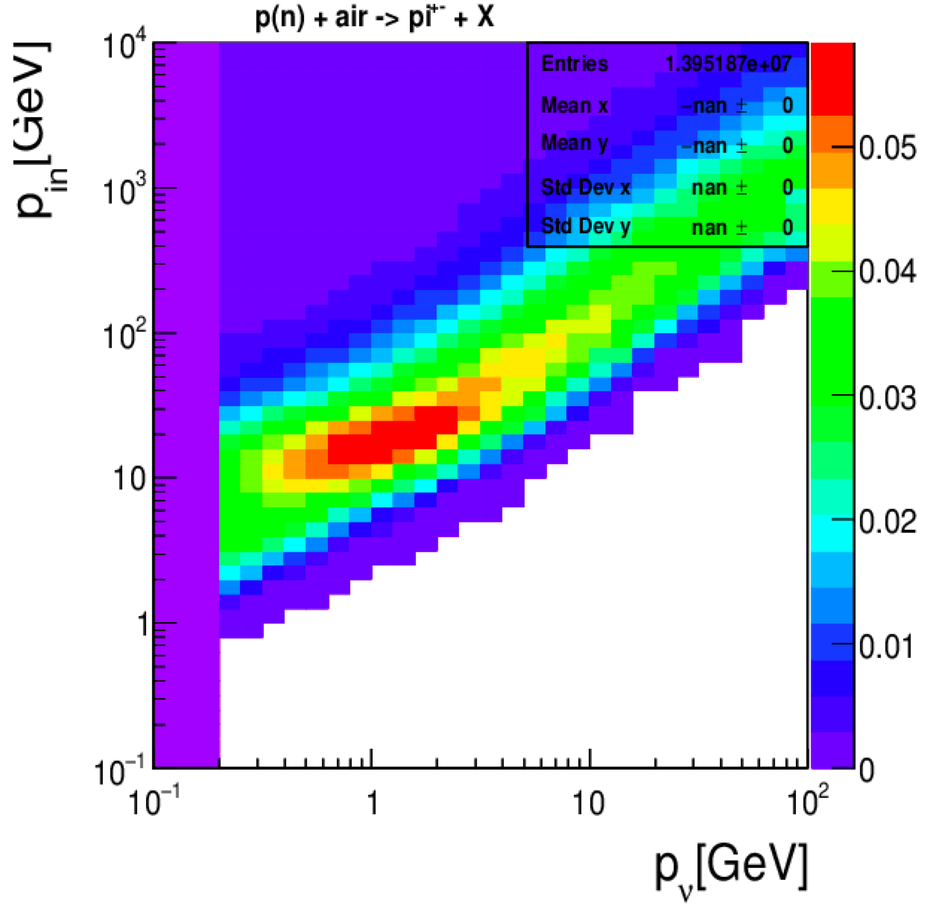


NA61 p+C → p + X



proton E vs neutrino E

p + Air → π^{+-} + X



fit result

fit (each)

- summary of p+ A → π + X
  - p_{beam} >= 17.5 GeV
    - SW can't fit with data
  - p_{beam} <= 12.3 GeV
    - generally SW is better than BMPT
    - both SW and BMPT are not good for HARP 8 GeV π^+, p+C & p+Be data

---

GeV/c , A	31 on C	450 on Be
p+A→**K⁺**+X	81.5 / 88	
p+A→**K⁻**+X	122 / 80	
p+A→**p**+X	193. / 177.	

fit χ^2 / NDF (fit for each beam data)

$p+A \rightarrow \pi^+ + X$

* **bold** : χ^2/NDF > ~2

BMPT

GeV	3	5	6.4	8	12	12.3	17.5	31	450
p+Be	3.5/3	21.7/18	10.1/23	**121/51**	**112/57**	64.8/64	**115/67**		
p+C	0.5/3	**34.0/17**		**113/49**	69.0/55			582/407	
p+Al	**3.0/1**	23.5/19		83.2/51	69.6/58				

Sandford-Wang

GeV	3	5	6.4	8	12	12.3	17.5	31	450
p+Be	—	23.8/21	10.3/23	**110/51**	87.3/57	89.2/64	**225/67**		
p+C	1.5/4	22.4/18		**99.5/49**	58.6/55			**2390 /407**	
p+Al	**3.2/1**	17.6/22		61.0/52	52.7/58				

fit χ^2 / NDF (fit for each beam data)

$p+A \rightarrow \pi^- + X$

* **bold** : χ^2/NDF > ~2

BMPT

GeV	3	5	6.4	8	12	12.3	17.5	31	450
p+Be	**4.1/2**	6.7/12	33.5/22	55.6/47	76.0/52	**126/57**	**125/67**		
p+C	1.29/1	10.2/14		52.1/49	65.5/51			**1185 /438**	
p+Al	0.84/2	6.6/13		58.0/47	75.6/56				

Sandford-Wang

GeV	3	5	6.4	8	12	12.3	17.5	31	450
p+Be	4.2/4	5.9/14	37.6/24	52.1/49	61.3/54	80.5/59	**143/69**		
p+C	1.7/3	10.7/16		55.4/47	56.1/53			**2870 /440**	
p+Al	0.84/2	6.9/15		50.0/49	77.5/58				

simultaneous fit over different beam P

$$\chi^2 = \sum_j^{beam} \{ N_j \times \sum_i^{point} \left(\frac{X_i - f\left(p_{beam;j}, p_i, \theta_i\right)}{\sigma_i} \right)^2 \}$$

p+**Be** → **π⁺** + X

GeV	HAPR 3,5,8,12	6.4	12.3	17.5
BMPT	**284.072 / 141**	x	x	x
BMPT	**614.741 / 315**			
SW	**265.575 / 147**	x	x	x
SW	**495 / 246**			x

p+**Be** → **π⁻** + X

HARP 3,5,8,12	6.4	12.3	17.5
142.015 / 131	x	x	x
366.479 / 303			
134.605 / 133	x	x	x
295.8 / 230			x

p+**C** → **π⁺** + X

GeV	HAPR 3,5,8,12	NA61 31
BMPT	855.375 / 551	

p+**C** → **π⁻** + X

GeV	HAPR 3,5,8,12	NA61 31
BMPT	899.087 / 577	

fit χ^2 / NDF (neighboring beam energy)

HARP p+(Be,C,Al) **BNLE910 p+Be** **NA61 p+C**

$p+A \to \boldsymbol{\pi^+} + X$

	3,5	5, 6.4 ,8	8,12, 12.3	12,12.3,17.5	17.5, 31
BMPT	114.268 / 91	490.398 / 265	**820.649 / 422**	**658.817 / 327**	**972.313 / 480**
SW	116.595 / 100	499.225 / 272	**922.322 / 422**	—	—

$p+A \to \boldsymbol{\pi^-} + X$

	3,5	5, 6.4 ,8	8,12, 12.3	12,12.3,17.5	17.5, 31
BMPT	40.8346 / 82	296.486 / 248	580.453 / 403	528.442 / 317	**1470. / 513**
SW	42.5501 / 84	265.481 / 250	524.49 / 405	—	—

$p+A \to \boldsymbol{K^+} + X$

	31
BMPT	81.5 / 88

$p+A \to \boldsymbol{K^-} + X$

	31
BMPT	122 / 80

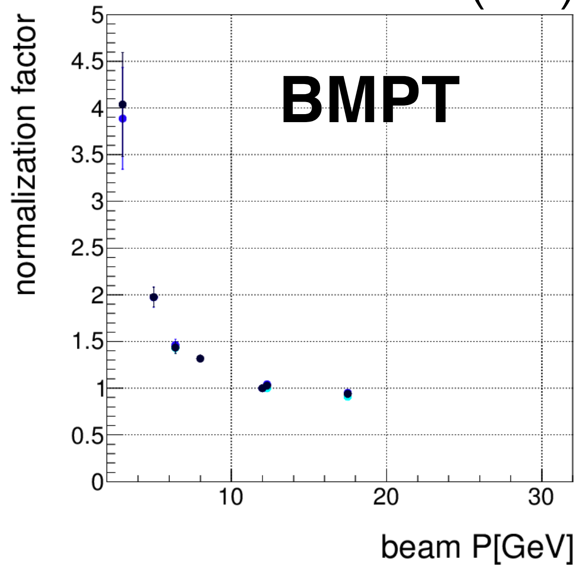
$p+A \to \boldsymbol{p} + X$

	31
BMPT	193 / 177

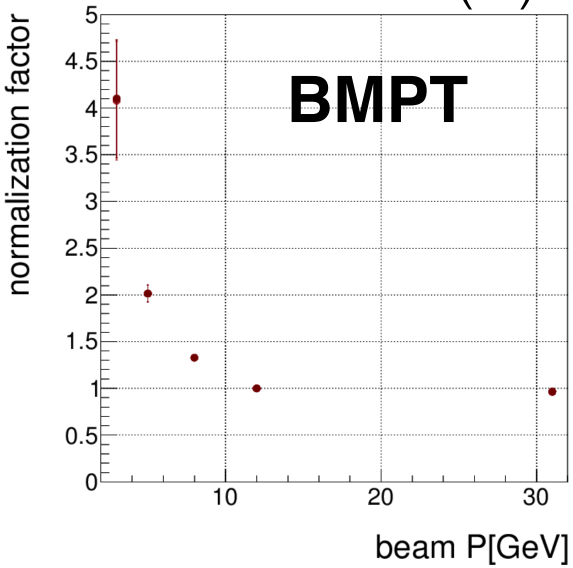
normalization factor

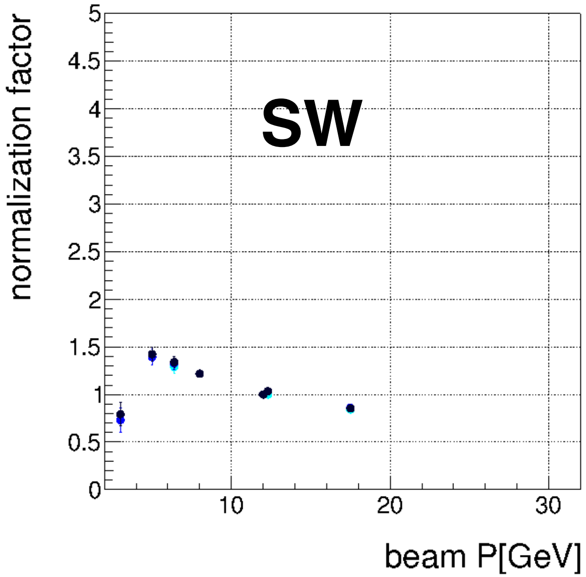
$$\chi^2 = \sum_{j}^{beam} \{N_j \times \sum_{i}^{\text{point}} \left(\frac{X_i - f\left(p_{beam;j}, p_i, \theta_i\right)}{\sigma_i} \right)^2 \}$$

HARP + BNL (Be)



HARP + T2K (C)





- 断面積の絶対値は合わない。

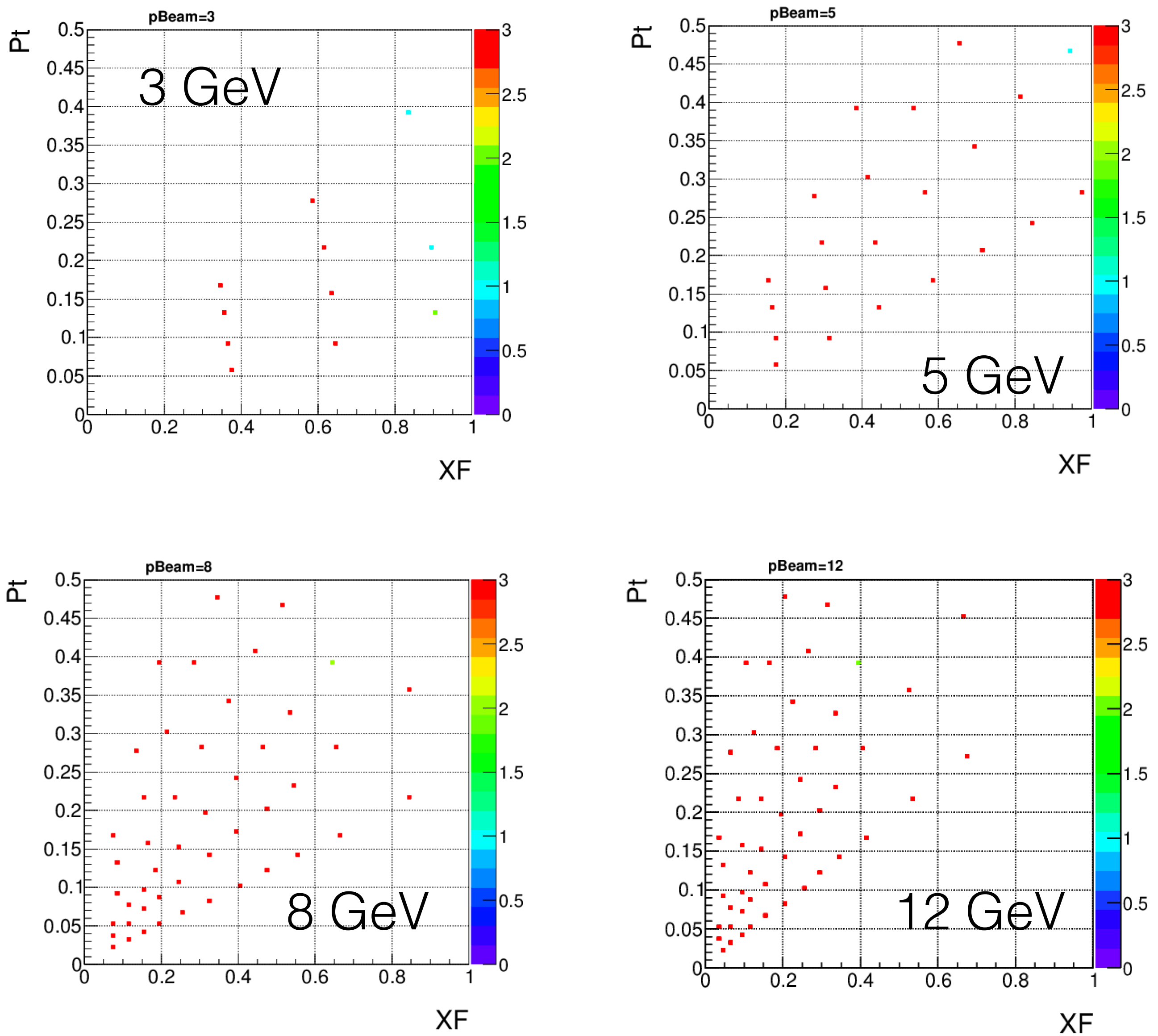
  → **横断的にfitするのは危険か。**

check for A-scaling with HARP data

HARPデータ

- p_{beam} = 3, 5, 8,12 GeVのBe, C, Alのデータを使用
  核種が2つ以上あるデータ点を使用

各p_{beam}ごとにfit



Fit

$$\chi^2 = \sum_i^{X_F,P_t,A} \left(\frac{\sigma_i - \alpha(X_F,P_t)\sigma_{Be}(X_F,P_t)}{\Delta\sigma_i} \right)^2$$

$\sigma_i \pm \Delta\sigma_i$: *i*-th data

$\sigma_{Be}(X_F,P_t)$: X_F PtにおけるBeの断面積

(nuisance parameters)

* (XF, Pt)の組み合わせごとにσ_{Be}がある

$$\alpha(x_F,p_T) = (a + bx_F + cx_F^2)(d + ep_T^2)$$

a,b,c,d,e: parameters we want

全データを使ってa~eを求める。

p_{beam}	chi2 / NDF	a	b	c	d	e
3	7.54 / 12	0.90±0.17	-0.9±0.4	-1.8±0.8	1.0 (fixed)	-2 ± 21
5	25.2 / 48	0.62±0.26	-0.2±1.3	0.1±1.5	1.0 (fixed)	-1.1±1.8
8	128 / 115	0.69±0.11	-1.0±0.7	2.8±1.2	1.0 (fixed)	0.8±0.4
12	68 / 128	0.82±0.07	-1.0±0.5	-0.6±0.9	1.0 (fixed)	0.4±0.4
ref:T2K (p_{beam} :19.2, 24)		0.75	-0.52	0.23	1.0 (fixed)	0.21

- chi2 /NDFを見る限りはデータを説明できている
- a以外のパラメータの精度は、ほぼ無い

difference of target atom

need to consider the difference of target atom

target atom in beam : **Be**, **C**, **Al**

target atom in air shower : **N**, **O** (mean of A = 14.5)

- robust thought :

  $\sigma_2 = \sigma_1 \times (A_2/A_1)^{2/3}$

- parameterization as a function of x_f and p_T is proposed

(M. Bonesini et al., Eur. Phys. J. C 20 (2001))

$$E\frac{d^3\sigma(A_1)}{dp^3} = \left[\frac{A_1}{A_0}\right]^{\alpha(x_F,p_T)} E\frac{d^3\sigma(A_0)}{dp^3}, \qquad (6)$$

where:

$$\alpha(x_F, p_T) = (a + bx_F + cx_F^2)(d + ep_T^2). \qquad (7)$$

	a	b	c	d	e
Bonesini *et al.* [43]	0.74	-0.55	0.26	0.98	0.21
Fit to π data	0.75	-0.52	0.23	1.0 (fixed)	0.21
Fit to K data	0.77	-0.32	0.0	1.0 (fixed)	0.25

$$\rightarrow \quad \frac{d\sigma}{dpd\Omega}(A) = \frac{d\sigma}{dpd\Omega}(A_{air} = 14.5)\left(\frac{A}{A_{air}}\right)^{\alpha(x_F,p_T)}$$

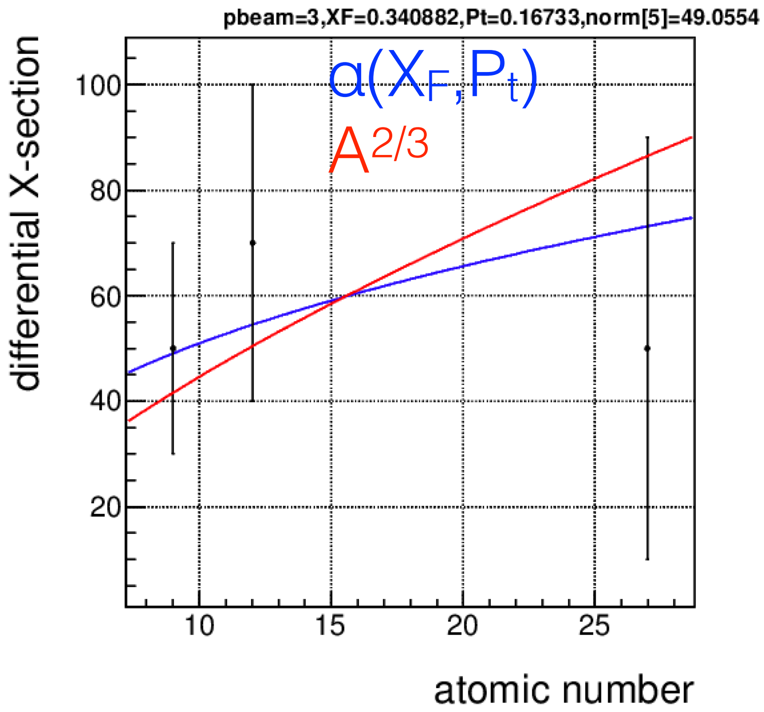
* I checked α parameterization with HARP Be, C, Al data (→ backup)

* generally, A-scaling effect is smaller than error bars of beam data

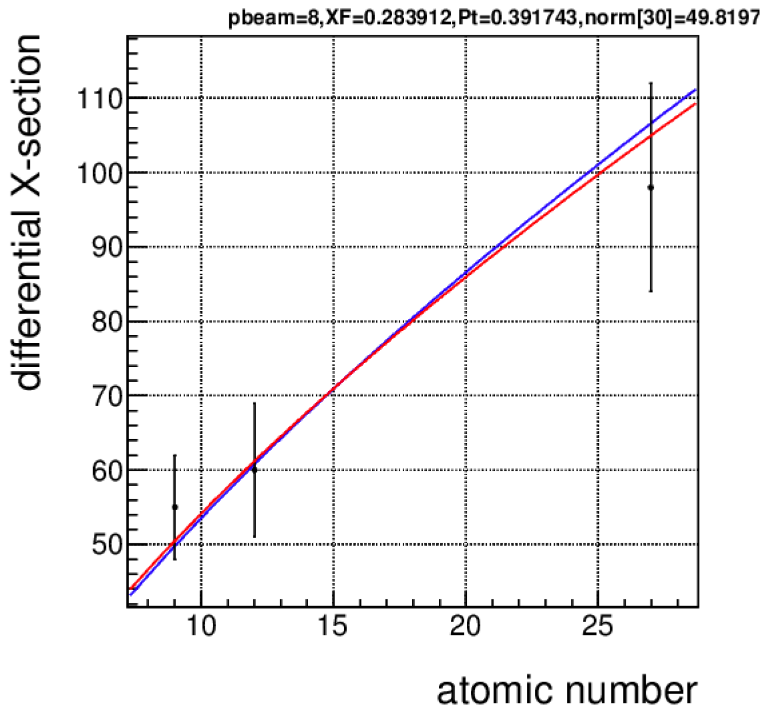
comparison with simple method

check by using HARP Be, C, and Al data

ex) p_{beam} = 3GeV



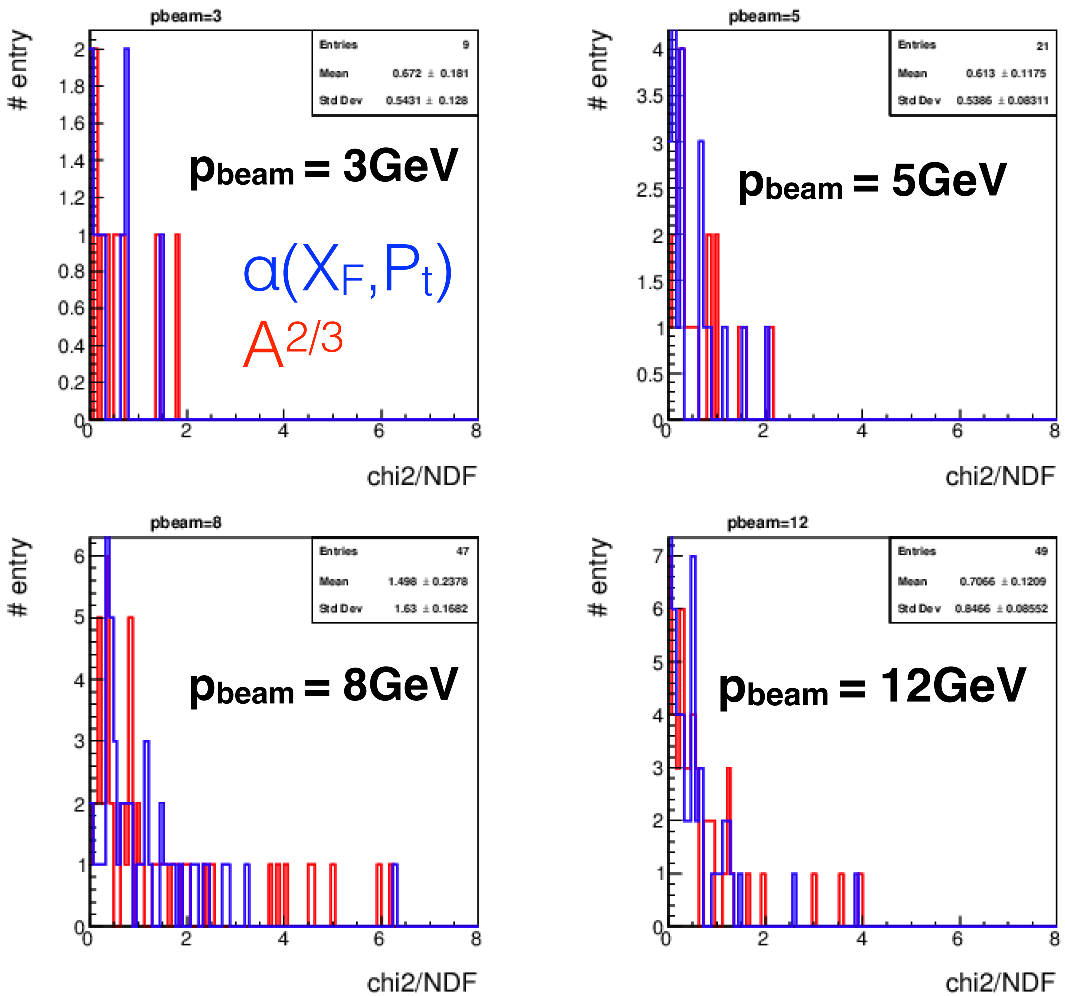
ex) p_{beam} = 8GeV



- generally speaking, the difference between $A^{2/3}$ and $\alpha(X_F,P_t)$ is small compared to the error bar.

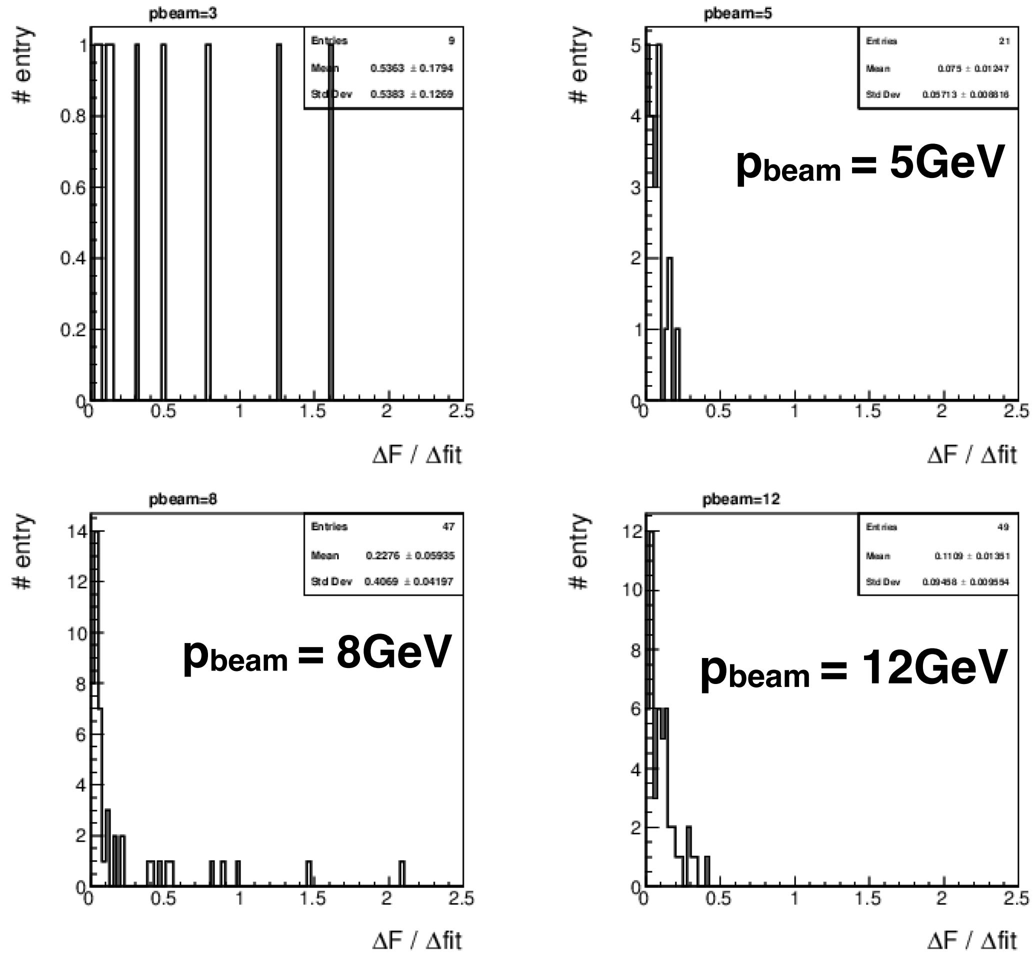
chi2, difference

chi2 / NDF for each (X_F, Pt)



ΔF / Δfit for each (X_F, Pt)

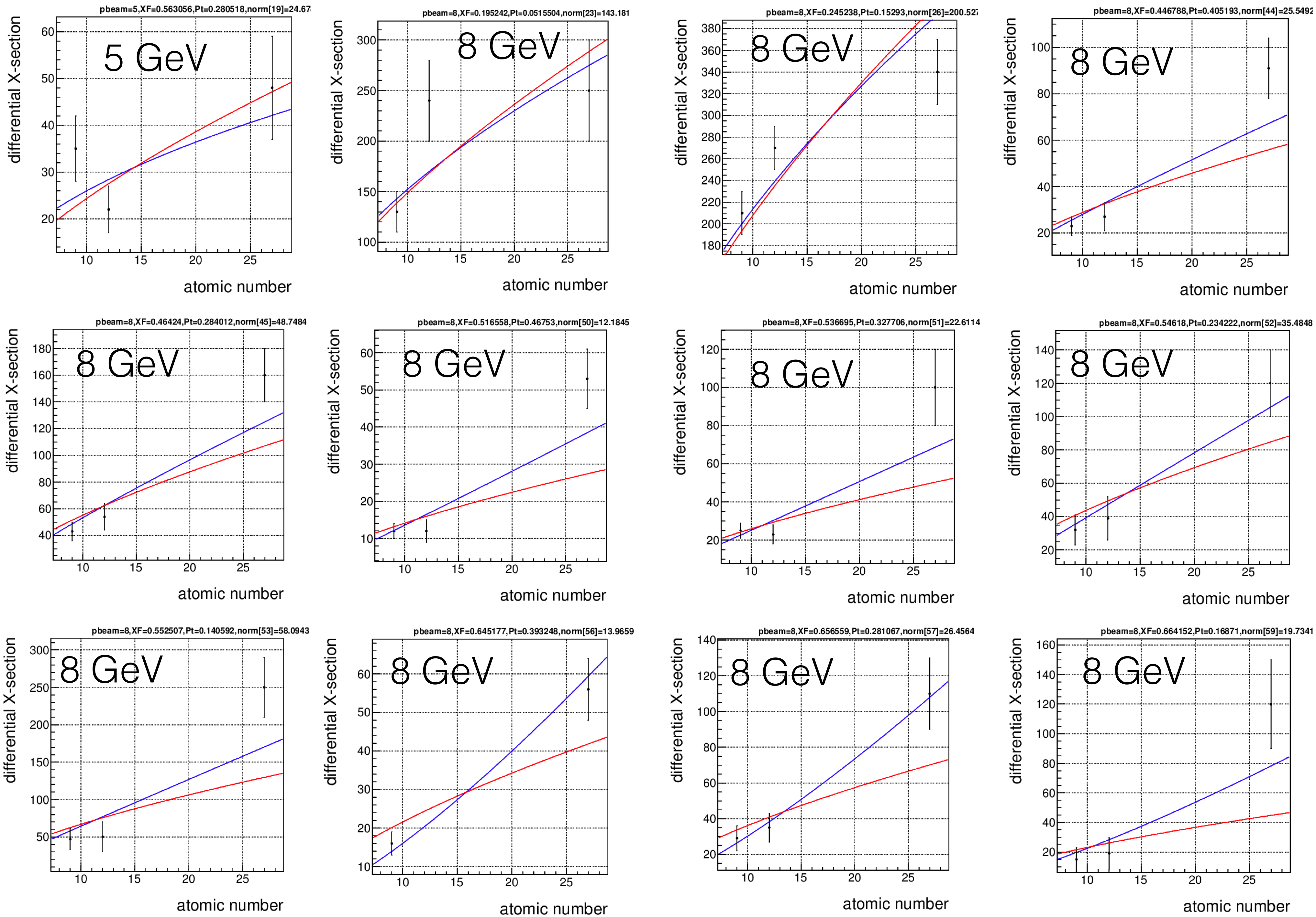
p_{beam} = 3GeV



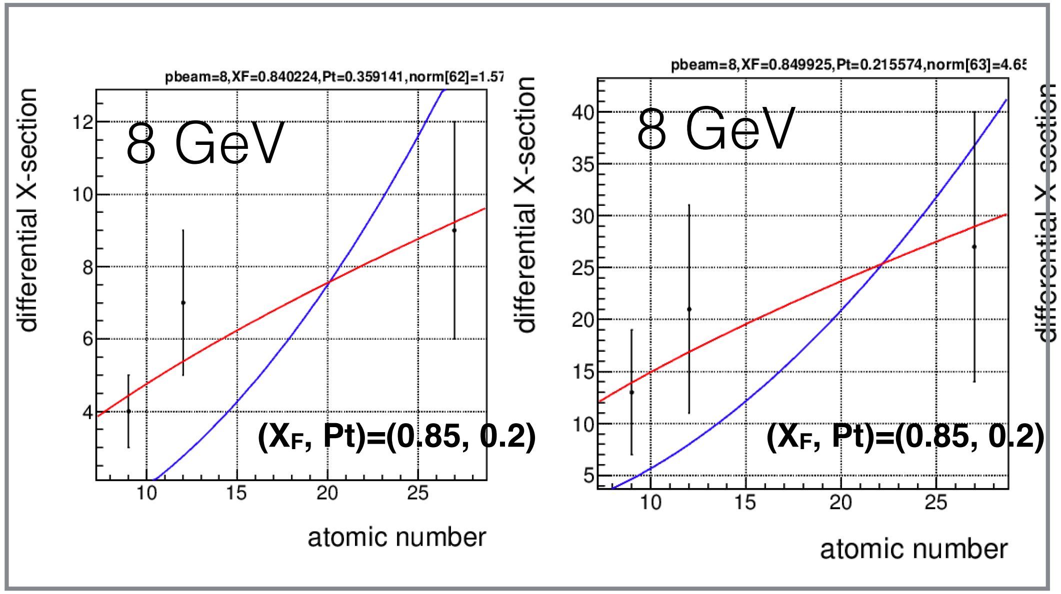
$$\Delta F \equiv |\sigma_{Be;1}\alpha(X_F, P_t) - \sigma_{Be;2}(A/A_{Be})^{2/3}|$$

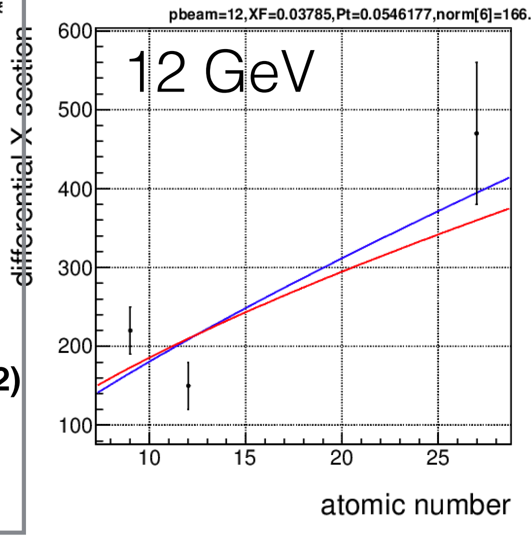
$$\Delta fit \equiv (A/A_{Be})^{2/3} \times \text{fitting error of } \sigma_{Be;2}$$

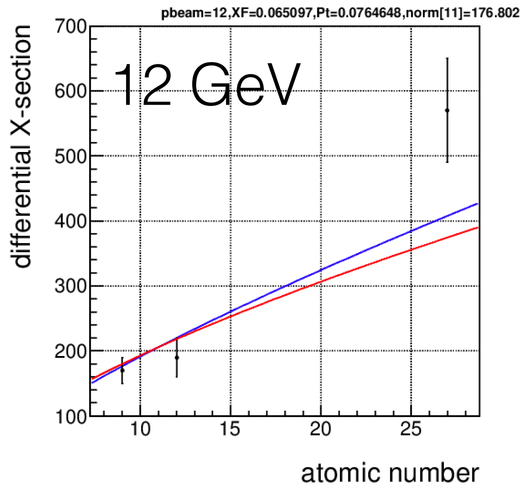
chi2の大きいデータ

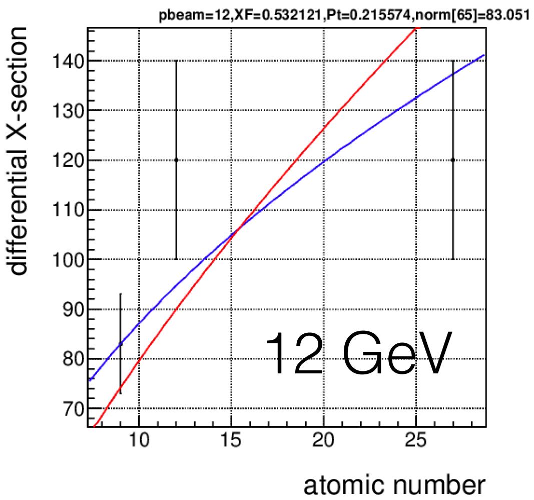


cont.



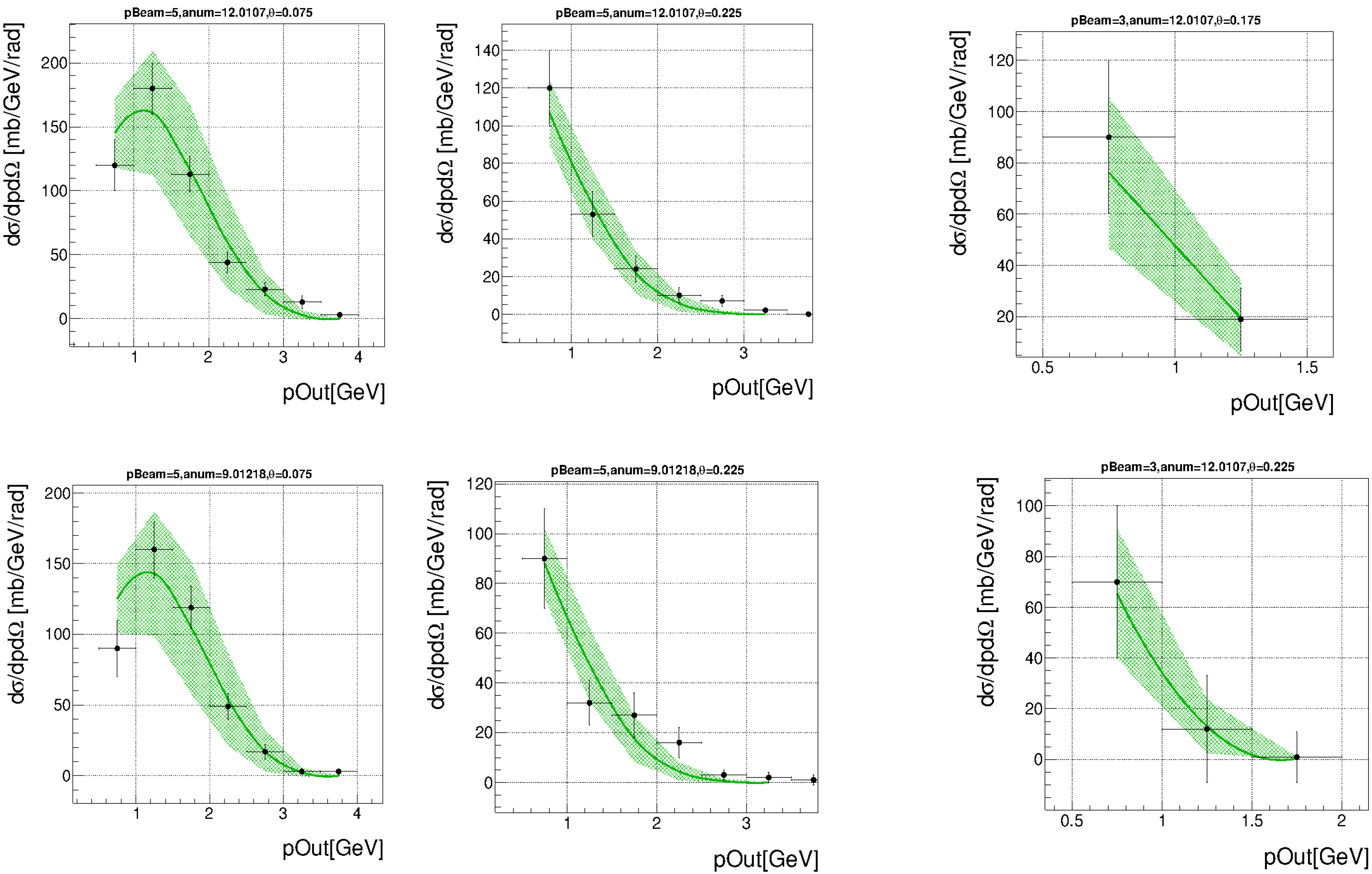




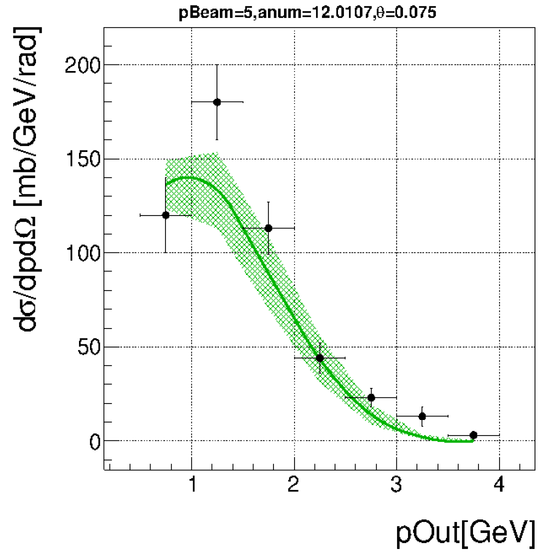


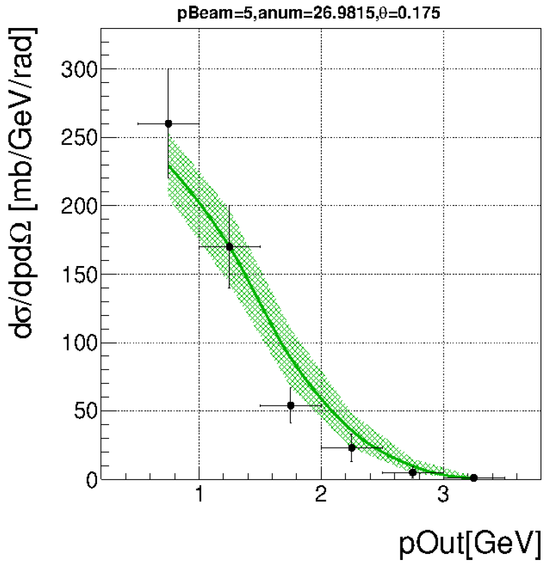
- X_Fが大きいと、Alの測定値が$A^{2/3}$則より明らかに大きい時がある
  - それらではα(X_F,Pt)の方が合いが良い
- 8 GeV/cのX_F > 0.8以上のデータではα(X_F ,Pt)は合わない

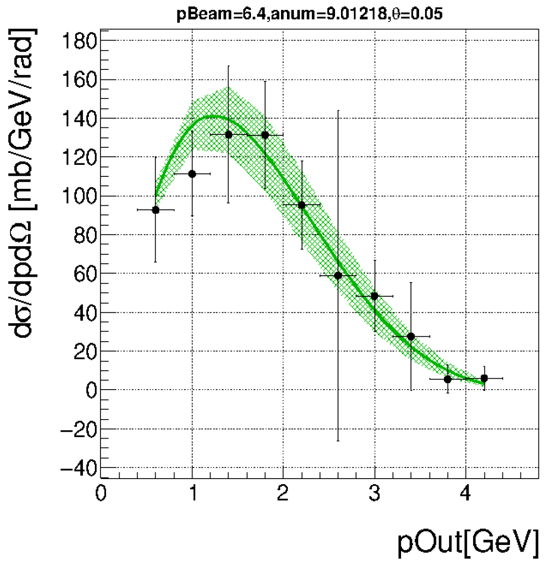
fit result (3-5 GeV)

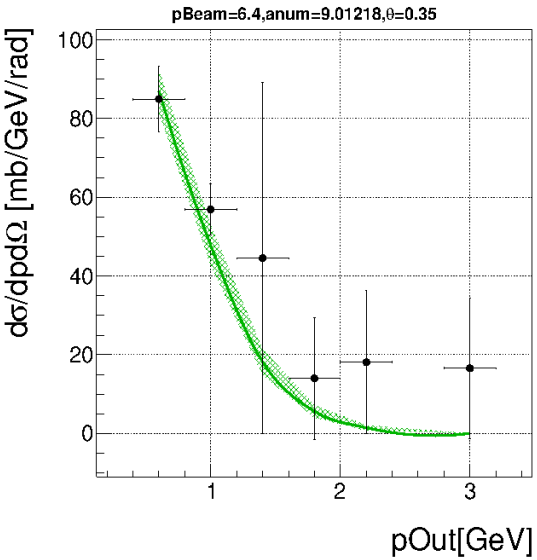


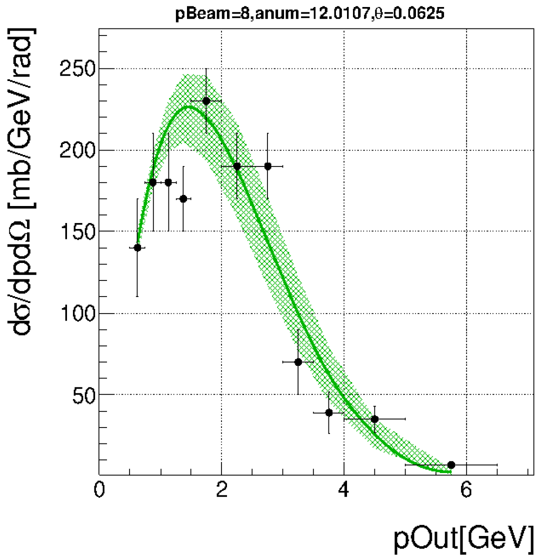
fit result (5, 6.4, 8 GeV/c)

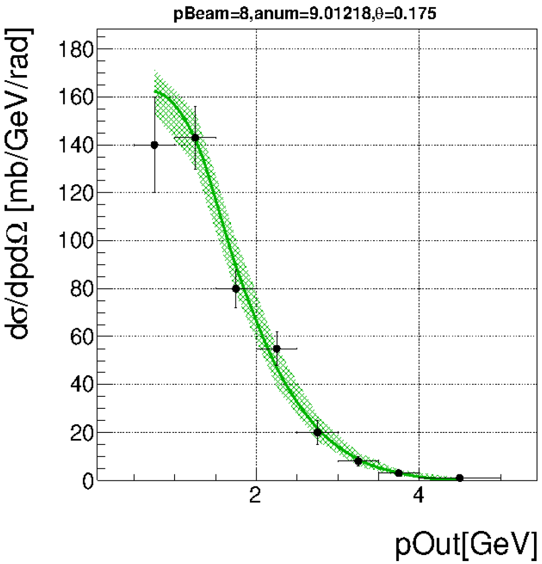












fit result (12,12.3,17.5 GeV)