ESIPAP March 1, 2021

Exercises "Superconducting Detector Magnets"

Exercise 1 "Sizing a practical superconductor for a 10 T magnet"

Assume you want the make a superconducting magnet of 10T with a bore of 2 m diameter and bore length of 8 m.

a) Calculate roughly the stored energy.

($E = B^2V/ 2\mu_0 \approx 1GJ$)

b) This stored energy ($0.5LI^2$) needs to be dissipated in and external resistor within 1 minute (assume linear decrease to zero) and the voltage (V=LI/t) is limited to 1000V.
What is the minimum safe operating current of this magnet.

($I = 2E / Vt = 33kA$)

c) So we need a cable, but how does it look? For a 10T magnet at 4.2K, which superconductor do we need?

(Nb_3Sn)

d) Assume a wire with 1.0 mm diameter, critical current density 4000 A/mm^2 at 10 T and 40 % superconductor in the wire.

What is the area of superconductor in the wire?

($0.4*pi/4\ mm^2 = 0.314\ mm^2$)

Which current can this wire take when we assume we take a margin of 70%?

(880A)

e) How many wires do I need in the cable to make 33 kA: ?

(about 38)

f) When we make a twisted Litze cable (3x3x3xetc.), what would be the size, the diameter of this cable?

(about 7-8 mm)

Exercise 2 "Understanding the effective resistivity of a superconductor"

R=0 in a superconductor when it is perfect, uniform, free of defects etc. In practice, however it is never perfect and we can take as a criterion 1 or 10 uV per meter of wire.

For MRI applications Philips Healthcare requires some 50 km of wire in one piece and perfectly connected in a persistent mode. A typical MRI magnet has 3 T in a volume of say 1 m^3 and the magnet operates at 200A.

a) What is the inductance of the magnet ?

($B^2V / \mu_0 I^2 = 180$ H)

b) What is the voltage drop across the coil with 50 km of wire operating at 1μV per meter ?

(50 mV)

c) What is the effective resistance of the magnet ?

(0.25 mΩ)

d) What is the current decaying time constant of the magnet ?

(L/R=180/0.00025= 720 000 s = 200 h)

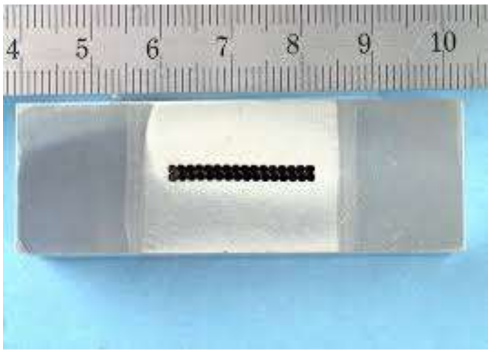
e) Is this a good value for an MRI magnet that should be stable in 1ppm/h ?

($\exp(-1/200) = 0.995 < 0.999999 \rightarrow$ not good enough)

f) What would be the maximum resistance tolerated across the wire and the maximum voltage per meter?

($5\ 10^{-8}$ Ω ; 0.2 nV/m)

Exercise 3 “Understanding a typical conductor for a Detector Magnet”



Write a half page summary on what we see in this picture.
Mention all relevant issues determining the shape and internal layout of this conductor.

Exercise 4 "Minimum propagation zone / Minimum quench energy / Adiabatic filament stability"

A 0.8 mm diameter superconducting wire used in the magnet of problem 1 has 40 % copper in it, and is electrically insulated with an epoxy resin, with the following properties at 4.2K and 15T:

- Electrical resistivity : $\rho = 3\ 10^{-10}\ \Omega$ m
- Thermal conductivity of Cu: $\lambda = 200$ W m^{-1} K^{-1}.
- Thermal conductivity of wire insulation: $\lambda = 0.03$ W m^{-1} K^{-1}.

The magnet is running at 4.2K with 2.6K temperature margin. Current is 187.5A at 15T.

What's the local current density in the Cu if the winding locally becomes normal (assume all current flows in the Cu)? Using the thermal margin ΔT work out the length L of the minimum propagation zone.

(J_{Cu} = 938 A mm^{-2}; L = 2 mm)

The specific heat of Cu and Nb3Sn under the magnet working conditions are given below.

- Specific heat copper : $c_{Cu} \approx 800$ J m^{-3} K^{-1}
- Specific heat Nb_3Sn : $c_{Nb3Sn} \approx 11\ 000$ J m^{-3} K^{-1}

Work out the minimum quench energy MQE, i.e. the energy required to establish a minimum propagation zone, the energy required to heat up the Minimum Propagation Zone MPZ.

(MQE = 6.8 mJ)

What is the Lorentz force F_L acting on the wire in the maximum-field zone of the magnet? If the wire slips over a distance dx under influence of this Lorentz force, how big would this distance need to be for the energy released to establish a minimum propagation zone? Is this a likely disturbance?

(F_L = 2.8 kN m^{-1} ; dx = 1.2 mm; No)

Under normal operation, what's the current density in the Nb_3Sn filaments? Still using the same thermal margin ΔT, work out the maximum allowable filament diameter d_{fil} to ensure adiabatic filament stability. How many filaments must the wire minimally contain? Is this realistic?

(J_{Nb3Sn} = 623 A mm^{-2} ; $d_{fil} < 330$ μm; $N_{fil} > 4$;
Easily, typical wires contain hundreds to thousands of filaments)