Daniel DZAHINI

LPSC => TIMA / Xdigit

ATLAS / ILC etc..

dzahini@univ-grenoble-alpes.fr

Xdigit is a start-up for specific ADC design

Mainly for sensors' array like pixels

www.xdigit.fr

OVERVIEW FOR SIGNAL READOUT FOR CAPACITIVE DETECTOR PULSE PROCESSING

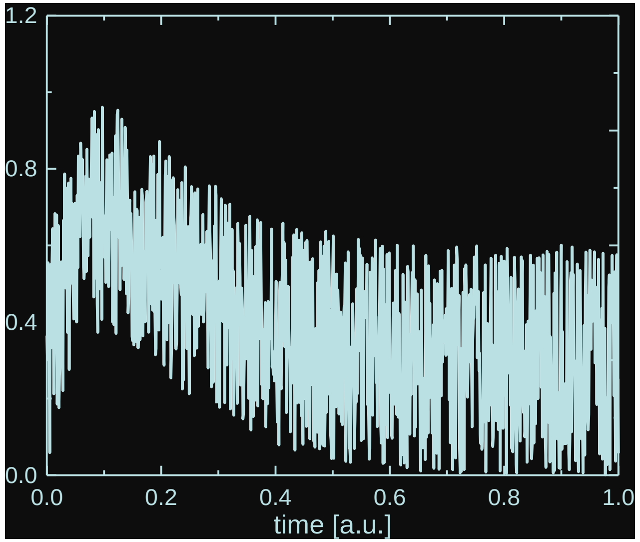
THANKS to many who provide slides and other documents online that I use for this lecturer

- Angelo Rivetti: *Front end electronics for radiation sensors*
- *Emilio Gatti & Manfredi (INFN)*
- Yan Kaplon (CERN)
- Christophe de la Taille (Omega lab)
- Glenn F. Knoll: *Radiation detection & measurement*
- Chiara Guazzoni; http://home.dei.polimi.it/guazzoni

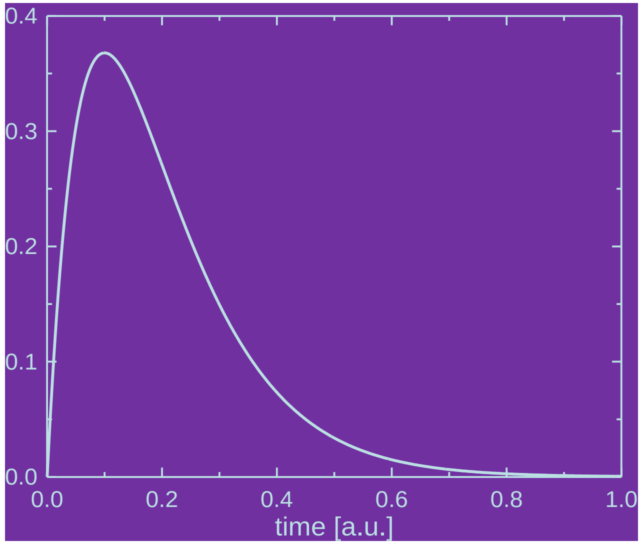
What signal processing means?

Sculpturing!

Designing?



signal processing

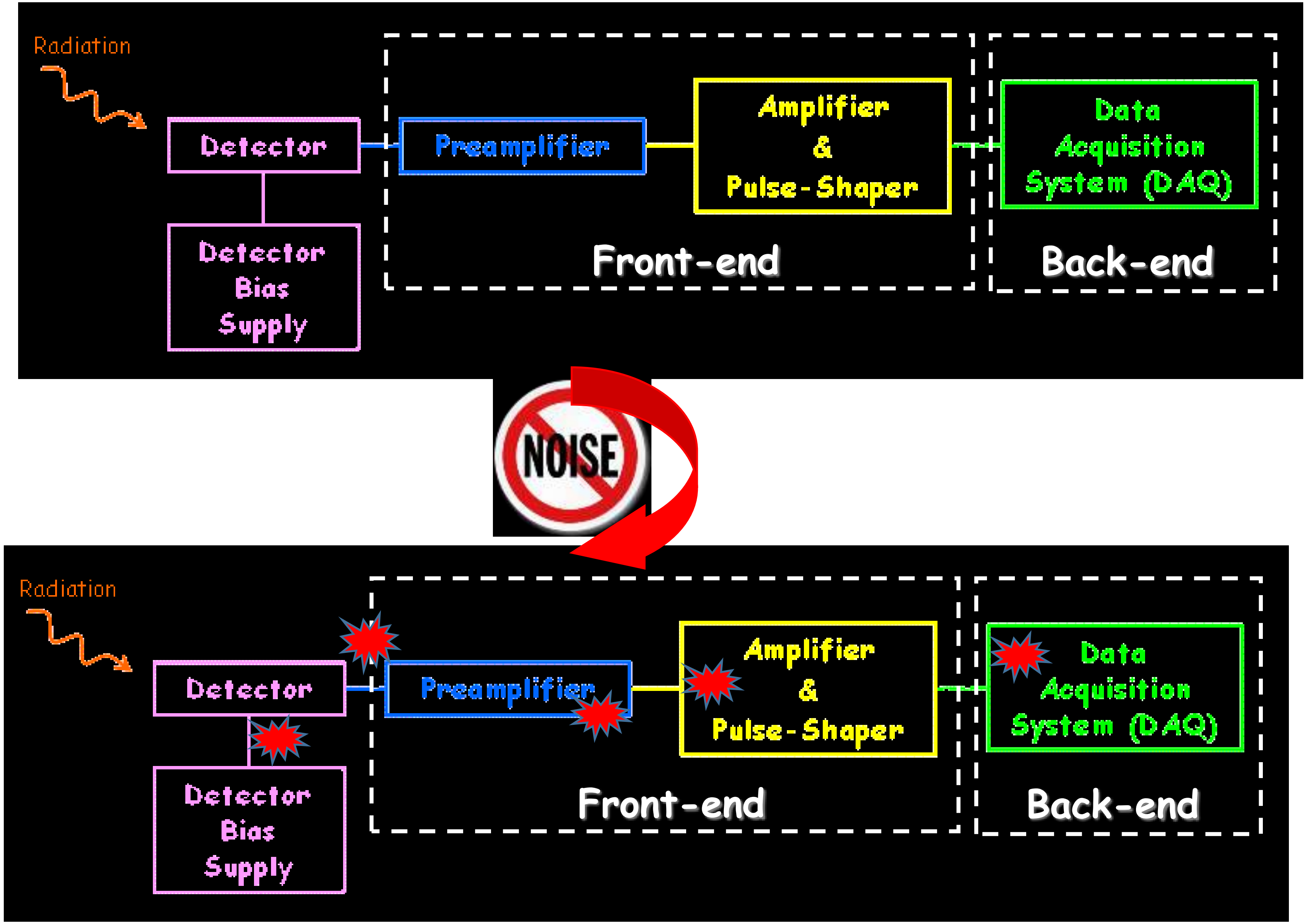


How noisy are flowers in your garden

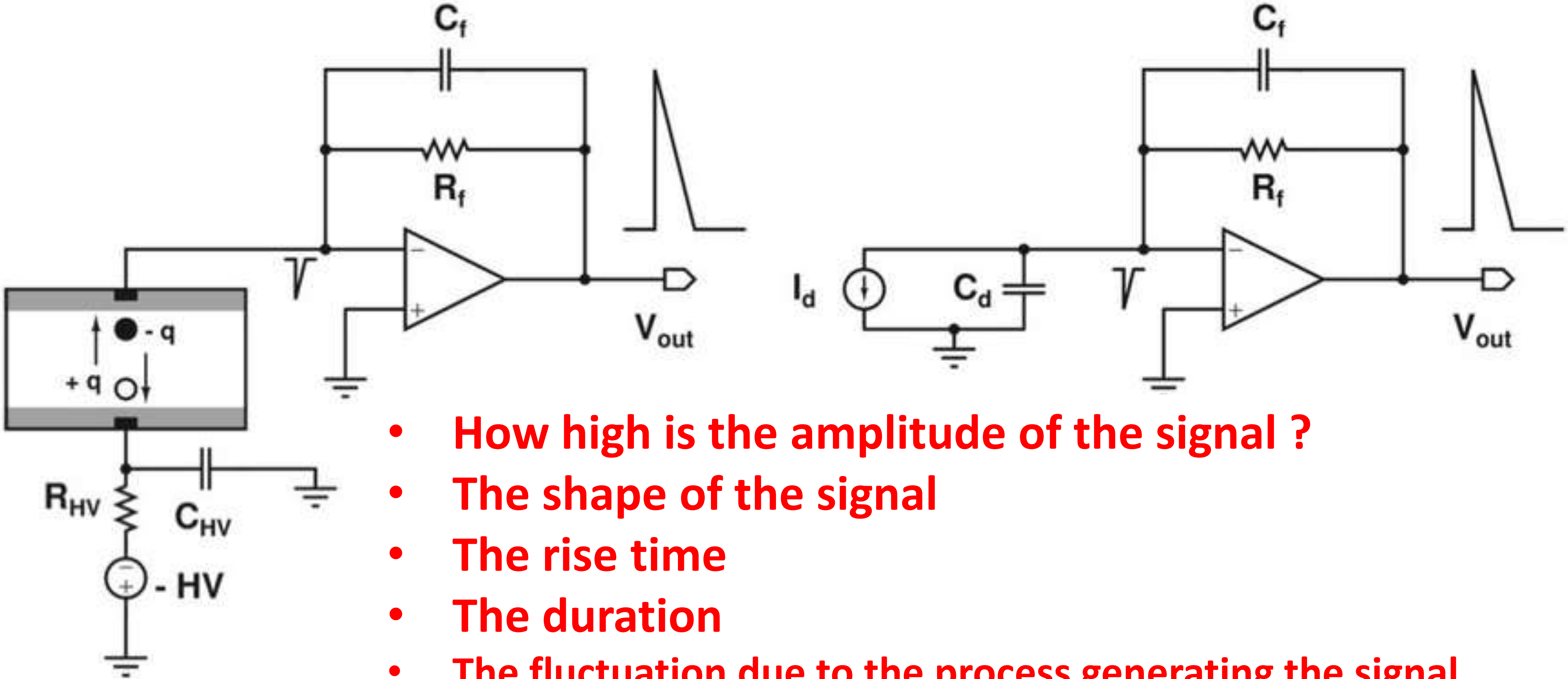
If plants can grow without ?? noise,

Why can't I amplify without adding noise

Actual system is unfortunately noisy



Signal polarity with negative High voltage



- How high is the amplitude of the signal ?
- The shape of the signal
- The rise time
- The duration
- The fluctuation due to the process generating the signal

Detector signal duration: **how short**?

The signal generally is a **short** current pulse

– thin silicon detector (10 –300 μm):	100 ps–30 ns
– thick (~cm) Si or Ge detector:	1 –10 μs
– proportional chamber:	10 ns –10 μs
– Microstrip Gas Chamber:	10 –50 ns
– Scintillator+ PMT/APD:	100 ps–10 μs

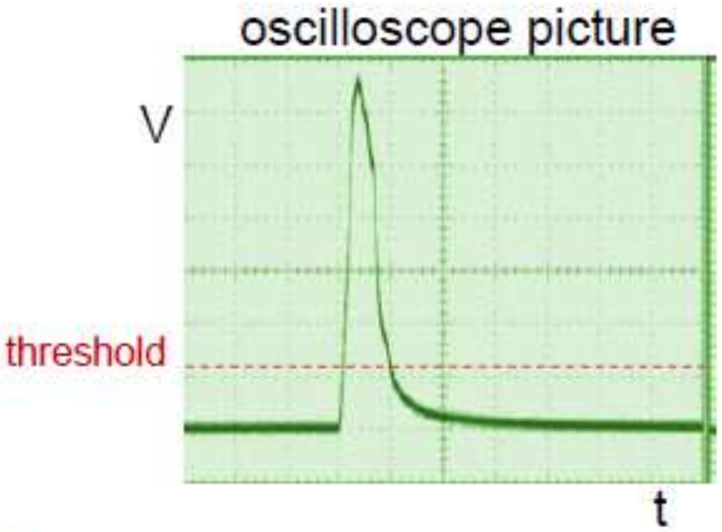
$$\boldsymbol{Energy} \sim \int \boldsymbol{i(t)\,dt}$$

What Information can one extract from a pulse?

Various measurements of this signal are possible
Depending on information required:

- Signal above threshold
  digital response / event count
- Integral of current = charge
  ➔ energy deposited
- Time of leading edge
  ➔ time of arrival (ToA) or time of flight (ToF)
- Time of signal above threshold
  ➔ energy deposited by TOT

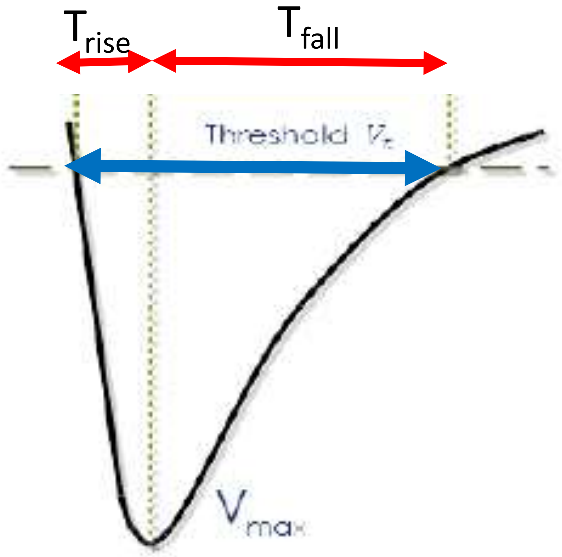
and many more …

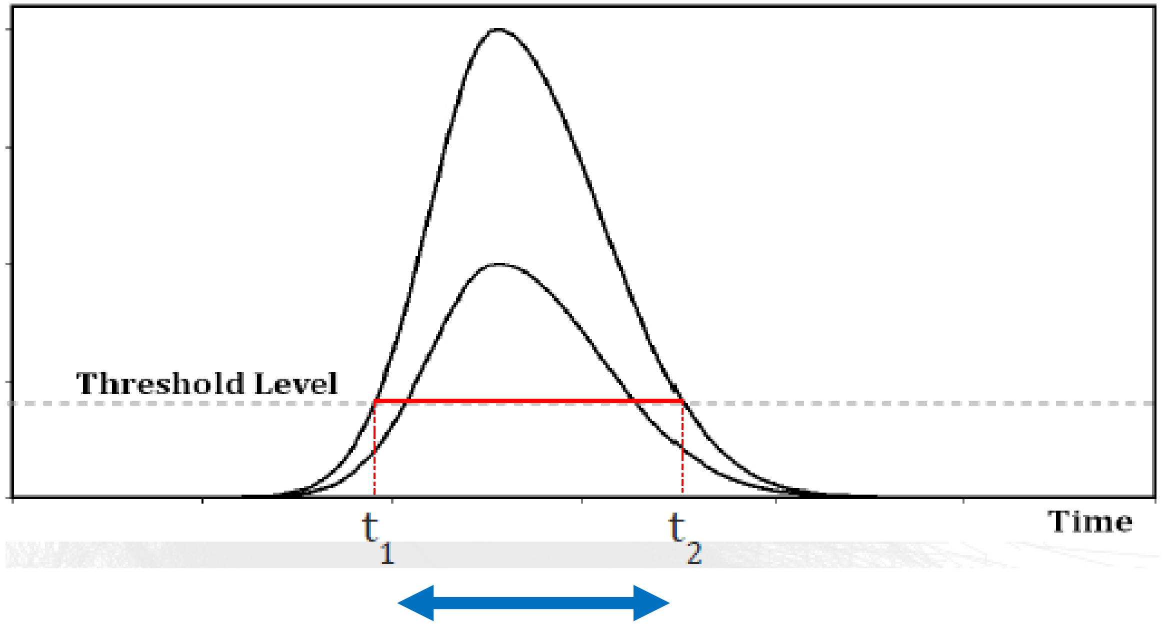


Counting and time over threshold

For **digital imaging** a counter is used after the comparator
Then one proceed by counting the number of events in a frame rate.

The comparator system could be used also to quatify the amplitude of an incoming signal.
The time spent over threshold by the amplifier output is somehow proportional to the amplitude of the incoming signal: TOT





Parameter impacting the pulse **amplitude**:

Ei = Minimum **ionization** energy (depends on the detector cristal, gaz, or liquide)
Ep (> Ei): *average* energy to generate a charges **pair**
E : Energy lost by an incoming particle =>
Np: *Average* **Number** of generated **Pairs**

Np = E/Ep => an *average* number

But instantly, the number follows a probabilistic law with a fluctuation from one event to another displaying a ***standard deviation*** $\sigma_{Np} = \sqrt{F * Np}$; F is the Fano factor

In many material F<1 then σ_{Np} is better than one could expect from the Poisson statistics ($\sqrt{Np}$);

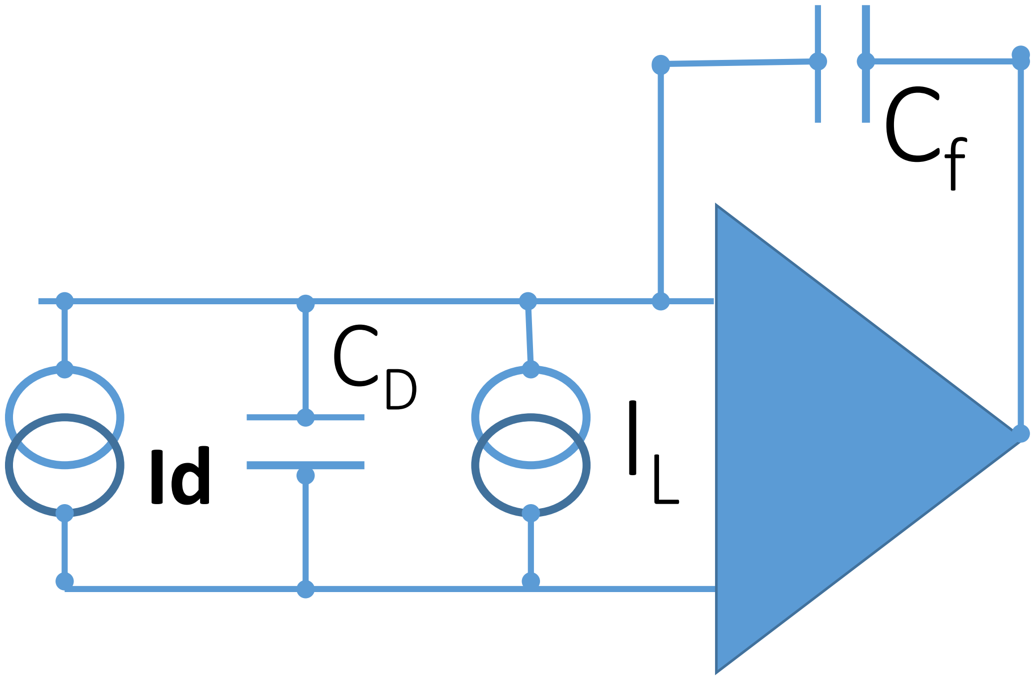
Detector's equivalent circuit: C_D an I_L

Detector = capacitance Cd

- Pixels : 0.1-10 pF
- PMs : 3-30 pF
- Ionization chambers: 10-1000 pF
- Sometimes effect of transmission line

Signal : current source

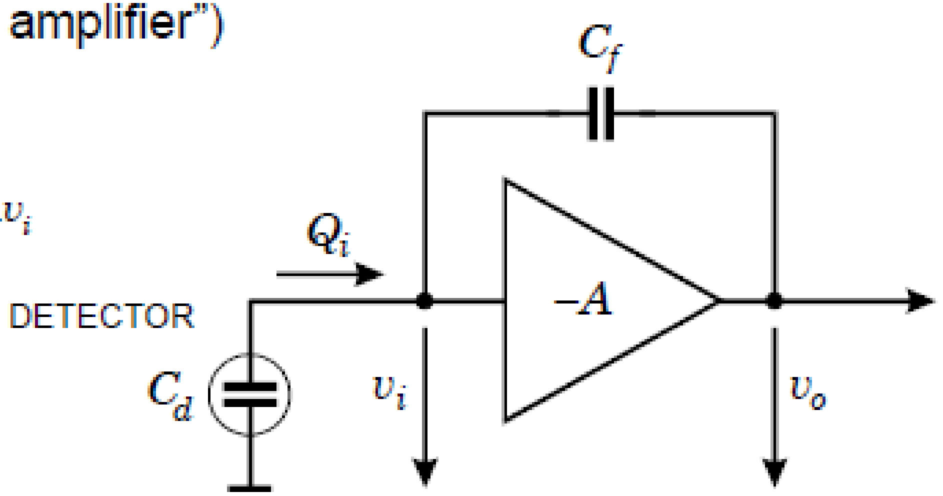
- Pixels : ~100 e^-/µm
- PMs : 1 photoelectron -> 10^5-10^7 e^-
- Modeled as an impulse (Dirac) : $i(t)=Q_0\delta(t)$



- C_D impact on **speed** and **noise** figures
- I_L impact output **DC level**, and on **noise**

Charge sensitive preamplifier

Active Integrator ("charge-sensitive amplifier")



Start with inverting voltage amplifier

Voltage gain $dv_o / dv_i = -A \quad \Rightarrow v_o = -Av_i$

Input impedance = ∞
(no signal current flows into amplifier input)

Connect feedback capacitor C_f between output and input.

so v0-vi =-Avi-vi =-(A+1)vi=-vf

Voltage difference across C_f: $v_f = (A+1)v_i$

⇒ Charge deposited on C_f: $Q_f = C_f v_f = C_f(A+1)v_i$

$Q_i = Q_f$ (since $Z_i = \infty$)

⇒ Effective input capacitance $C_i = \frac{Q_i}{v_i} = C_f(A+1)$ ("dynamic" input capacitance)

Gain $A_Q = \frac{dV_o}{dQ_i} = \frac{A \cdot v_i}{C_i \cdot v_i} = \frac{A}{C_i} = \frac{A}{A+1} \cdot \frac{1}{C_f} \approx \frac{1}{C_f} \quad (A \gg 1)$

Set by a well-controlled quantity, the feedback capacitance.

Helmuth Spieler
LBNL

Charge preamplifier: exemple of typical values

So finally the fraction of charge signal measured by the amplifier is:

$$\frac{Q_i}{Q_s} = \frac{C_i v_i}{v_i(C_i + C_{\det})} = \frac{1}{1 + C_{\det}/C_i} \qquad C_f \approx \frac{A}{C_i} \quad (A >> 1)$$

Example: $A = 10^3$

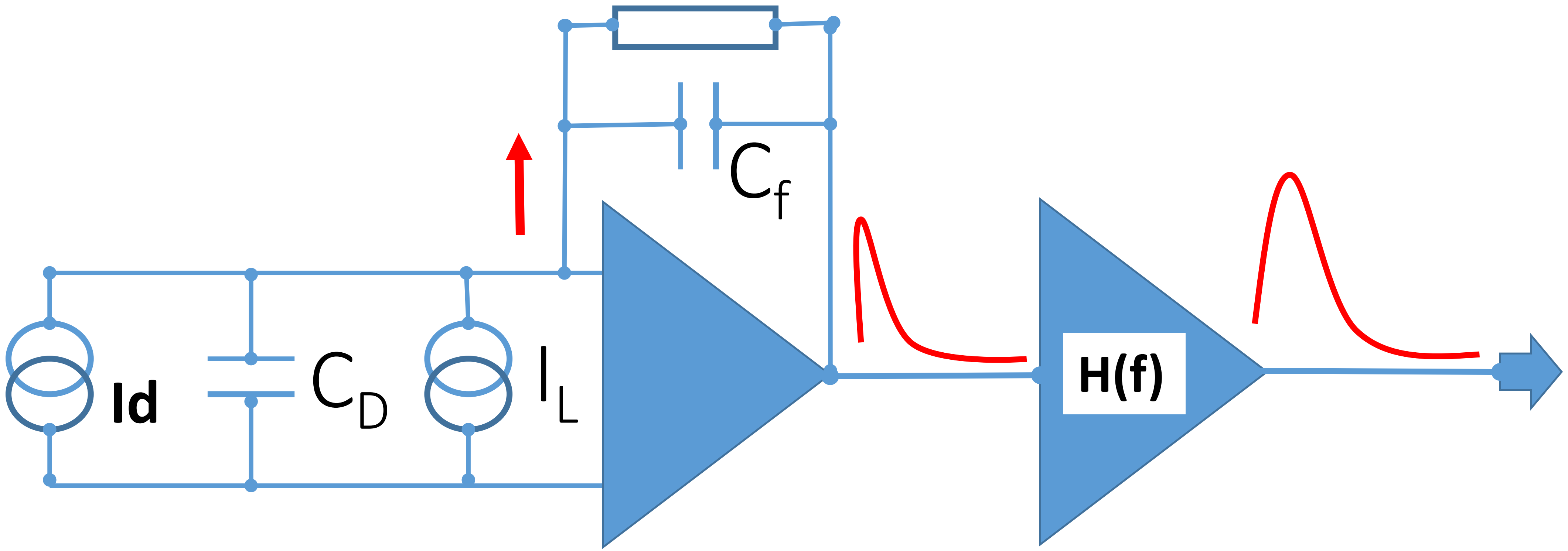
$C_f = 1pF$ ➔ $C_i = 1nF$

$C_{det} = 10pF$ ➔ $Q_i/Q_s = 0.99$ $(C_i >> C_{det})$

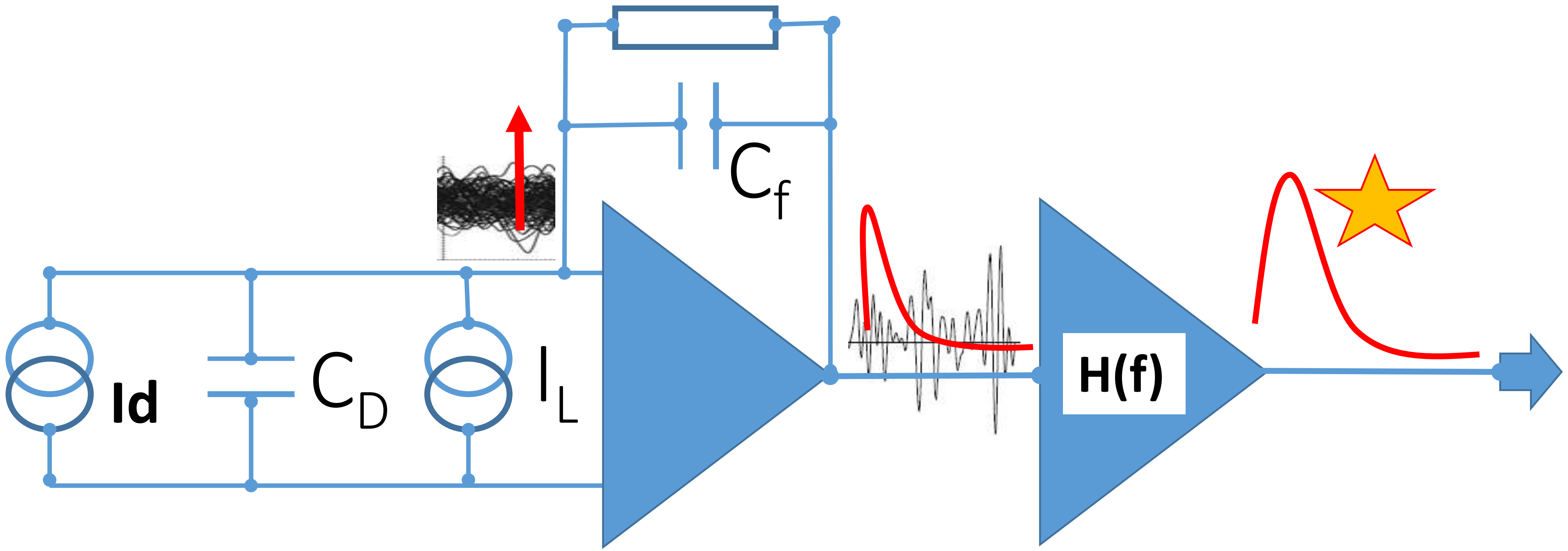
$C_{det} = 500pF$ ➔ $Q_i/Q_s = 0.67$ $(C_i \sim C_{det})$

↑ Si det: 50um thick, 500mm^2 area

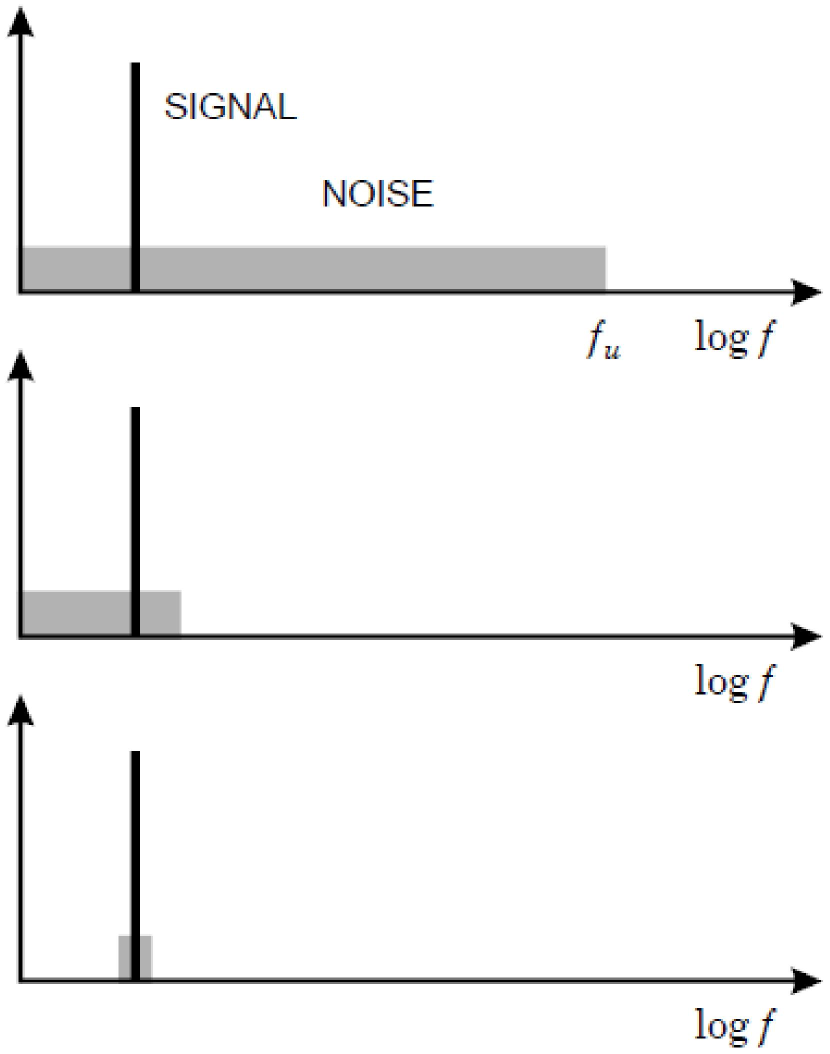
Front end amplifier and shaper circuit



Noise and Front end amplifiers



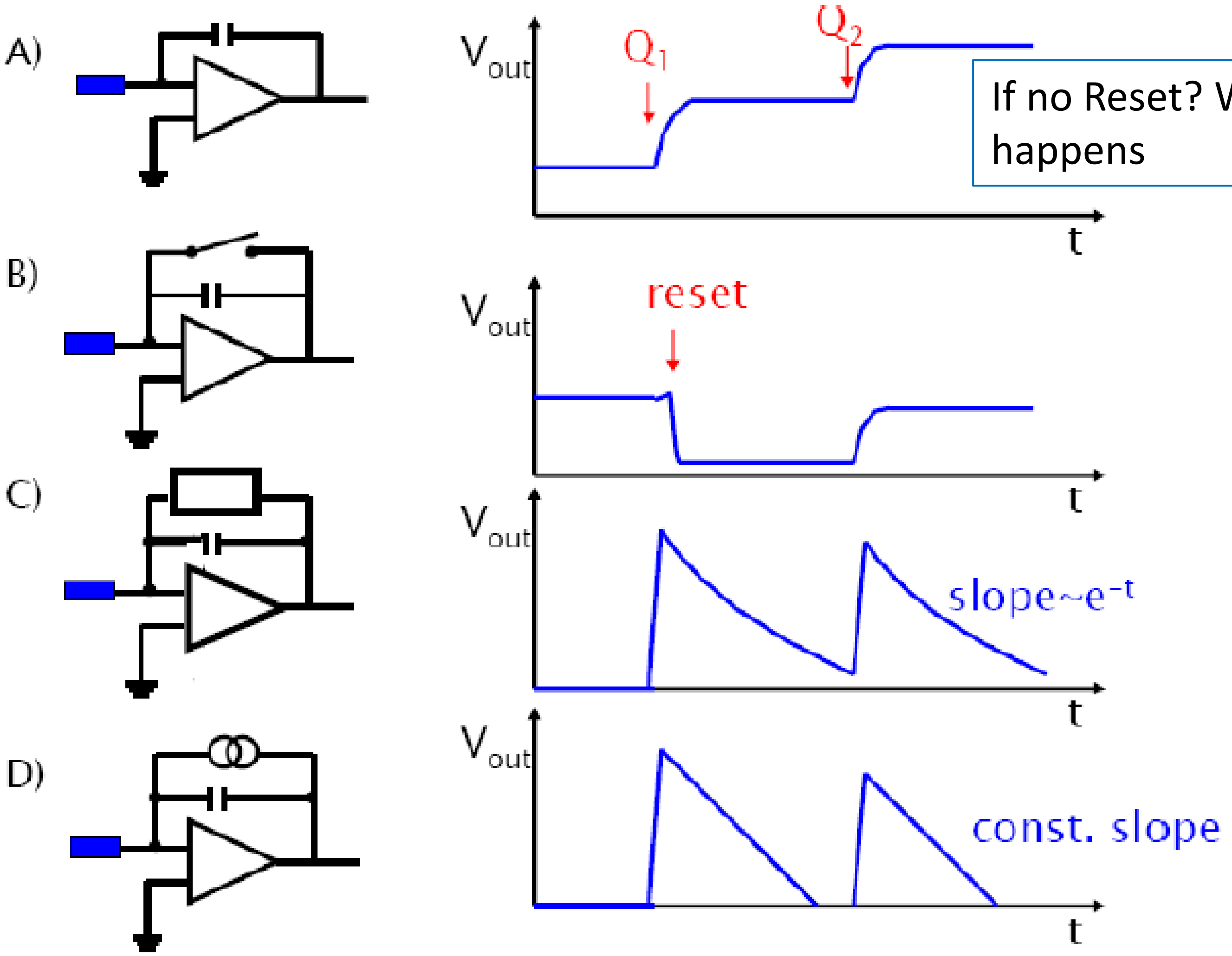
Signal & noise bandwidth



Helmuth Spieler
LBNL

The total noise will be the integral over the bandwidth
Then Signal/Noise could be improved by « optimizing » the noise bande close to the signal’s.

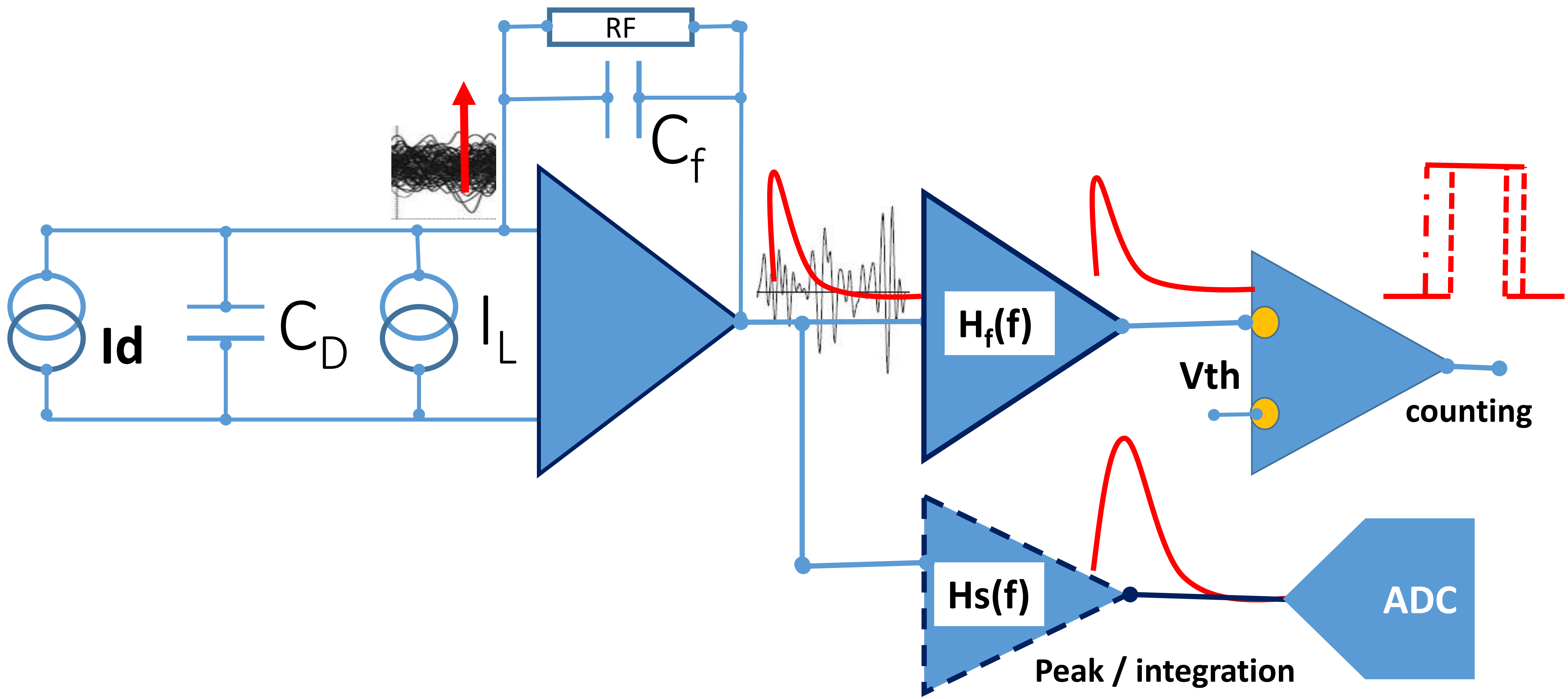
Different options of reset systems



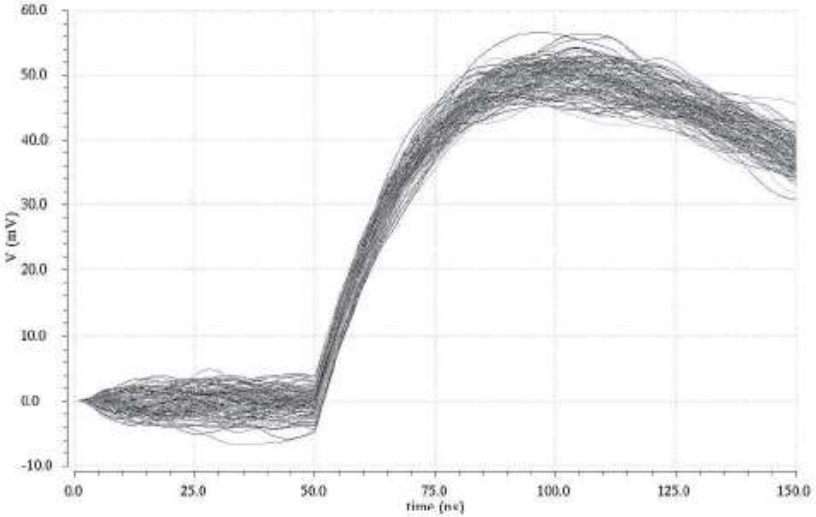
If no Reset? What happens

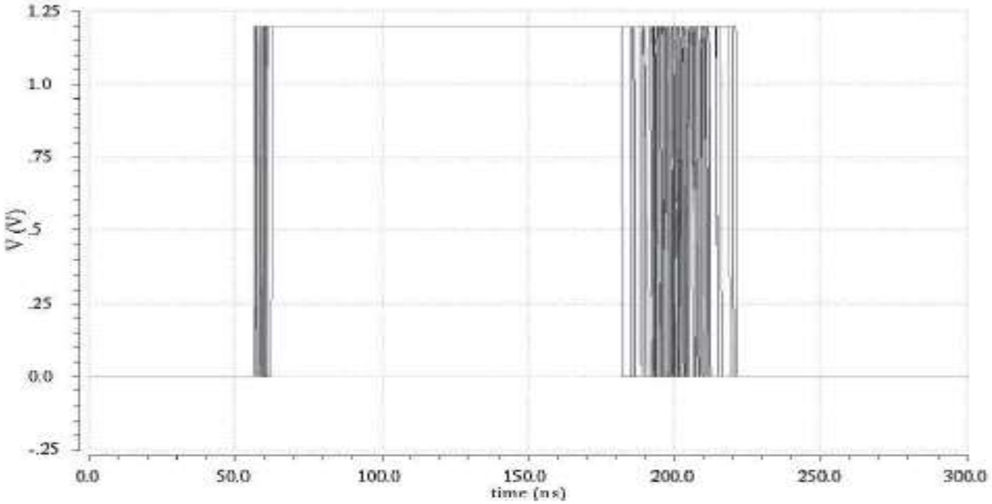
Ref LBL + de La Taille

Noise and Front end amplifiers and comparators

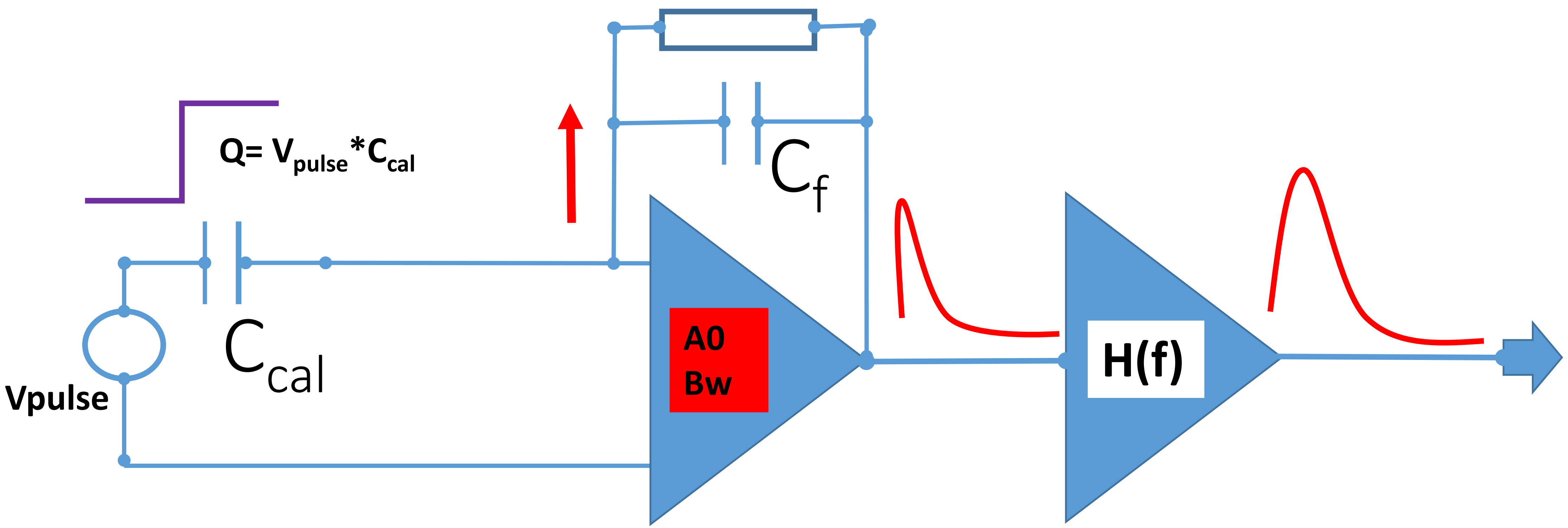


Even for counting, beware of the noise: why using a shorter peaking time for counting readout?



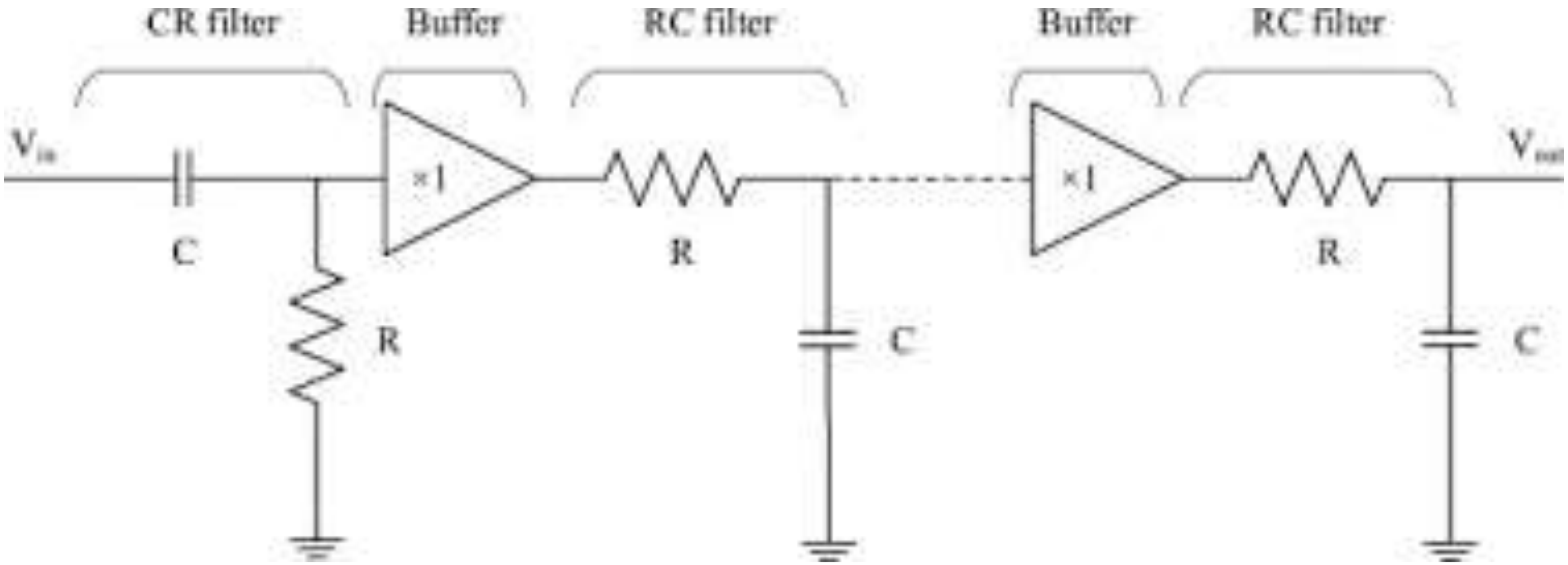


Calibration and simulation of Front end amplifiers

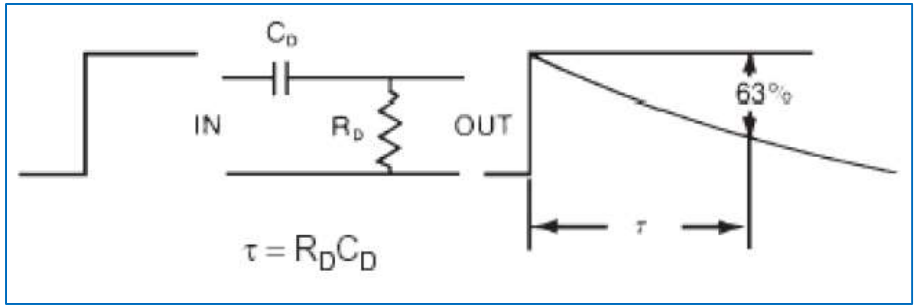


Before been so happy that you find a noise very low!!,
make sure your circuit is still amplifying the signal

CR RC shapers (filters)



CR Filter after the preamp



$$V_{in}(t) = \frac{Q(t)}{C} + V_{out}(t) \;\Rightarrow\; \frac{dV_{in}(t)}{dt} = \frac{i(t)}{C} + \frac{dV_{out}(t)}{dt}$$

by $V_{out}(t) = i(t)R$ and $\tau = RC$, $\quad \tau\frac{dV_{in}(t)}{dt} = V_{out}(t) + \tau\frac{dV_{out}(t)}{dt}$

Assuming the zero initial condition, taking Laplace transform leads to

$$V_{out}(s) = \frac{\tau s}{1+\tau s}V_{in}(s) = G_{CR}(s)V_{in}(s)$$

For the step function input

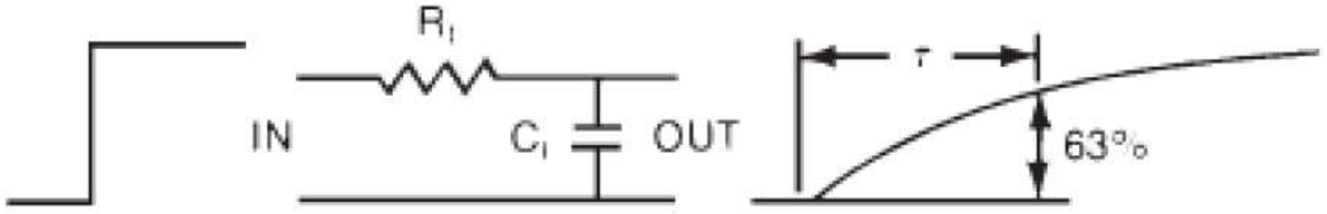
$$V_{in}(t) = \begin{matrix} V_0 & (t > 0) \\ 0 & (t \le 0) \end{matrix} \;\Rightarrow\; V_{in}(s) = L[V_{in}(t)] = \frac{V_0}{s}$$

the output signal becomes

$$V_{out}(s) = \frac{\tau}{1+\tau s}V_0 \;\Rightarrow\; V_{out}(t) = V_0 e^{-t/\tau}$$

$$G_{CR}(i\omega) = \frac{i\omega\tau}{1+i\omega\tau} \;\Rightarrow\; |G_{CR}(i\omega)| = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}$$

RC stage of the shaper



$\tau = R_I C_I$

$$V_{in}(t) = i(t)R + V_{out}(t) \text{ and } i(t) = \frac{dQ(t)}{dt} = C\frac{dV_{out}(t)}{dt}$$

$$\Rightarrow \quad V_{in}(t) = \tau\frac{dV_{out}(t)}{dt} + V_{out}(t) \quad \Rightarrow \quad V_{out}(s) = \frac{1}{1+\tau s}V_{in}(s) = G_{RC}(s)V_{in}(s)$$

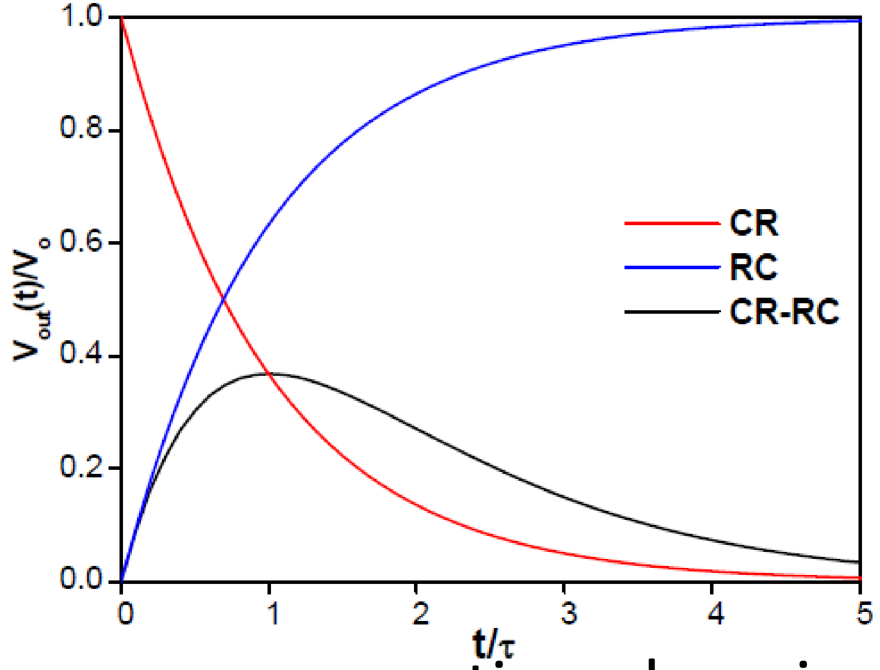
Output signal for the step function input:

$$V_{out}(s) = \frac{1}{1+\tau s}\frac{V_0}{s} \quad \Rightarrow \quad V_{out}(t) = V_0(1 - e^{-t/\tau})$$

Frequency domain transfer function:

$$G_{RC}(i\omega) = \frac{1}{1+i\omega\tau} \quad \Rightarrow \quad \left|G_{RC}(i\omega)\right| = \frac{1}{\sqrt{1+\omega^2\tau^2}}$$

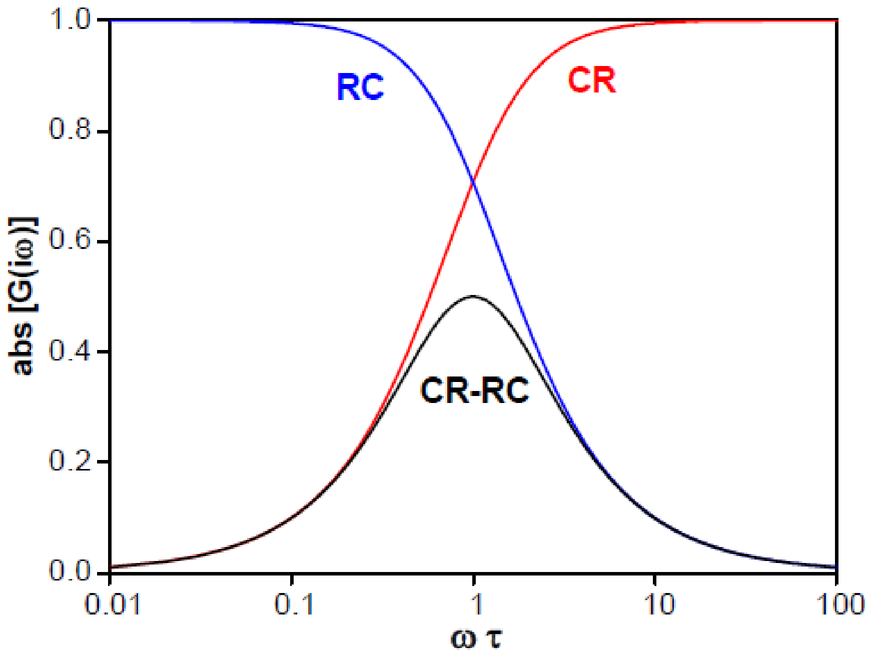
CR+RC transfert functions for a step input
This step stand for the integrator output signal



time domaine

$$V_{out}(t) = \frac{V_0 \tau_1}{\tau_1 - \tau_2}\left(e^{-t/\tau_1} - e^{-t/\tau_2}\right)$$

if $\tau_1 = \tau_2$ ⇨ $V_{out}(t) = \frac{V_0}{\tau} t e^{-t/\tau}$



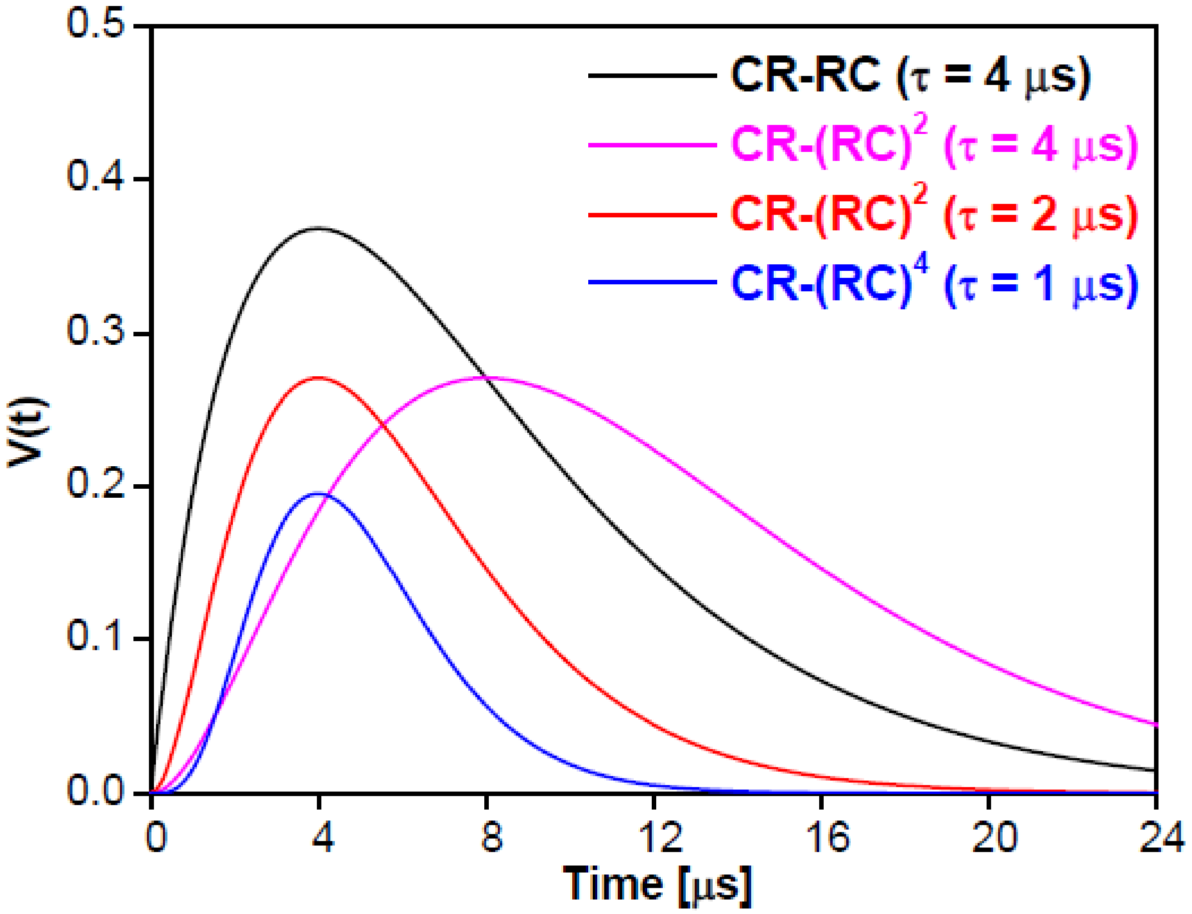
Frequency domaine:

Pass high * pass low

$$G_{CR-RC}(s) = \frac{1}{(1+\tau_2 s)} \frac{\tau_1 s}{(1+\tau_1 s)}$$

Filter / Shaper
first order? second? Why

CR*RC^n filters or Semi-Gaussian pulse shaping

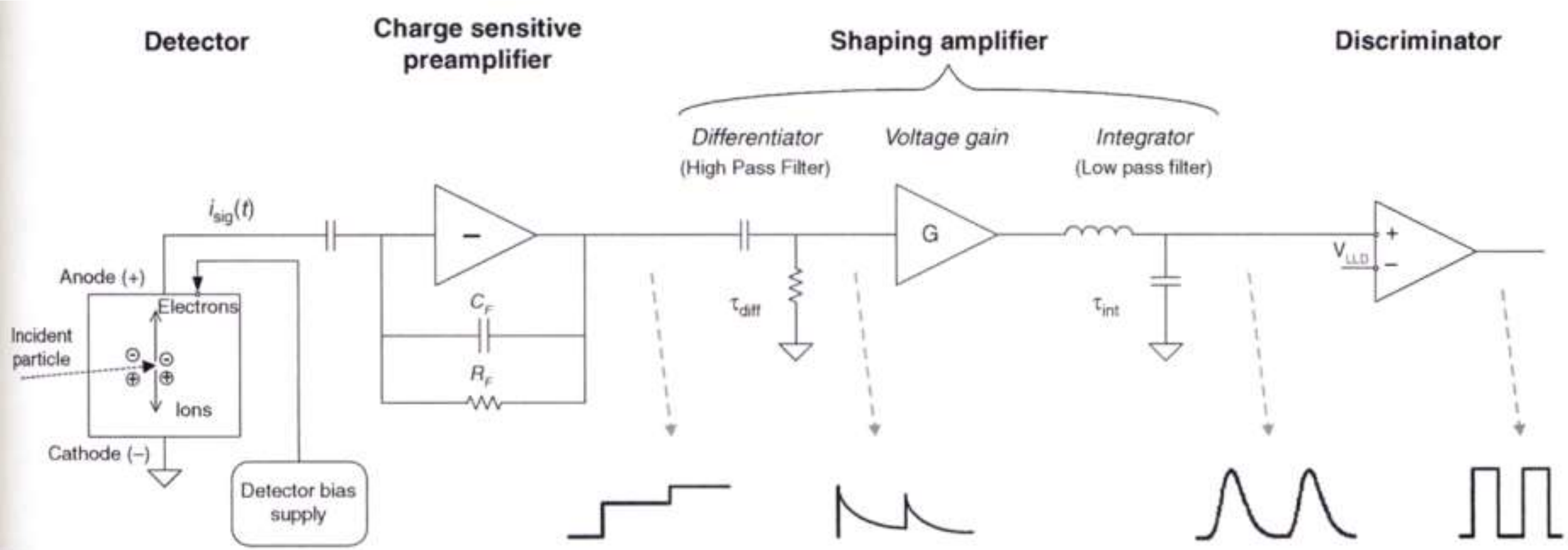


If a single CR high-pass filter is followed by several stages of RC integration, the output pulse shape becomes close to Gaussian amplifiers shaping, in this way are called **semi-Gaussian shaper**. Its output pulse is given by:

$$V_{out}(t) \propto \left(\frac{t}{\tau}\right)^n e^{-t/\tau}$$

The peaking time in this case is equal to $n*\tau$.

Signal shape following read out steps when 2 successive pulses (II)

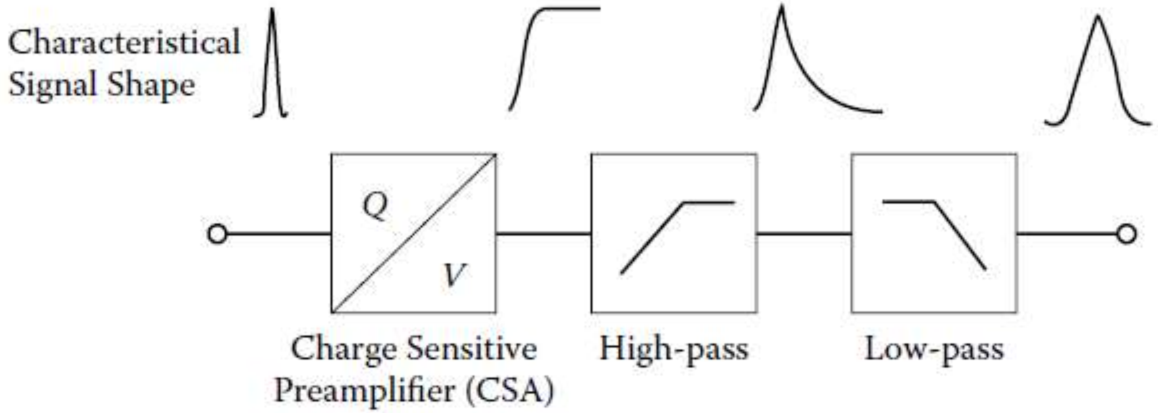


Schematic of simple signal processing electronics. This circuit is suitable for use, as shown, in many applications and is conceptually similar to more complex circuits. These elements are discussed in greater detail in Chapter 17. (Courtesy of R. Redus, Amptek, Inc.)

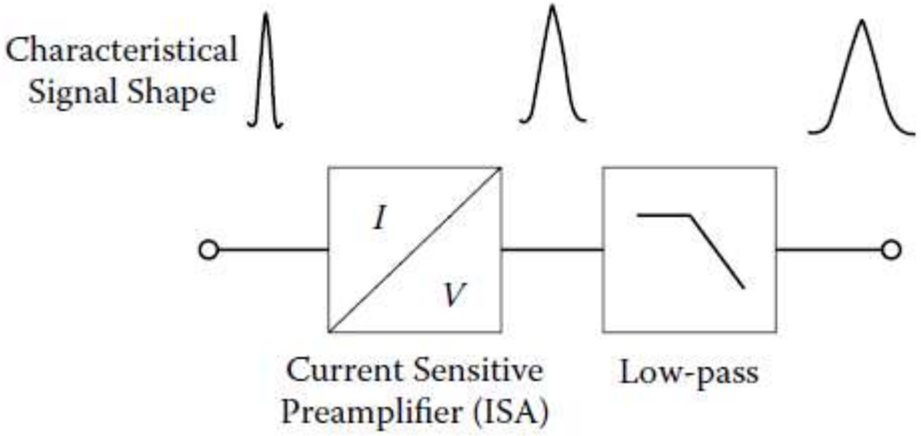
Courtesy, Glenn F. « radiation detection »

Simplified Signal shape throughout electronic readout

Charge sensitive Signal conditioning



Current sensitive Signal conditioning



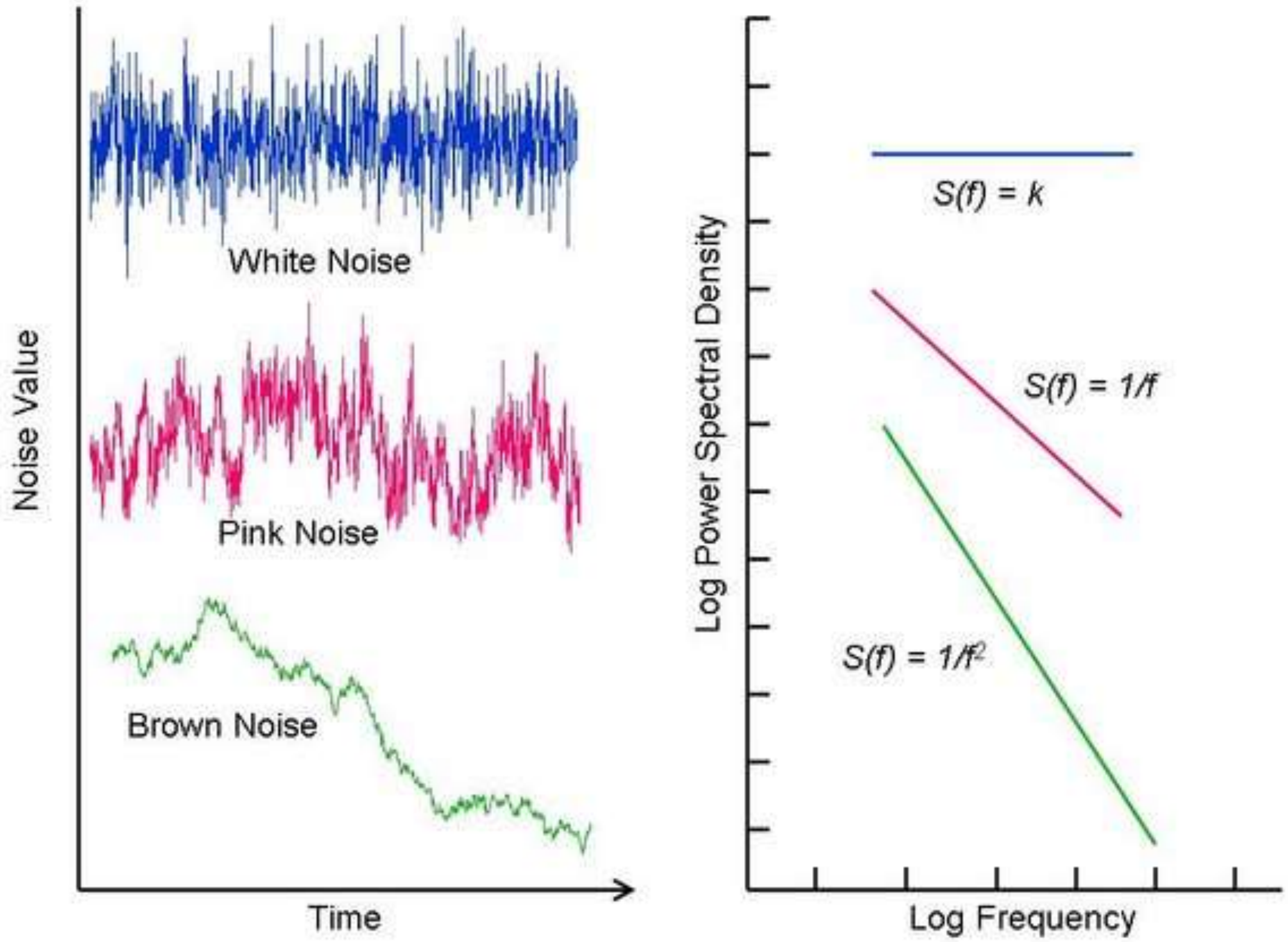
WHAT IS NOISE ?

- **What is noise?**
  - **Noise is any undesired signal that masks the signal of interest.**
    - Unwanted disturbance that interferes with a desired signal
    - External: power supply & substrate coupling, crosstalk, EMI, etc.
    - Internal: random fluctuations that result from the physics of the devices or materials
    - Smallest detectable signal, signal-to-noise ratio (SNR), and dynamic range are determined by noise

$$SNR = \frac{P_{signal}}{P_{noise}} = \frac{V^2_{rms,\,signal}}{V^2_{rms,\,noise}}$$

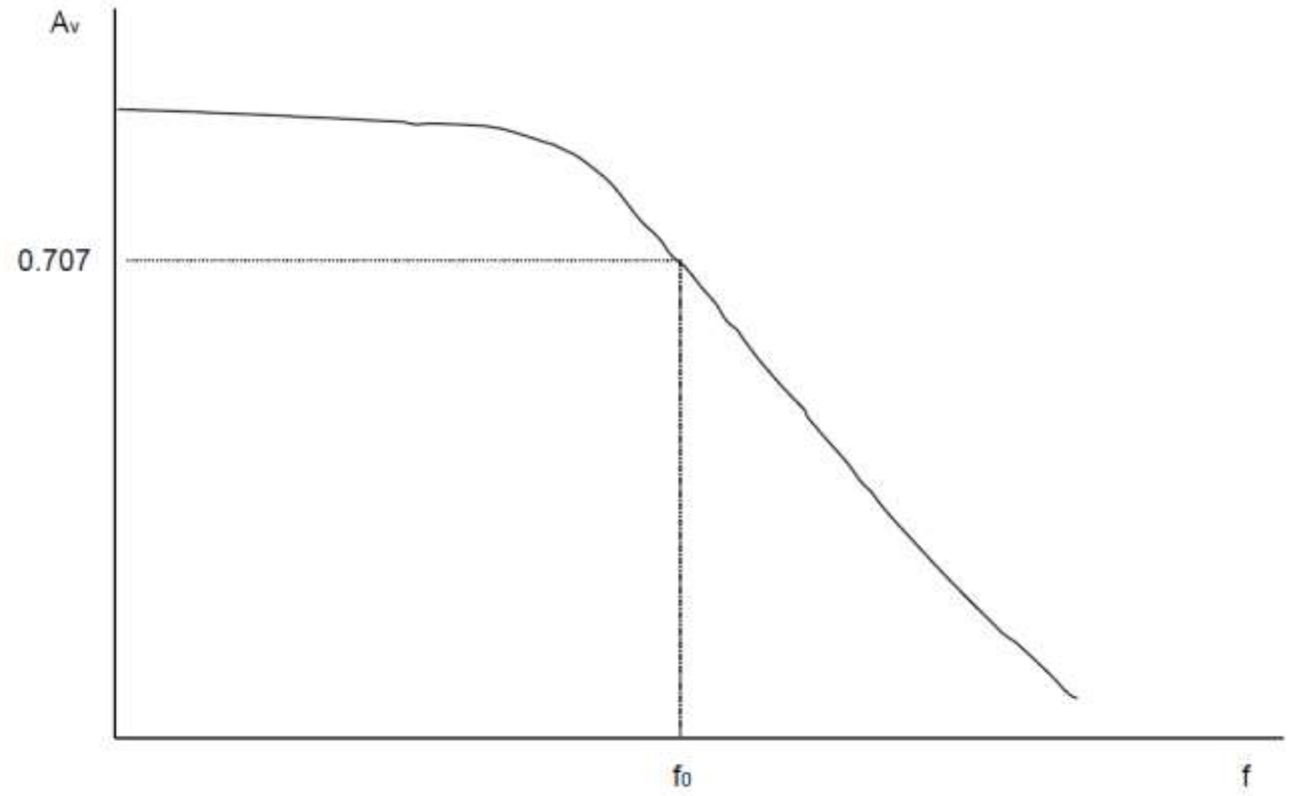
We will look at internal noise sources and how they affect key performance metrics.

Noise in general: time & frequency domain



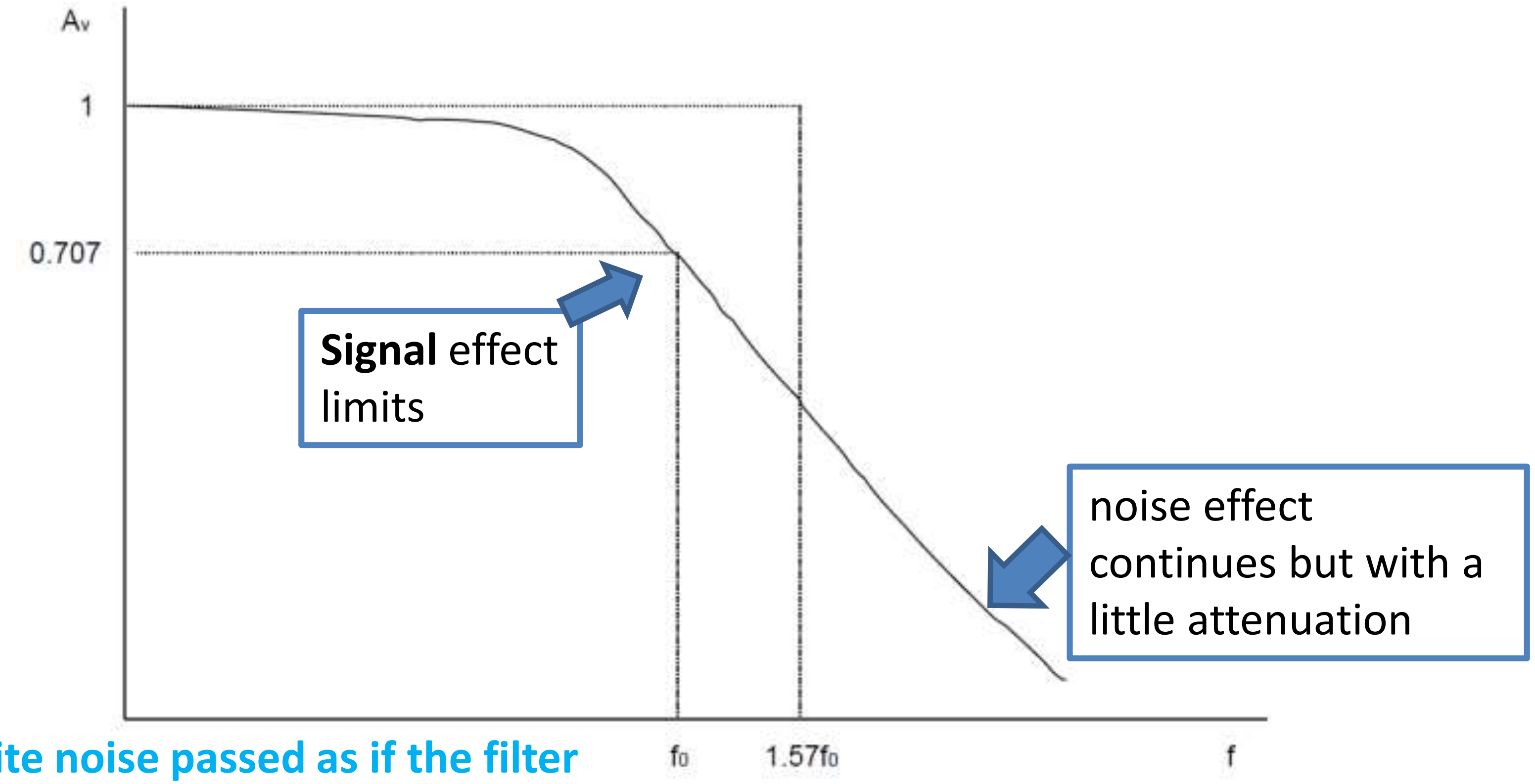
Noise Effective BW (NEB) of an amplifier (filter)

Lets consider a general *low pass* amplification (filter) system;
What happen to a white noise located at the input of such amplifier?
Is it amplified exactly as the signal?



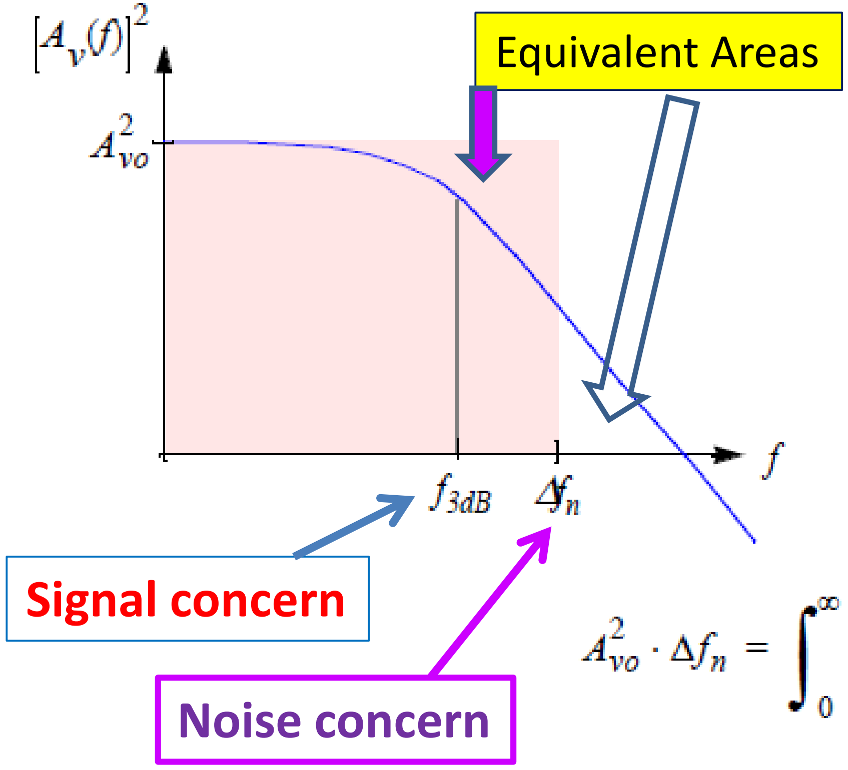
Noise bandwidth

Lets consider a low pass amplification system;
What is the effect of a larger bandwidth white noise located at the input of such amplifier?



The white noise passed as if the filter were a brick wall type but with a cutoff freq 1.57 times larger

Noise Bandwidth # signal bandwidth



No. Poles	NOISE BW
1	$\Delta f_n = \frac{\pi}{2} \cdot \Delta f_{3dB}$
2	$\Delta f_n = 1.22 \cdot \Delta f_{3dB}$
3	$\Delta f_n = 1.16 \cdot \Delta f_{3dB}$
2nd-order BPF	$\Delta f_n = \frac{\pi}{2} \cdot \Delta f_{3dB}$

$$A_{vo}^2 \cdot \Delta f_n = \int_0^\infty [A_v(f)]^2 df$$

- Noise bandwidth is defined for a brickwall transfer function
- Noise bandwidth is not the same as 3dB bandwidth

Noise Bandwidth improves when number of poles increase

Keep it in mind and make the link later with CRRCn filtering

Optimizing Filtering <=> Optimize NEB

For Maximally flat (Butterworth) where $f_0=f_{3dB}$

$$NEB = \left(\int_0^\infty \frac{df}{1+(f/f_0)^{2n}}\right)$$

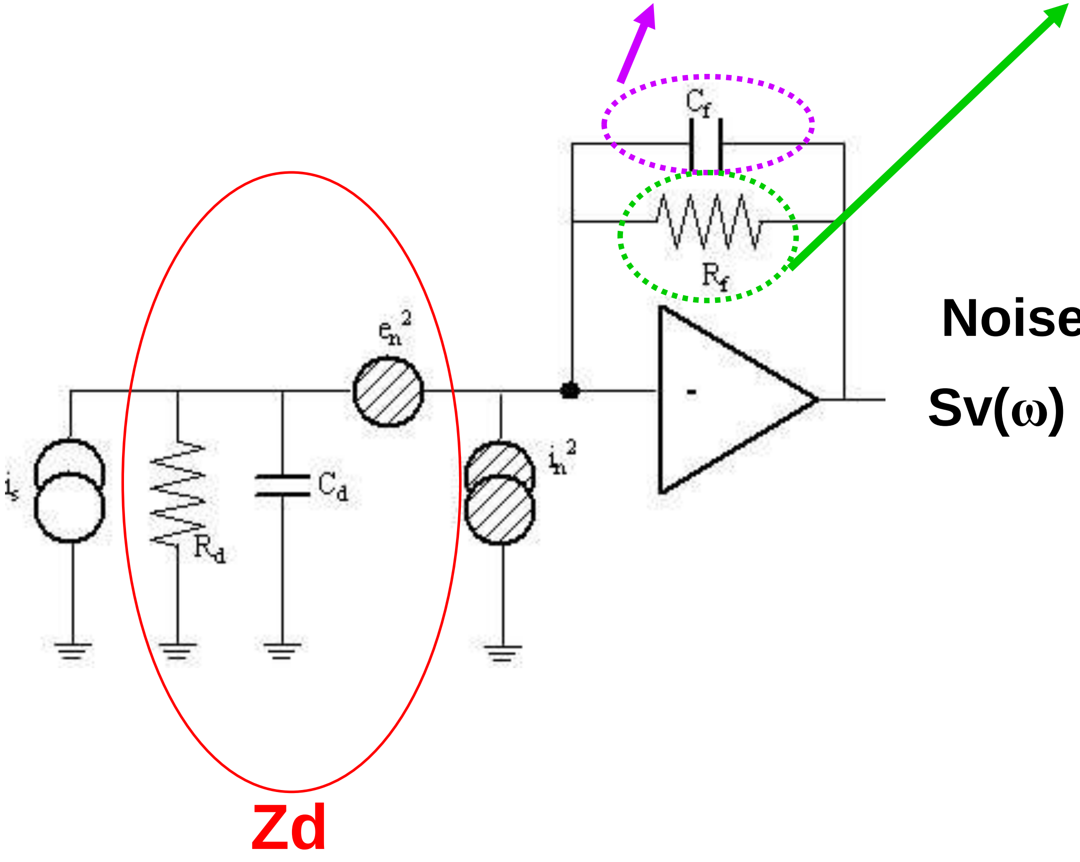
n	NEB
1	$1.57f_0$
2	$1.11f_0$
3	$1.05f_0$
4	$1.025f_0$

High order filter is good(0.5dB improvement)

But higher order n means:
More complicated and/or
more power budget

In spectroscopie,
One considers
in general n=2

Preamplifier: charges / Courant output noise



Noise spectrum at the output

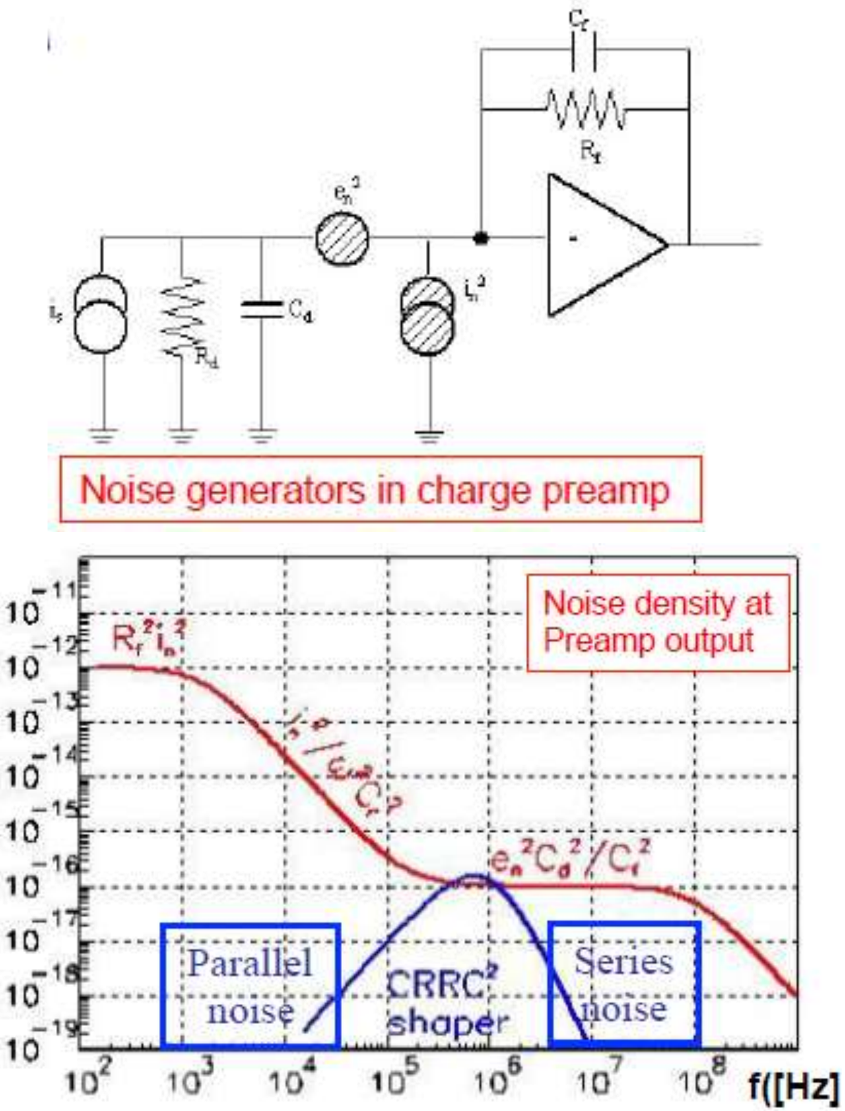
$$Sv(\omega) = (in^2 + en^2/|Zd|^2) / \omega^2 Cf^2$$

$$= in^2 / \omega^2 C_f^2 + en^2 C_d^2 / Cf^2$$

If we neglect Rd and
we translate en to a current
By doing en/Zd

Noise issues for charge preamp: frequency domaine

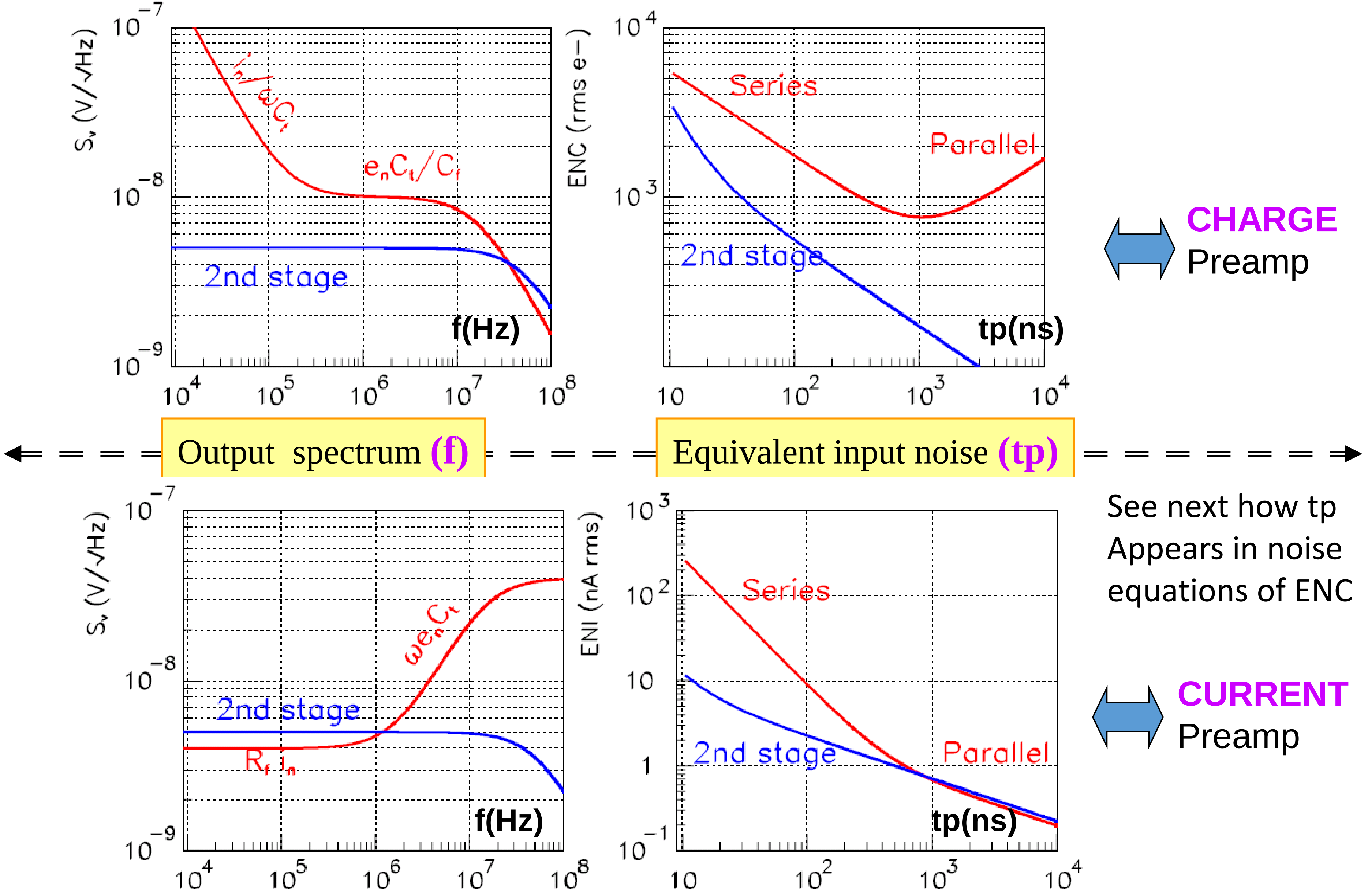
- 2 noise generators at the input
  - Parallel noise : (i_n^2) (leakage currents)
  - Series noise : (e_n^2) (preamp)
- Output noise spectral density :
  - $S_v(\omega) = (i_n^2 + e_n^2/|Z_d|^2) / \omega^2 C_f^2$
    $= i_n^2 / \omega^2 C_f^2 + e_n^2 C_d^2/C_f^2$
  - Parallel noise in $1/\omega^2$
  - Series noise is flat, with a « noise gain » of C_d/C_f
- *rms* noise V_n
  - $V_n^2 = \int Sv(\omega)\, d\omega/2\pi \to \infty$ (!)
  - Benefit of shaping…



The methode here is to Transfert each input To current domain, then Multiply by the feedback

Ref. C. de la Taille

Output spectrum and it's equivalent input

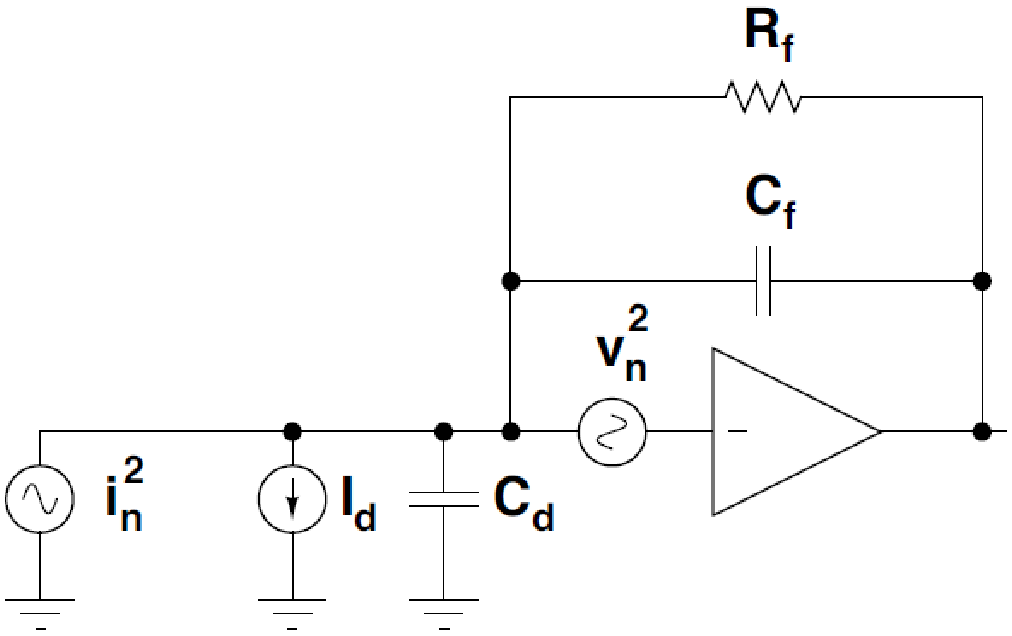


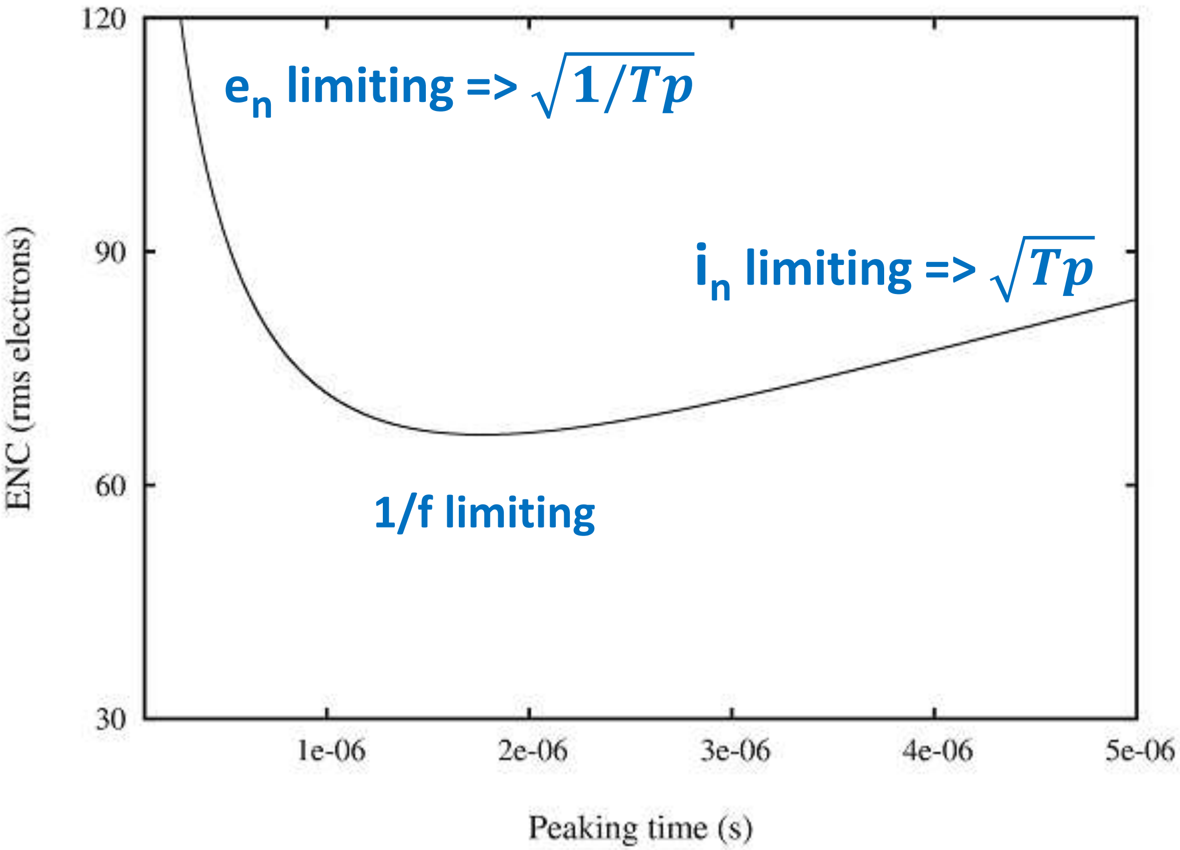
CHARGE Preamp

See next how tp Appears in noise equations of ENC

CURRENT Preamp

Noise issues in charge preamp: *time domaine*

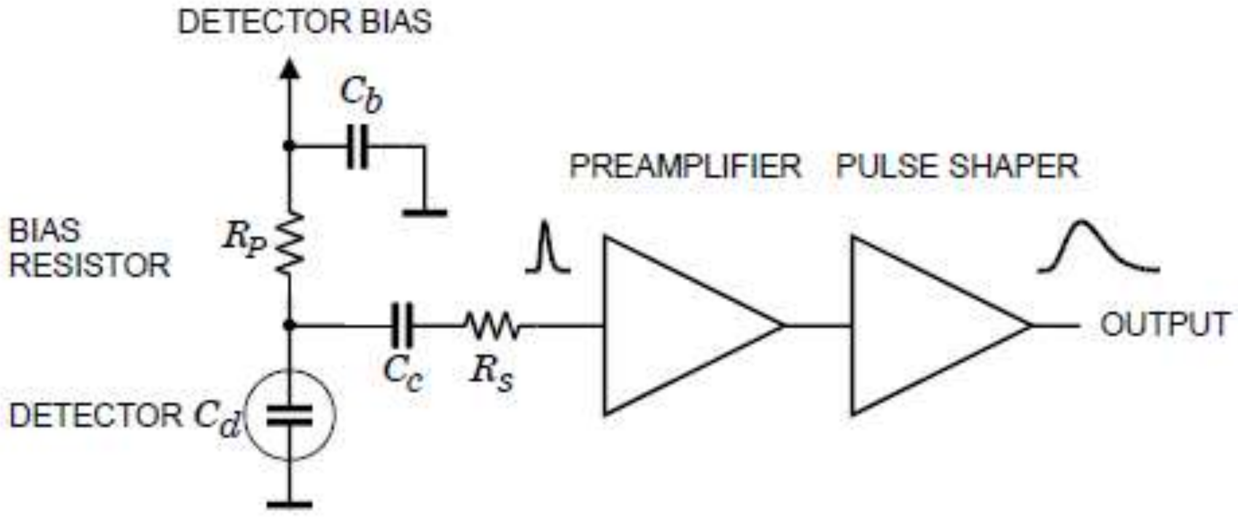


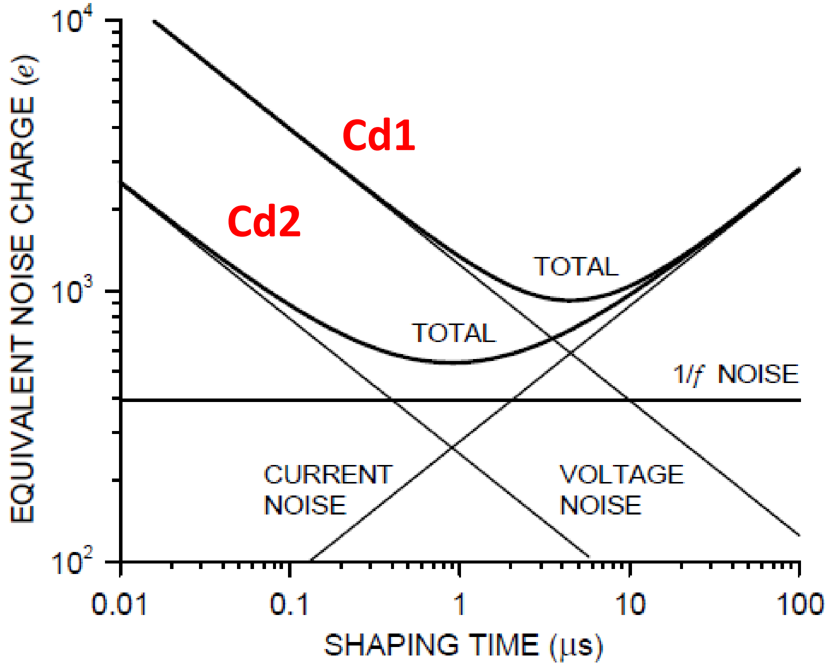


$$ENC^2 = (C_d + C_{in})^2 \left(A_w v_n^2 \frac{1}{T_p} + A_f K_f \right) + A_p i_n^2 T_p$$

Increase with Cd

Practical summary about noise in charge integrator





ENC using a simple CR-RC shaper with peaking time T:

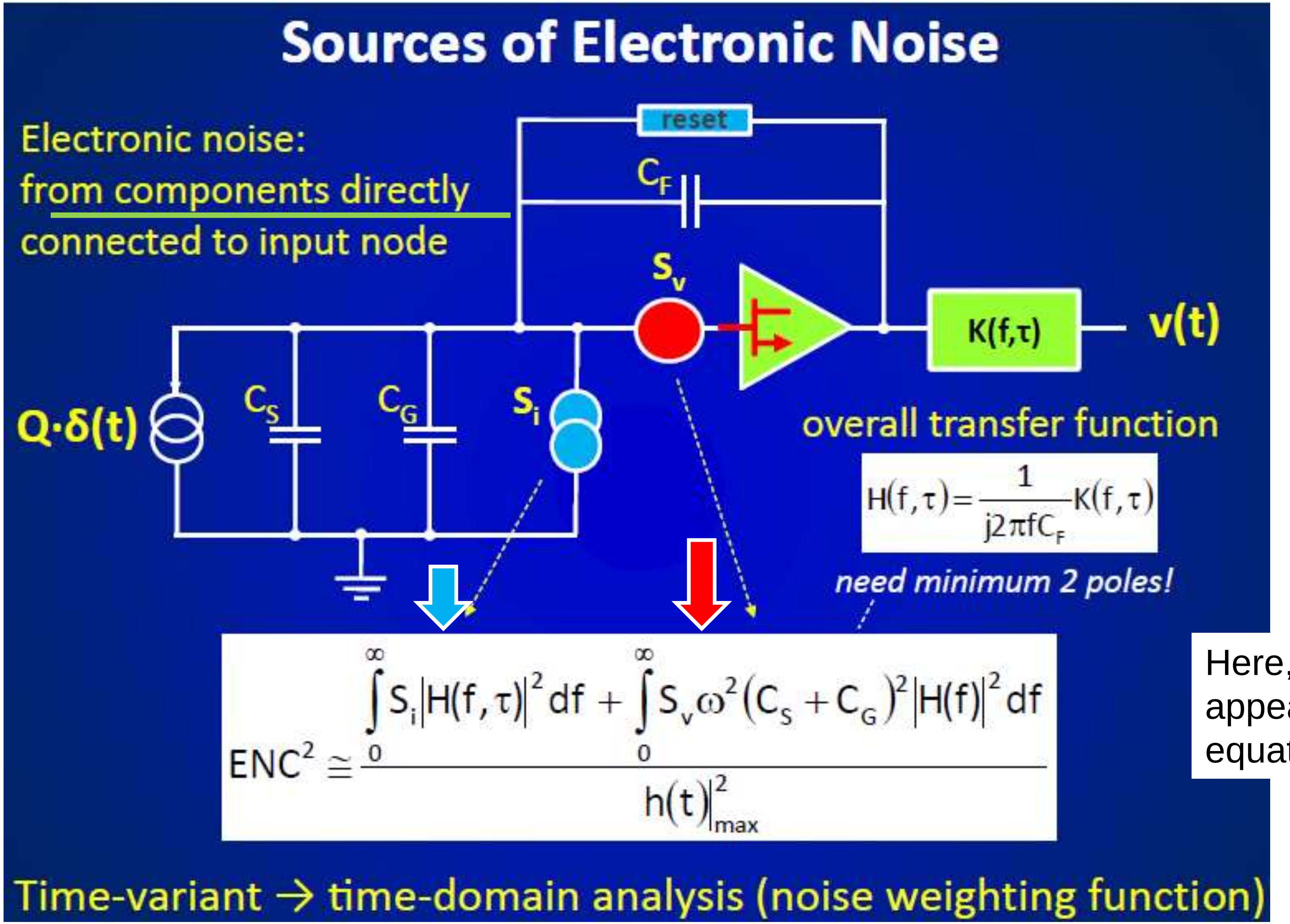
$$Q_n^2 \approx \left[\left(2q_e I_d + \frac{4kT}{R_P} + i_{na}^2\right)\cdot T + \left(4kTR_S + e_{na}^2\right)\cdot\frac{C_d^2}{T} + 4A_f C_d^2\right]$$

current noise	voltage noise	$1/f$ noise
$\propto T$	$\propto 1/T$	independent of T
independent of C_d	$\propto C_d^2$	$\propto C_d^2$

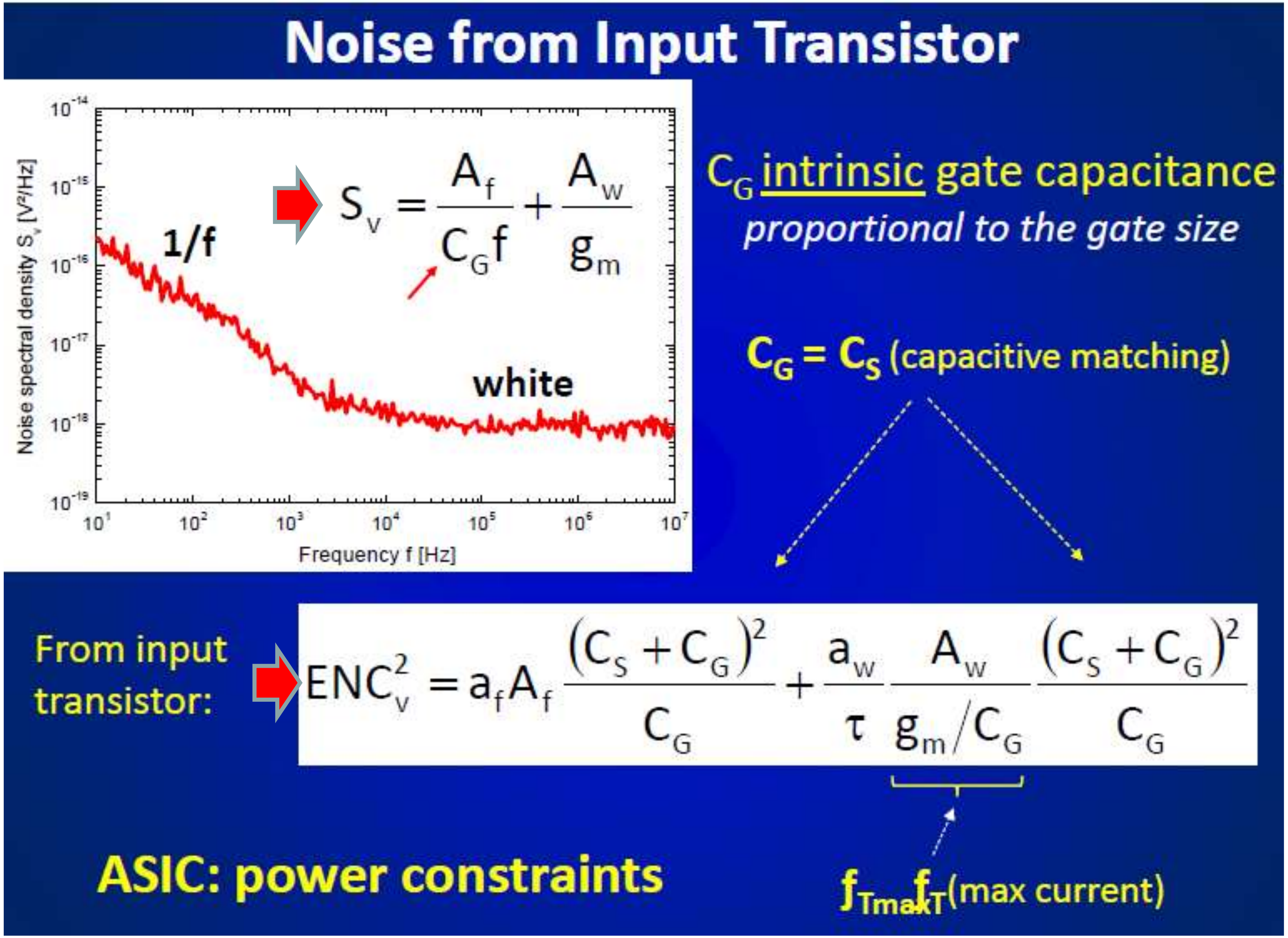
Helmuth Spieler
LBNL

One stage preamplifier scheme

- The main contributor to the total noise is the **preamp input transistor**. We consider next the contribution of this transistor to the equivalent noise $\mathbf{e_n}$ and $\mathbf{i_n}$



Here, time appears in equations


Noise from Input Transistor
Noise spectral density S_v [V²/Hz]
Frequency f [Hz]
1/f
white
$S_v = \frac{A_f}{C_G f} + \frac{A_w}{g_m}$
C_G intrinsic gate capacitance
proportional to the gate size
$C_G = C_S$ (capacitive matching)
From input transistor:
$ENC_v^2 = a_f A_f \frac{(C_S + C_G)^2}{C_G} + \frac{a_w}{\tau} \frac{A_w}{g_m / C_G} \frac{(C_S + C_G)^2}{C_G}$
ASIC: power constraints
f_{Tmax} f_T (max current)

Input Transistor in CMOS

From transistor's white noise:

$$ENC_{vw}^2 \approx \frac{a_w}{\tau}\frac{A_w}{g_m(I_D)/C_G}\frac{(C_S+C_G)^2}{C_G}$$

Fix power = fix drain current I_D
→ **size (W,L) ?**

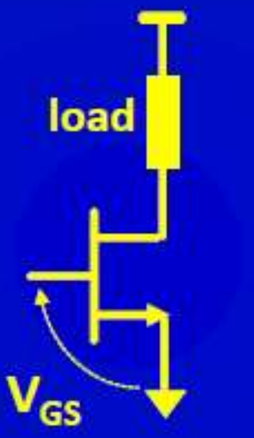
V_{GS} >> V_{th} (strong inversion)

$$g_m(I_D) \approx \sqrt{\frac{2\mu C_{ox}}{n}\frac{W}{L}I_D} \propto \sqrt{\frac{C_G I_D}{L^2}}$$

↓

$$ENC_{vw}^2 \propto \frac{L}{\sqrt{I_D}}\frac{(C_S+C_G)^2}{\sqrt{C_G}}$$

- minimum L
- $C_G = C_S/3$



V_{GS} << V_{th} (weak inversion)

$$g_m(I_D) \approx \frac{I_D}{nV_T} \propto I_D$$

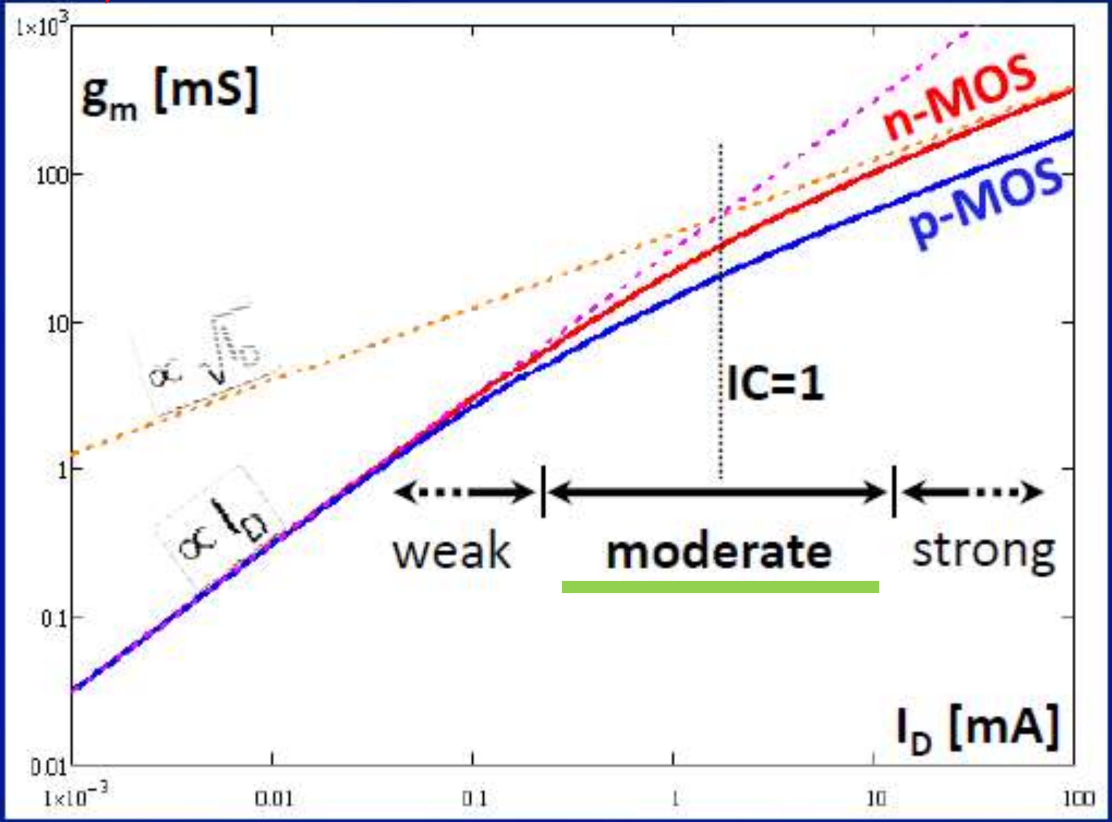
↓

$$ENC_{vw}^2 \propto \frac{(C_S+C_G)^2}{I_D}$$

- independent of L
- $C_G = 0$ pushes back towards strong inversion

→ $V_{GS} \approx V_{th}$ (moderate inversion): model?

Moderate Inversion



Notice gm/Id efficiency

From EKV model

$$g_m(I_D) \approx \frac{I_D}{nV_T} \frac{\sqrt{1+4\cdot IC}-1}{2\cdot IC}$$

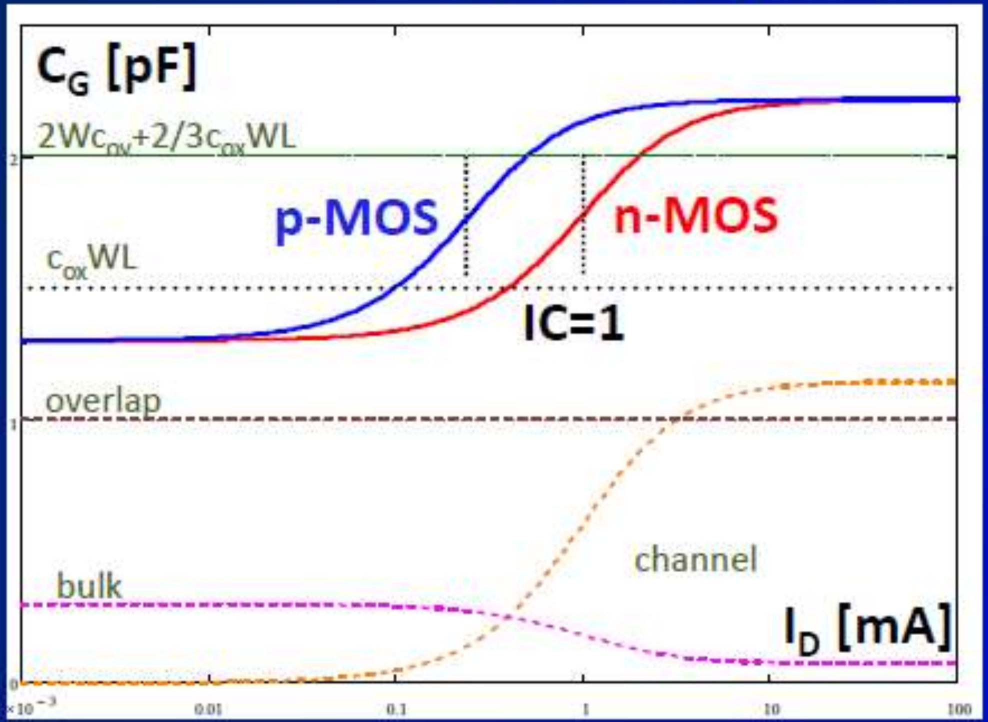
$$IC = \frac{L}{W}\frac{I_D}{2nV_T^2\mu c_{ox}}$$

inversion coefficient

alternative: extract from simulators (BSIM)

De Geronimo, IEEE TNS 52, 2005

Gate Capacitance



~~$\propto C_{ox}WL$~~

$$C_G(I_D) \approx 2c_{ov}W + C_{ox}WL\left(\gamma_C(IC) + \frac{n-1}{n}\left[1-\gamma_C(IC)\right]\right)$$

$$\gamma_C(IC) \approx \left(\frac{3}{2} + \frac{1}{3}\frac{\sqrt{1+4\cdot IC}+1}{IC^2}\right)^{-2/3}$$

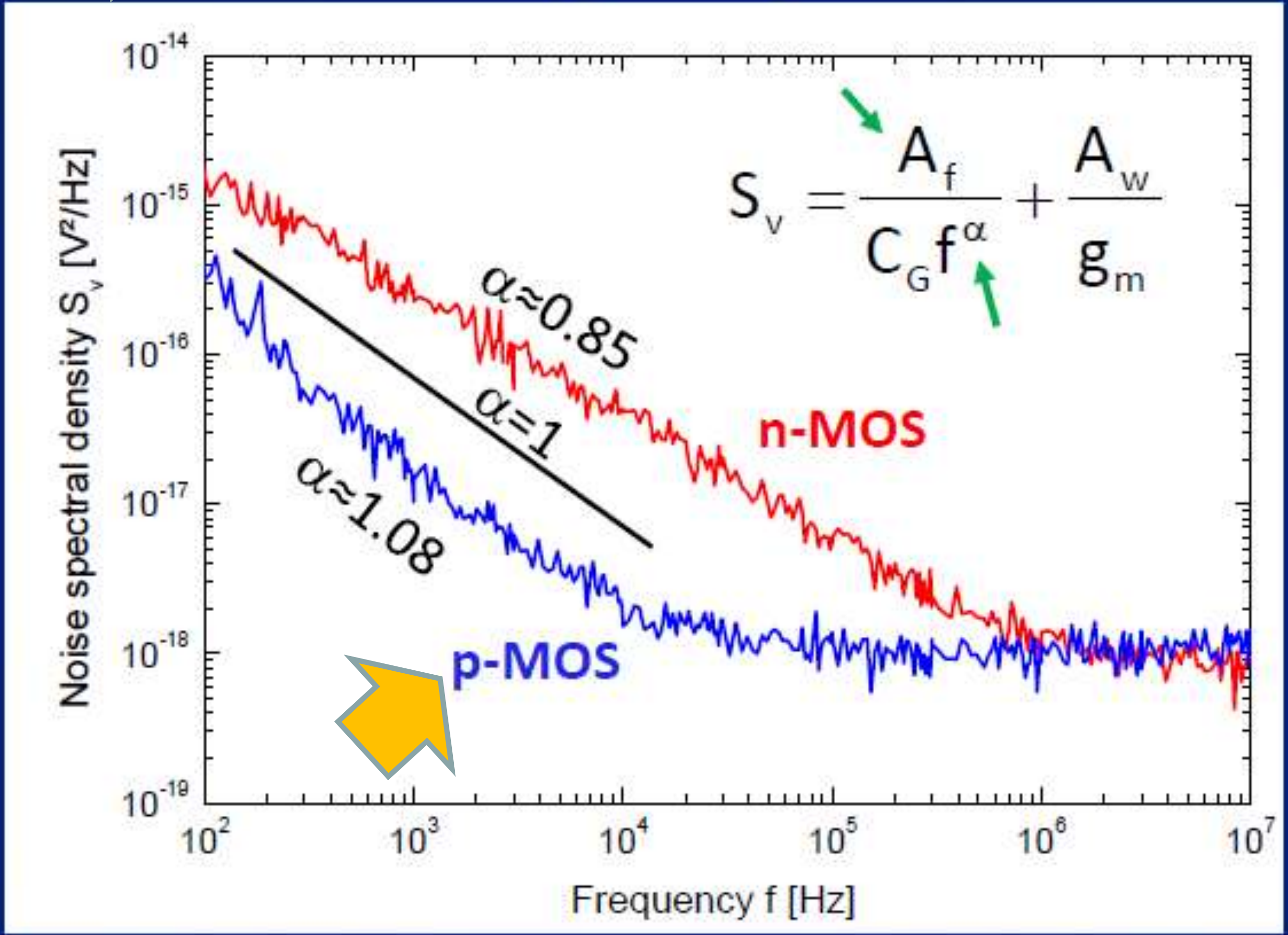
Both g_m and C_G push towards using n-channel and **$L = L_{min}$**

Conclusion 1: considering (g_m and Cg)

De Geronimo, IEEE TNS 52, 2005

BUT,
now let add the 1/f noise contribution

Low-Frequency Noise



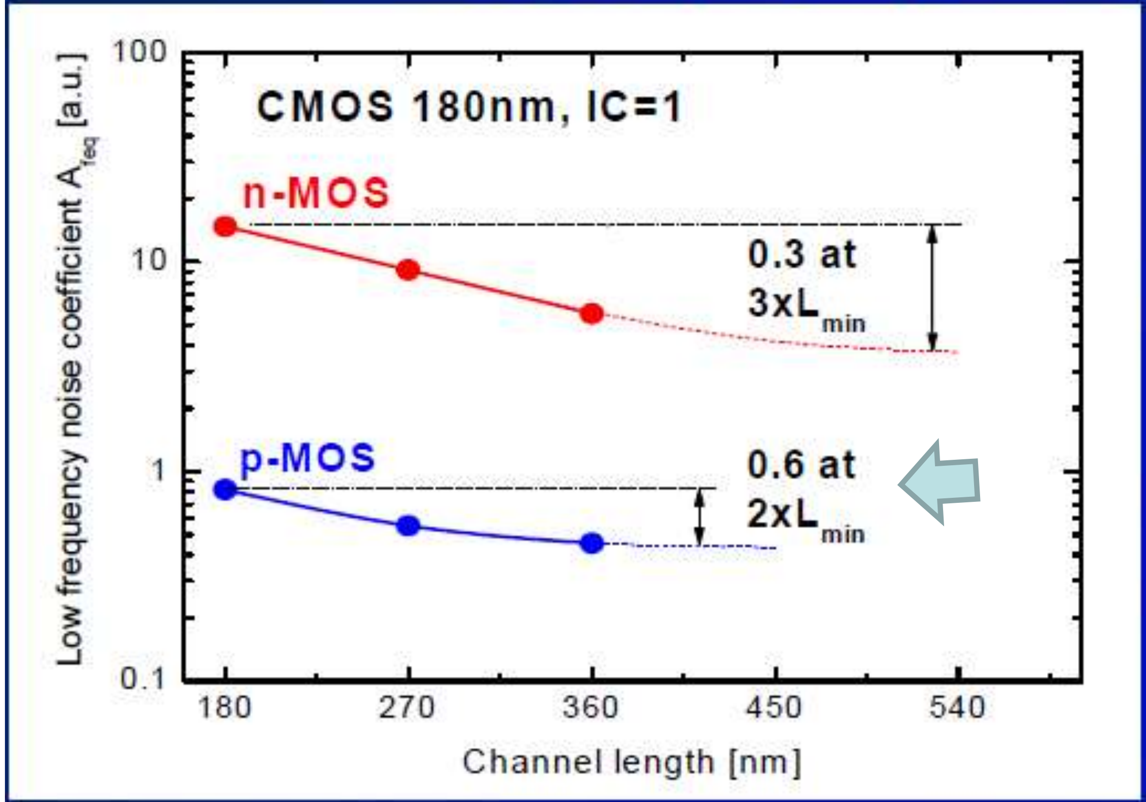
From transistor's low-freq. noise:

$$ENC_{vf}^2 = a_f(\alpha)\frac{A_f}{\tau^{1-\alpha}}\frac{(C_S + C_G)^2}{C_G}$$

depends on τ

De Geromino

Low-Frequency Noise vs L



$$S_V = \frac{A_f(L)}{C_G f^{\alpha}} + \frac{A_W}{g_m}$$

1/f equivalent, IEEE TNS 58, 2011

From transistor's low-freq. noise:

$$ENC_{vf}^2 = a_f(\alpha)\frac{A_f(L)}{\tau^{1-\alpha}}\frac{(C_S + C_G)^2}{C_G}$$

Conclusion 2

LF noise pushes towards p-channel & L > L_{min}

Preamp trends with agressive process

Preamp design & ‘Scaling’

One may consider 2 prospection studies (old now !!)

1) Paul O’Connor => Brookhaven Lab, Upton , New York: 1,5µ au 180n

At constant power one save 23% in term of noise per generation: λ=0.7

$$ENC' = \lambda^{3/4} * ENC$$

At constant noise: one save 60% of power per generation

$$P' = \lambda^{3} * P$$

But in dynamique range, one lose 10% of SNR per generation

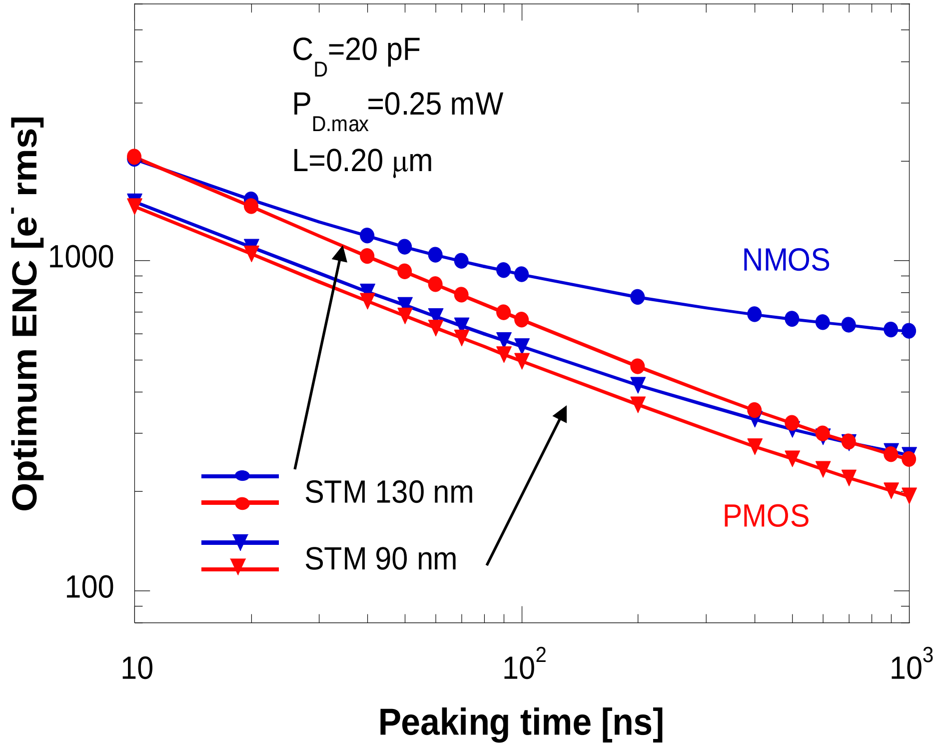
$$\mathbf{SNR' = \lambda^{1/4} * SNR}$$

2) L. Rattia NSS 2007 => Università degli Studi di Pavia : 100n et 90n

Next slides show some of L. Rattia’ work

ENC *vs* peaking time, @ Pd=cte

L. Rattia NSS 2007 =>The 90nm leads to less ENC (noise) than the 130nm

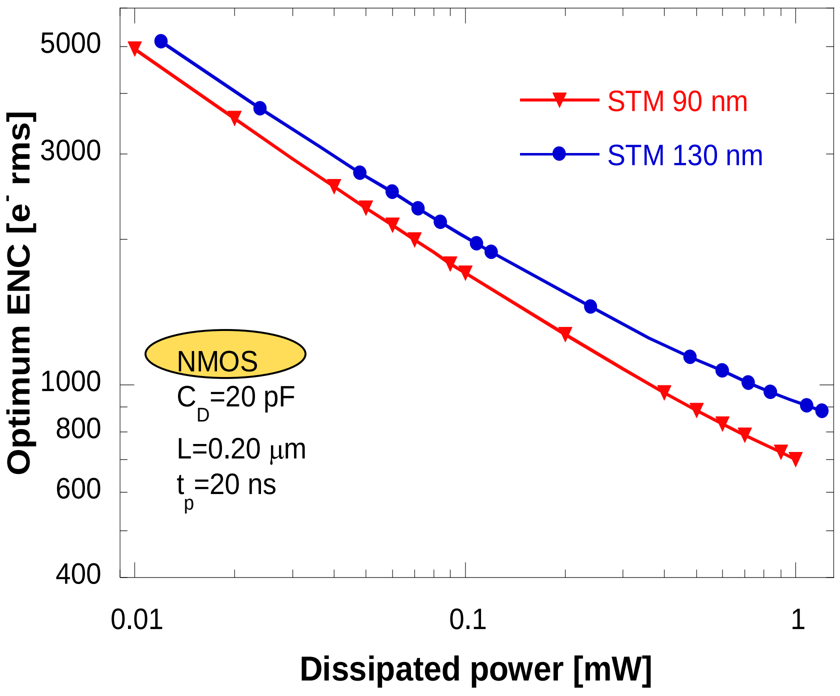


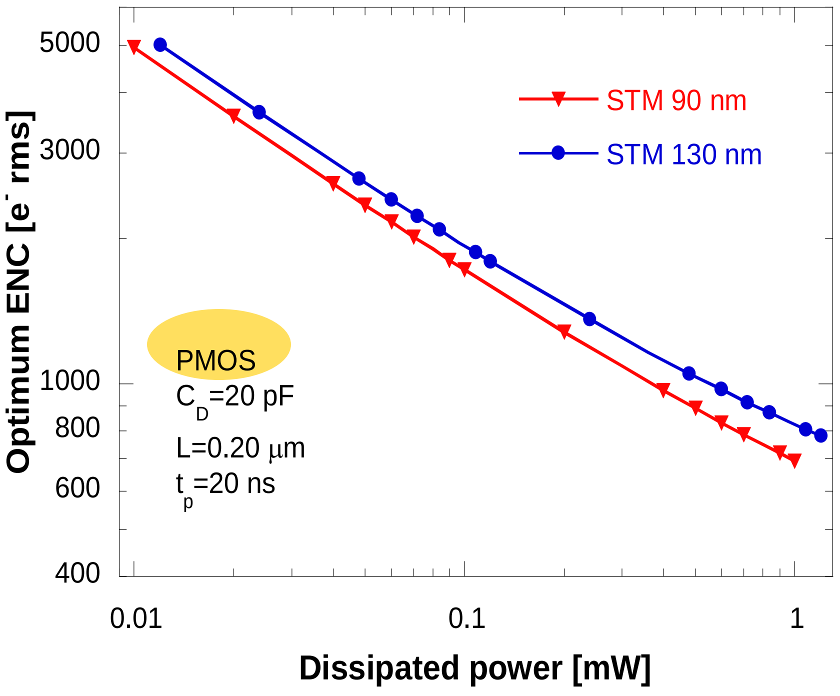
ENC was evaluated in the case of a second order, unipolar (RC^2-CR) shaping processor

- In the explored peaking time and power range, **PMOS input device** always provides better noise performances than NMOS input (*except for the 130 nm process at t_p close to 10 ns*)
- Using the 90 nm process may yield quite significant improvement with respect to the 130 nm technology, especially when NMOS input charge preamplifiers are considered

ENC *as function of power necessary (P et N)* ***for given tp***

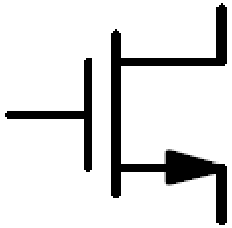
L. Rattia NSS 2007 **=> The 90nm needs less power at a constant noise level**





- At t_p=20 ns, noise performances provided by NMOS and PMOS input devices in the 90 nm technology are comparable
- Better noise-power trade-off can be achieved by using the 90 nm technology

Figure of Merit for a MOS Process



- Transit Frequency

$$\omega_T = \frac{g_m}{C_{gs}}$$

- Transconductor Efficiency

$$\frac{g_m}{I_D}$$

- Intrinsic Gain

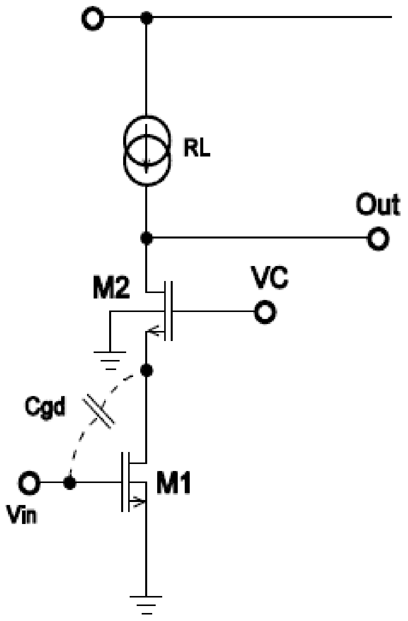
$$g_m r_o$$

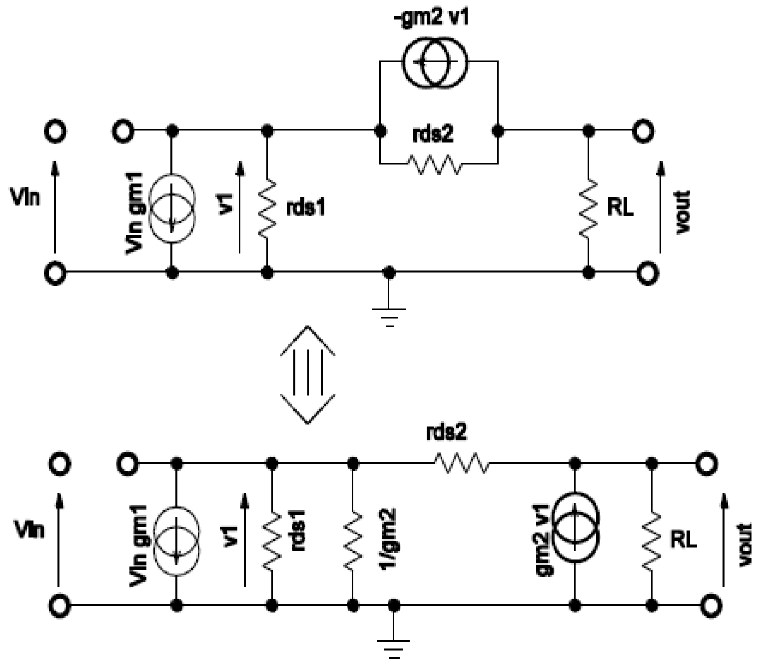
DO NOT FORGET: You may optimize the *bias point of input transistor* (moderate inversion, for a gm/Id efficiency)

HOW THE AMPLIFIER STAGE IS BUILT USING TRANSISTORS

Exemple of single stage amplifier

Telescopic Cascode: Common source-common gain





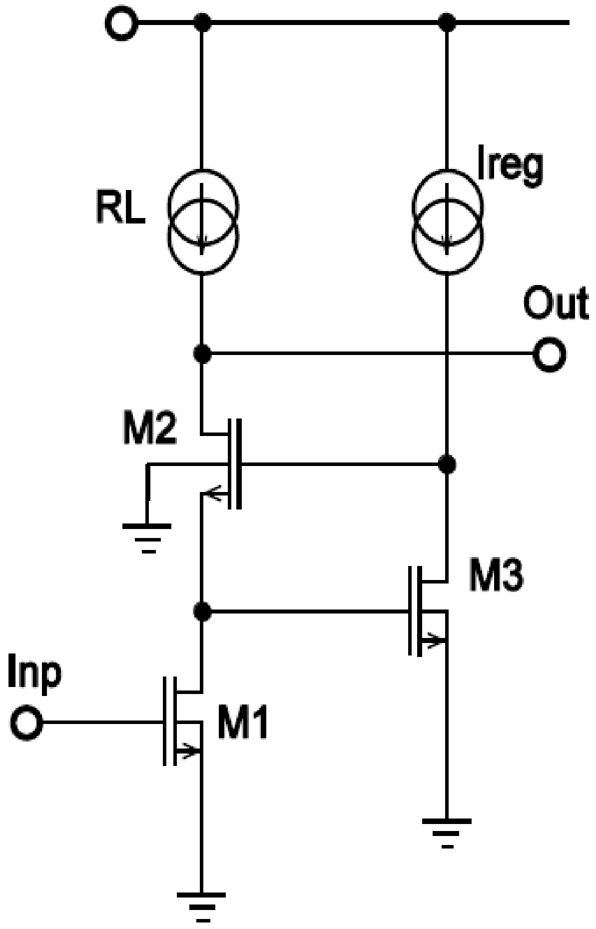
$$K_U \cong \frac{-g_{m1}}{g_{DS1}\dfrac{g_{DS2}}{g_{m2}} + g_L}$$

$$GBW = \frac{g_{m1}}{2\pi\, C_{OUT}}$$

Courtesy Y. Kaplon, Cern

- single stage amplifier; one dominant pole
- no Miller effect (low gain of common source stage)

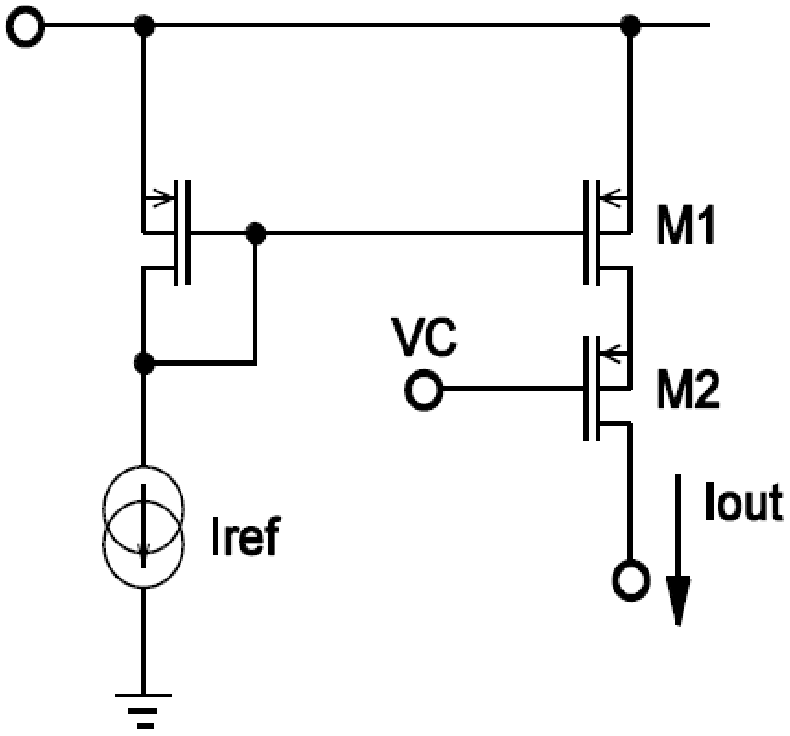
Regulated Cascode: **A0** improvement by acting on the cascode



$$K_U \cong \frac{-g_{m1}}{\frac{g_{DS1} \cdot g_{DS2}}{g_{m2}} \cdot \frac{g_{DS3}}{g_{m3}} + g_L}$$

- Cascode transistor controlled with common source amplifier
- Higher output conductance of cascode; possible higher gain
- GBW the same as for simple cascode

Cascode current: gain bosting via the load

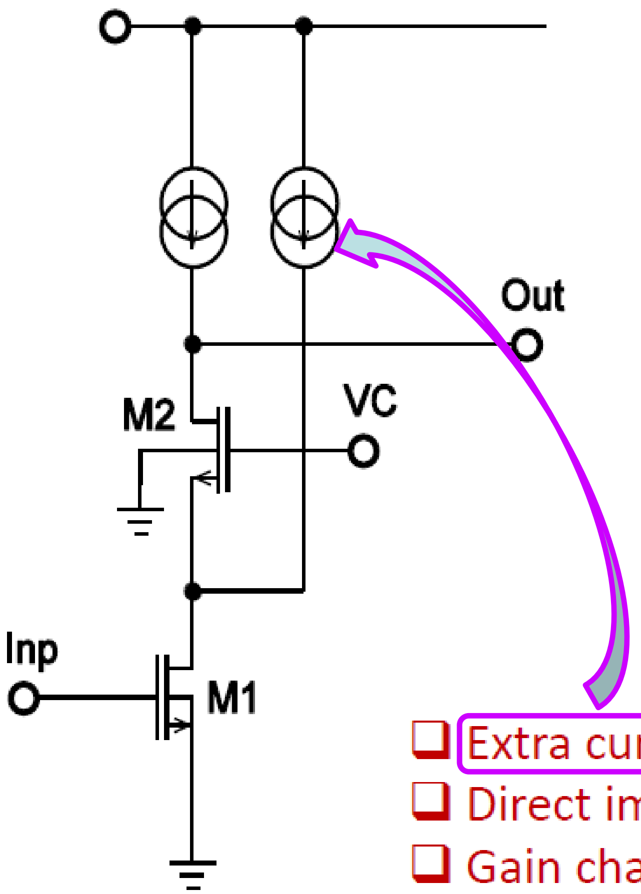


$$r_{OUT} \cong g_{m2} \cdot r_{DS2} \cdot r_{DS1}$$

But he dynamic range is reduced by 2*Vdssat

- Amplification of r_{DS1} by g_{m2}
- For short SSD application; OK for 250nm, not sufficient for 130 & 90nm

Boosting **both** Gain & bandwidth



$$GBW = \frac{g_{m1}}{2\pi \, C_{OUT}}$$

$$K_U \cong \frac{-g_{m1}}{\frac{g_{DS1} \cdot g_{DS2}}{g_{m2}} + g_L}$$

- Extra current source to drain of M1 → increase of g_{m1}
- Direct impact on gain bandwidth
- Gain changed according to output conductance of cascode and active load

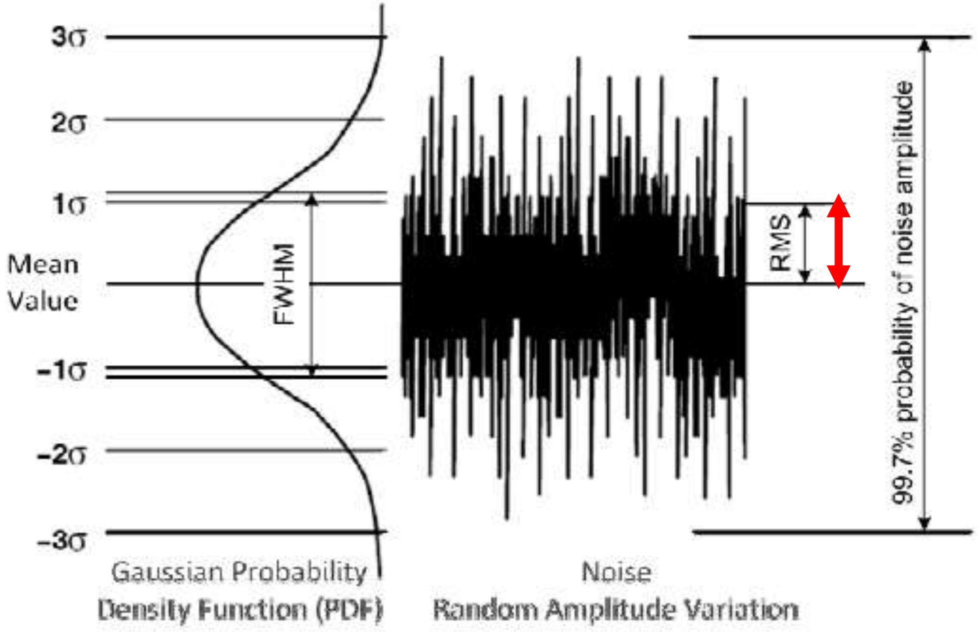
Courtesy Y. Kaplon, Cern

After the amplification schemes, let introduce how electronics noise looks like in the readout flow

NOISE EFFECT IN THE FLOW

Noise: Time domain characteristics

Noise: Time Domain Characteristics



- Noise amplitudes vary randomly with time.
- Noise can only be specified by a Probability Density Function (PDF).
- Thermal noise and shot noise have Gaussian PDFs.
- Theoretically Gaussian noise amplitudes may have values approaching infinity.

Topics for later sessions

How can one characterize a comparator for counting accurately?

In couting flow: Frequency of noise hits (**fn**); Threshold **Vth**; input noise (**vn**) …

S. O. Rice *Mathematical analysis of random noise* [1945] Bell System Technical journal, 24; 46-156

- How often are noisy events counted?
- Noise at your comparator input?
- Threshold value above the baseline?
- Counting rate and so your bandwidth

  or τ?

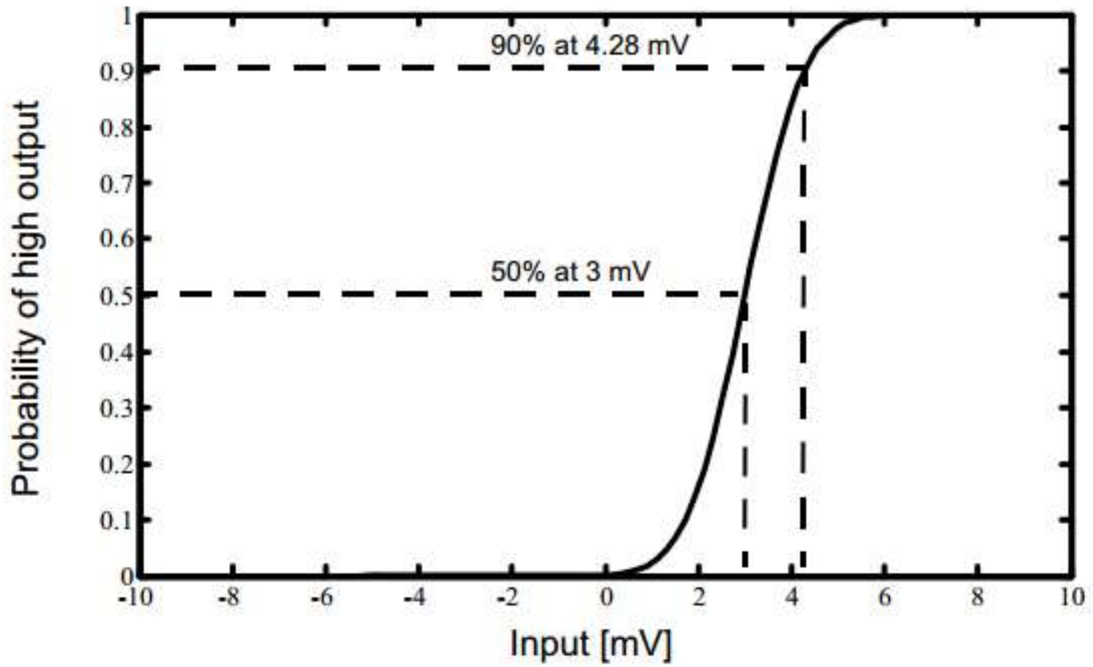
$$f_n = \frac{1}{2\pi\tau} e^{-(Vth^2/2Vn^2)}$$

Or Threshold over noise ratio

$$\frac{Vth}{Vn} = \sqrt{-2\ln(fn * 2\pi\,\tau)}$$

EXO: estimate f_n for V_{th}/V_n =7 for different τ

Cumulative distribution function (CDF) The comparator issue using S-curve



Probability of a high output logic level with varing input level for a particular comparator

The 90% confidence interval for
A normal distribution is at **1.3** (or 1.2816) times its standard deviation (rms)
In this exemple 90% correspond to 4.28mV; hence the rms input noise will be
(4.28mV – 3mV)/1.2816 =1mV rms

Cumulative Distribution Function

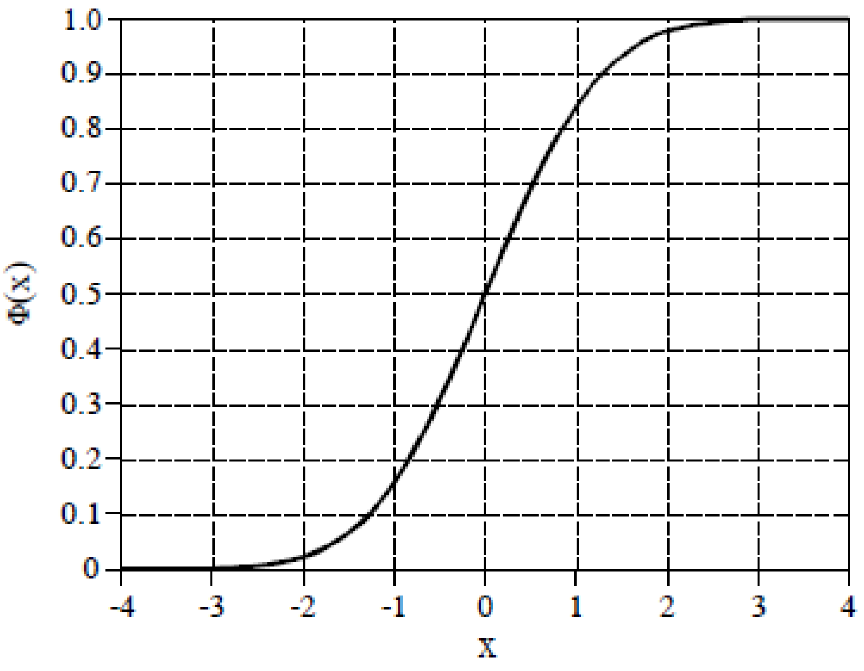
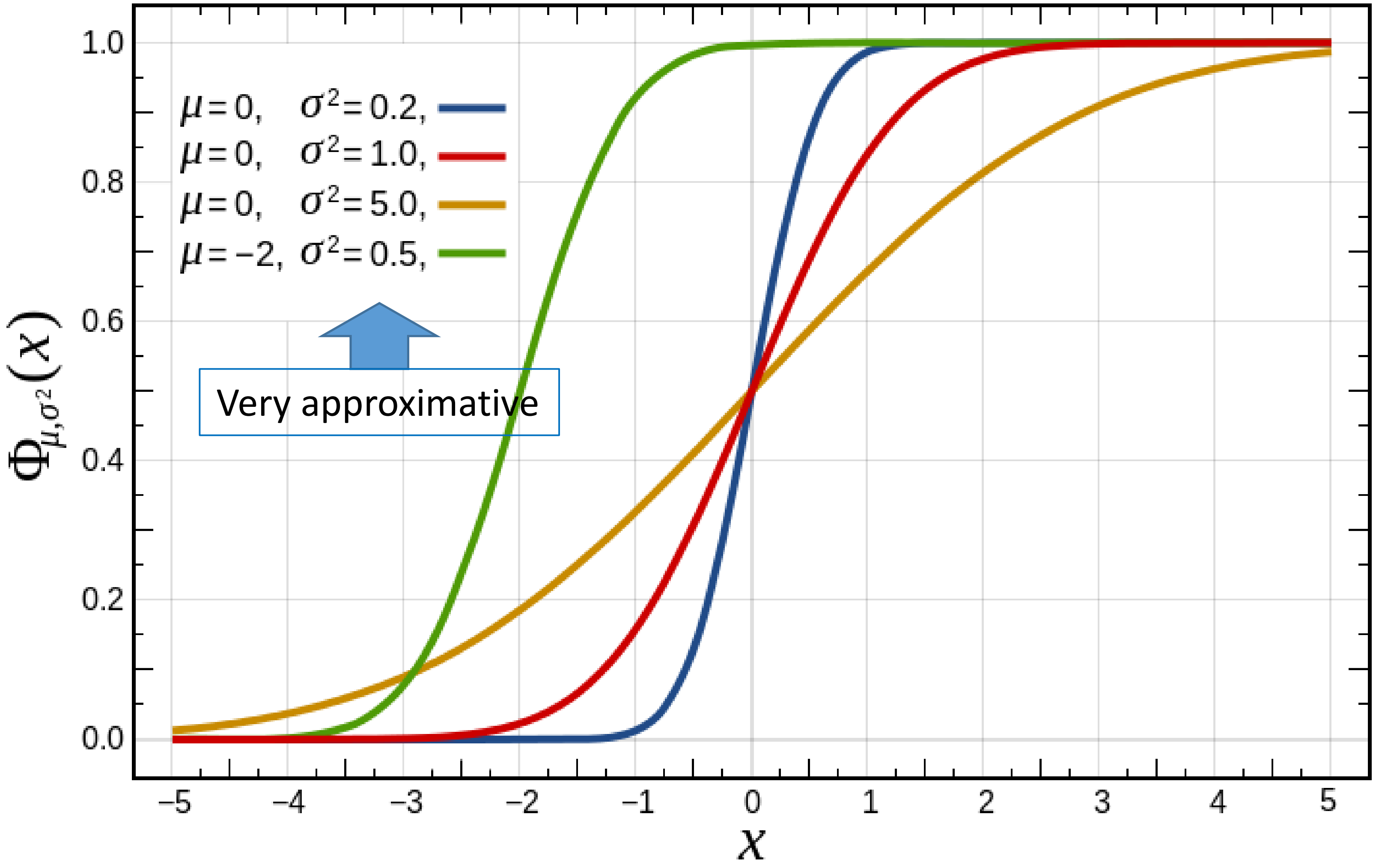


FIGURE 2-9 & TABLE 2-5
$\Phi(x)$, the cumulative distribution function of the normal distribution (mean = 0, standard deviation = 1). These values are calculated by numerically integrating the normal distribution shown in Fig. 2-8b. In words, $\Phi(x)$ is the probability that the value of a normally distributed signal, at some randomly chosen time, will be less than x. In this table, the value of x is expressed in units of standard deviations referenced to the mean.

x	Φ(x)	x	Φ(x)
-3.4	.0003	0.0	.5000
-3.3	.0005	0.1	.5398
-3.2	.0007	0.2	.5793
-3.1	.0010	0.3	.6179
-3.0	.0013	0.4	.6554
-2.9	.0019	0.5	.6915
-2.8	.0026	0.6	.7257
-2.7	.0035	0.7	.7580
-2.6	.0047	0.8	.7881
-2.5	.0062	0.9	.8159
-2.4	.0082	1.0	.8413
-2.3	.0107	1.1	.8643
-2.2	.0139	1.2	.8849
-2.1	.0179	1.3	.9032
-2.0	.0228	1.4	.9192
-1.9	.0287	1.5	.9332
-1.8	.0359	1.6	.9452
-1.7	.0446	1.7	.9554
-1.6	.0548	1.8	.9641
-1.5	.0668	1.9	.9713
-1.4	.0808	2.0	.9772
-1.3	.0968	2.1	.9821
-1.2	.1151	2.2	.9861
-1.1	.1357	2.3	.9893
-1.0	.1587	2.4	.9918
-0.9	.1841	2.5	.9938
-0.8	.2119	2.6	.9953
-0.7	.2420	2.7	.9965
-0.6	.2743	2.8	.9974
-0.5	.3085	2.9	.9981
-0.4	.3446	3.0	.9987
-0.3	.3821	3.1	.9990
-0.2	.4207	3.2	.9993
-0.1	.4602	3.3	.9995
0.0	.5000	3.4	.9997

Cumulative Distribution Function (CDF)
The comparator issue using S-curve

The mean value µ could correspond to:
The Vth ± offset



EXO: Can you calculate a more accurate value using 90% ->1.3