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Reaction mechanisms for deuteron production in HICs

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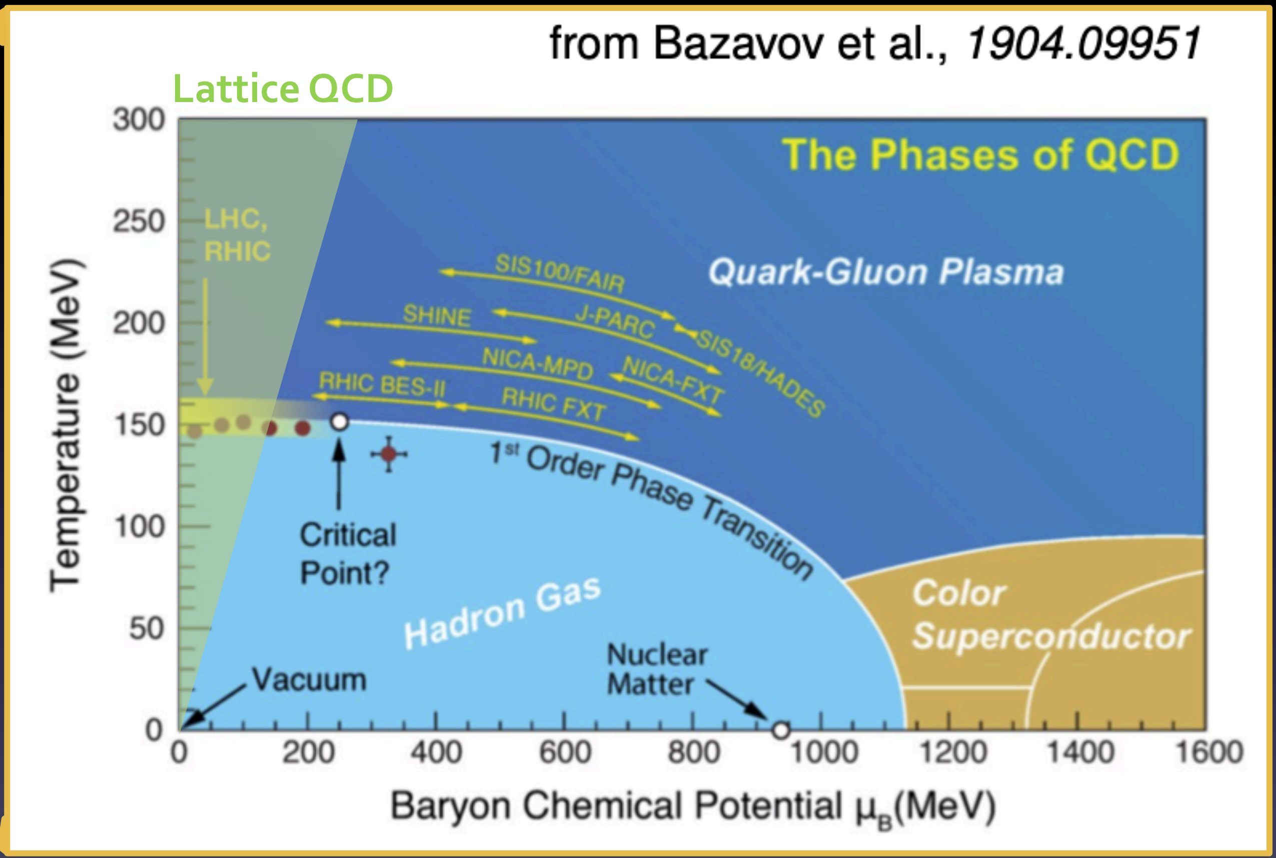
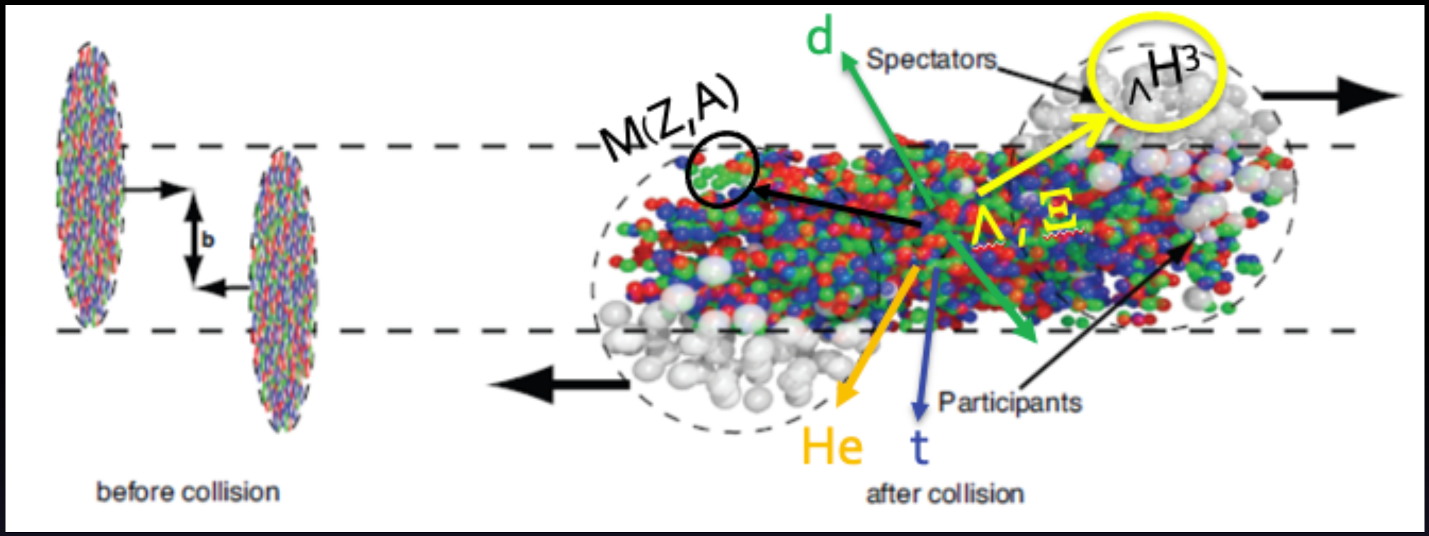
PHSD PHQMD

HIPSTARS - Workshop on Heavy-Ion Physics and Compact Stars (online)
December 3rd 2020

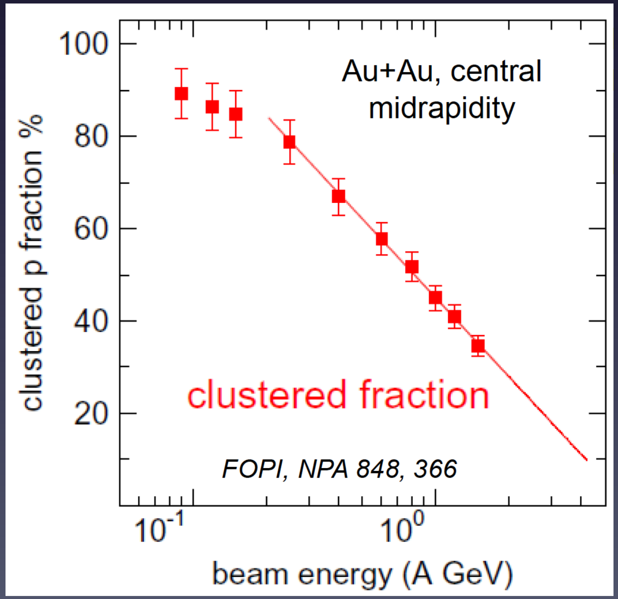
Motivations



Clusters can be formed both in the overlap and in the target/projectile region

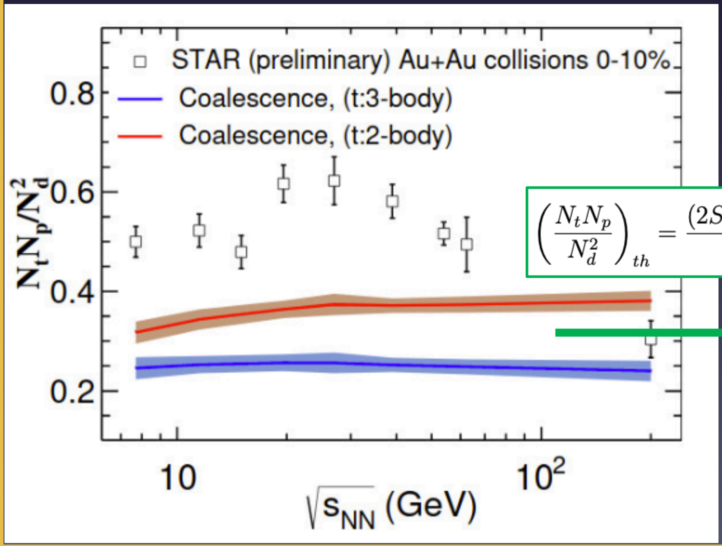


Large abundance affects bulk observables



Density fluctuations can enhance light nuclei multiplicities

→ L.N. Ratios are sensitive to QCD EoS



$$N_d \propto \int d^3\vec{x}\rho_n(\vec{x})\rho_p(\vec{x})$$

$$N_t \propto \int d^3\vec{x}\rho_n^2(\vec{x})\rho_p(\vec{x})$$

$$\left(\frac{N_t N_p}{N_d^2}\right)_{th} = \frac{(2S_t+1)(2S_p+1)}{(2S_d+1)^2}\frac{m_t^2 m_p^2}{m_d^4}\frac{K_2(m_t/T)K_2(m_p/T)}{K_2(m_d/T)^2}$$

~ 0.32 at T~155 MeV

Sun et al. PLB77 (2017)

Zhao et al. PRC102 (2020)

Theoretical models for clusters

- Static approaches:

  - **Thermal Model: [Andronic et al. PLB 697(2011)]**
    Produced at mid-rapidity from equilibrium source
    Yields unchanged after chemical freezout.

    T_{CFO}~155 MeV >> E_B ~ 2-8 MeV
    "Create Ice cubes in a Fire"

  - **Coalescence Model: [Li,Ko PRC93(2013); Sun et al. PLB781(2018); Sombun et al. PRC99 (2019)]**
    Numbers of d (t) calculated from overlap of $f_{p,n}(x,p)$
    with Wigner function of d (t) internal wave-function.

- Dynamical approaches:

  - **Kinetic approach: Cluster production and breakup by hadronic reactions**

  - **Recognition Algorithms: [PHQMD PRC101 (2020)]**

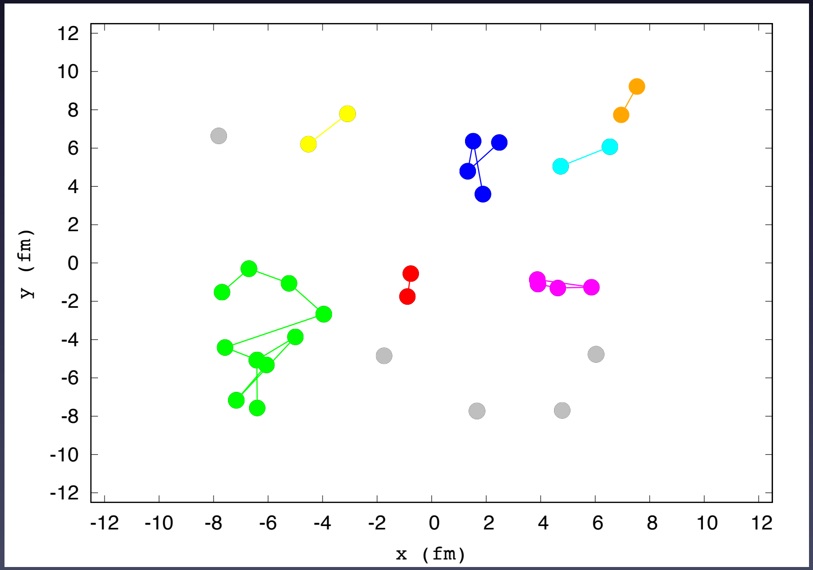
Minimum Spanning Tree

[Phys. Rep. 510 (2012) 119-200]

- **Algorithms for network's design and other applications.**
- Cluster recognition method which is applied in the asymptotic final state looking at **correlations of particle "i" and particle "j" in coordinate space.**

$$\left| \vec{r}_i - \vec{r}_j \right| \leq 2.5\ fm$$

- Algorithm can be improved adding constraints also in the momentum space → Coalescence



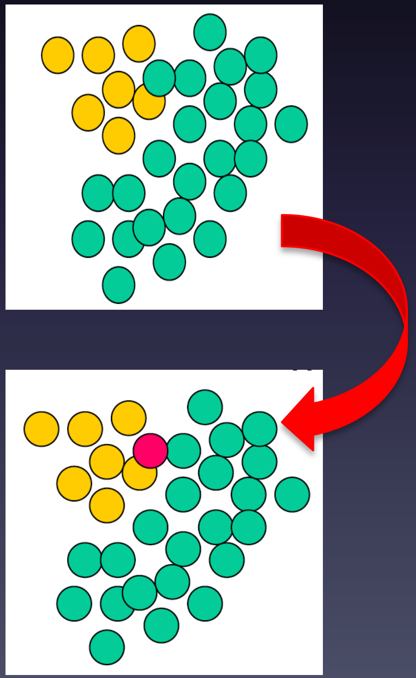
Example: Clustering pf particles in a "box" with Kruskal's algorithm.

Simulated-Annealing-Clusterization-Algorithm (SACA)

- **Simulated Annealing is an optimization procedure based on Metropolis' algorithm with Boltzmann's probability distribution**
  [Van Laarhoven and Aarts – SA: Theory and Applications (1987)]

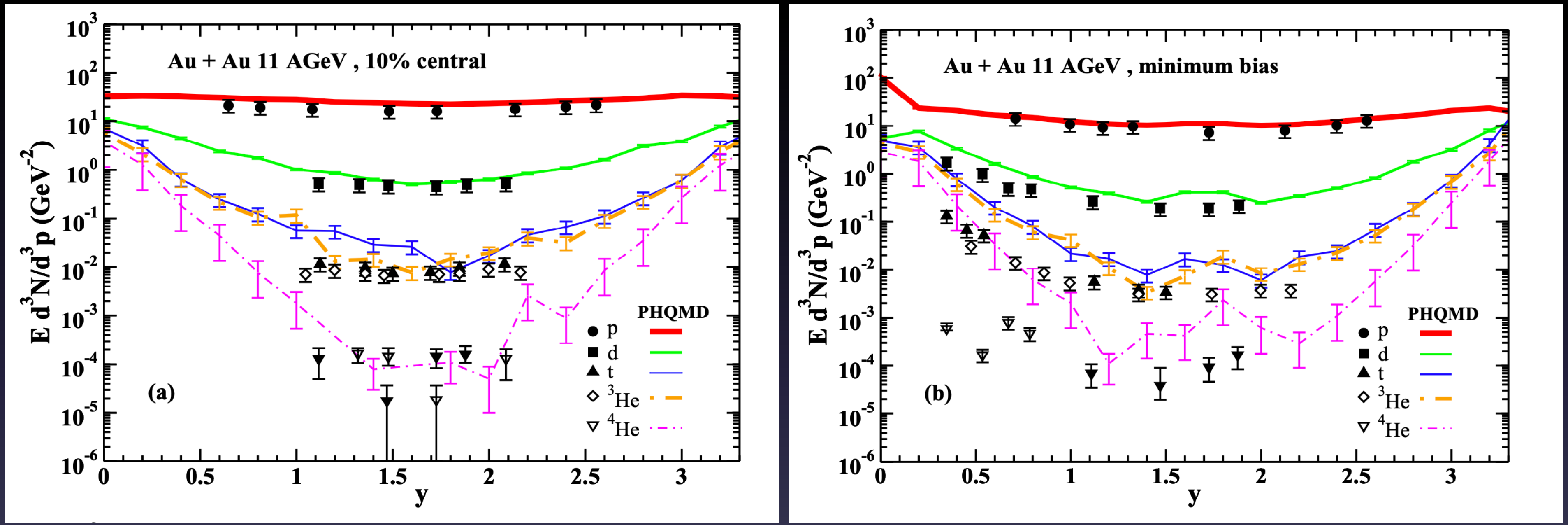
- **Compute the total energy E of an initial configuration of free and bound nucleons.**
  (E = Kinetic En. + Potential Interaction in each cluster)

- **Exchange nucleons within a cluster to find the configuration with the minimum energy.**



Ref. Aichelin, Bratkovskaya et al. PRC101 (2020)
Kireyeu et al. Bull.Russ.Acad.Sci.Phys. 84 (2020)
Puri and Aichelin J. Comp. Phys.162 (2000)
Dorso and Randrup PLB301 (1993)

Invariant light nuclei distributions with p<0.1 GeV at AGS energy
MST recognition



[Ref. Aichelin, Bratkovskaya et al. PRC101 (2020)]

Kinetic approach

$$p_{1,\mu}\partial^{\mu}_{x} f_i(x,p_1) = I^{i}_{coll}$$

- Boltzmann Eq. describes dynamical evolution of (on-shell) phase-space distribution function $f_i(x,p_1)$

- *Collision Integral* accounts variation of $f_i(x,p_1)$ due to dissipative processes

$$I^{i}_{coll} = \sum_{n}\sum_{m} I^{i}_{coll}[n \leftrightarrow m]$$

$$= \sum_{n}\sum_{m}\frac{1}{2}\frac{1}{N_{id}!}\sum_{\nu}\sum_{\lambda}\left(\frac{1}{(2\pi)^3}\right)^{n+m-1}\int\left(\prod_{j=2}^{n}\frac{d^3\vec{p}_j}{2E_j}\right)\left(\prod_{k=1}^{m}\frac{d^3\vec{p}_k}{2E_k}\right)$$

$$\times W_{n,m}(p_1,p_j;i,\nu \mid p_k;\lambda)\ (2\pi)^4\ \delta^4(p_1^{\mu}+\sum_{j=2}^{n}p_j^{\mu}-\sum_{k=1}^{m}p_k^{\mu})$$

$$[\prod_{k=1}^{m} f_k(x,p_k) - f_i(x,p_1)\prod_{j=2}^{n} f_j(x,p_j)]$$

Transition amplitude

- d+π , d+N elastic scattering
- Inelastic production/breakup

d+π <-> N+N

d+N <-> N+N+N

d+π <-> N+N+π

Etc…

Example: d+π <-> N+N (two-body) reactions:

- The scattering rate in a small volume dV:

$$\frac{dN_{coll}[1(i)+2 \leftrightarrow 3+4]}{dtdV} = \int \frac{d^3p_1}{2E_1} I^i_{coll}[2 \leftrightarrow 2]$$

- For forward reaction (→) the scattering rate can be expressed in terms of total cross section for d breakup by incident pion into N+N pair

$$\frac{dN_{coll}[1(i)+2 \rightarrow 3+4]}{dtdV} = \int \frac{d^3p_1}{(2\pi)^3 2E_1} f_1(x,p_1) \int \frac{d^3p_2}{(2\pi)^3 2E_2} f_2(x,p_2)$$

$$\frac{1}{N_{id}!} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} W_{2,2}(p_1,p_2 \mid p_3,p_4)(2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$

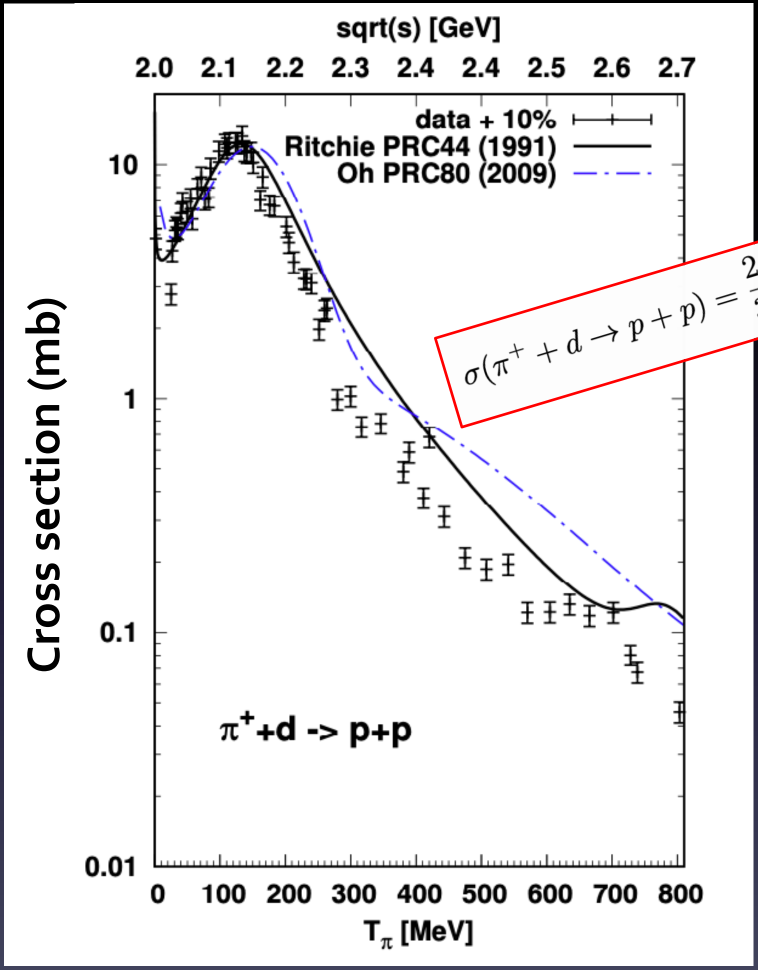
$$4E_1E_2v_{rel}\sigma^{2,2}_{tot}$$

Using Test-Particle representation of *f(x,p)* the collision integral is numerically solved dividing the coordinate space in ΔV_{cell} where the reactions at each time step Δt are sampled stocastically with probability:

$$P_{2,2}\left(\sqrt{s}\right) = \frac{1}{N_{id}!}\sigma^{2,2}(\sqrt{s})v_{rel}(\sqrt{s})\frac{\Delta t}{\Delta V_{cell}}$$

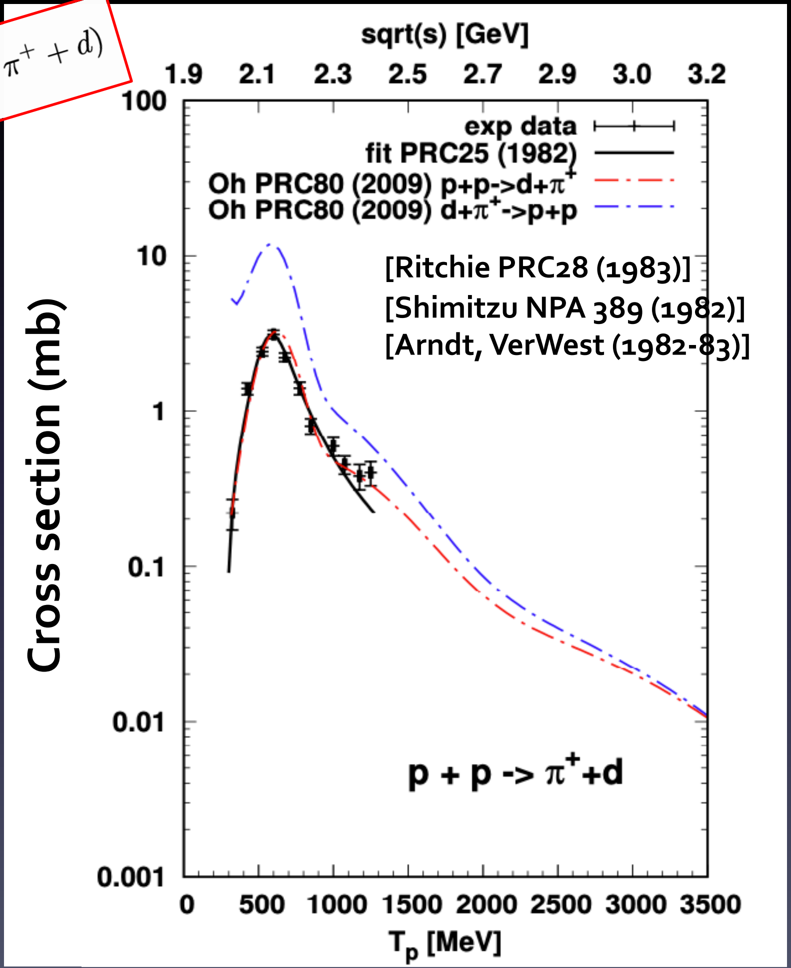
Example: d+π <-> N+N (two-body) reactions:

- For forward reaction (→) the scattering rate can be expressed in terms of total cross section for d breakup by incident pion into N+N pair



- For backward reaction (←) theory/exp. analysis proved validity of detailed balance

$$\sigma(\pi^+ + d \to p + p) = \frac{2}{3}\frac{(p_N^*)^2}{(p_\pi^*)^2}\sigma(p + p \to \pi^+ + d)$$

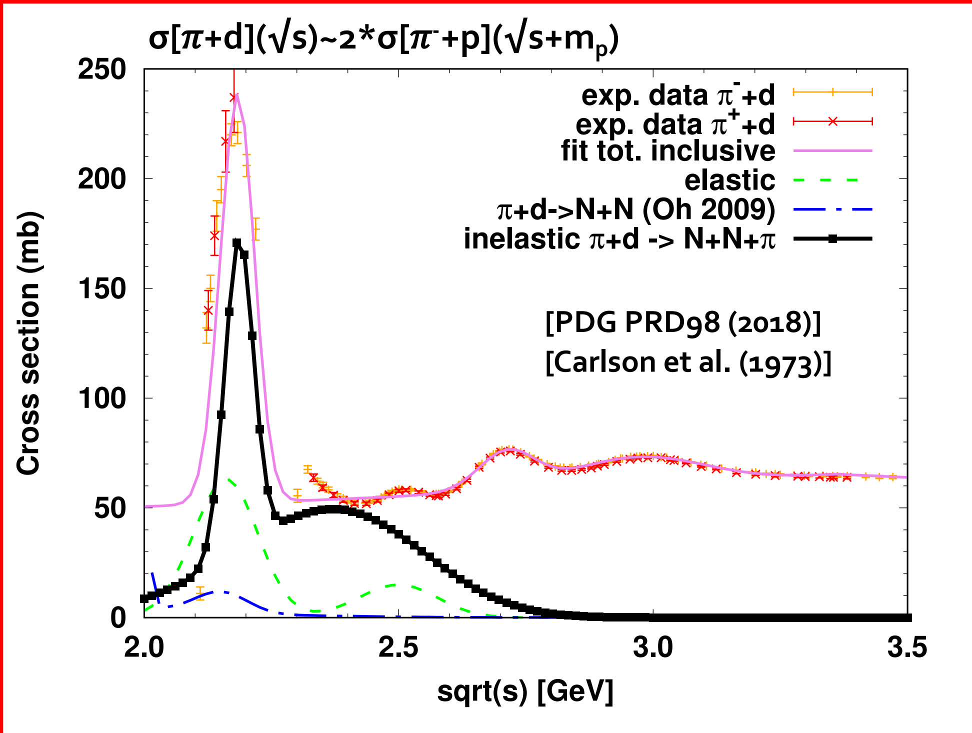


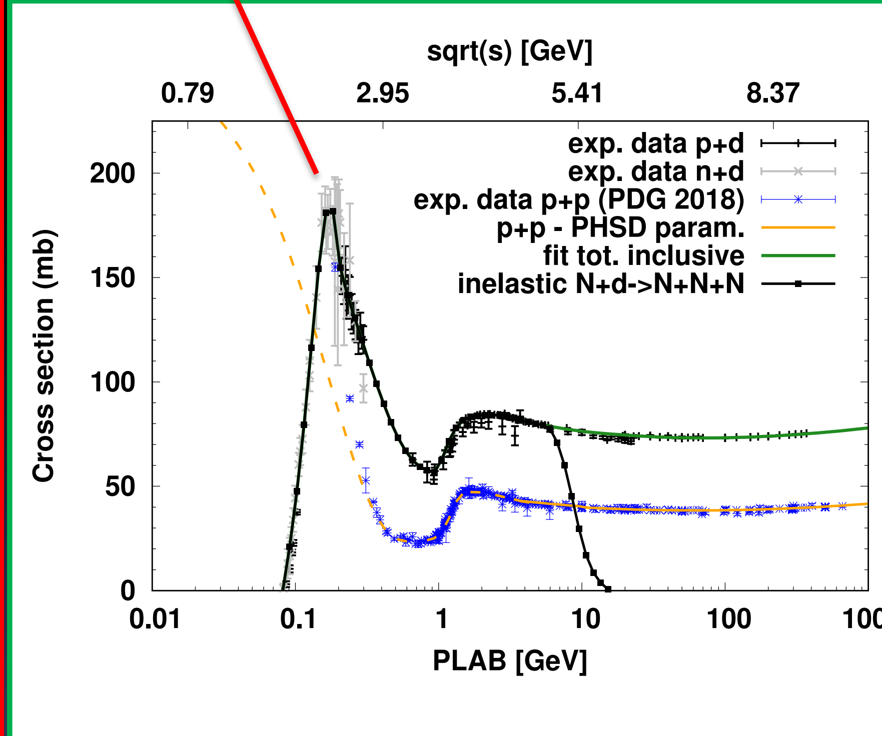
Cross section param. sqrt(s) from **[Oh, Lin, Ko (2009)]** implemented in had. transport model and compared to deuteron production by coalescence.

Inelastic cross sections for deuteron breakup

π+d→N+N exhaust less than 5% of tot. inclusive $\pi^{\pm}$+d cross section

[Kievsky et al. PLB(2000)]
σ(N+d) narrow peak at sqrt(s)~2.8 GeV





- Subtract elast. and N+N inel. from Fit σ(tot. incl.)
- Account inel. final n-body (n>3) contribution with a smooth cut on the phases-space.
- Matching low to high energy param. as done in PHSD for σ(pp) [Cugnon et. al. (1996)]

[Sibirtsev et al. Z. Phys. A358 (1997)]

NUMERICAL SOLUTIONS FOR THREE-BODY REACTIONS:

- The scattering rate for forward reaction (2 → 3) can still be expressed in terms of total cross section:

$$\frac{dN_{coll}[1(i)+2\to 3+4+5]}{dtdV} = \frac{1}{N_{id}!}\int \frac{d^3p_1}{2(\pi)^3 2E_1} f_1(x,p_1) \int \frac{d^3p_2}{2(\pi)^3 2E_2} f_2(x,p_2) \int \prod_{k=3}^{5} \frac{d^3p_k}{(2\pi)^3 2E_k} W_{2,3}(p_1,p_2\mid p_k)(2\pi)^4 \delta^4(p_1+p_2-\sum_{k=3}^{5} p_k)$$

$$P_{2,3}\left(\sqrt{s}\right) = \frac{1}{N_{id}!}\sigma^{2,3}(\sqrt{s})v_{rel}(\sqrt{s})\frac{\Delta t}{\Delta V_{cell}}$$

- BUT the same is not possible FOR BACKWARD REATION: N+N+N(π)→ N(π)+d
  - Effective approach to solve THREE-BODY SYSTEM

$$\sigma_{Xd\to Xd'}(s) \propto \frac{1}{s}\int_{2m_N}^{\sqrt{s}-m_X} dm\, \mathcal{A}_{d'}(m)\, \frac{p_f^*(\sqrt{s},m,m_X)}{p_i^*(\sqrt{s},m_X,m_d)}$$

[Oliinychenko et al. PRC99 (2019)]

$$\mathcal{A}_{d'}(m) = \frac{2\mathcal{N}}{\pi}\frac{m^2\Gamma_{d'}(m)}{(m^2-M_{d'}^2)^2+m^2\Gamma_{d'}(m)^2}$$

M_{d}' ~ m_d + 10-100 MeV
Γ_{d}' ~ 50-200 MeV

✓ Proved to fulfill detailed balance
x Introduce artificial resonance (M_d' , Γ_d') which must be tuned to exp. cross section

baryon anti-baryon annihilation ←→ baryon antibaryon production by meson fusion

[Seifert & Cassing PRC97 (2018)]

- General approach for any I_{coll} **[Cassing NPA 700 (2002)]**
- ASSUMPTION: $W(p_k \mid p_1, p_2) = W(\sqrt{s})$
- Covariant Rate for 3 → 2 d production :

$$\frac{dN_{coll}[1(i) + 2 \leftarrow 3 + 4 + 5]}{dtdV} = \int (\prod_{k=1}^{3} \frac{d^3 p_k}{(2\pi)^3} f_k(x, p_k)) P_{3,2}(\sqrt{s})$$

$$P_{3,2}(\sqrt{s}) = P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

$$P_{2,3}(\sqrt{s}) = \frac{1}{N_{id}!} \sigma^{2,3}(\sqrt{s}) v_{rel}(\sqrt{s}) \frac{\Delta t}{\Delta V_{cell}}$$

✓ No artificial resonance or ad-hoc parameters

✓ Compute the final-state in advance

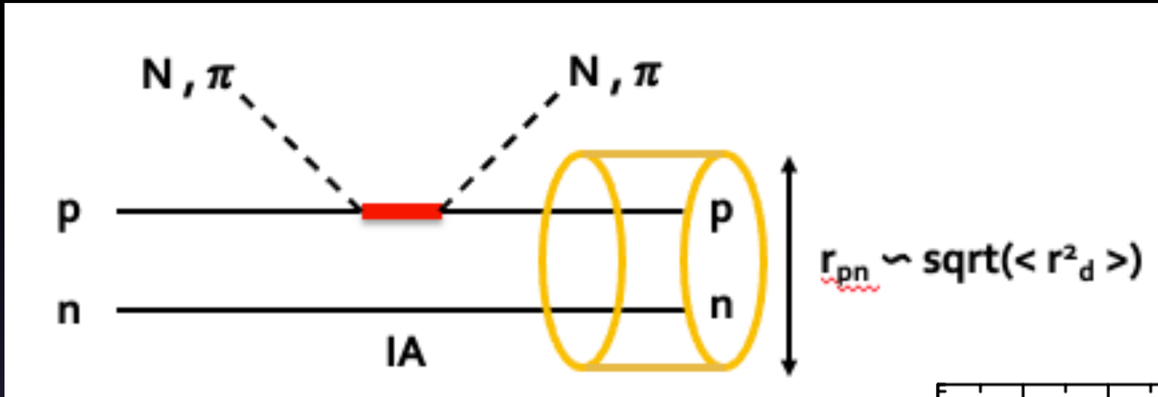
Implemented in PHSD also for:

N+N+π<-> N+N

N+ π +π<-> N+ π

$$P_{3,2}(\sqrt{s}) = P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3E_4E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

- Production probability from 3 → 2 process is valid only for pointlike particles.



Adapted from
[Haidelbauer,Uzikov PLB 562(2003)]
[Hoftiezer et al. PRC23 (1981)]

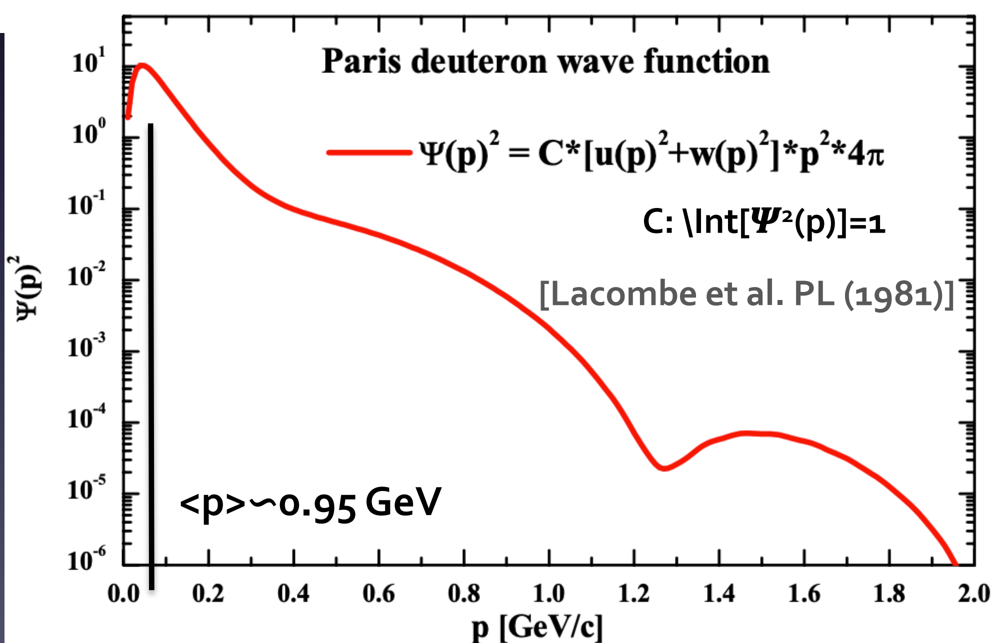
$$p \sim 1/\sqrt{< r_d^2 >} \sim 0.1\, GeV$$

- "p" is the relative momentum of the (pn)-pair in the **rest frame** of final deuteron

- "p" is extracted according to the deuteron wave-function **$\Psi^2(p)$**
  Production probability:

$$P_{3,2}(\sqrt{s}) \rightarrow P_{3,2}(\sqrt{s}) * \Psi(p)^2$$

Solution of Schrodinger eq. with Paris-V_{NN}



Box Simulations

BUU-type Box to study deuteron production and breakup in "infinite" nuclear matter

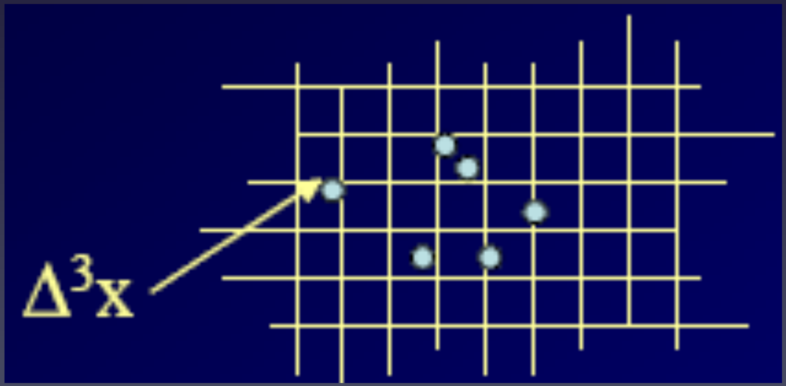
[Bratkovskaya, Cassing et al. NPA 675 (2000)]

BOX PARAMETERS:

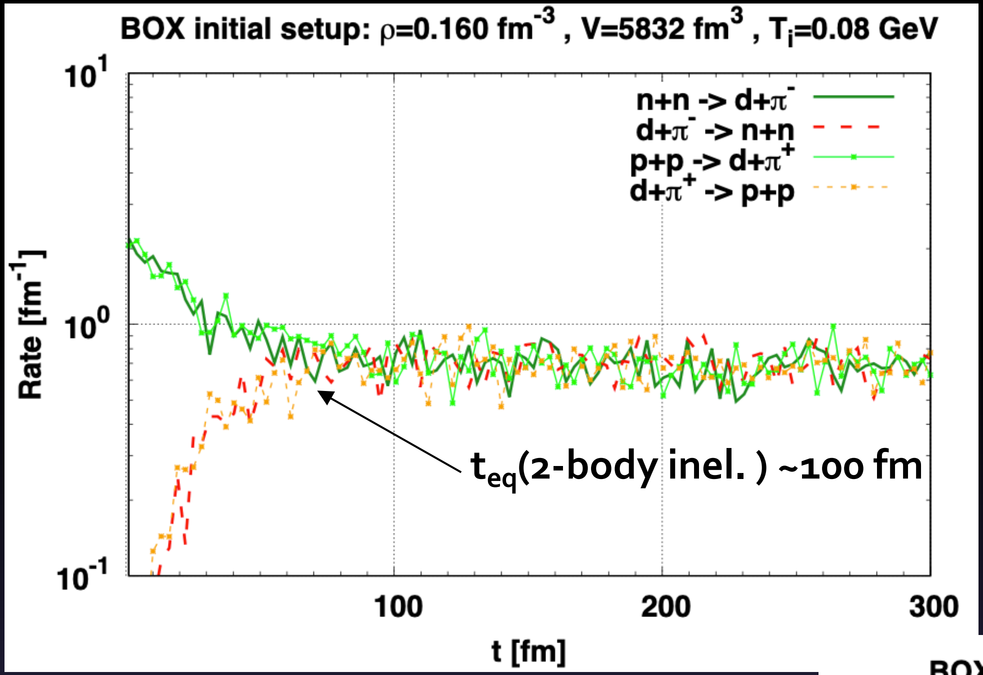
- Volume V ∽ 5.10^3 (fm^3)
- Density ρ ∽1-3 ρ_0 (ρ_0=0.16 fm^{-3}
- Temperature T ∽ 80 – 120 MeV

✓ Verify detailed balance condition looking at the collision rate

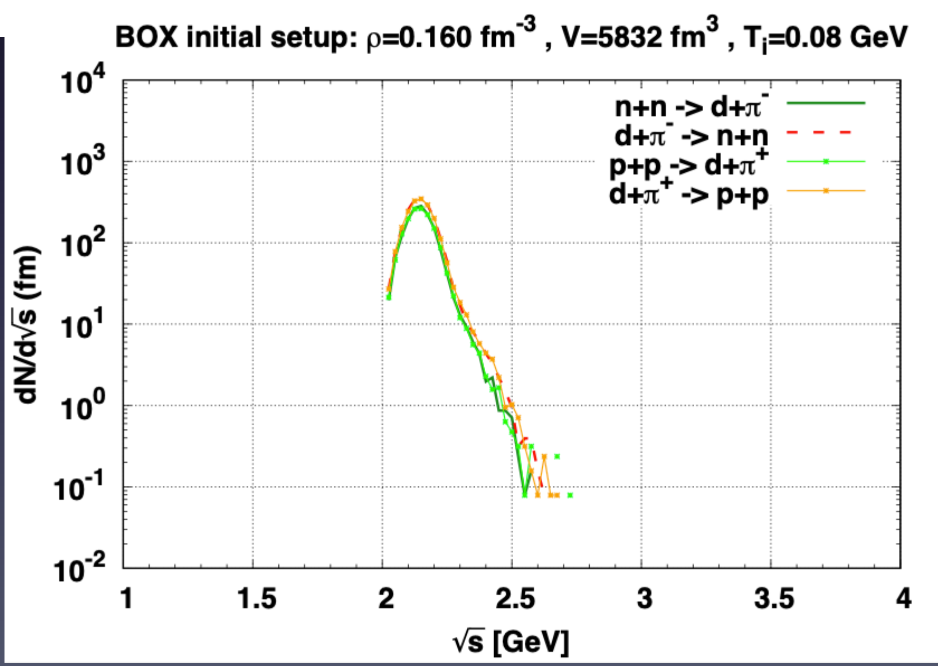
✓ Particles' spectra and densities



Detailed Balance for inelastic 2-body reactions with channel decomposition

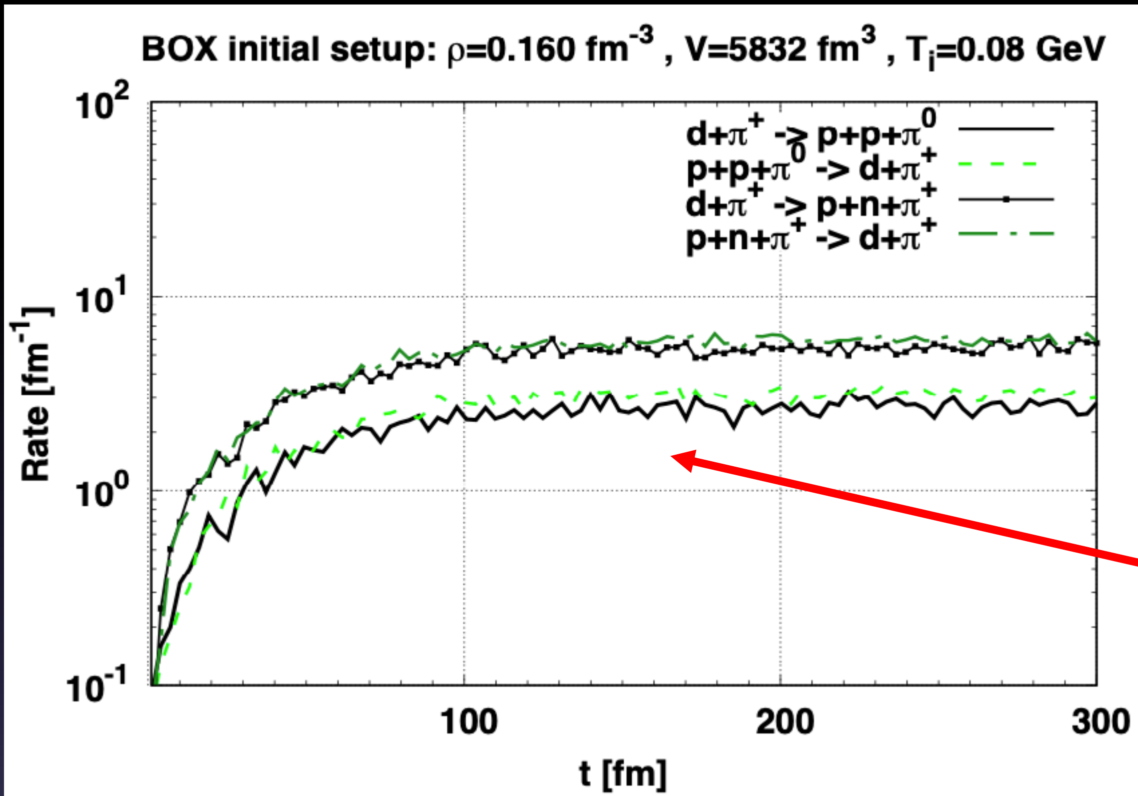


Rate distribution as function of sqrt(s) integrated over the full time evolution



Deuteron production/breakup from inelastic 3-body reactions

✓ Detailed Balance Condition



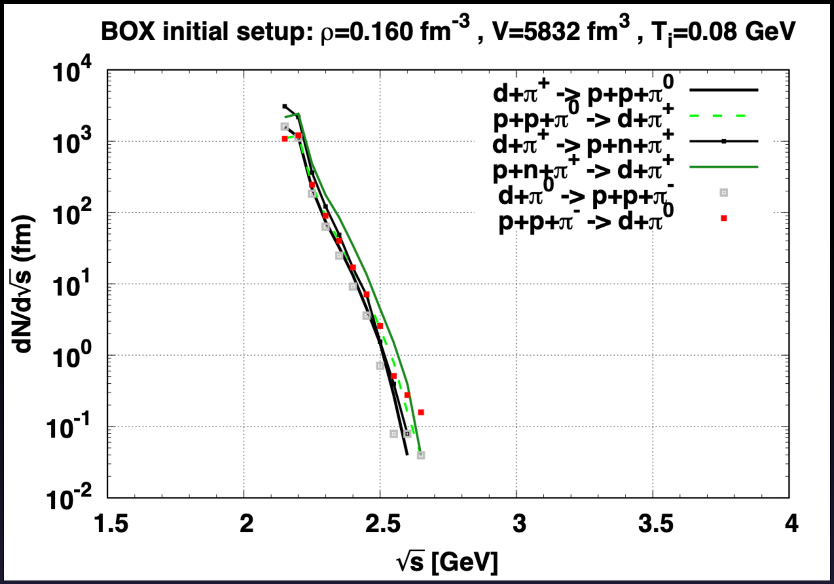
$(i)\quad d+\pi$	Q^i_{tot}	$(f)\quad N+N+\pi$
$\pi^- + d$	0	$p(n)+n(p)+\pi^-$
......	0	$n+n+\pi^0$
$\pi^0 + d$	1	$p(n)+n(p)+\pi^0$
......	1	$p+p+\pi^-$
......	1	$n+n+\pi^+$
$\pi^+ + d$	2	$p(n)+n(p)+\pi^+$
......	2	$p+p+\pi^0$

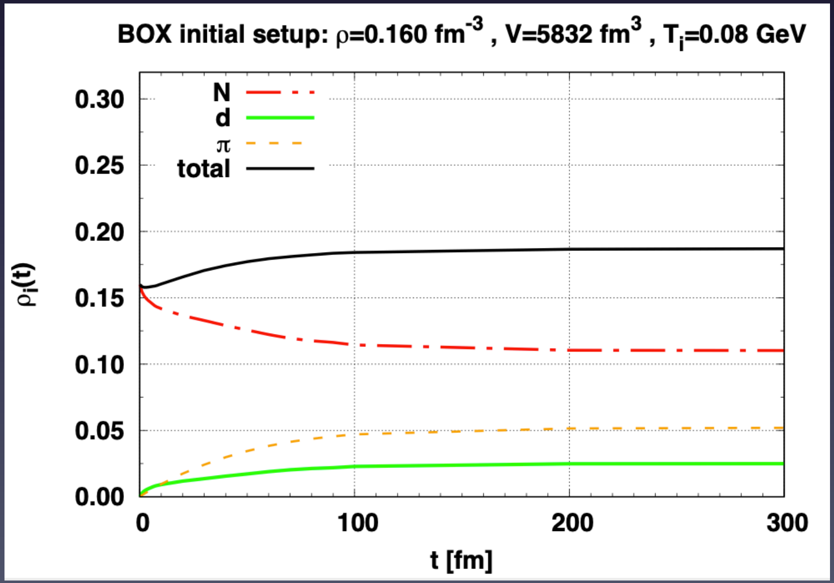
- All final breakup's channel implemented with the proper isospin factor
- Deuteron production from N+N+π

1) Induced by N+N->d+π
2) Box initialization with eq. ρ_π(T)

Rate distribution as function of sqrt(s) integrated over the full time evolution

N+N+π ←→ d+ π

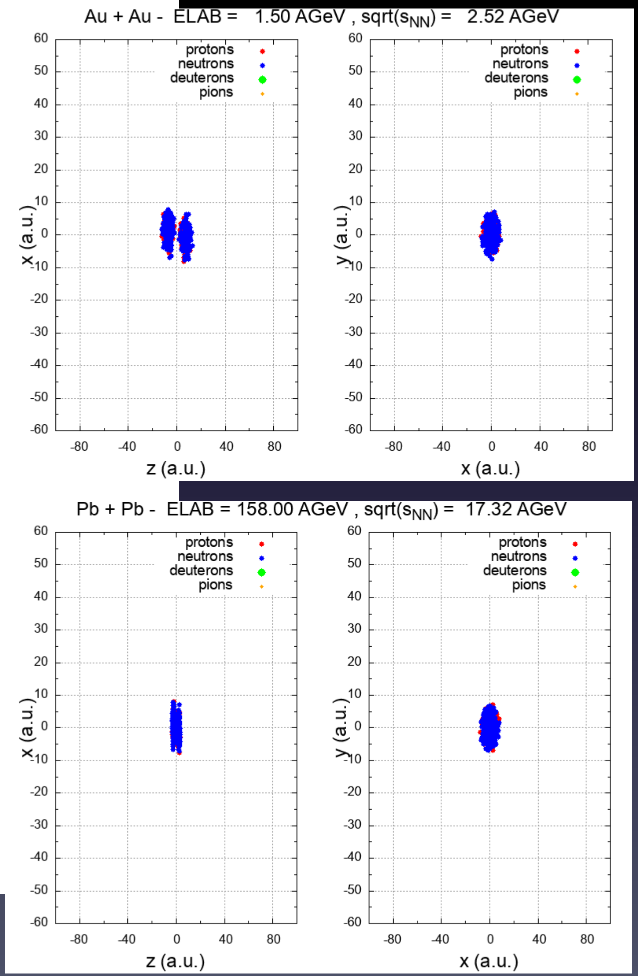




Parton-Hadron-String-Dynamics

Ref. Cassing and Bratkovskaya PRC78 (2008) 034919; NPA831 (2009) 215
Cassing EPJ ST 168 (2009) 3; NPA856 (2011) 162

PHSD is a non-equilibrium microscopic transport approach for the description of strongly-interacting hadronic (& partonic) matter created in HICs.



- **Initialization** → String formation, subsequent decay into pre-hadrons
  → ($\varepsilon > \varepsilon_c$) Formation of QGP

- **Dynamics** → Generalized off-shell transport eqs. from Kadanoff-Baym many-body theory for the QGP[1] and the hadronic phase with mean field (+ E.M. fields[2]) and collision integral

- **Hadronization**

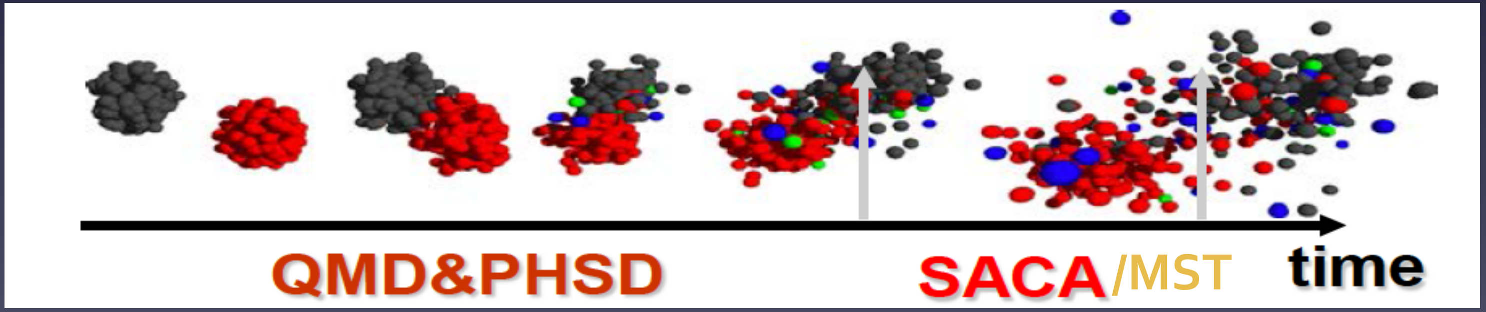
For details see 1) O. Soloveva's and 2) L. Oliva's talks.

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

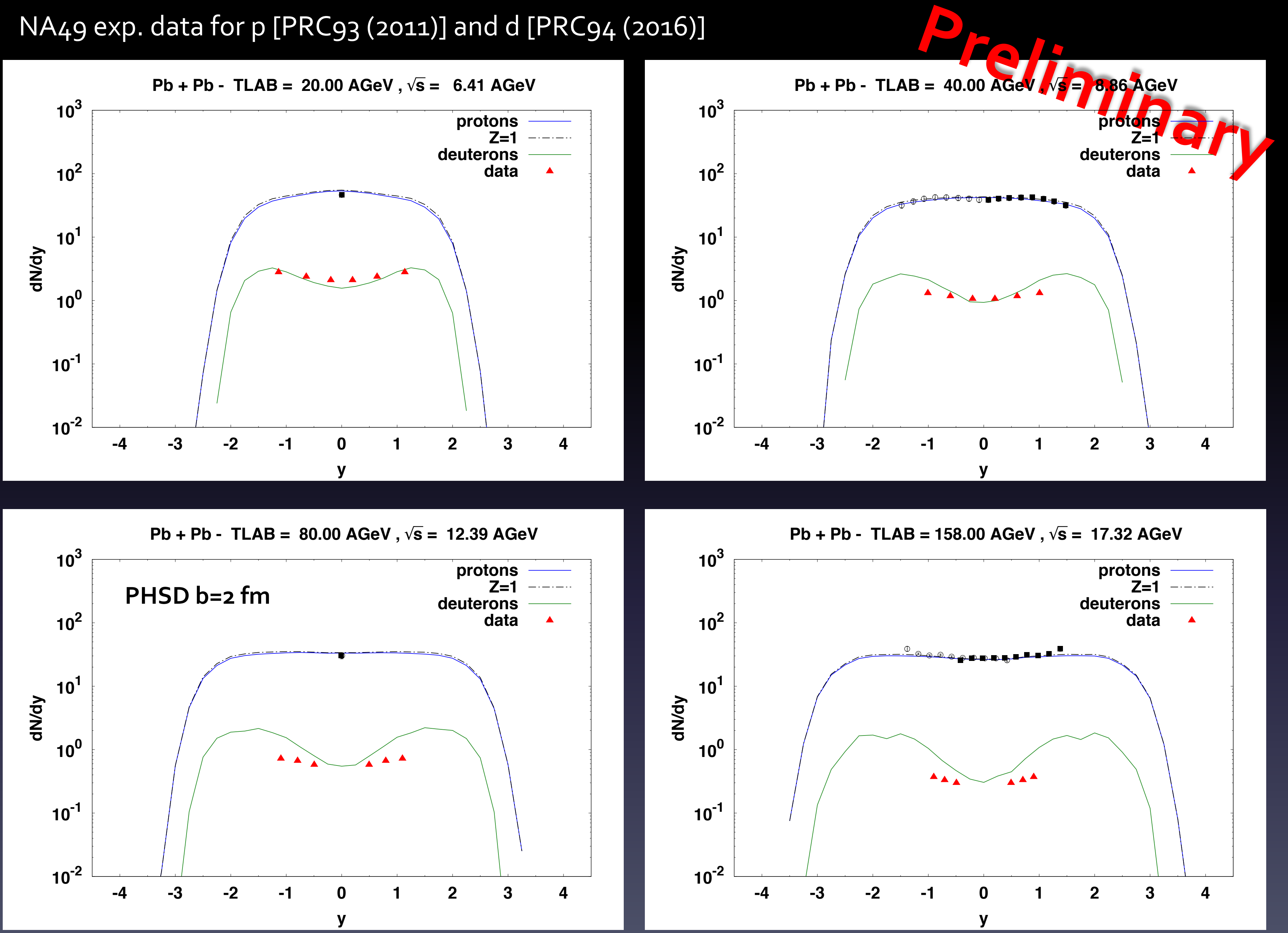
Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

Clusters recognition:
SACA (Simulated Annealing Clusterization Algorithm)
vs. MST (Minimum Spanning Tree)

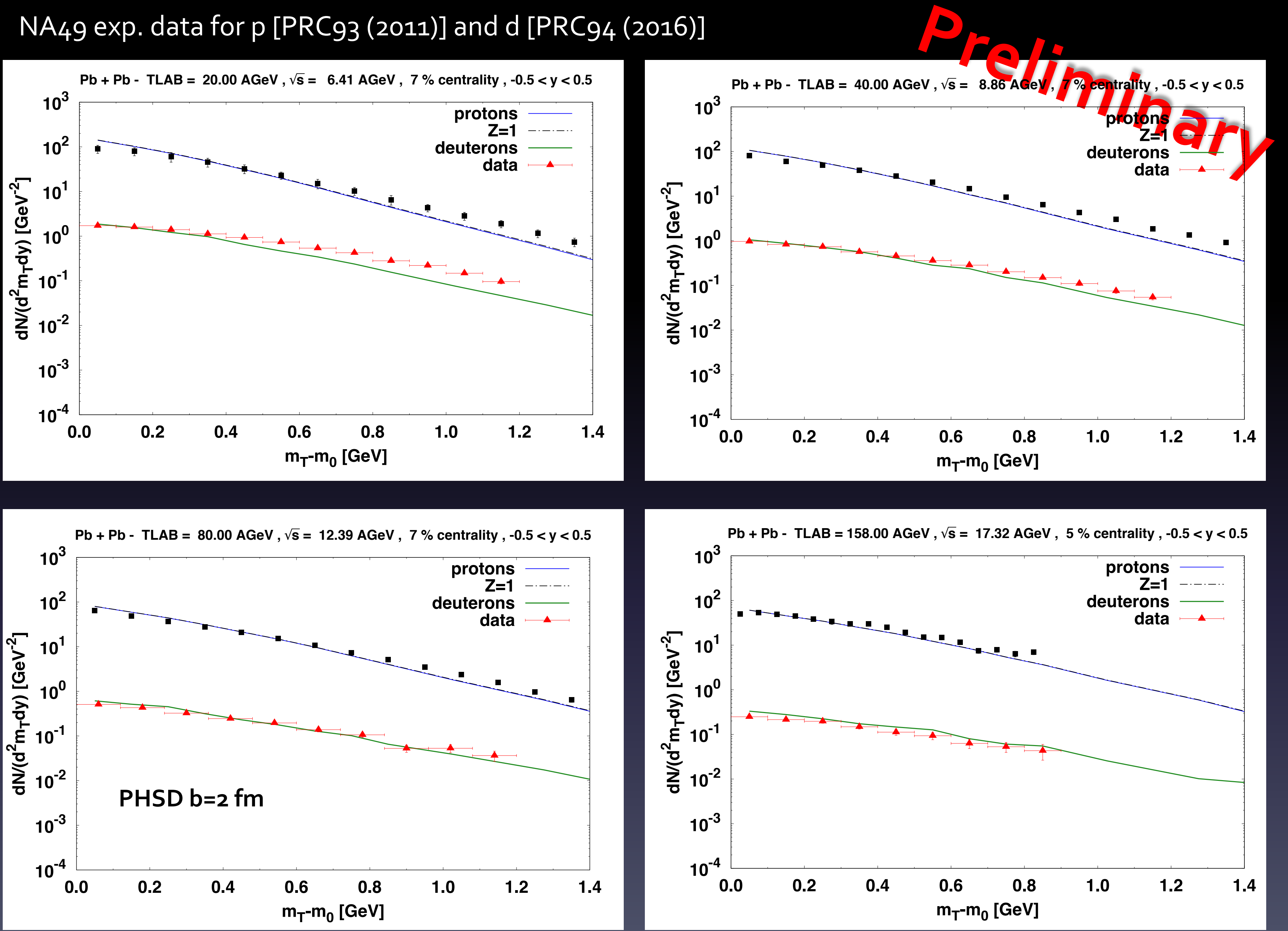


[Ref. Aichelin, Bratkovskaya et al. PRC101 (2020)]

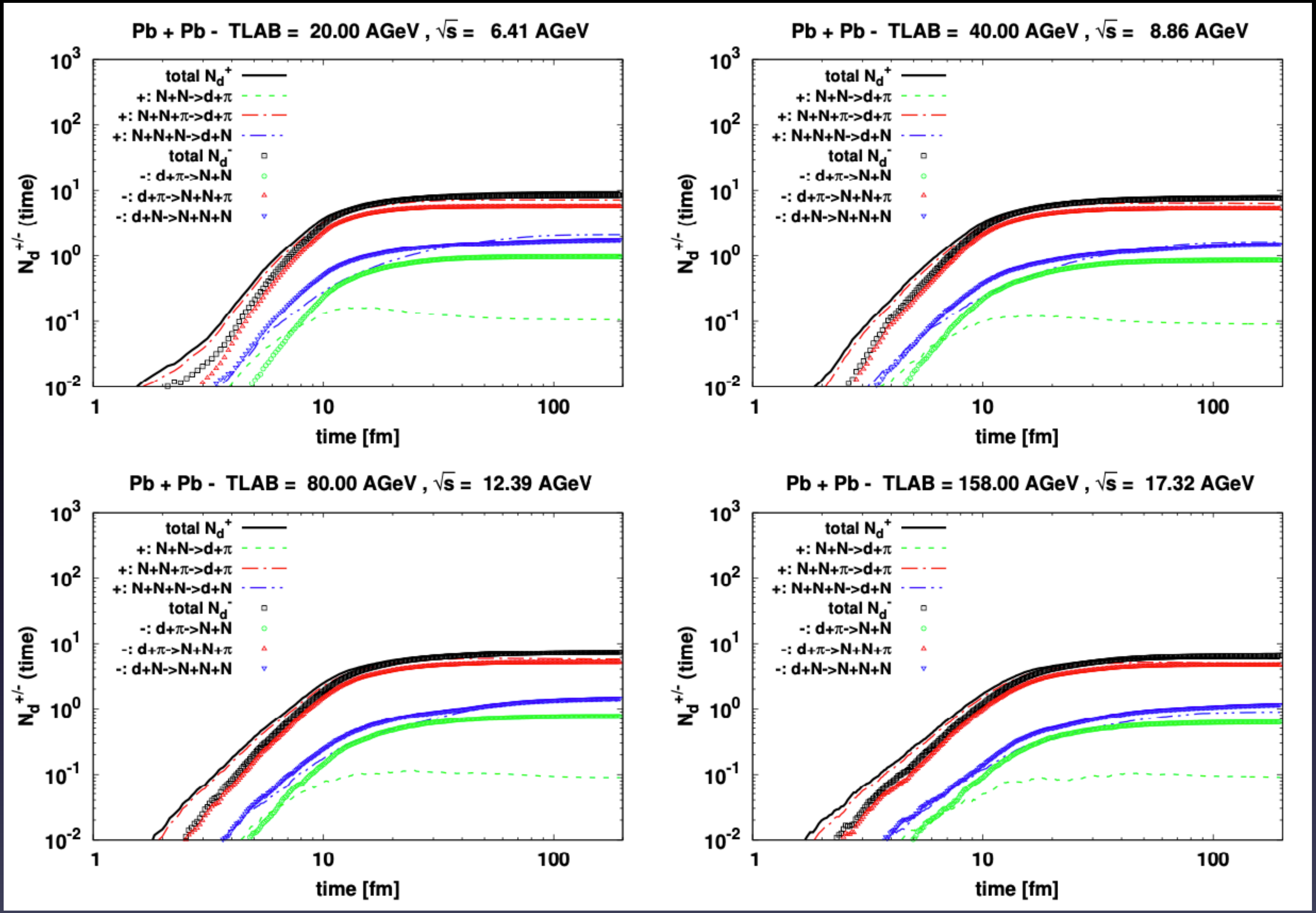
NA49 exp. data for p [PRC93 (2011)] and d [PRC94 (2016)]



NA49 exp. data for p [PRC93 (2011)] and d [PRC94 (2016)]

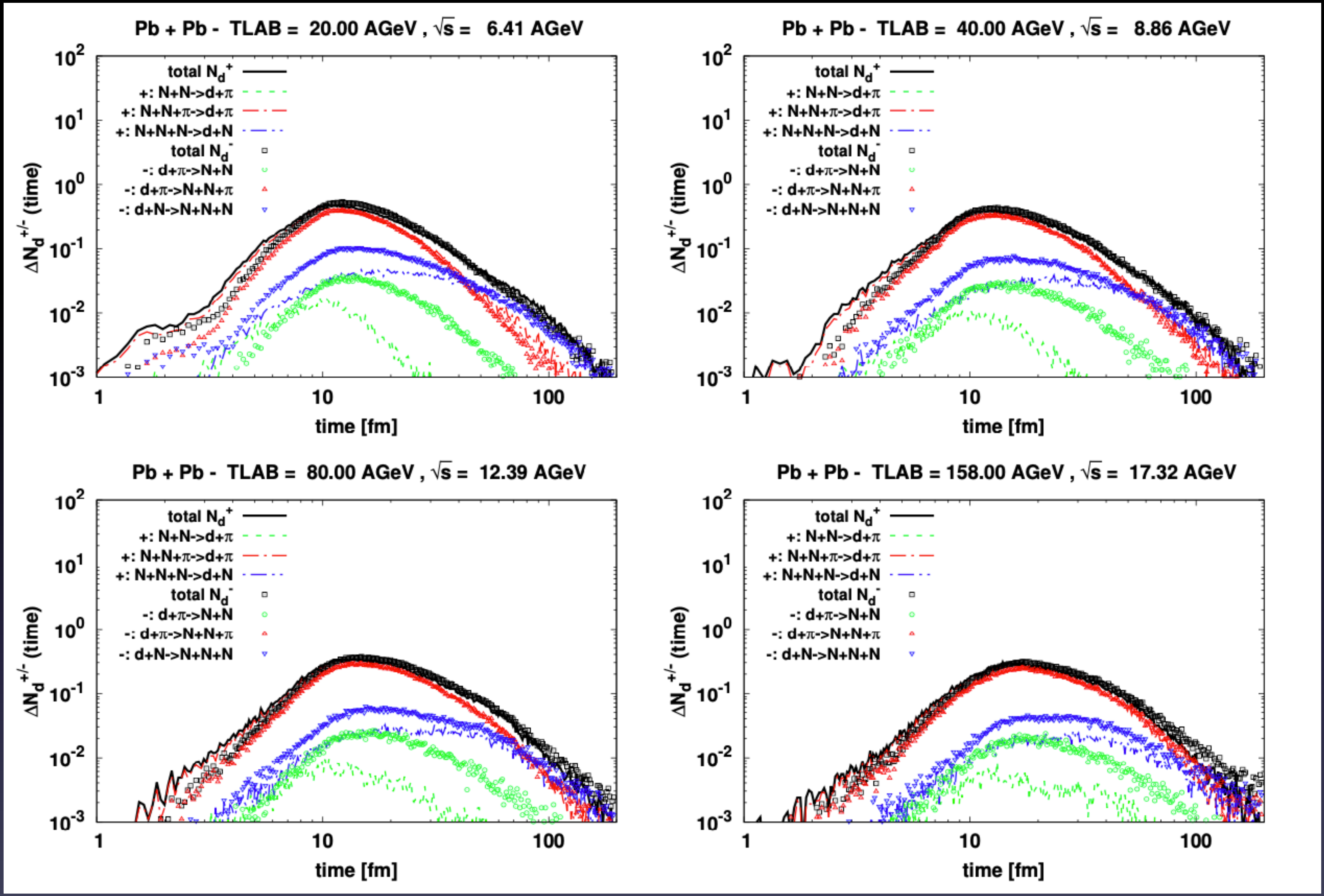


Channel dominance , time-scale of reactions and equilibration

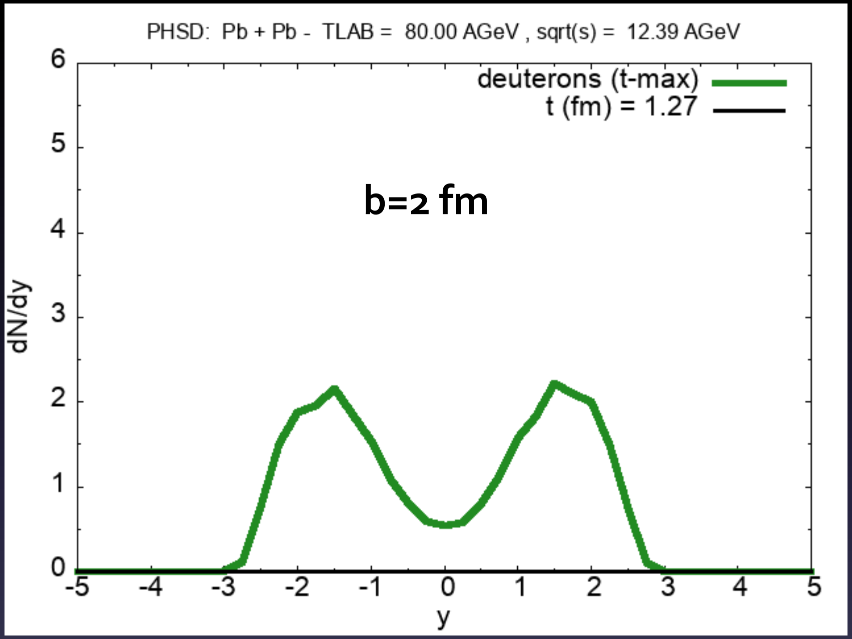


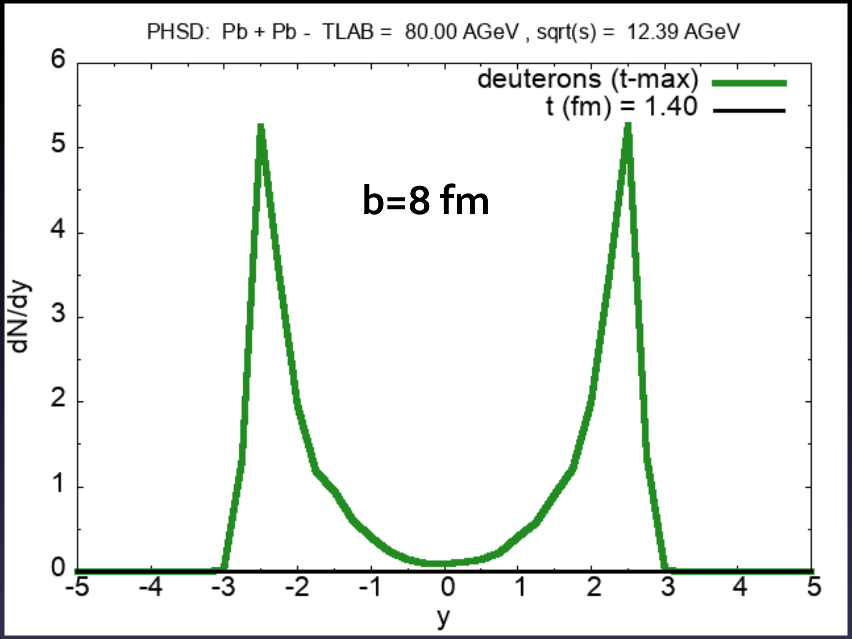
- Net separation at SPS energies: produced N_d (N+N) << N_d(N+N+N) < N_d(π+N+N)

Channel dominance , time-scale of reactions and equilibration

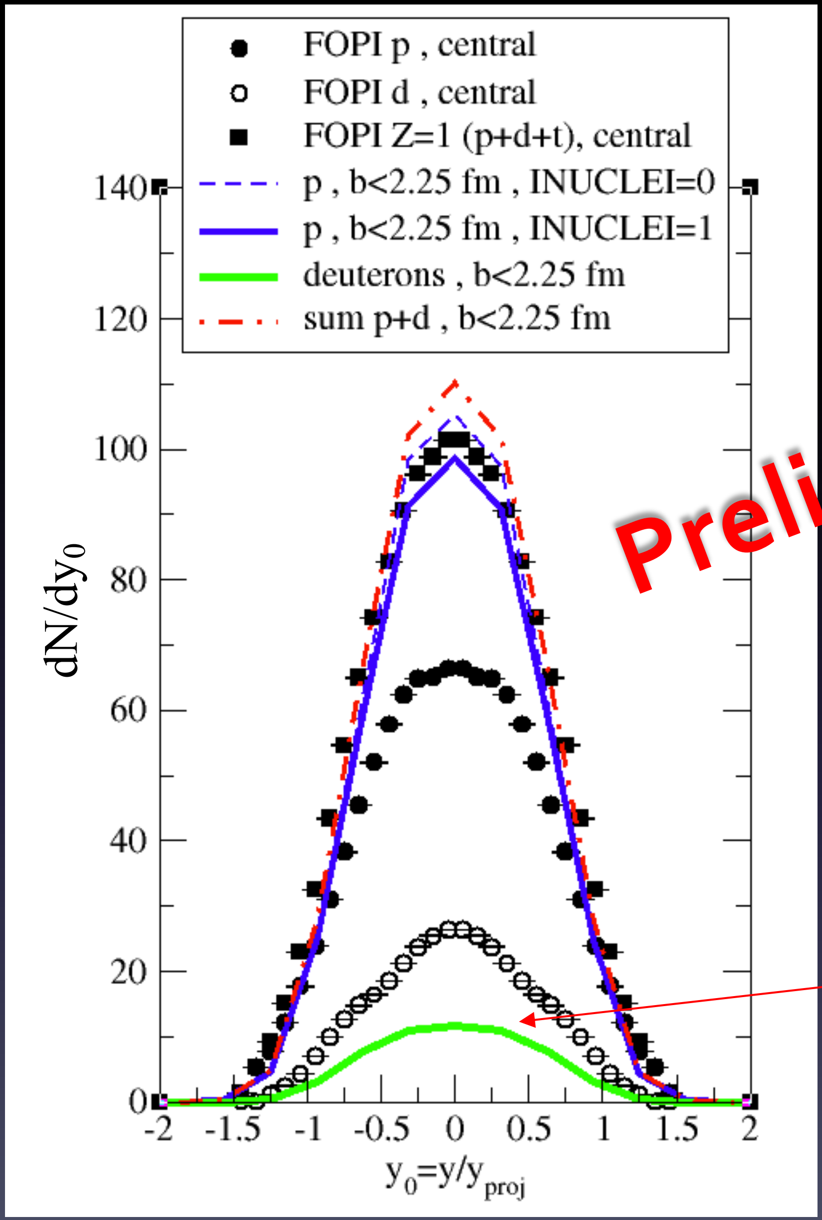


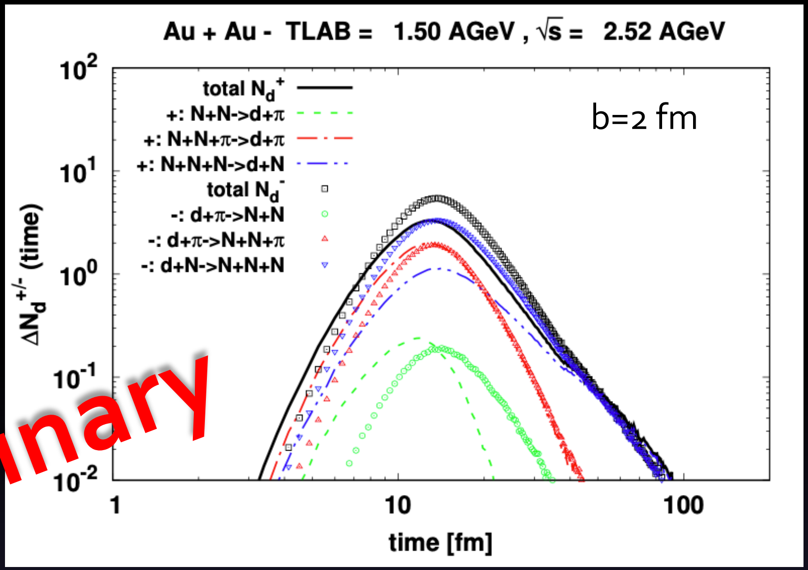
- Equilibration observed only for 3-body: σ(N+N+N(π)->d+N(π)) >> σ(N+N->d+π)
- Larger abundance of π brings to faster equilibration


PHSD: Pb + Pb - TLAB = 80.00 AGeV , sqrt(s) = 12.39 AGeV
deuterons (t-max)
t (fm) = 1.27
b=2 fm
dN/dy
y


PHSD: Pb + Pb - TLAB = 80.00 AGeV , sqrt(s) = 12.39 AGeV
deuterons (t-max)
t (fm) = 1.40
b=8 fm
dN/dy
y

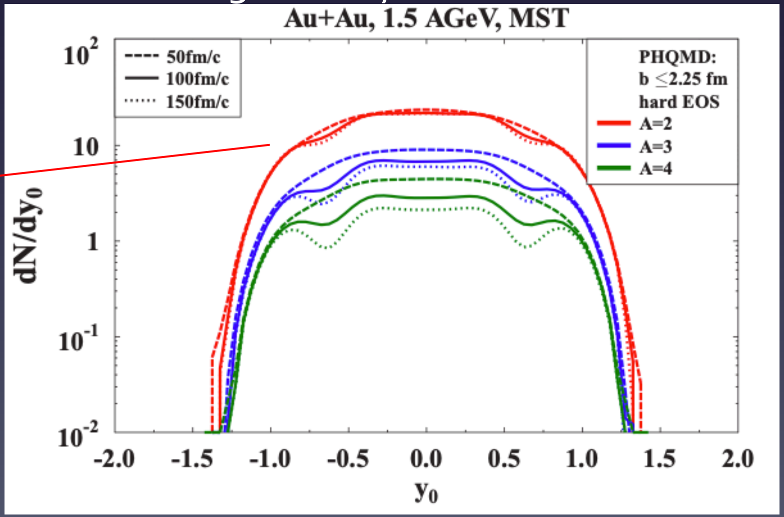
Production of deuterons at SIS energy: exp. data [FOPI PRC58(1998)]





Preliminary

- Forward-backward time shift
  → Different dynamics (PHSD-PHQMD)
- 50% d from other mechanisms
- → Recognized by MST or SACA



[PHQMD PRC101 (2020)]

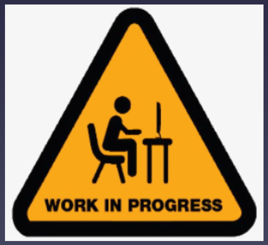
Summary & Outlooks

Light nuclei are a novel -pressing- probe to study hot-dense nuclear matter:

- Formation of light nuclei affects bulk observables.
- Enhanced light nuclei production could be a signature of 'criticality': $N_t N_p / N_d^2$ ...
- Description of light nuclei observables is theoretically challenging.

Deuteron reactions have been consistently included in a kinetic approach:

- Implement reactions for other light nuclei, hypernuclei:
  → Measured or computed hadronic cross section: t+p , t+π , Λ+p , ...
- PHSD vs PHQMD: Impact of different dynamical description on light nuclei.



- PHQMD: Combine formation of deuterons through reactions within clusterization algorithms.

Thank you!