Introduction to the Gauge/Gravity Duality and its Application to Heavy-Ion Physics

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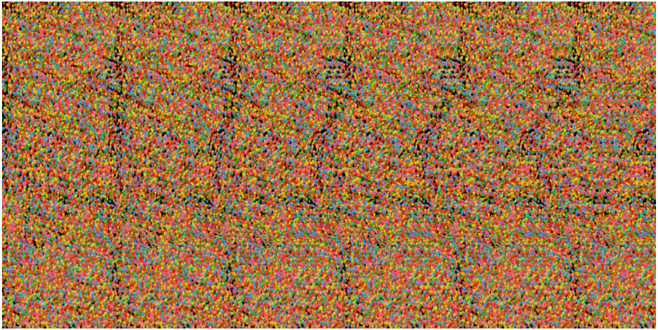
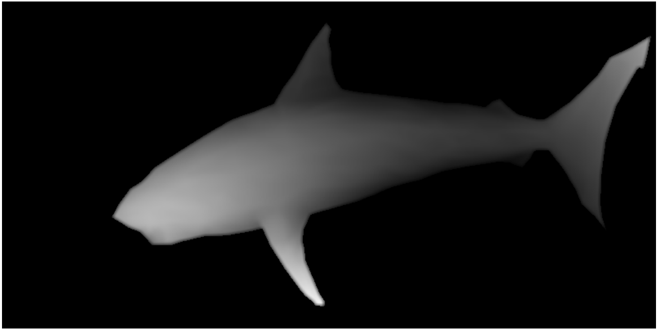
M.F. Wondrak, M. Kaminski, M. Bleicher, Phys. Lett. **B811** (2020) 135973, 2002.11730 [hep-ph].
M.F. Wondrak, M. Kaminski, M. Bleicher, Nucl. Phys. **A1005** (2021) 121880, 2010.10575 [hep-ph].

A Holographic Duality



[https://upload.wikimedia.org/wikipedia/commons/8/8f/Stereogram_Tut_Random_Dot_Shark.png, accessed: 29 Nov 2020]

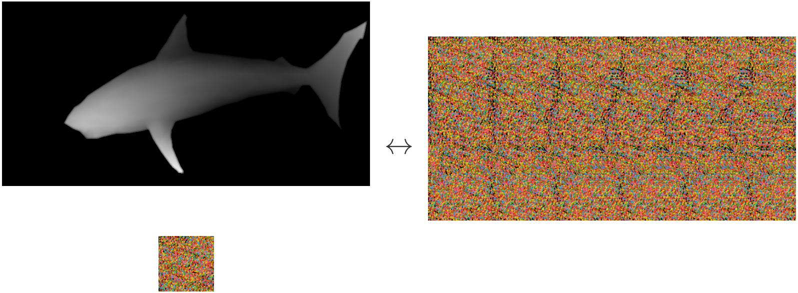
A Holographic Duality



[`https://upload.wikimedia.org/wikipedia/commons/3/3e/Stereogram_Tut_Shark_Depthmap.png`, accessed: 29 Nov 2020]

A Holographic Duality

Equivalent representations of the same information.

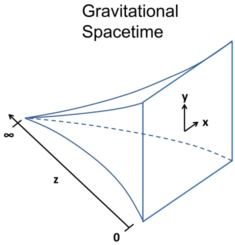


3-dim representation ↔ 2-dim representation

[`https://upload.wikimedia.org/wikipedia/commons/8/8f/Stereogram_Tut_Random_Dot_Shark.png`, `https://upload.wikimedia.org/wikipedia/commons/3/3e/Stereogram_Tut_Shark_Depthmap.png`, accessed: 29 Nov 2020]

Gauge/Gravity Duality

Equivalent representations of the same information.



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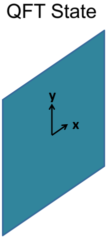


+ matter content

$(d+1)$-dim representation

weakly curved SUGRA spacetime

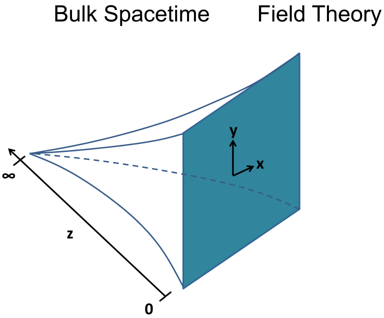
↔

d-dim representation

strongly coupled field theory at large gauge rank

Gauge/Gravity Duality

Field theory manifold can be identified with the conformal boundary of the bulk spacetime.



Nomenclature: Gauge/gravity duality aka holography aka AdS/CFT correspondence

Gauge/Gravity Duality

Two views on the same information: The gauge/gravity glasses!

Outline

- Gauge/Gravity Duality
- Heavy-Ion Physics
    - Specific Shear Viscosity near Equilibrium
    - Specific Shear Viscosity far from Equilibrium

Gauge/Gravity Duality

Dictionary

- Dictionary: One-to-one map between operators $\mathcal{O}$ in the field theory and fields ϕ in the bulk spacetime
  (transforming in the same representation of the symmetry group).
- Boundary expansion ($z = 0$):
  $\phi(x, z) = \phi_{(0)}(x)\, z^{d-\Delta} + \phi_{(+)}(x)\, z^{\Delta} + \dots$
- Generating functional for connected n-point functions

$$W[J]\big|_{J=\phi_{(0)}} = S_{\mathrm{SUGRA}}[\phi]\big|_{\lim\limits_{z\to 0}(\phi(z,x)z^{\Delta-d})=\phi_{(0)}(x)}$$

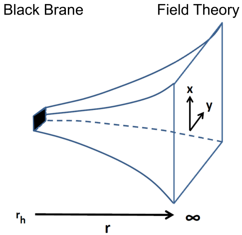
- Operator expectation value: $\langle \mathcal{O}(x)\rangle_J = -\frac{\delta W}{\delta J(x)} \propto \phi_{(+)(x)}$
- Correlation functions: $\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_J = -\frac{\delta^2 W}{\delta J(x)\delta J(y)} \propto \frac{\delta\langle \mathcal{O}(y)\rangle_J}{\delta J(x)}$

E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253; S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. **B428** (1998) 105; M. Ammon, J. Erdmenger, Gauge/Gravity Duality, 2015

Black p-Branes and Thermodynamics

- Thermal n-point-functions
  → Non-vanishing boundary energy-momentum tensor $\langle T^{\mu\nu}(x)\rangle_{g_{(0)}}$.
  → Non-trivial bulk metric configuration.
- Black p-brane
  - Spatially extended black hole with a planar horizon topology.
  - Spacetime still asymptotically AdS.
  - Thermodynamic quantities of the field theory state and the black brane match:

Field theory	Black brane
T	$T_{\mathrm{H}} = \kappa/2\pi$
s	$S_{\mathrm{BH}} = A/4G$
μ	$A_0\big\|_{r\to\infty}$



E. Witten, Adv. Theor. Math. Phys. **2** (1998) 505

Applications

Versatile tool, used e.g. in the areas of

- Information theory:
  Complexity and entanglement entropy, connection to tensor networks, ...
- Quantum gravity:
  Black hole entropy, black hole evaporation, ...
- Condensed matter physics:
  Characterization of new materials, topological insulators, superconductivity, ...
- Hydrodynamics:
  magnetic fields, applicability (divergence of gradient expansion, non-hydro modes, critical momenta), thermalization, ...
- QCD & Heavy-Ion Physics:
  access to the time-dependent non-dilute regime, QCD phase transition, jet quenching, quarkonium yields, elliptic flow, ...

Specific Shear Viscosity near Equilibrium

Hydrodynamics

- Effective field theory of conserved charges.
- Set of coupled equations: conservation equations and constitutive relations (energy-momentum tensor, currents). Closed by equation of state.
- Conventional validity regime: small frequencies and large wavelength (small Knudsen number), requires local thermodynamic equilibrium. However, in several cases violating these constraints, hydrodynamics yields correct results.

e.g. Landau, Lifschitz, Bd. VI Hydrodynamik, 2007; Baier et al., J. High Energy Phys. **04** (2008) 100; Kovtun, J. Phys. A: Math. Theor. **45** (2012) 473001; Ammon et al., J. High Energy Phys. **04** (2017) 067; Romatschke, Phys. Rev. Lett. **120** (2018) 012301; Grozdanov et al., Phys. Rev. Lett. **122** (2019) 251601; Heller et al., 2007.05524 [hep-th]; Baggioli, 2010.05916 [hep-th]

Transport Coefficients

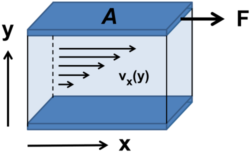
- Constitutive relation for the energy-momentum tensor at first order in derivative expansion:

$$\begin{aligned} T_{\mu\nu} &= T^{\mathrm{IF}}_{\mu\nu} + T^{\mathrm{NIF}}_{\mu\nu} \\ &= \left(\epsilon\, u_\mu u_\nu + P\,\Delta_{\mu\nu}\right) + \left(-\eta\,\sigma_{\mu\nu} - \zeta\left(\nabla_\rho u^\rho\right)\Delta_{\mu\nu} + \mathcal{O}\left(\nabla^2\right)\right) \end{aligned}$$

$$\text{with} \qquad \Delta_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu \qquad \sigma_{\mu\nu} \equiv 2\nabla_{\langle\mu}u_{\nu\rangle}$$

- Transport coefficients defined by fundamental theory, here: by the holographic field theory.
- Shear viscosity: Efficiency of transverse momentum diffusion. Local rest frame with $u_x = v_x(y) \ll 1$:

$$-T_{yx} = \frac{F}{A} = \eta\,\sigma_{yx} = \eta\,\partial_y v_x$$



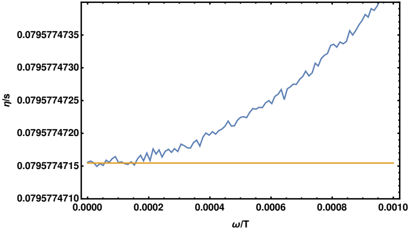
e.g. Landau, Lifschitz, Bd. VI Hydrodynamik, 2007; Baier et al., J. High Energy Phys. **04** (2008) 100

Near-Equilibrium η/s

- Holographic corresponding pair: g_{mn}, $\langle T^{\mu\nu}\rangle$
- Insert metric perturbation h_{xy} and solve linearized Einstein equations.
- Kubo formula for the shear viscosity from the retarded Greens function in frequency space:

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\,\Im\left[G_{\mathrm{R}}^{xy,xy}\left(\omega,\ \vec{k}=0\right)\right]$$

- Normalization by entropy density ($\sim$ number of degrees of freedom).

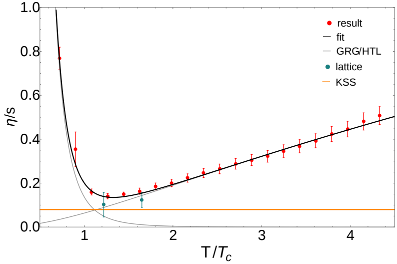


- Analytic result by Policastro, Son, Starinets: $\eta/s = 1/4\pi$

Policastro, Son, Starinets, Phys. Rev. Lett. **87** (2001) 081601

Near-Equilibrium η/s

- KSS value for gauge theories dual to Einstein-Hilbert theory, $\eta/s = 1/4\pi \approx 0.08$.
- Exceptions:
  systems with anisotropy, finite gauge rank N, finite 't Hooft coupling.
- Temperature-dependent calculation of η/s for quark-gluon plasma from lattice QCD (blue) and functional renormalization group (red).



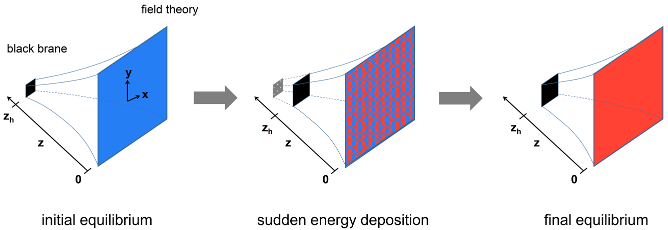
Kovtun, Son, Starinets, Phys. Rev. Lett. **94** (2005) 111601; Christiansen et al., Phys. Rev. Lett. **115** (2015) 112002

Specific Shear Viscosity far from Equilibrium

- Consider shear transport far from equilibrium.
- Challenges:
  - Out of local thermodynamic equilibrium
    → Generalize the definition of s.
  - No time translation symmetry
    → Generalize the definition of η.

Far from Equilibrium in Bulk and Boundary $\rightarrow s$

$\Rightarrow$ Calculate the effective η/s for far-from-equilibrium systems, especially for the heating-up phase in a collision.



- Sudden energy deposition @ boundary
  $\leftrightarrow$ Rapid mass infall (Reissner-Nordström Vaidya spacetime).
- Holographic equation of state $\rightarrow$ time-dependent s.

M.F. Wondrak et al., Nucl. Phys. **A1005** (2021) 121880

Far-from-Equilibrium Model

- Theory (bulk): Einstein-Maxwell:
  $S = \int \mathrm{d}^4 x \sqrt{-g} \left(\frac{1}{16\pi\, G_{\mathrm{N}}} (R+6) - \frac{1}{4} F^{mn} F_{mn} \right) + S_{\mathrm{matter}}$
- Configuration: Highly concentrated matter infall into black brane, presence of U(1) gauge field.
  Arbitrary functions $M(v)$ and $Q(v)$.

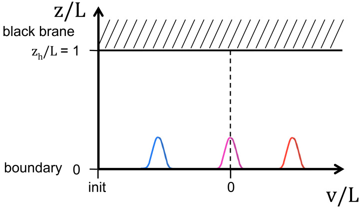
$$\begin{aligned} ds^2 &= \frac{1}{z^2} \left(-f(v,z)\,\mathrm{d}v^2 - 2\,\mathrm{d}v\,\mathrm{d}z + \mathrm{d}x^2 + \mathrm{d}y^2 \right) \\ A(v) &= \left(\mu(v) - Q(v)\,z \right) \mathrm{d}v \\ f(v,z) &= 1 - 2G_{\mathrm{N}}\, M(v)\, z^3 + 4\pi\, G_{\mathrm{N}}\, Q(v)^2\, z^4 \\ \Delta M &\propto 1 + \tanh(v/\Delta t) \end{aligned}$$

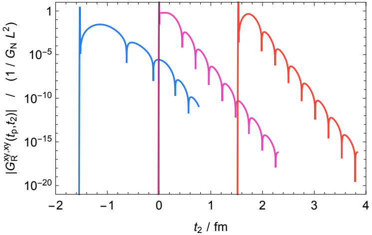
- v denotes the ingoing Eddington-Finkelstein coordinate.
  It coincides with t at the boundary.

V. Balasubramanian et al., J. High Energy Phys. **04** (2013) 069; E. Caceres et al., J. High Energy Phys. **01** (2014) 084; S. Banerjee et al., J. High Energy Phys. **08** (2016) 048; M.F. Wondrak, Phys. Lett. **B811** (2020) 135973

System Response to Perturbations → η

- Bulk: Evolution of the geometry perturbation h_{mn}.
  → Boundary: Damped oscillations of $\langle T^{\mu\nu} \rangle$.





- Linear response:

$$\langle T^{xy}(t_2)\rangle_h = \int \mathrm{d}\tau \; G_R^{xy,xy}(\tau,\, t_2) \; \underbrace{h^{(0)}_{xy}(\tau)}_{=\delta(\tau - t_p)} = G_R^{xy,xy}(t_p,\, t_2)$$

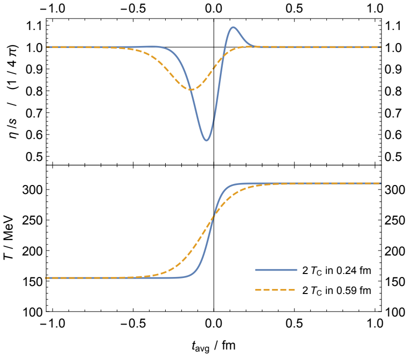
- Wigner transformation → Kubo formula → time-dependent η.

M.F. Wondrak et al., Nucl. Phys. **A1005** (2021) 121880

Far-from-Equilibrium η/s: Time Dependence

Typical RHIC temperature rise:

$T = 155\,\mathrm{MeV} \rightarrow T = 310\,\mathrm{MeV}$ over $\Delta t = 0.3\,\mathrm{fm}$.

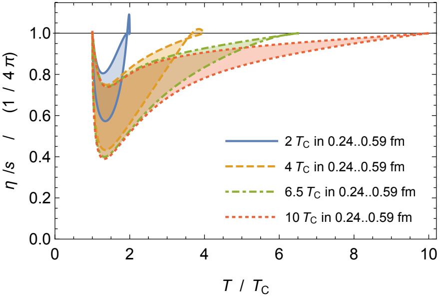


- Shorter heating-up time: smaller & more pronounced minimum.
- Shorter heating-up time: overshoot.

Adare et al. (PHENIX), Phys. Rev. **C81** (2010) 034911; M.F. Wondrak et al., Nucl. Phys. **A1005** (2021) 121880; M.F. Wondrak, Phys. Lett. **B811** (2020) 135973

Far-from-Equilibrium η/s: Temperature Dependence

Regime relevant to heavy-ion collisions at RHIC and LHC:

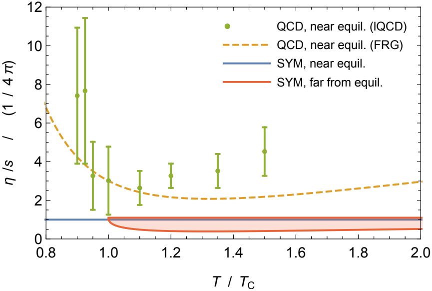


- Universal trend in $\eta/s(T)$.
- Impact of temperature increase saturates.
- Time span is the dominant out-of-equilibrium parameter to determine minimum value.

M.F. Wondrak, Phys. Lett. **B811** (2020) 135973

Comparison

Many efforts from lattice QCD and FRG to determine the shear viscosity being a key property of the QGP in the hydrodynamic phase.



Reduction of η/s is a general trend as found in holography (e.g. in presence of anisotropies) and effective kinetic theory (higher order viscous corrections).

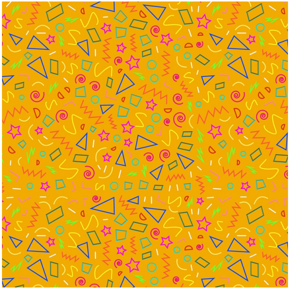
Romatschke, Phys. Rev. Lett. **120** (2018) 012301; Denicol, Noronha, Phys. Rev. Lett. **124** (2020) 152301; Behtash et al., Phys. Lett. **B797** (2019) 134914; Rebhan, Steineder, Phys. Rev. Lett. **108** (2012) 021601

Conclusions and Outlook

- Holographic calculation of η/s far from equilibrium by strong time dependence.
- Marked modification of η/s by $-60\%/+10\%$ in the regime relevant for RHIC and LHC.
- Impact on the simulation of heavy-ion collisions and on the extraction of viscosity from data.
- Applicability of this procedure to further holographic setups and further transport coefficients.

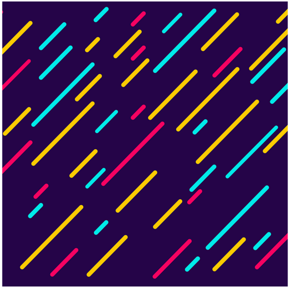
The Hipst~~a~~ers' View

Equivalence of descriptions



Boundary field theory:

Highly excited strongly coupled plasma



Bulk spacetime:

Matter infall at the speed of light

When looking through the gauge/gravity glasses . . .

. . . do you have questions?