COLLISIONS

LECTURE 2

CONTENT

REFERENCES

- Particle Data Group reviews
  - http://pdg.lbl.gov/2017/reviews/rpp2017-rev-kinematics.pdf

- Mark Thomson, 'Modern Particle Physics', 2013
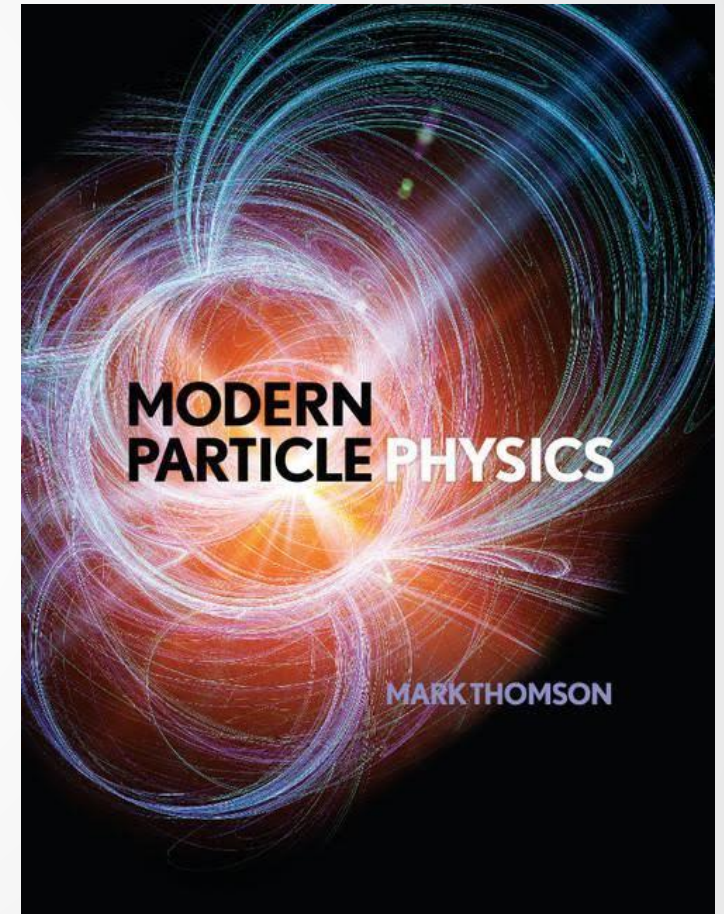
1. INTRODUCTION

PARTICLE CONTENT OF THE STANDARD MODEL

- Very few elementary particles are needed to describe everyday life and collider results from experimental point of view

- However a staggering level of detail, theoretical calculations and subtleties is needed to do so accurately and for all energies.

- In colliders, low-mass elementary particles and/or nuclei collide and produce intermediate heavy particles (e.g. top quark, H) which immediately (cascade-)decay to lighter particles

- Many discoveries have been made using hadron colliders.

- Much of the precision information on the SM parameters is from lepton colliders.

RECENT DISCOVERIES OF FUNDAMENTAL PARTICLES

- **Higgs boson**: LHC, ATLAS and CMS in 2012 in proton-proton collisions

- **top quark**: Tevatron, D0 and CDF, 1995 in proton-antiproton collisions

- **W and Z boson**: SppS, UA1 and UA2, 1983 in proton-antiproton collisions

- **g (gluon):** DESY, PETRA collider, TASSO experiment, 1979, e^+/e^- collisions by gluon radiation

- **b quark**: Fermilab, E288 experiment, in proton-nucleus collisions, 1977, indirectly in a dimuon resonance, the 'Upsilon'.

- **τ lepton**: SPEAR e^+/e^- at SLAC with the MARK detector, 1975, $e\mu$ events from e^+/e^- collisions

A RECENT EXAMPLE: OBSERVATION OF THE H →μμ DECAY

- The SM predicts the Higgs boson decays into a muon pair. This is the 'signal' hypothesis. As the alternative 'background' hypothesis choose: The Higgs boson never decays to a muon pair.

- What part of the SM is needed to describe & understand this hypothesis fully in the context of an experiment?

electroweak interaction: how the top quark and the W boson decay

perturbative QCD: how the top quark and the gluon interact

non-perturbative QCD: How the high energetic quarks create showers of particles, turn into hadrons, and decay

electroweak symmetry breaking: how the Higgs couples to the t

low energy interactions with the detector: electromagnetic interaction of charged particles, nuclear interactions with the detector material, the propagation of a heavy charged particle (μ) through matter

Quantum Chromo Dynamics (QCD) describes initial state bound state (e.g. protons) and the probability to find a quark or gluon inside with a specific momentum (non-perturbative)

B-physics (part of QCD) how metastable hadrons are formed, how far they travel and how they decay

→ The whole SM is involved at low & high energies!
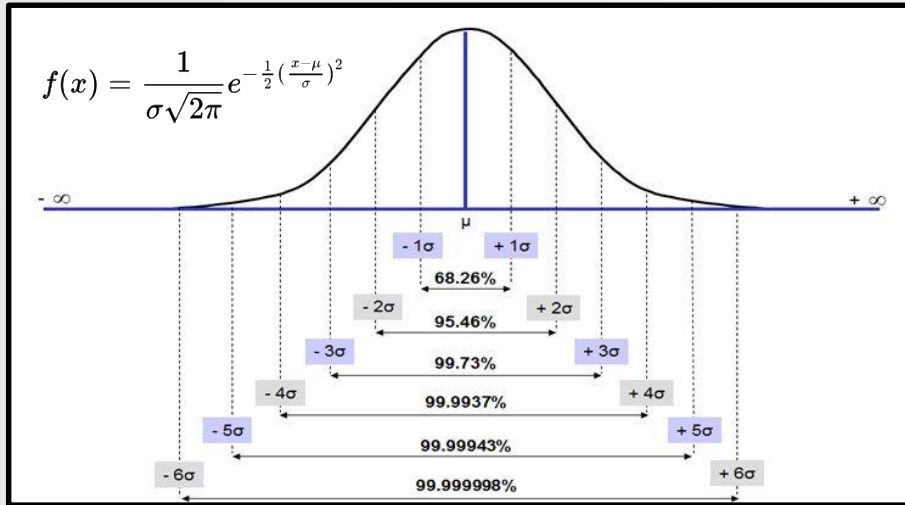
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

- We translate probabilities into units of **σ** using the Normal distribution

- Shown on the left is the probability of the background hypothesis for the measured data as a function of the assumed mass of the Higgs boson.
  - We know that $m_H = 125.35 \pm 0.15$ GeV from H→$\gamma\gamma$ and H→ZZ*

8

OBSERVATION OF THE H→μμ DECAY

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- The remainder of this lecture is devoted to establishing the main roads of the interplay between theory and experiment

- The remainder of the course is devoted discussing the ingredients of such measurements

  a. the physical processes in detectors,
  b. detector concepts (how these processes are exploited),
  c. how detectors are arranged in experiment,
  d. how data is taken and how data analysis is performed and
  e. how the result is statistically interpreted (e.g. what **3σ** mean)
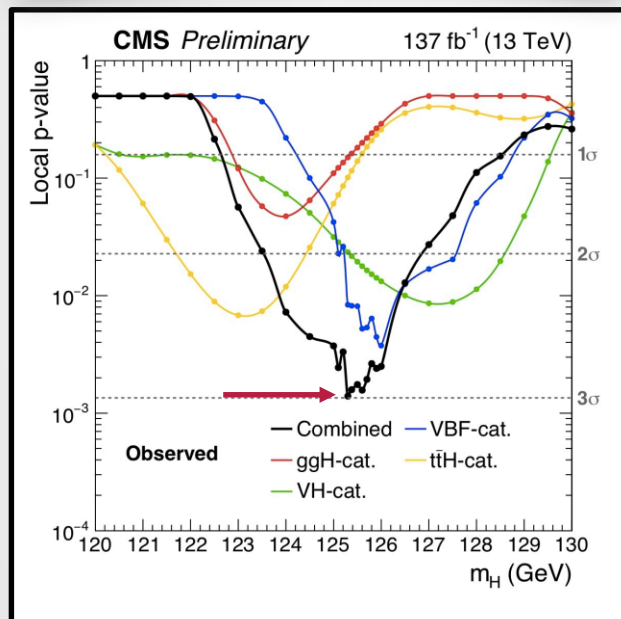
STRONG INTERACTIONS AND THE QUARK MODEL

- In order to deal with the rich phenomenology in collider experiments, M. Gell-Mann proposed to organize the new particles in a spin-1/2 meson octet and a spin-3/2 baryon decuplet (the 8-fold way), later understood as representation of SU(3) flavor symmetry.

- This is the quark model of hadrons.

- The theory of the strong interaction describes how quarks and gluon interact and how the gluon binds the quarks in nuclei

- "Quantum Chromo Dynamics" (QCD) took decades to develop, it is a non-abelian gauge theory that is part of the SM.

quarks in the proton
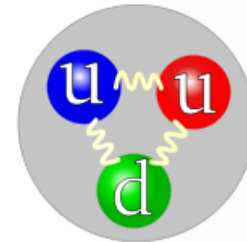
Murray Gell-Mann
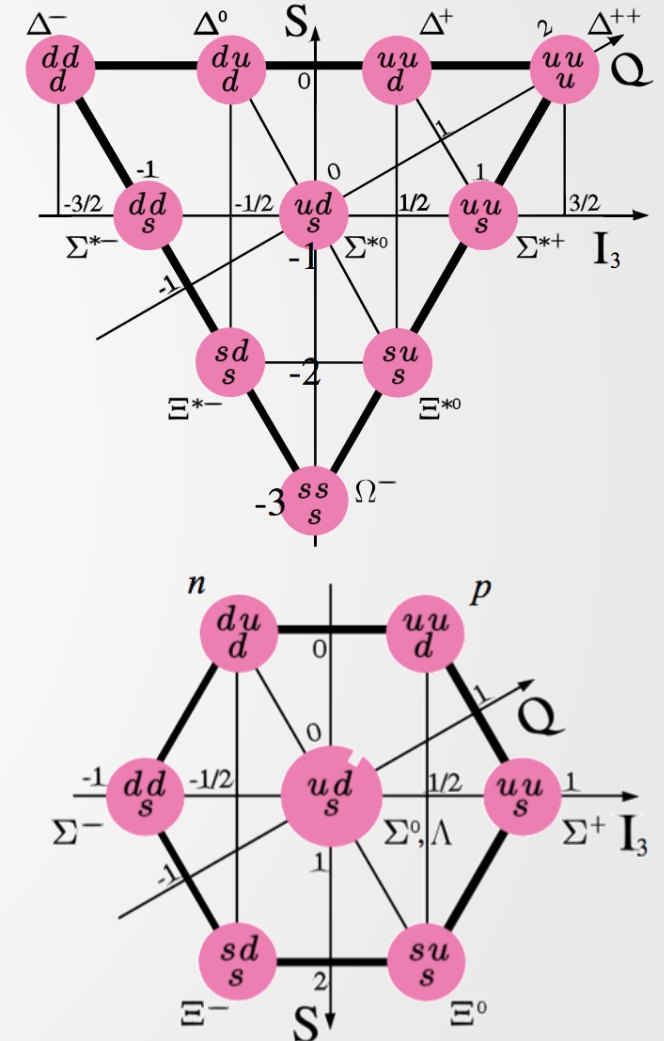
STRONG INTERACTIONS AND THE QUARK MODEL

- A central implication, already known experimentally early on, is that the strength of the strong interaction (α_s) decreases with energy because of *quantum corrections*.

- This 'asymptotic freedom' is a crucial key to the interplay of collider experiments among themselves and with theoretical calculations!

1. High energetic collisions with hadrons with $\alpha_s \ll 1$ can be calculated in perturbative QCD (=Feynman amplitudes).

2. Consider pair production of colored particles. At larger distances, the coupling increases. Effectively, and in a (very!!) simplified picture, a color flux tube of O(1GeV/fm) is created that quickly exceeds the pair production threshold.
   - The ensuing hadronisation to the colorless bound states (mesons, baryons) must be described in effective models.

3. The description of QCD bound states (e.g. the proton) in terms of quarks and gluons can not be derived from first principles.

simplified picture of hadronisation
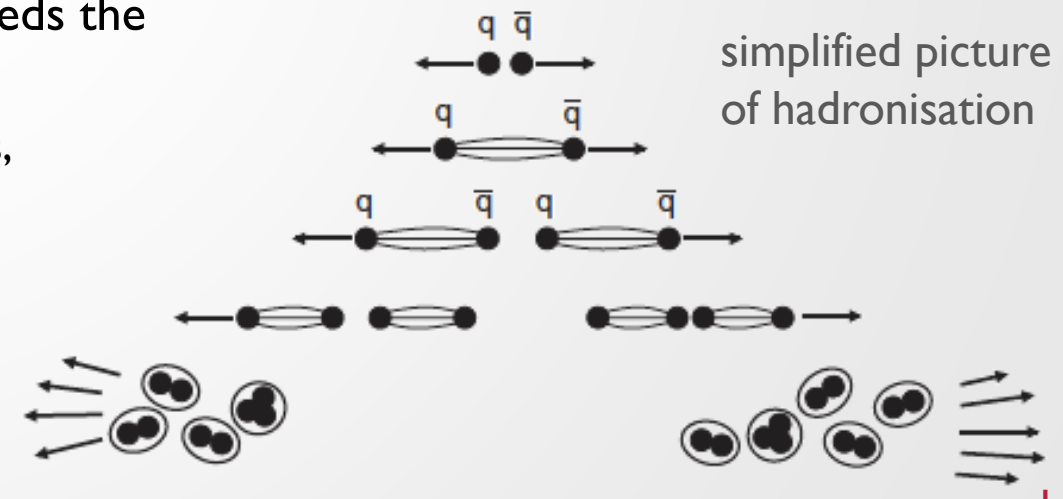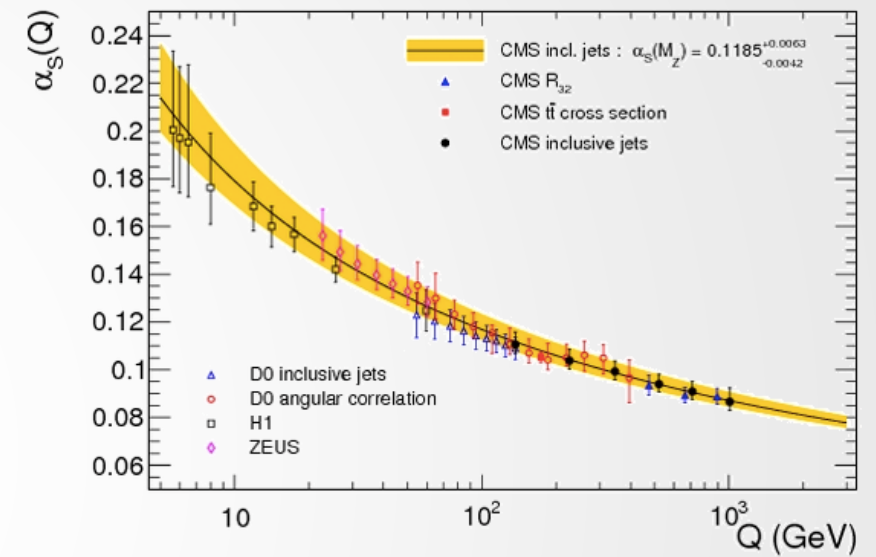
2. PREREQUISITES

2.1 RELATIVISTIC PARTICLE KINEMATICS

- In every rest frame we have $p_{1,2}^2 = E_{1,2}^2 - \vec{p}_{1,2}^{\;2} = m_{1,2}^2$

  or $E_{1,2} = \sqrt{\vec{p}_{1,2}^{\;2} + m_{1,2}^2}$.

- We define the velocity as $\vec{\beta} = \dfrac{\vec{p}}{E}$.

- A Lorentz transformation of the energy, the momentum parallel to the direction of motion and perpendicular to it is given by

$$\begin{pmatrix} E' \\ p'_\parallel \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}$$

$$p'_\perp = p_\perp$$

  where $\gamma = (1 - \beta^2)^{-1/2}$.

- We can use these equations to relate measurements in different rest systems.

centre-of-mass (c.m.) frame

m₁   m₂

$$p_{1,\text{c.m.}} = \begin{pmatrix} E_{1,\text{c.m.}} \\ \vec{p}_{\text{c.m.}} \end{pmatrix} \qquad p_{2,\text{c.m.}} = \begin{pmatrix} E_{2,\text{c.m.}} \\ -\vec{p}_{\text{c.m.}} \end{pmatrix}$$

lab frame

m₁   m₂

$$p_{1,\text{lab}} = \begin{pmatrix} E_{1,\text{lab}} \\ \vec{p}_{1,\text{lab}} \end{pmatrix} \qquad p_{2,\text{lab}} = \begin{pmatrix} m_2 \\ 0 \end{pmatrix}$$
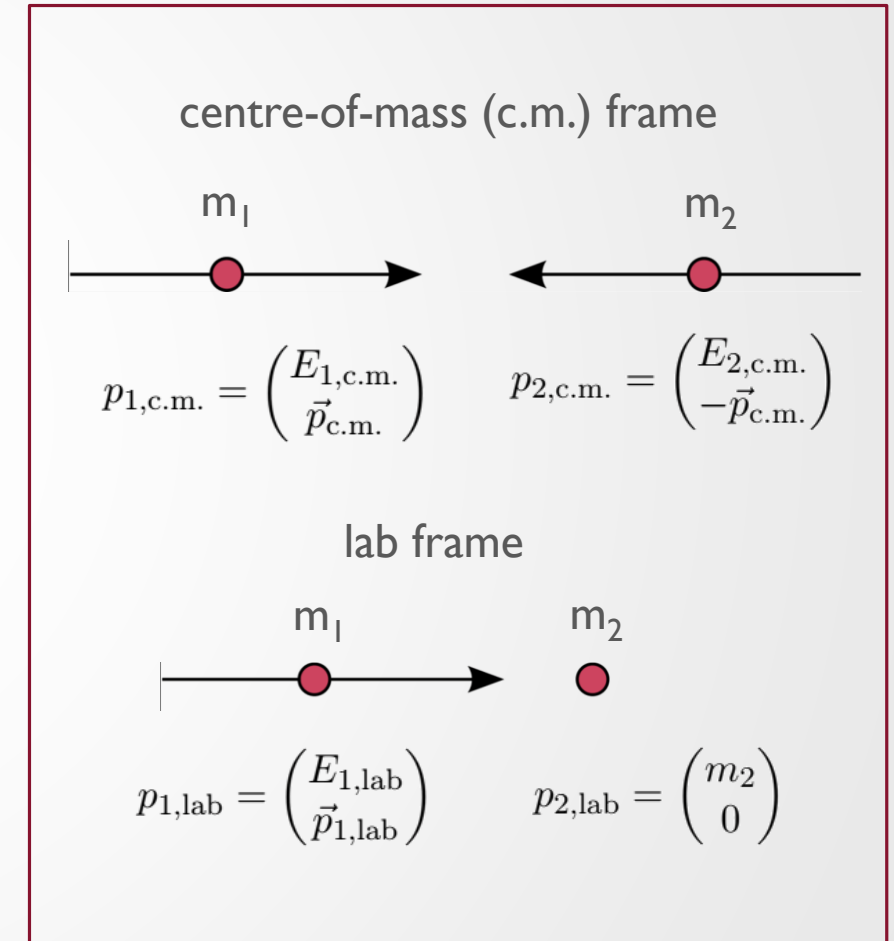
2.1 RELATIVISTIC PARTICLE KINEMATICS

- We relate the energies by computing the Lorentz invariant total momentum squared and approximate for m/E≪1.

$$\sqrt{s} = \left((E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2\right)^{1/2}$$

$$= \left(m_1^2 + m_2^2 + 2E_1 E_2(1 - \vec{\beta}_1 \cdot \vec{\beta}_2)\right)^{1/2}$$

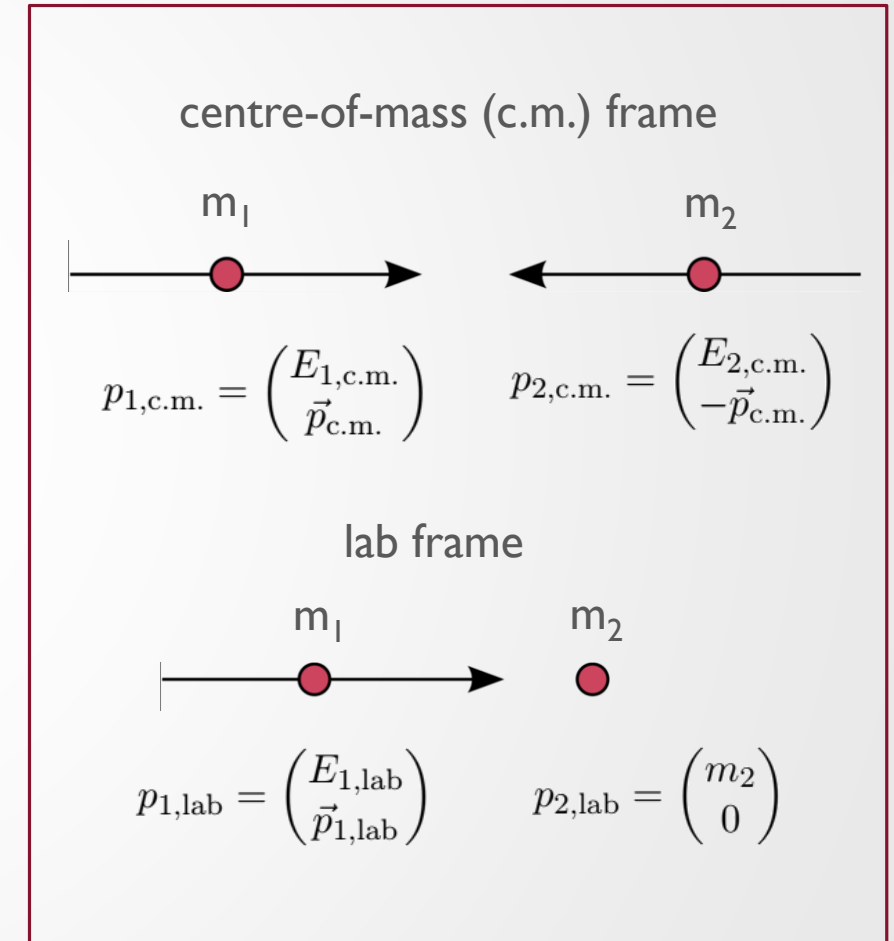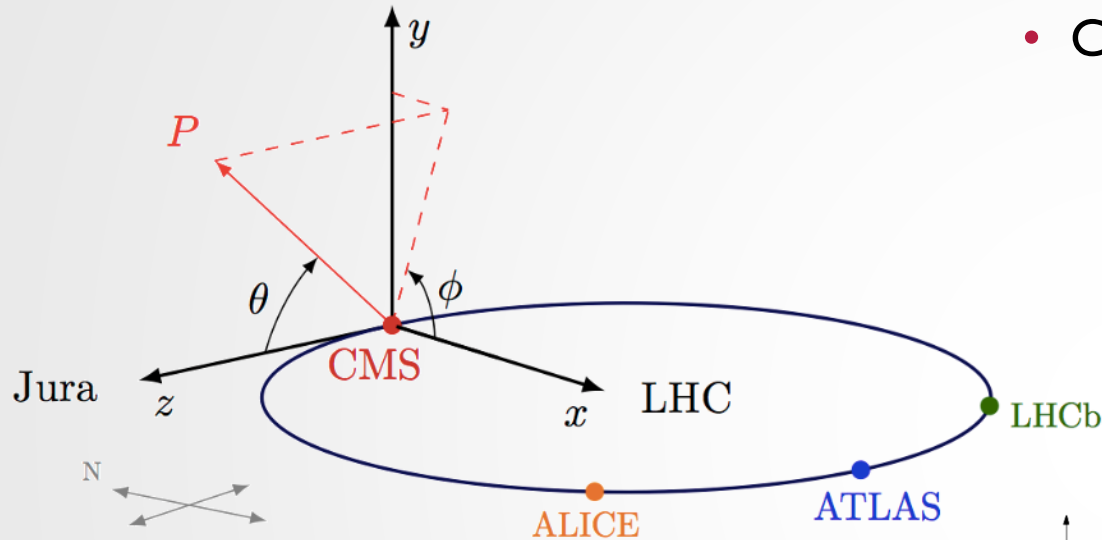$$= \left(m_1^2 + m_2^2 + 2E_1 E_2(1 - \beta_1 \beta_2 \cos\theta)\right)^{1/2}$$

$$\overset{\text{lab.}}{=} \left(m_1^2 + m_2^2 + 2E_{1,\text{lab}} m_2\right)^{1/2} \approx \sqrt{2E_{1,\text{lab}} m_2}$$

$$\overset{\text{c.m.}}{=} E_{1,\text{c.m.}} + E_{2,\text{c.m.}} = E_{\text{c.m.}} \overset{m_1 = m_2}{\approx} 2E$$

- Example 1: Beam of K$^{\pm}$ (0.497 GeV) with p = 0.8 GeV/c on a proton target p(0.936 GeV): √s = 1.698 GeV

- Example 2: NA48 fixed target 450 GeV protons on Be nucleus (8.39 GeV) becomes √s = 87.31 GeV

  - (N.B.: NA48 used secondary Kaons)

centre-of-mass (c.m.) frame

m$_1$ m$_2$

$$p_{1,\text{c.m.}} = \begin{pmatrix} E_{1,\text{c.m.}} \\ \vec{p}_{\text{c.m.}} \end{pmatrix} \qquad p_{2,\text{c.m.}} = \begin{pmatrix} E_{2,\text{c.m.}} \\ -\vec{p}_{\text{c.m.}} \end{pmatrix}$$

lab frame

m$_1$ m$_2$

$$p_{1,\text{lab}} = \begin{pmatrix} E_{1,\text{lab}} \\ \vec{p}_{1,\text{lab}} \end{pmatrix} \qquad p_{2,\text{lab}} = \begin{pmatrix} m_2 \\ 0 \end{pmatrix}$$

2.2 COLLIDER EXPERIMENT COORDINATE SYSTEM

- Coordinate systems are conveniently chosen such that
  - z points in the beam direction
  - x points inside the accelerator ring
  - Instead of **θ**, the pseudo-rapidity **η** is used

$$\eta = -\ln \tan \frac{\theta}{2}$$

- jargon for collider experiments
  - central : $|\eta| \approx 0$, $\theta \approx \pi$
  - endcap: outside tracking, $|\eta| \gtrsim 2.5 - 3$
  - forward: $|\eta| \gtrsim 3 - 5$, close to the beam

ATLAS inner detector in the r/z view
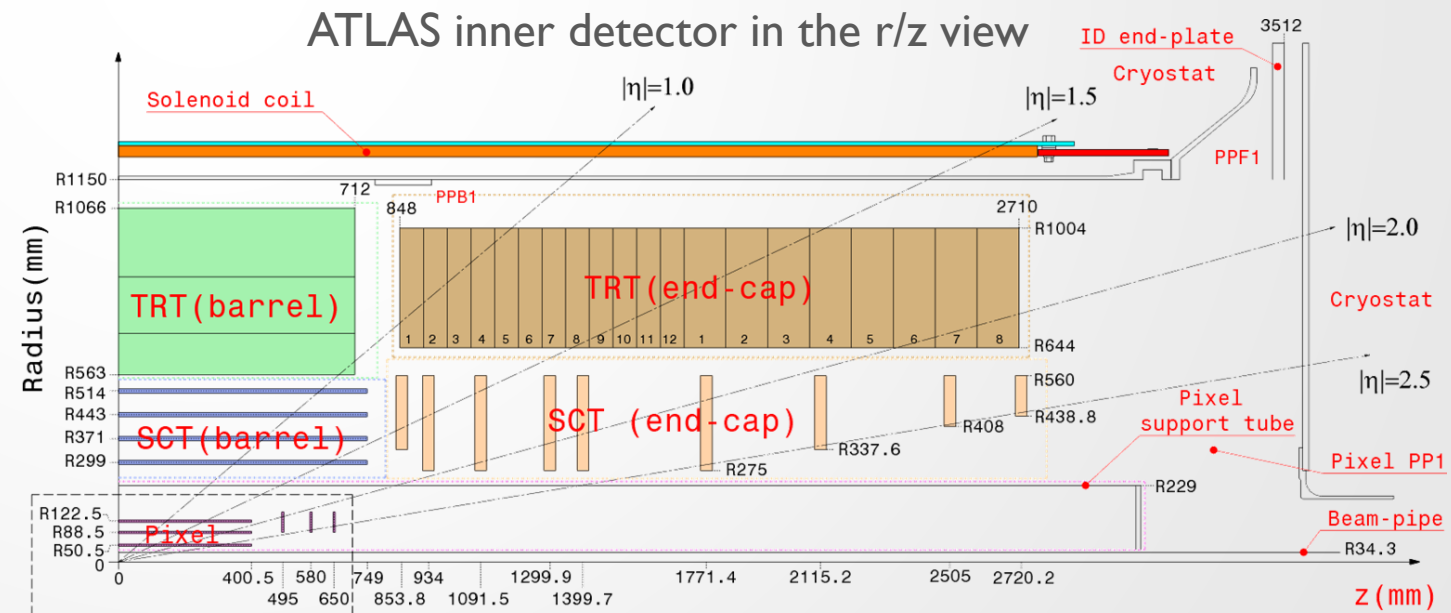
2.3 LORENTZ BOOST IN THE DETECTOR SYSTEM

- We can write the 4-momentum as

$$p^\mu = \begin{pmatrix} E \\ \vec{p} \end{pmatrix} = \begin{pmatrix} \sqrt{m^2 + |\vec{p}|^2} \\ \vec{p} \end{pmatrix} = m\gamma \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix} = |\vec{p}| \begin{pmatrix} \sqrt{1 + \frac{m^2}{|\vec{p}|^2}} \\ \cos\phi\sin\theta \\ \sin\phi\sin\theta \\ \cos\theta \end{pmatrix} = |\vec{p}_T| \begin{pmatrix} \sqrt{\cosh^2\eta + \frac{m^2}{|\vec{p}_T|^2}} \\ \cos\phi \\ \sin\phi \\ \sinh\eta \end{pmatrix} \overset{m=0}{=} |\vec{p}_T| \begin{pmatrix} \cosh\eta \\ \cos\phi \\ \sin\phi \\ \sinh\eta \end{pmatrix}$$

- It follows that $\sinh\eta = \dfrac{1}{\tan\theta}$ or $\eta = -\ln\tan\dfrac{\theta}{2}$. After a boost in z direction $L_\mu^{(z)\nu} = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix}$
we find

$$\tanh\eta' = \frac{\beta\cosh\eta + \sinh\eta}{\cosh\eta + \beta\sinh\eta} = \frac{\beta + \tanh\eta}{1 + \beta\tanh\eta} = \frac{\tanh(\tanh^{-1}\beta) + \tanh\eta}{1 + \tanh(\tanh^{-1}\beta)\tanh\eta}$$

$$= \tanh(\eta + \tanh^{-1}\beta)$$

$$\eta' = \eta + \underbrace{\tanh^{-1}\beta}_{\Delta\eta}$$

- Thus, $\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$ is a boost-invariant (along the z-direction) angular distance measure.

- Moreover, $\theta \approx \dfrac{\pi}{2} - \eta + \mathcal{O}(\eta^3)$ and therefore φ and η have the same 'unit' for low η.

2.4 MANDELSTAM VARIABLES

- When modeling a collision as a 2→2 process with a force carrier exchange, there are three topologies:

's-channel'   't-channel'   'u-channel'
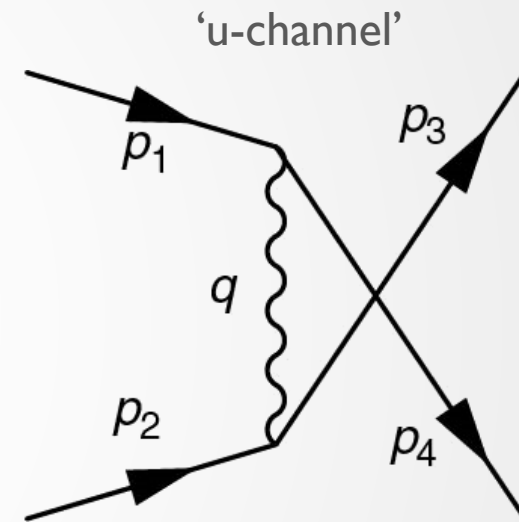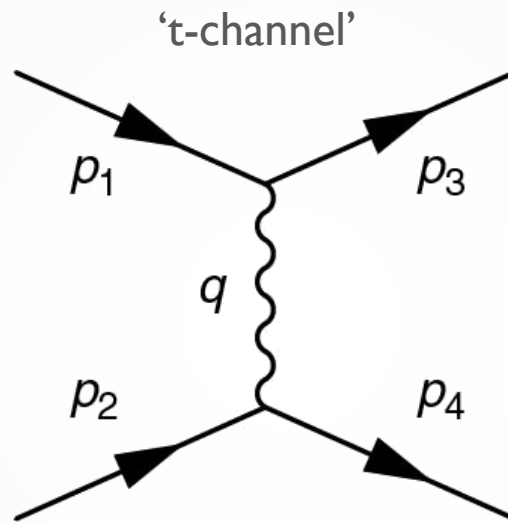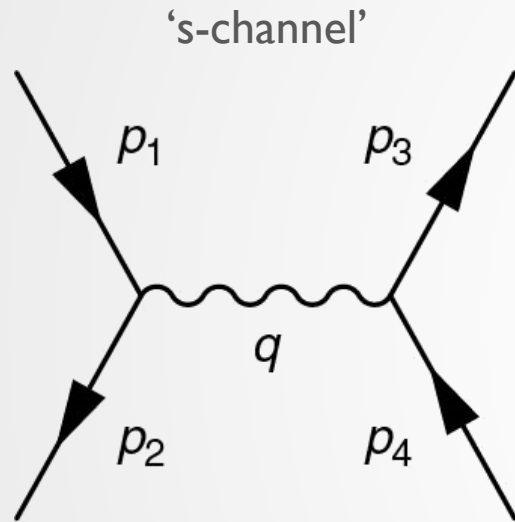
$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \qquad t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \qquad u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

- However, from the 8 d.o.f of the final state particles, 2 are removed from on-shell conditions and 4 more by energy-momentum conservation. Two d.o.f. remain. Therefore, the s, t and u are not independent:

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

2.5 IMPORTANT TYPES OF COLLISIONS

e^- → ← e^+

clean collision of (elementary) electrons

e^- → ← | p

e/h collisions to probe hadron structure

p | → ← | $p/\bar{p}$
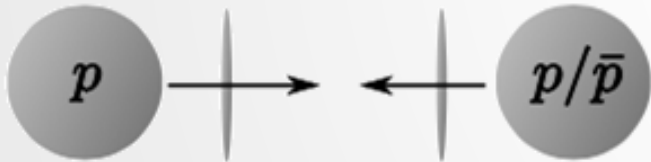
pp (e.g. LHC) or p/anti-p (e.g. Tevatron). The hadron constituents react.

Ion

$e^-, p, He^{++}, \ldots$ →
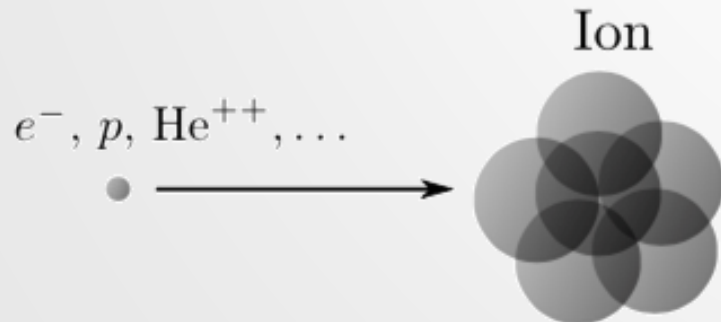
fixed target collision. penetrate a nucleus with projectile, or trigger photon-nucleus interactions.
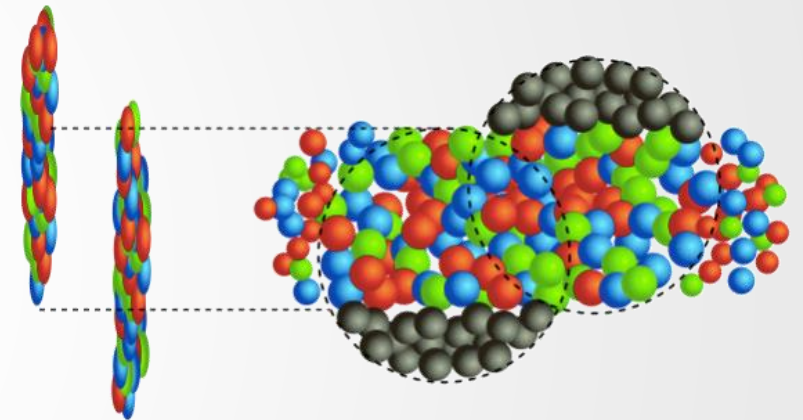
Heavy Ion collision.
Study collective behavior
for signs of e.g. quark-gluon plasma.

3. ELECTRON-POSITRON COLLISIONS

3.1 COLLISIONS OF ELEMENTARY PARTICLES
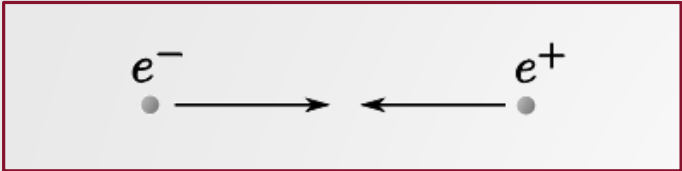

e^- ————————→ ←———————— e^+

- The collision of an e± pair at is described by the Standard Model, a renormalizable quantum field theory.

- Scattering amplitude are represented by Feynman diagrams.

- Diagrams are visualizations of transition matrix elements according to the Feynman rules (here: QED)

$$e^- \quad p_1 \searrow \quad \gamma \quad \nearrow p_3$$
$$e^+ \quad p_2 \nearrow \quad \searrow p_4$$
$$s = (p_1 + p_2)^2$$

$$\bar{f} \quad \bar{v}(p_2)[-ie\gamma^\nu]u(p_1)$$
$$\frac{-ig_{\mu\nu}}{q^2}$$
$$f \quad \bar{u}(p_3)[-ie\gamma^\nu]v(p_4)$$

-i M$_{fi}$
=
product of
all terms

The event kinematics fully determined by matrix element and kinematical phase space.

initial-state particle:	$u(p)$	————————→●
final-state particle:	$\bar{u}(p)$	●————————→
initial-state antiparticle:	$\bar{v}(p)$	————◀————●
final-state antiparticle:	$v(p)$	●————◀————
initial-state photon:	$\varepsilon_\mu(p)$	～～～～～●
final-state photon:	$\varepsilon_\mu^*(p)$	●～～～～～
photon propagator:	$-\frac{ig_{\mu\nu}}{q^2}$	●～～～～～●
fermion propagator:	$-\frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2}$	●————————●
QED vertex:	$-iQe\gamma^\mu$	

… for full derivation see QFT lecture

3.1 COLLISIONS OF ELEMENTARY PARTICLES

- Diagrams with the same initial and final state must be added up

's-channel'   't-channel'

… add, when f=e

$$M_{fi} = M_{fi}^{(1)} + M_{fi}^{(2)} + \ldots$$
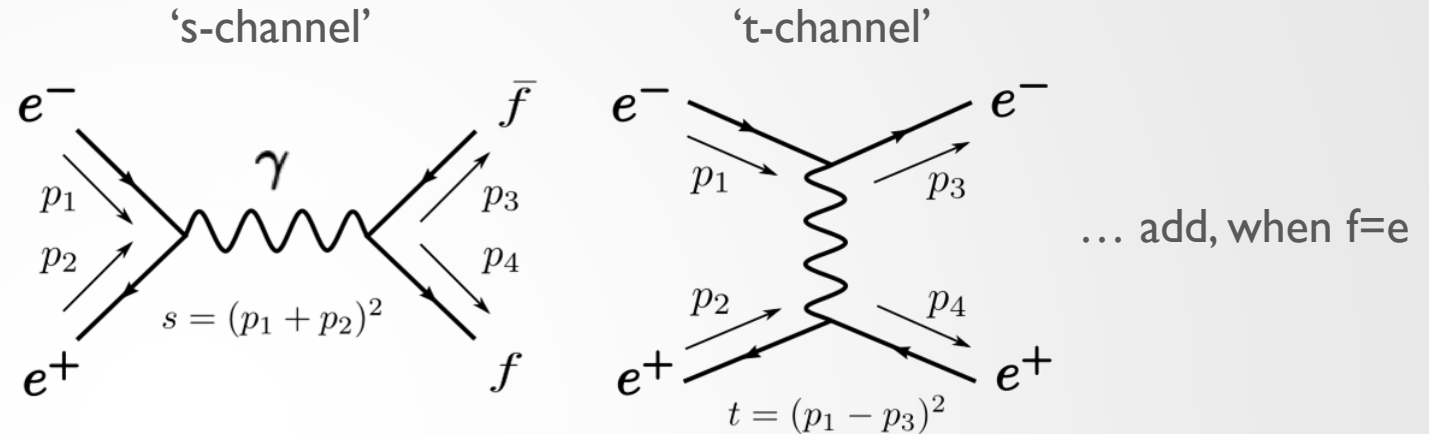
- differential cross-section: number of interactions per unit time, per target particle, and per incident flux.

- The general structure of a differential cross-section of a $2 \longrightarrow 2$ process is

$$\mathrm{d}\sigma = \underbrace{\frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}}_{\text{flux}} \underbrace{(2\pi)^4 |\mathcal{M}_{fi}(p_i)|^2}_{\text{matrix element}} \underbrace{\delta^{(4)}\left(\sum_{i=1}^{4} p_i\right)}_{\text{momentum cons.}} \underbrace{\frac{\mathrm{d}\vec{p}_3}{(2\pi)^3 2E_3} \frac{\mathrm{d}\vec{p}_4}{(2\pi)^3 2E_4}}_{\text{Lorentz-inv. phase space}}$$

3.3 EXAMPLE: LEP AT THE Z POLE MASS

- colliders operate at a fixed energy

  - Because electrons are fundamental particles,
    $\sqrt{s} = (2E_{beam})^2$ is given by the beam energy

  - In order to investigate energy dependent effects,
    the beam energy must be varied – 'scanned'

  - requires an estimate of interesting energy regime
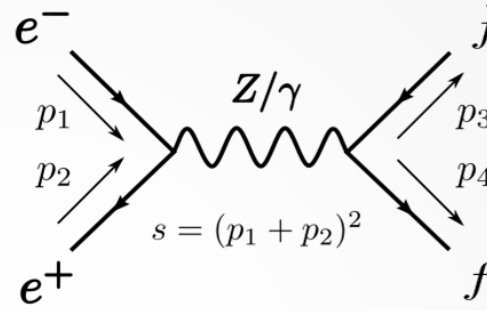
  - can be very time consuming!

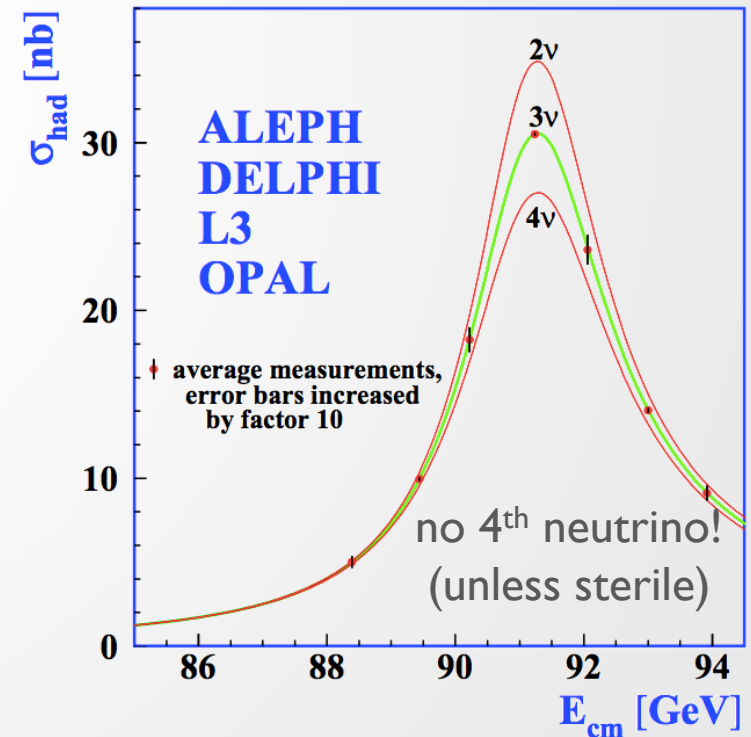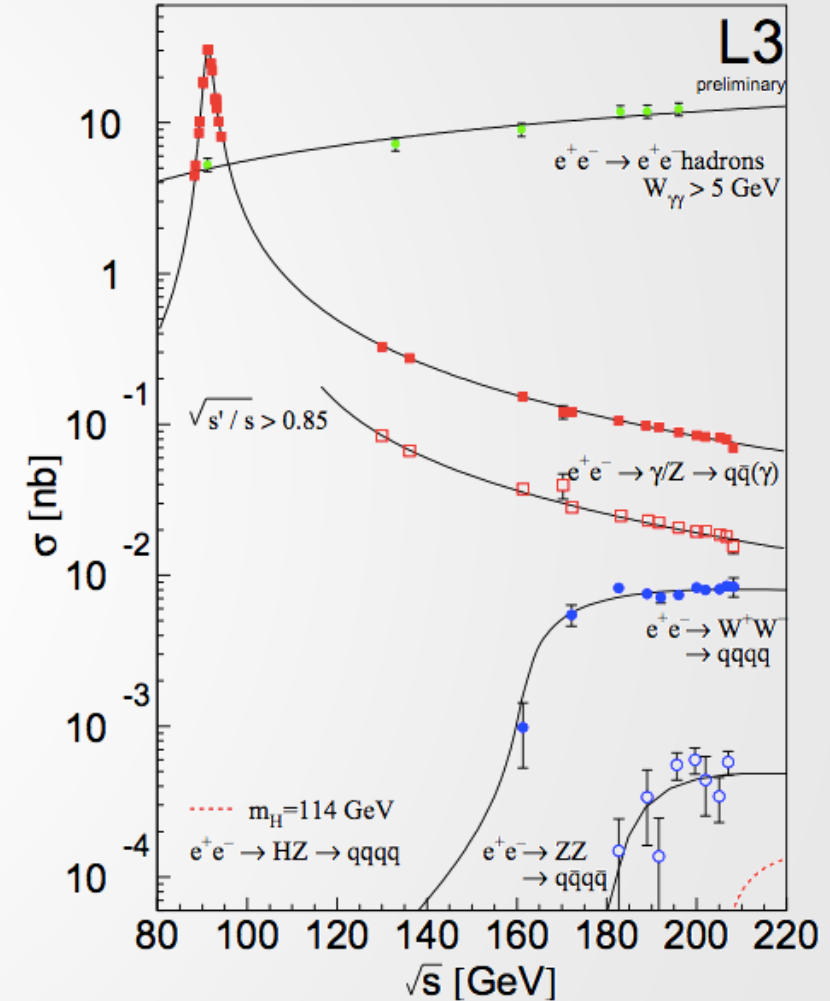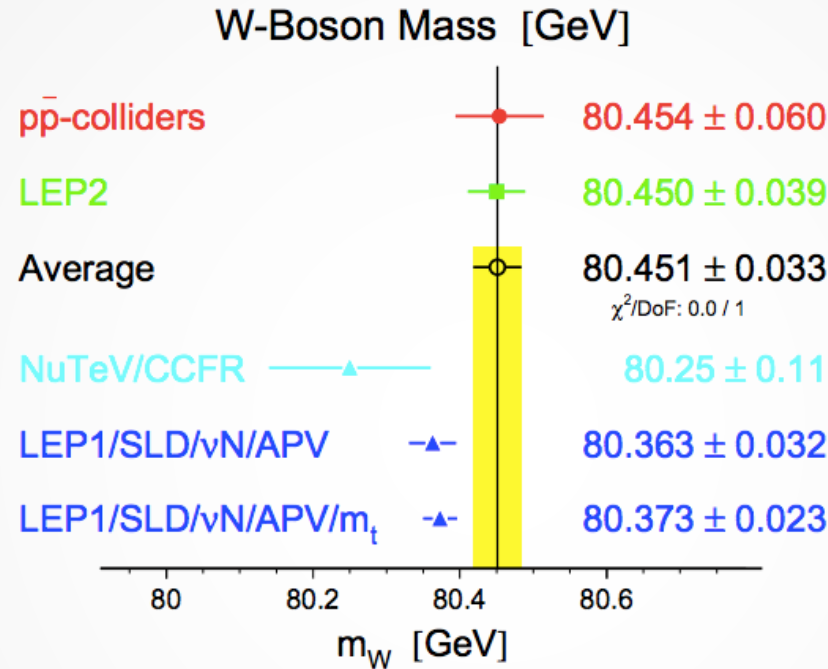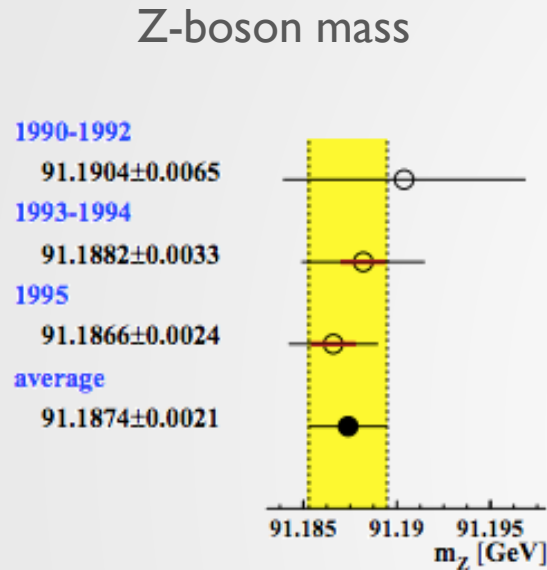| Year	Centre-of-mass energy range [GeV]	Integrated luminosity [pb^{-1}]
1989	88.2 – 94.2	1.7
1990	88.2 – 94.2	8.6
1991	88.5 – 93.7	18.9
1992	91.3	28.6
1993	89.4, 91.2, 93.0	40.0
1994	91.2	64.5
1995	89.4, 91.3, 93.0	39.8
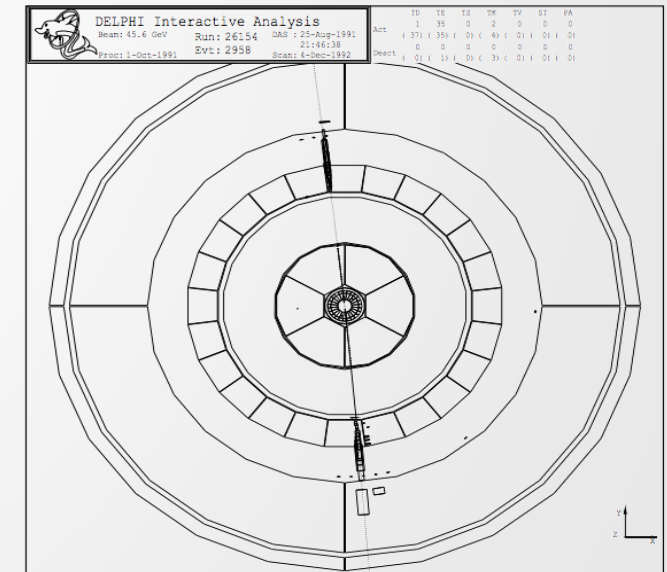
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  - requires an estimate of interesting energy regime

  - can be very time consuming!

- Example: LEP at the Z pole

  - energy scan in ~1GeV steps around the Z pole mass between 1992 and 1995, combined with data from Stanford's SLD/SLC detector

  - Because $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{had} + N_\nu \Gamma_{\nu\bar{\nu}} \approx 2.5$ GeV is calculable with high precision from the SM amplitudes, a precise measurement of the Z mass can constrain the number of neutrino families.

$$\sigma_Z \propto \left| \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2 = \frac{1}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$
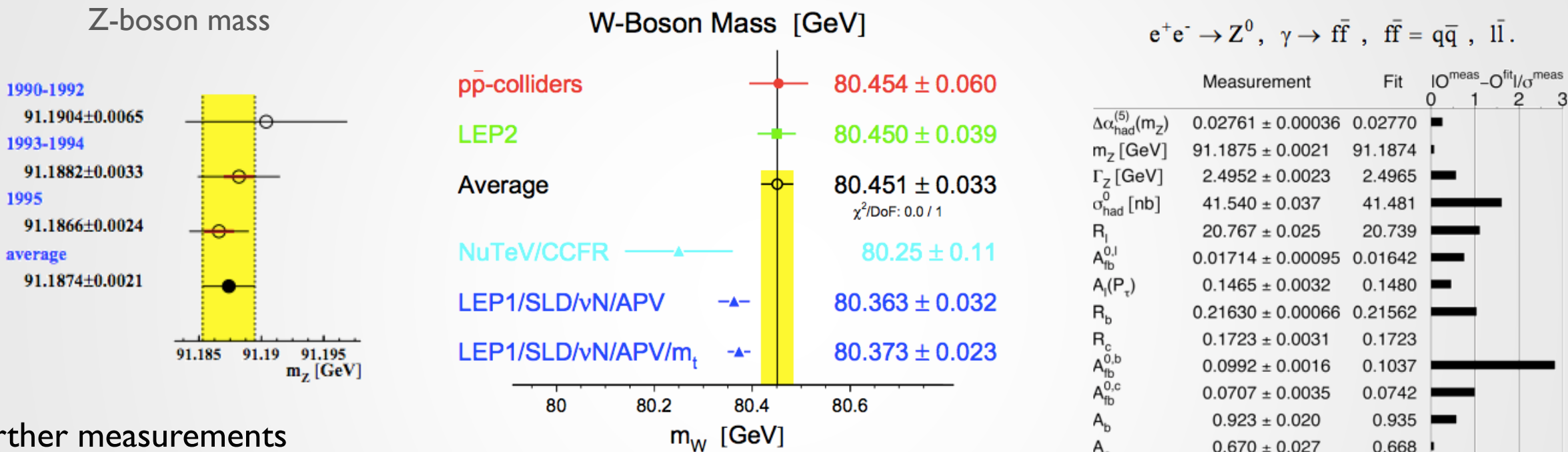
$s = (p_1 + p_2)^2$

ALEPH
DELPHI
L3
OPAL

average measurements, error bars increased by factor 10

no 4th neutrino! (unless sterile)

σ_{had} [nb]

E_{cm} [GeV]

3.4 LEP AT AND BEYOND THE Z POLE

Z-boson mass

W-Boson Mass [GeV]

Further measurements

- couplings of the Z to b and c quarks
- forward/backward asymmetries
- WW production, ZZ production

- Higgs mass limit $m_H > 114$ GeV
- τ polarisation
- limits on supersymmetry
 (charginos, top & bottom squark)

3.4 LEP AT AND BEYOND THE Z POLE

Z-boson mass

1990-1992
91.1904 ± 0.0065

1993-1994
91.1882 ± 0.0033

1995
91.1866 ± 0.0024

average
91.1874 ± 0.0021

$91.185 \quad 91.19 \quad 91.195$

m_Z [GeV]

W-Boson Mass [GeV]

$p\bar{p}$-colliders $\qquad$ 80.454 ± 0.060

LEP2 $\qquad$ 80.450 ± 0.039

Average $\qquad$ 80.451 ± 0.033

χ^2/DoF: 0.0 / 1

NuTeV/CCFR $\qquad$ 80.25 ± 0.11

LEP1/SLD/νN/APV $\qquad$ 80.363 ± 0.032

LEP1/SLD/νN/APV/m_t $\qquad$ 80.373 ± 0.023

$80 \quad 80.2 \quad 80.4 \quad 80.6$

m_W [GeV]

$e^+e^- \rightarrow Z^0, \gamma \rightarrow f\bar{f}, \quad f\bar{f} = q\bar{q}, \; l\bar{l}.$

	Measurement	Fit	$|O^{meas} - O^{fit}|/\sigma^{meas}$
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02761 ± 0.00036	0.02770	
m_Z [GeV]	91.1875 ± 0.0021	91.1874	
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965	
σ_{had}^0 [nb]	41.540 ± 0.037	41.481	
R_l	20.767 ± 0.025	20.739	
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642	
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480	
R_b	0.21630 ± 0.00066	0.21562	
R_c	0.1723 ± 0.0031	0.1723	
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037	
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742	
A_b	0.923 ± 0.020	0.935	
A_c	0.670 ± 0.027	0.668	
$A_l(SLD)$	0.1513 ± 0.0021	0.1480	
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	
m_W [GeV]	80.425 ± 0.034	80.390	
Γ_W [GeV]	2.133 ± 0.069	2.093	
m_t [GeV]	178.0 ± 4.3	178.4	

$0 \quad 1 \quad 2 \quad 3$

Further measurements

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- forward/backward asymmetries
- WW production, ZZ production

- Higgs mass limit $m_H > 114$ GeV
- τ polarisation
- limits on supersymmetry
  (charginos, top & bottom squark)

4. DEEP INELASTIC SCATTERING

# 4.1 LENGTH SCALES IN ELECTRON PROTON COLLISIONS

$\lambda \gg r_p$

$\lambda \sim r_p$

$\lambda < r_p$
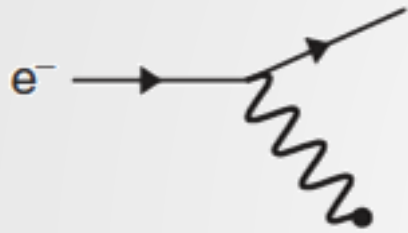
$\lambda \ll r_p$

- Wavelength of photon much larger than proton.

- Elastic scattering of e⁻ on a point-like proton with a Coulomb force.
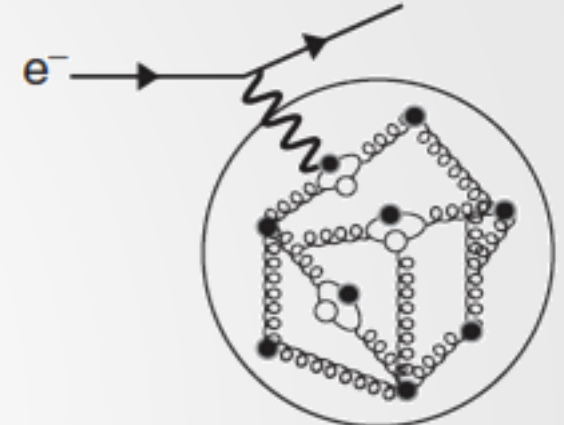
- Rutherford (or Mott) scattering applies.
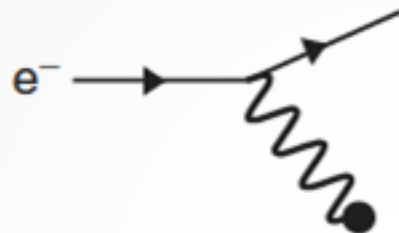
- Cross-section calculation needs to take into account extended charge and magnetic moment distribution.

- Parameterized with structure functions, related to Fourier transformation of the potential
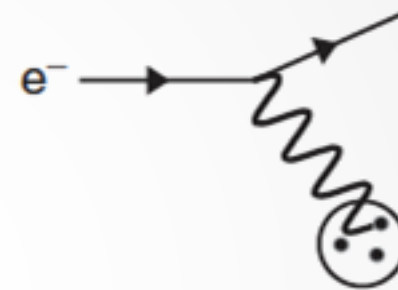
- The elastic cross-section becomes small and inelastic processes with constituent quarks dominate.

- The proton appears to be a sea of strongly interacting quarks and gluons.

- the detailed structure of the proton is resolved.
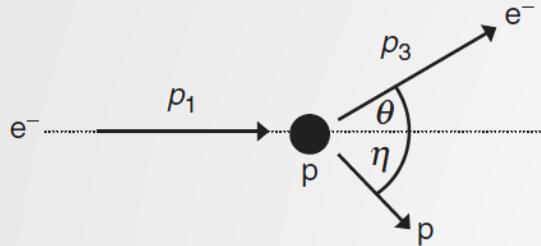
- Even without full matrix-element calculations we can infer the behavior:

$$\mathcal{M}_{fi} = \frac{Q_q e^2}{q^2} \left[\bar{u}(p_3)\gamma^\mu u(p_1) \right] g_{\mu\nu} \left[\bar{u}(p_4)\gamma^\nu u(p_2) \right]$$

In the rest-frame of the initial proton, the momentum transfer is

$$q^2 = (0, \mathbf{p}_1 - \mathbf{p}_3)^2 = -2p^2(1 - \cos\theta) = -4p^2 \sin^2(\theta/2)$$

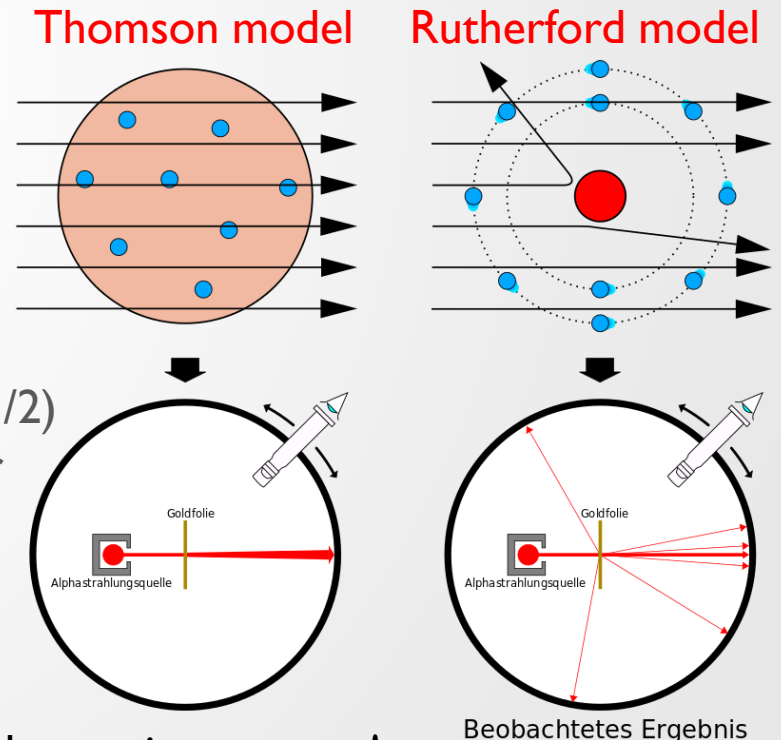- $|M^2| \propto \sin^{-4}(\theta/2)$ agrees with classical Rutherford scattering. Large θ possible!

- The spin-averaged matrix element is

$$\langle |\mathcal{M}_{fi}^2| \rangle = \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4(\theta/2)} \left[1 + \beta_e^2 \gamma_e^2 \cos^2\frac{\theta}{2} \right]$$

Rutherford scattering (scalar interaction)

Mott scattering (spin-1/2) e.g. a μ in a detector with $\beta\gamma \gg 1$

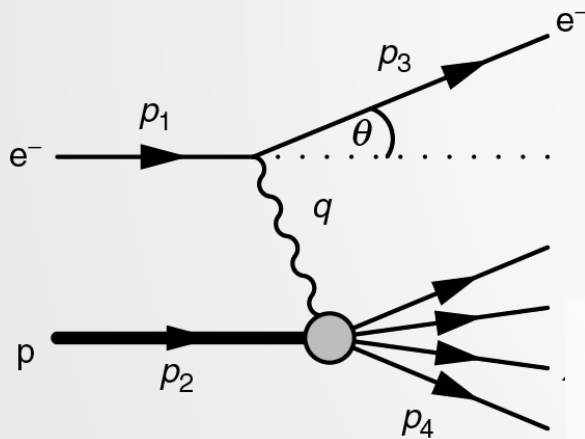- The classical Rutherford scattering amplitude corresponds to single-photon exchange.

  - Differential scattering cross-section reveals sub-atomic structure!
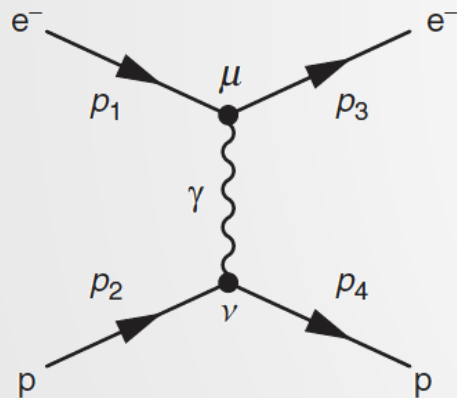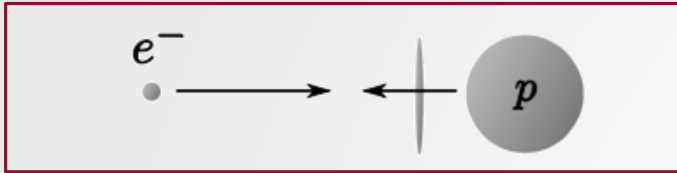
elastic scattering of relativistic particles is well described by single photon exchange!

Thomson model   Rutherford model

Goldfolie

Alphastrahlungsquelle

Goldfolie

Alphastrahlungsquelle

Beobachtetes Ergebnis

4.3 INELASTIC SCATTERING

kinematics of deep inelastic scattering

Note: Bjorken-x is a kinematic degree of freedom not present in elastic scattering!

- For the L.I.-invariant description of a colliding $e^{\pm}$-p system we first define the invariant mass of the hadronic system as

$$W^2 \equiv p_4^2 = (q + p_2)^2 = q^2 + 2p_2 \cdot q + p_2^2$$

- Next, we write the momentum transfer $Q^2 = -q^2$ and expand

$$Q^2 = -(p_1 - p_3)^2 = -2m_e^2 + 2p_1 \cdot p_3 = -2m_e^2 + 2E_1 E_3 - 2p_1 p_3 \cos \theta$$

- Neglecting m_e we have: $Q^2 \approx 2E_1 E_3 (1 - \cos \theta) = 4E_1 E_3 \sin^2 \frac{\theta}{2} \geq 0.$

- Next, define 'Bjorken' x as $x \equiv \dfrac{Q^2}{2p_2 \cdot q}$ and use $W^2 + Q^2 - m_p^2 = 2p_2 \cdot q$

  to write $x = \dfrac{Q^2}{Q^2 + W^2 - m_p^2}$. Because the final state must contain

  a proton too, we must have $W^2 \equiv p_4^2 \geq m_p^2 \longrightarrow 0 \leq x \leq 1$

- x measures the 'elasticity'. Elastic collisions ($W^2 = m_p^2$) have x=1.

4.3 (NOT SO DEEP) INELASTIC SCATTERING
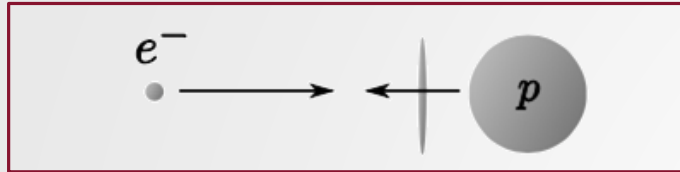
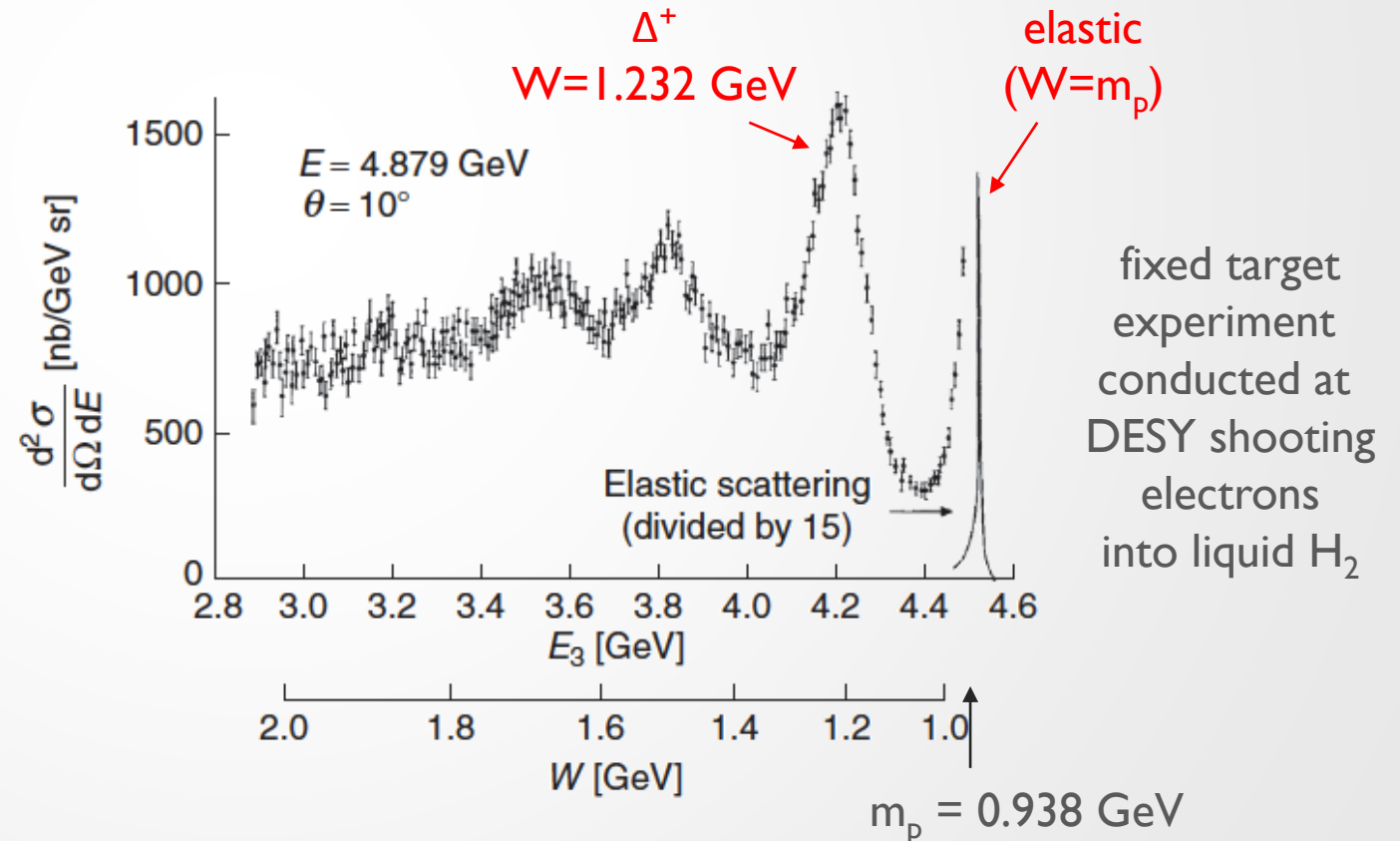- Consider E_1 = 4.879 GeV electrons on a liquid hydrogen target, **θ=10°**

$$W^2 = (p_2 + q)^2 = p_2^2 + 2p_2 \cdot q + q^2 = m_p^2 + 2p_2 \cdot (p_1 - p_3) + (p_1 - p_3)^2$$

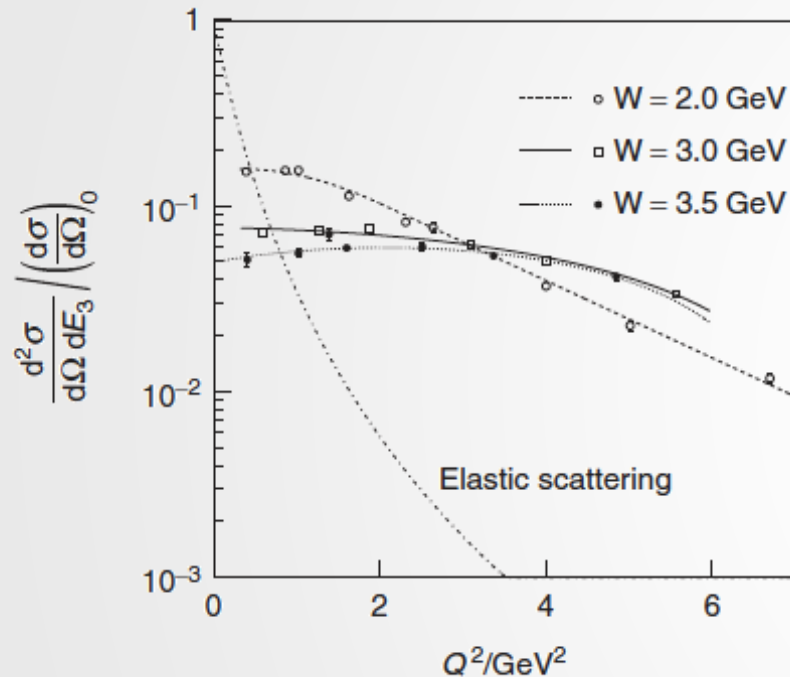$$\approx \left[m_p^2 + 2m_p E_1 \right] - 2 \left[m_p + E_1 (1 - \cos\theta) \right] E_3.$$
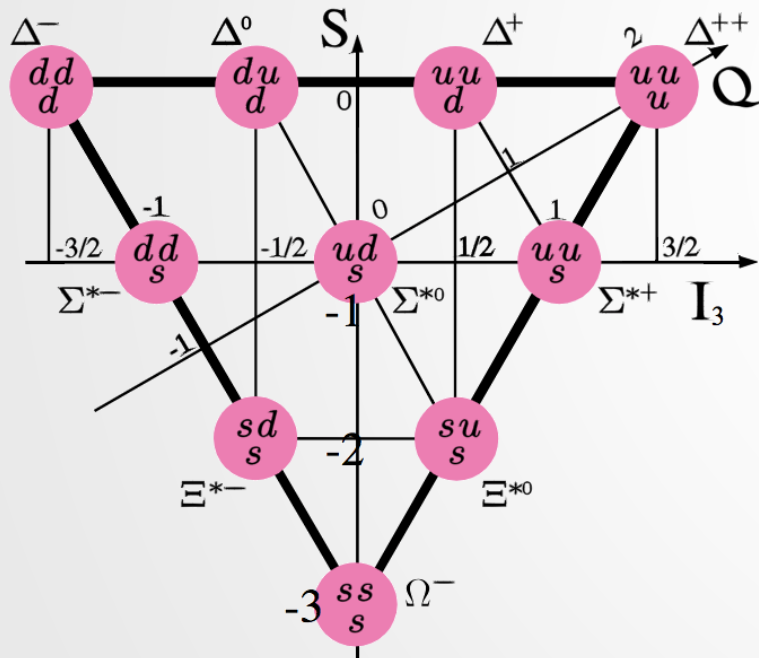
Δ^+ W=1.232 GeV

elastic (W=m_p)

E = 4.879 GeV
θ = 10°

Elastic scattering (divided by 15)

fixed target experiment conducted at DESY shooting electrons into liquid H_2

spin 3/2 baryon decuplet

m_p = 0.938 GeV

4.4 DEEP INELASTIC SCATTERING

first SLAC DIS results:

- Study the e^-/p cross-section ratio wrt. point-like Mott scattering

1. the elastic cross-section decreases strongly with Q^2

2. at higher inelasticity, the cross-section approaches a constant

- Q^2 dependence is related to potential by Fourier transformation
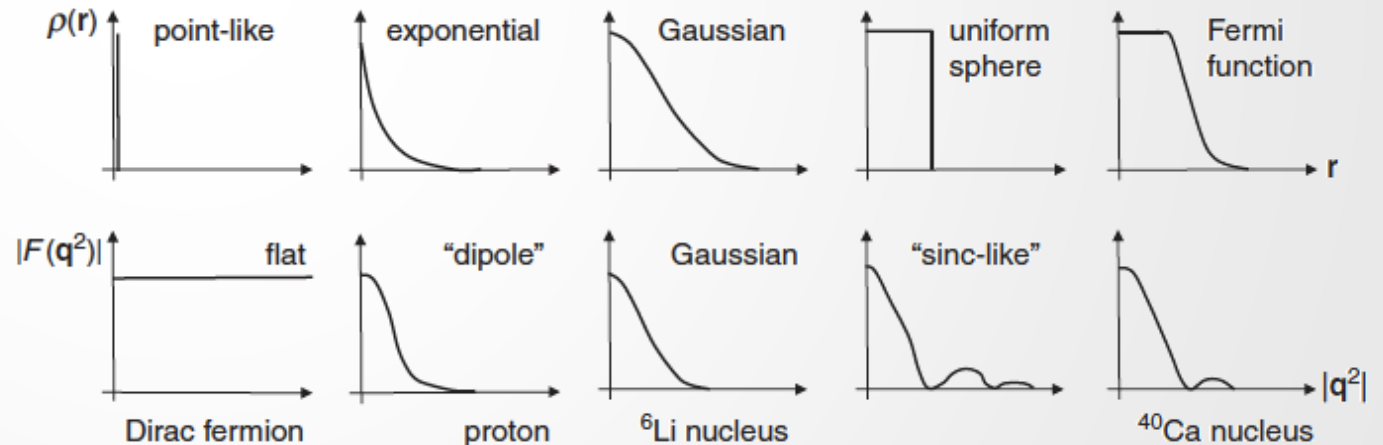
> There have been a number of different theoretical approaches in the interpretation of the high-energy inelastic electron-scattering results. One class of models,[6–9] referred to as parton models, describes the electron as scattering incoherently from pointlike constituents within the proton.

Evidence for point like scattering constituents inside the proton!
No 'form-factor-like' drop off at high Q^2!
Confirmation of the quark model!

4.5 THE PARTON MODEL
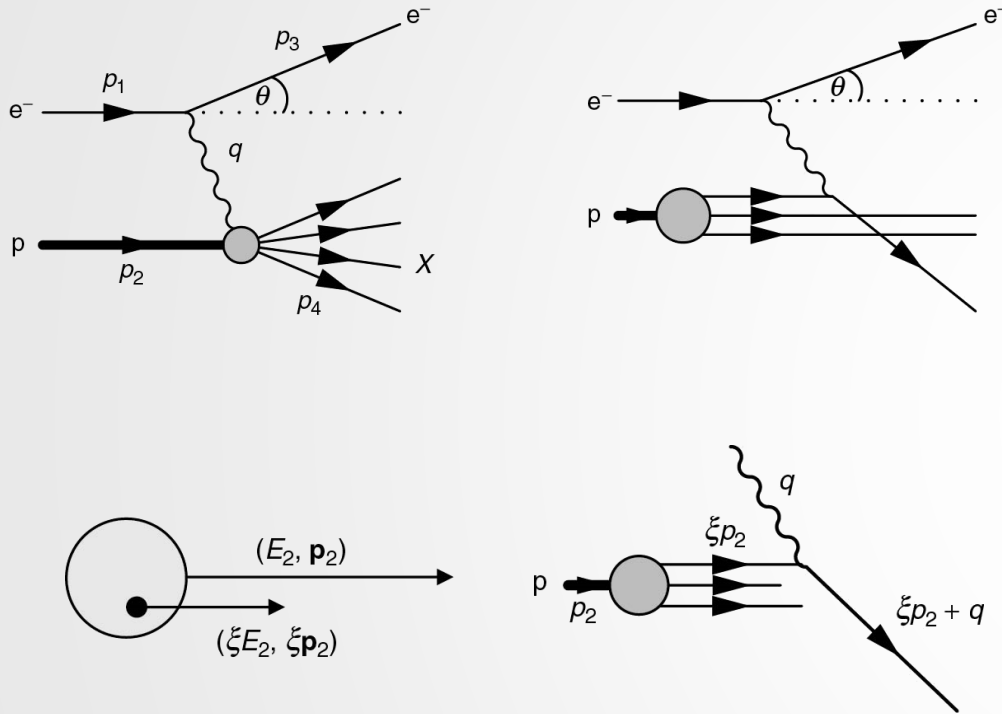
Bjorken-x can be identified as the fraction of the momentum carried by the struck parton!

- Before quarks and gluons were accepted, Feynman had proposed that the proton was made from point-like partons which take part in the reactions.

- Neglecting the mass of the proton and also neglecting the momentum of the partons transverse to the proton direction have $p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2)$, where ξ is the partons momentum fraction.

- After the interaction, the 4-momentum of the quark satisfies

$$(\xi p_2 + q)^2 = \xi^2 p_2^2 + 2\xi p_2 \cdot q + q^2 = m_q^2$$

- However, for the initial quark it also holds that $\xi^2 p_2^2 = m_q^2$

  and therefore $\xi = \dfrac{-q^2}{2p_2 \cdot q} = \dfrac{Q^2}{2p_2 \cdot q} \equiv x$.

- Therefore, the x-dependence of cross-sections informs us about the momentum fractions of the partons in the proton.
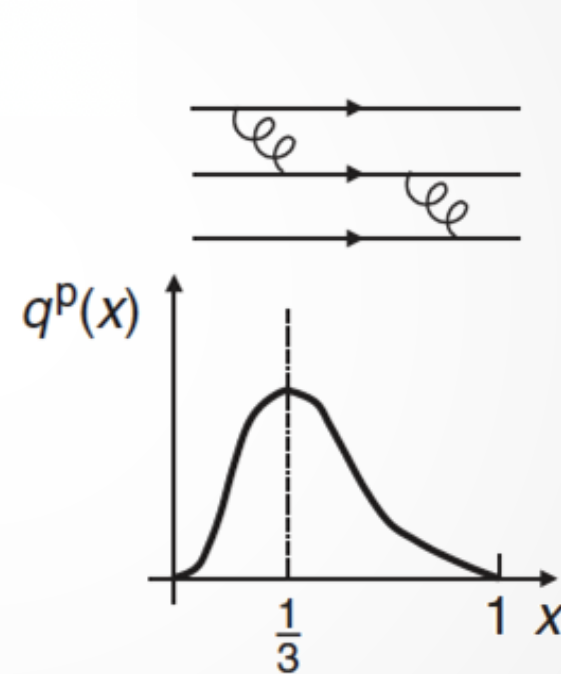
4.5.1 PARTON DISTRIBUTION FUNCTIONS

- The dynamics in the quark result in a distribution of momentum fractions of quarks in the proton.

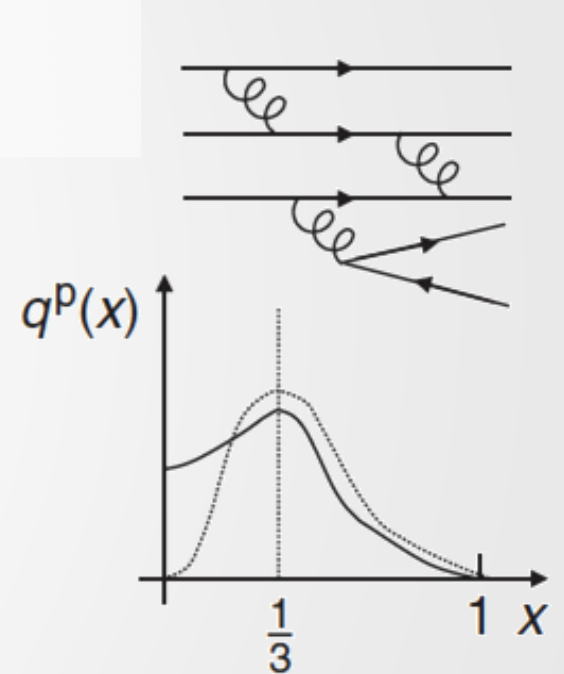- Need to be determined from experiment!
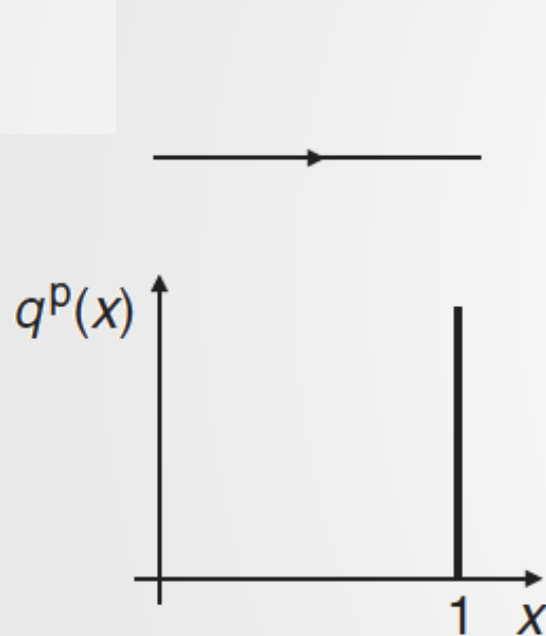
a single
point-like
particle
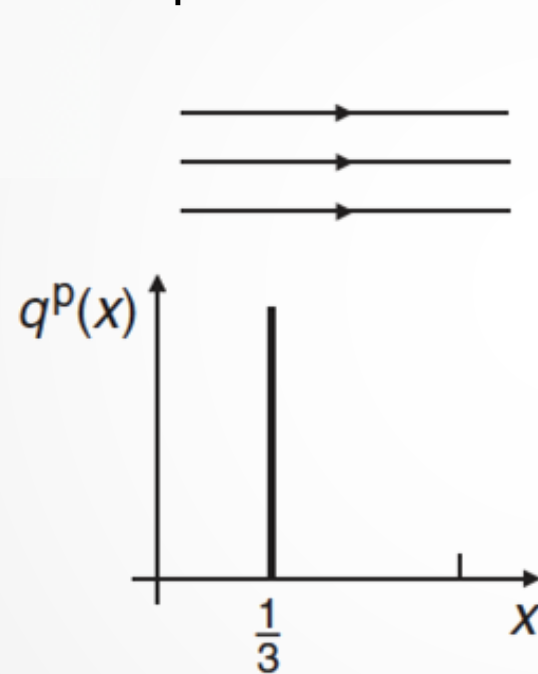
three static quarks
without interactions

three quarks that
exchange momentum
and smear the momentum

three interacting quarks
with higher-order gluon
processes producing
low energetic 'sea'-quarks
Also antiquarks & 5 flavors!

4.5.2 DETERMINATION OF PDFS

- the most general cross-section formula for e⁻/p scattering:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x,Q^2)}{x} + y^2 F_1^{ep}(x,Q^2)\right]$$

Note: y is a simple function of x, s and Q^2

single photon exchange (Rutherford)

electric +magnetic

electric

possible modifications from internal structure

- parton model prediction specializes & simplifies structure functions

$$F_2^{ep}(x,Q^2) = 2xF_1^{ep}(x,Q^2) = x\sum_i Q_i^2 q_i^p(x)$$

Callen-Gross relation: electric and magnetic properties are both calculated from the quarks, and therefore not independent

charge²

Parton distribution functions (PDFs)

structure function in ep scattering

no dependence on Q^2 because the quarks are treated point-like!

HERA F_2
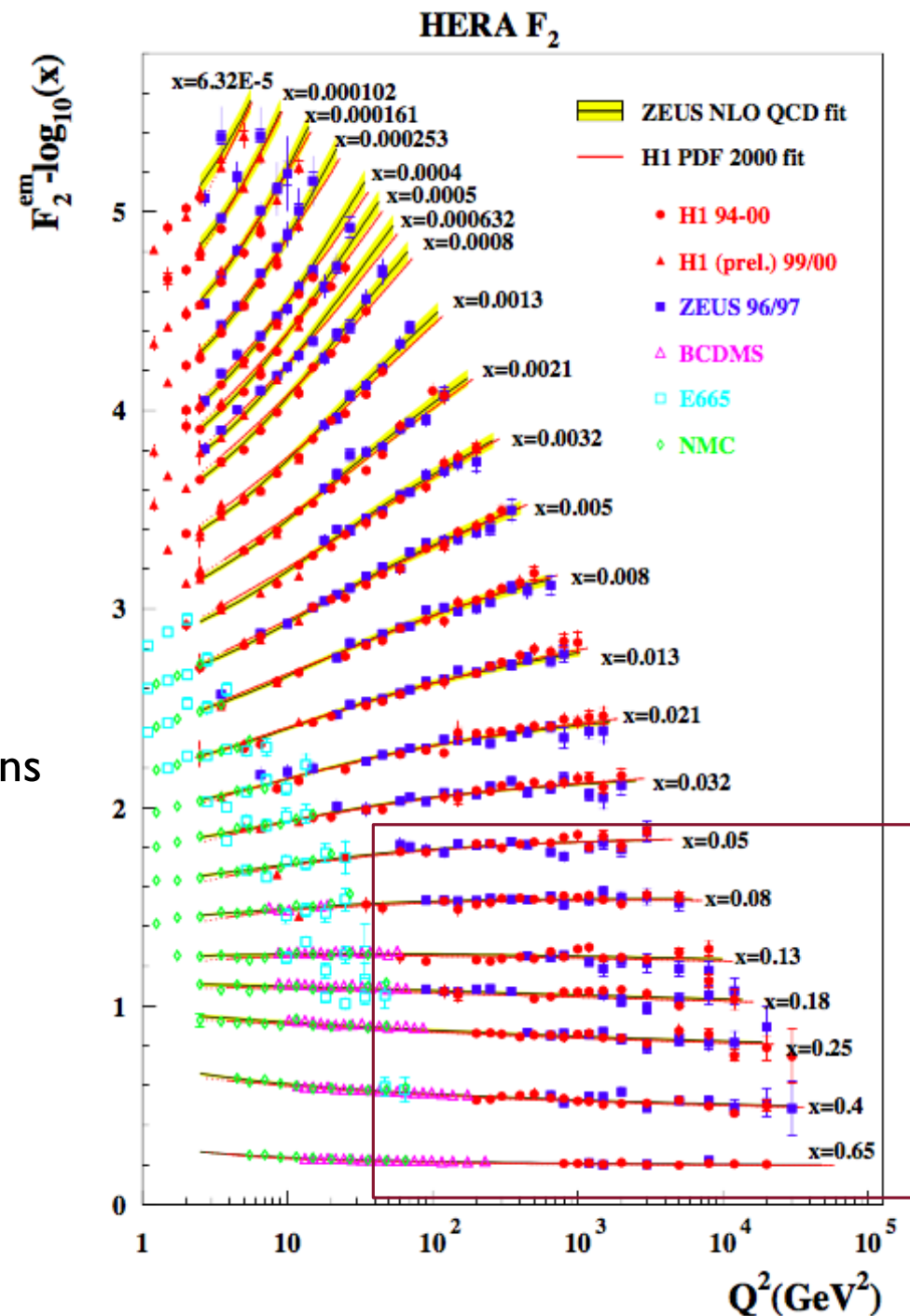
4.5.2 DETERMINATION OF PDFS

- A complex procedure! Write neutron and proton pdfs (including antiquarks)

$$F_2^{en}(x) = x \sum_i Q_i^2 q_i^n(x) \approx x \left(\frac{4}{9} u^n(x) + \frac{1}{9} d^n(x) + \frac{4}{9} \bar{u}^n(x) + \frac{1}{9} \bar{d}^n(x) \right)$$

$$F_2^{ep}(x) = x \sum_i Q_i^2 q_i^p(x) \approx x \left(\frac{4}{9} u^p(x) + \frac{1}{9} d^p(x) + \frac{4}{9} \bar{u}^p(x) + \frac{1}{9} \bar{d}^p(x) \right)$$

- Approximate isospin symmetry: $d^n(x) = u^p(x), \ u^n(x) = d^p(x)$

- A first check: integrate the PDFs over all momenta: Write

$$\int_0^1 F_2^{ep}(x)\, dx = \frac{4}{9} f_u + \frac{1}{9} f_d \quad \text{and} \quad \int_0^1 F_2^{en}(x)\, dx = \frac{4}{9} f_d + \frac{1}{9} f_u$$

where $f_u = \int_0^1 [xu(x) + x\bar{u}(x)]\, dx$ and $f_d = \int_0^1 [xd(x) + x\bar{d}(x)]\, dx$.

  - Measure $f_u = 0.36$ and $f_d = 0.18$ from proton and deuterium DIS.
  - Muon beam from CERN/SPS on fixed H_2 and deuterium target

- The valence quarks carry 54% of the proton momentum!
  - The rest is carried by the electrically neutral gluon

$2 \leq Q^2/\text{GeV}^2 \leq 30$
measured at SLAC

$F_2^{ep}(x, Q^2)$

x
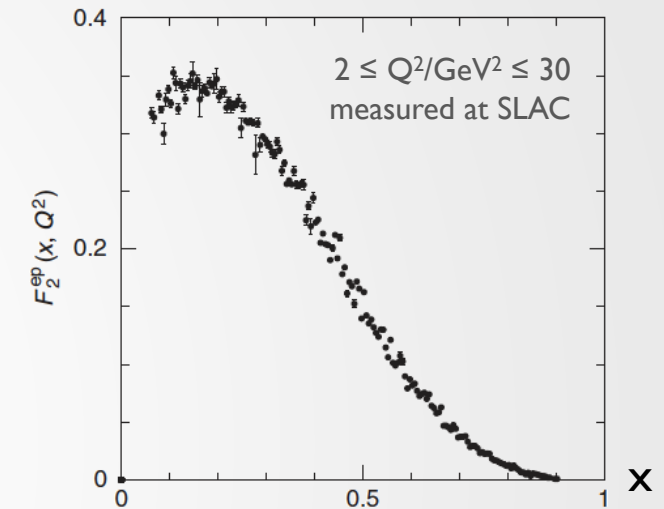
- How to determine the gluon PDF?
- Assume gluon distribution, take into account the $g \longrightarrow qq$ and $u \longrightarrow qg$ 'splitting' and fit this system to all experimental data.
- $g \longrightarrow qq$ also contains antiquarks & 5 flavors
- Fitting collaborations: CTEQ, MRST, NNPDF

4.5.2 DETERMINATION OF PDFS

- The result is a large gluon PDF at low x

- 'sea' quarks and anti-quarks enhanced at low x from g→qq splitting

  - We can split off the 'sea' quark PDF from the valence quarks
    $$d(x) = d_V(x) + d_S(x), u(x) = u_V(x) + u_S(x)$$
    and because the valence quark proton content is uud, there are
    only 'sea'-antiquarks: $\overline{d}(x) \equiv \overline{d}_S(x), \ \overline{u}(x) \equiv \overline{u}_S(x)$

  - Up and down quark masses are similar and the gluon interaction
    does not depend on isospin and flavor. Therefore, we can approximate
    $$u_S(x) = \overline{u}_S(x) \approx d_S(x) = \overline{d}_S(x)$$

- u(x) and d(x) can not be integrated (~ x$^{-1.25}$), however,
  $$\int dx\,(u(x) - \overline{u}(x)) = 2\,, \qquad \int dx\,(d(x) - \overline{d}(x)) = 1$$
  expresses the quark content in the language of PDFs and is finite.

- There is a small strange, charm and bottom quark
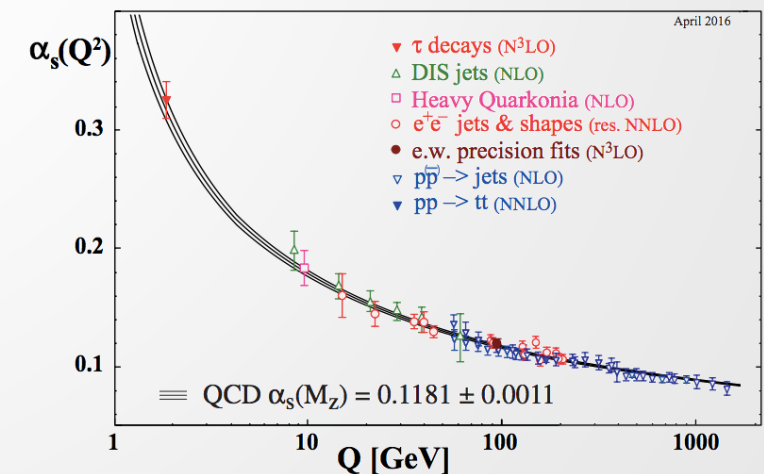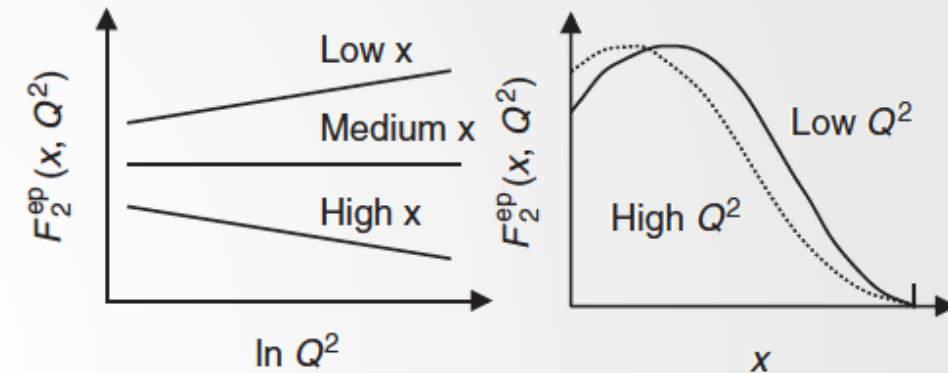 contribution in the proton.
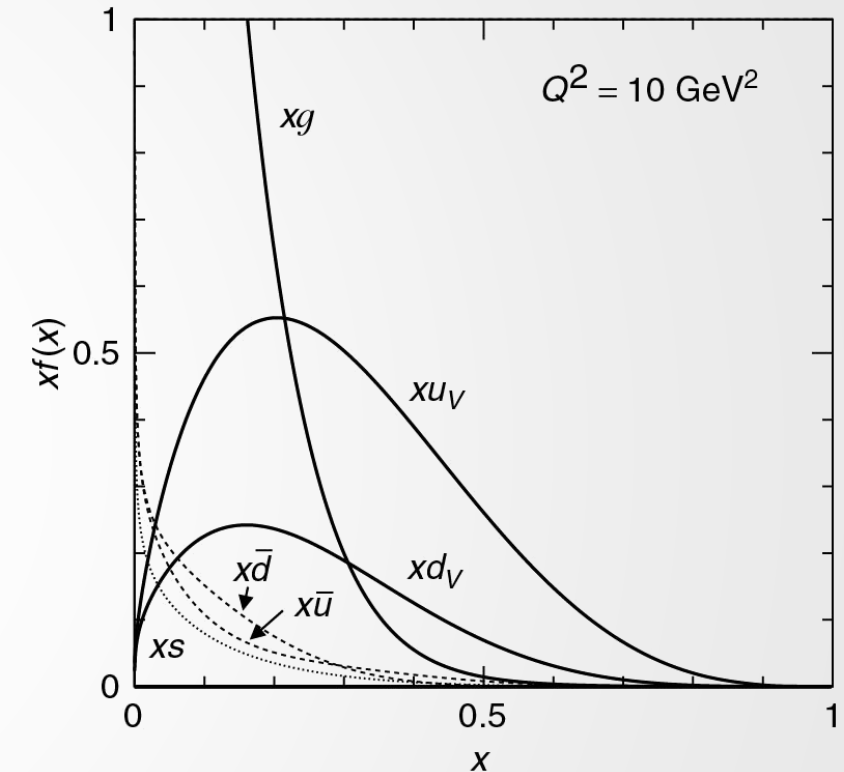
Generally, if $f_{a/h}(x)$ is the PDF of parton a
in hadron h, then the momentum sum rule is

$$\sum_{a \in q_i, \overline{q}_i, g} \int_0^1 dx\, x f_{a/h}(x) = 1$$

4.5.3* SCALING VIOLATION AND ASYMPTOTIC FREEDOM

- Bjorken scaling is the independence of F_2^{ep} from Q^2

  - only approximately satisfied (see p.24)

  - At high (low) x: structure function decreases (increases) with Q^2

  - At high Q^2: structure function shifted to lower x-values.

  - Interpretation: Higher probability to observe lower-x quark, because of valence quarks radiating gluons

  - Q^2 behavior calculable and serves as a powerful validation!

- low energetic phenomena $\Lambda \lesssim$ (few GeV) determine the proton structure and are not perturbatively calculable because $O(\alpha_s) \approx 1$.

  - However, the strong coupling α_s decreases with Q and at high energy perturbative calculations are possible (asymptotic freedom).

  - In order to make predictions for hadron-collisions, it is necessary to introduce a factorization scale μ_F that separates the long-distance hadron physics (PDF) from the short distance hard scatter matrix element.

Low x

Medium x

High x

Low Q^2

High Q^2

$F_2^{ep}(x, Q^2)$

$\ln Q^2$

$F_2^{ep}(x, Q^2)$

x

low Q^2

high Q^2

April 2016

$\alpha_s(Q^2)$

▼ τ decays (N³LO)
△ DIS jets (NLO)
□ Heavy Quarkonia (NLO)
○ e⁺e⁻ jets & shapes (res. NNLO)
● e.w. precision fits (N³LO)
▽ p$\bar{\text{p}}$ —> jets (NLO)
▼ pp —> tt (NNLO)

0.3

0.2

0.1

≡ QCD $\alpha_s(M_Z) = 0.1181 \pm 0.0011$

1   10   100   1000

Q [GeV]

5. HADRON-HADRON COLLISIONS
(AND THEIR INTERPLAY WITH DIS AND ELECTRON/POSITRON COLLISIONS)
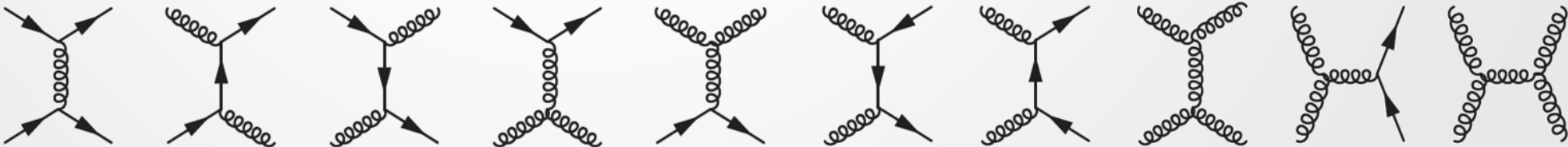
5.1 PROTON (ANTI)PROTON COLLISIONS
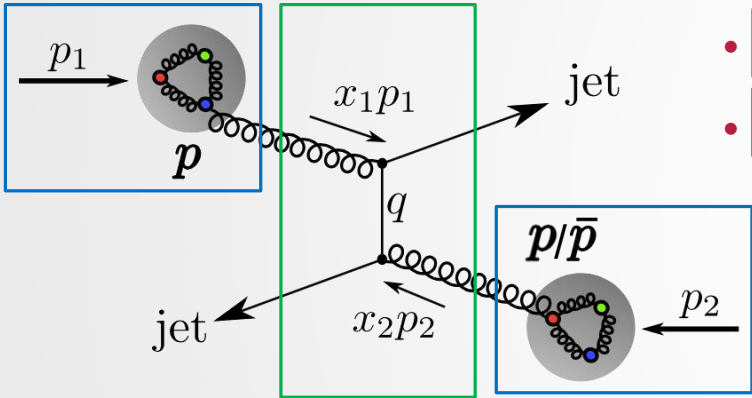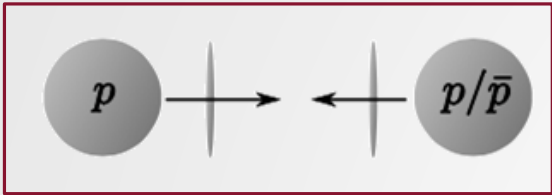
- The large gluon PDF is particularly important and can be measured in $p\bar{p}/p$ collisions

- Consider proton-antiproton collisions. Model as two colliding partons.

- Assume Factorization at a scale μ_F

  - hard (=high-energetic) parton process – perturbatively calculable in QCD

  - soft non-perturbative process-independent parton distribution functions

General formula relating hadron with parton-level cross sections:

$$\sigma = \sum_{a,b} \int \mathrm{d}x_1 \int \mathrm{d}x_2 f_{a/p}(x_1, \mu_F) f_{b/\bar{p}}(x_2, \mu_F) \hat{\sigma}^{ab}(x_1 p_1, x_2 p_2, \mu_F)$$

- Need to sum over all parton pairs and processes and integrate over momenta.

- Even simple hadron collider predictions necessitate sophisticated numerical tools.

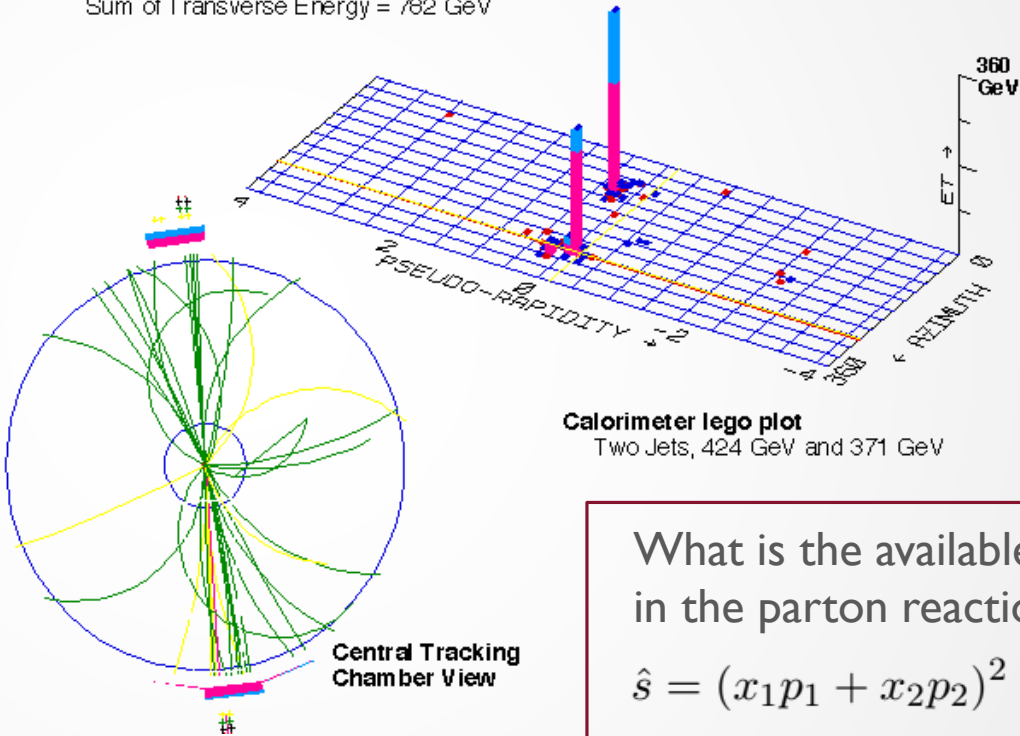leading order QCD processes for 'hard' dijet production in hadron-hadron collisions:

5.1 PROTON (ANTI)PROTON COLLISIONS

$p \quad + \quad \longrightarrow \quad \longleftarrow \quad + \quad p/\bar{p}$

$p_1 \rightarrow \quad p$

$x_1 p_1 \rightarrow \quad$ jet

q

jet $\quad x_2 p_2 \leftarrow \quad p/\bar{p} \quad p_2 \leftarrow$
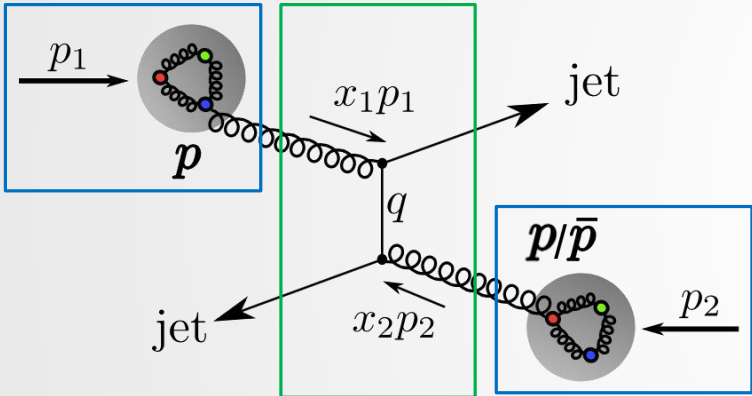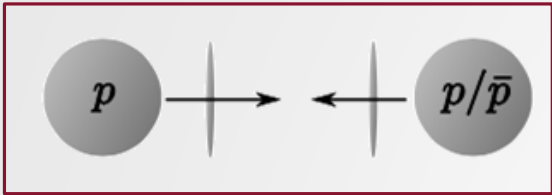
CDF: Highest Transverse Energy Event from the 1988-89 Collider Run

Sum of Transverse Energy = 782 GeV

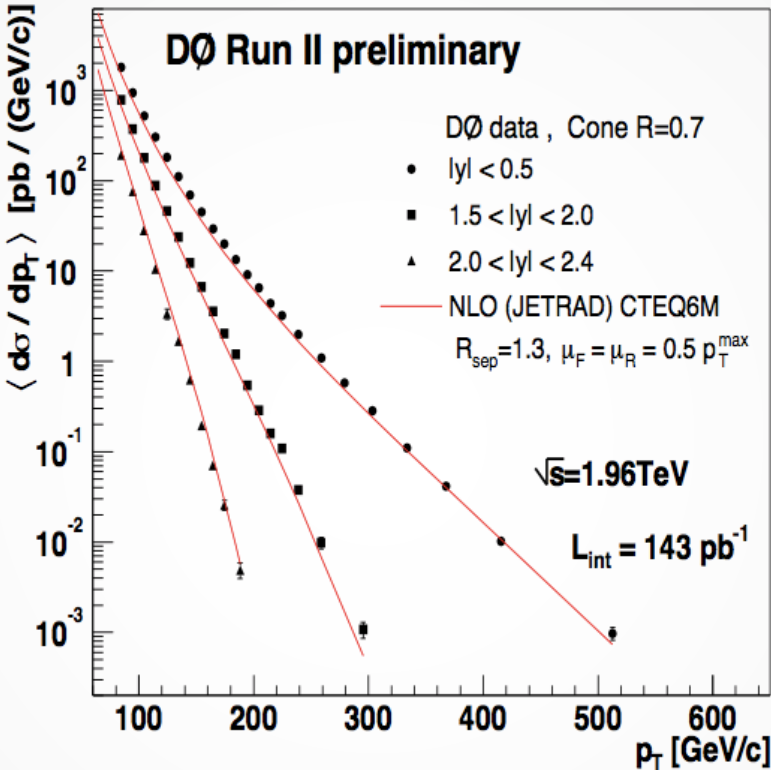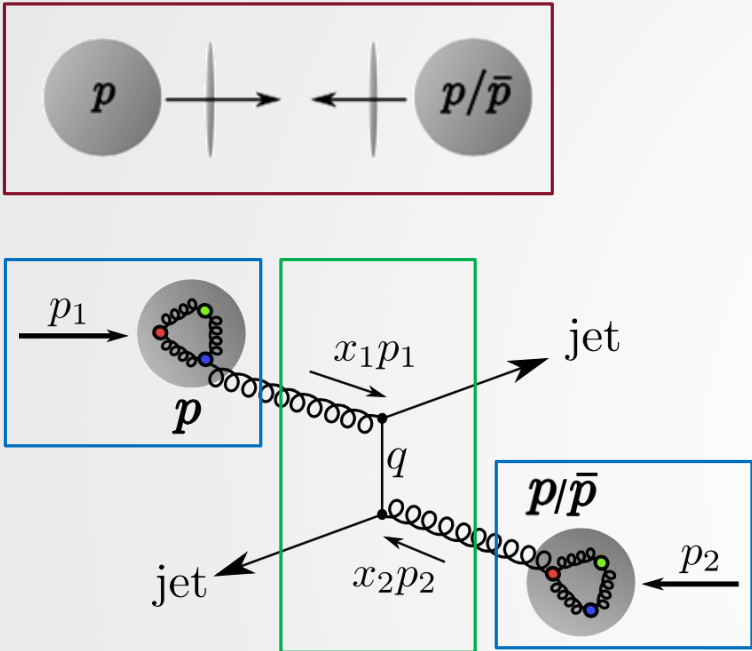Calorimeter lego plot
Two Jets, 424 GeV and 371 GeV

Central Tracking Chamber View

What is the available energy in the parton reaction?

$$\hat{s} = (x_1 p_1 + x_2 p_2)^2 \approx x_1 x_2 (2 p_1 p_2) = x_1 x_2 s$$

leading order QCD processes for 'hard' dijet production in hadron-hadron collisions:

5.1 PROTON (ANTI)PROTON COLLISIONS

D∅ Run II preliminary

D∅ data , Cone R=0.7
- $|y| < 0.5$
- $1.5 < |y| < 2.0$
- $2.0 < |y| < 2.4$

NLO (JETRAD) CTEQ6M
$R_{sep}=1.3$, $\mu_F = \mu_R = 0.5\ p_T^{max}$

$\sqrt{s}=1.96\,\text{TeV}$

$L_{int} = 143\ \text{pb}^{-1}$

$\langle d\sigma / dp_T \rangle$ [pb / (GeV/c)] p_T [GeV/c]
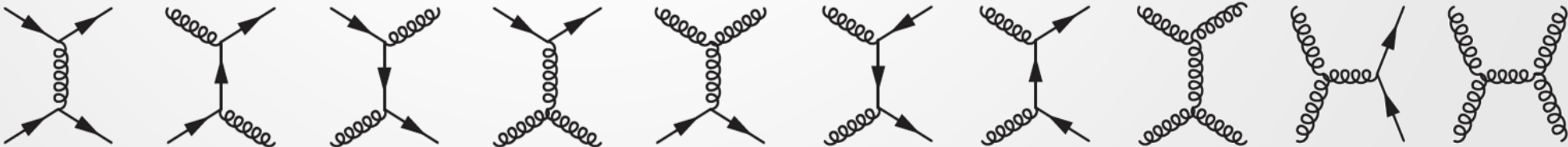
- quantitative prediction for multi-jet events: beautiful agreement!

- PDFs determined from global fit of 25 parameters to ~4000 data before the experiment.

- Contemporary PDFs differ in:
  - treatment of finite quark masses
  - treatment of b quark PDF
  - assumptions, e.g. $s(x)=\bar{s}(x)$
  - which part of the enourmous wealth of LHC jet data is used in the fit

- all PDF fits use various sum rules

- dominant uncertainty always: low-x gluon PDF

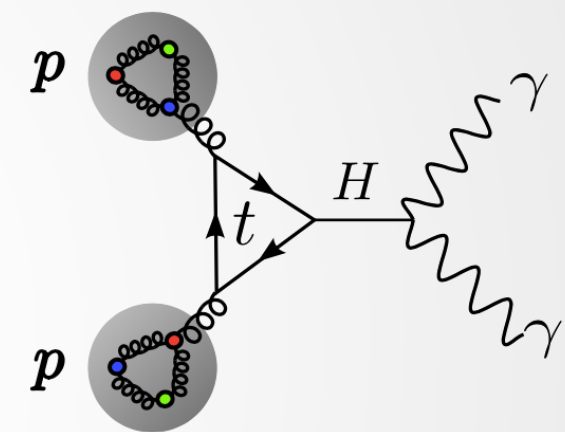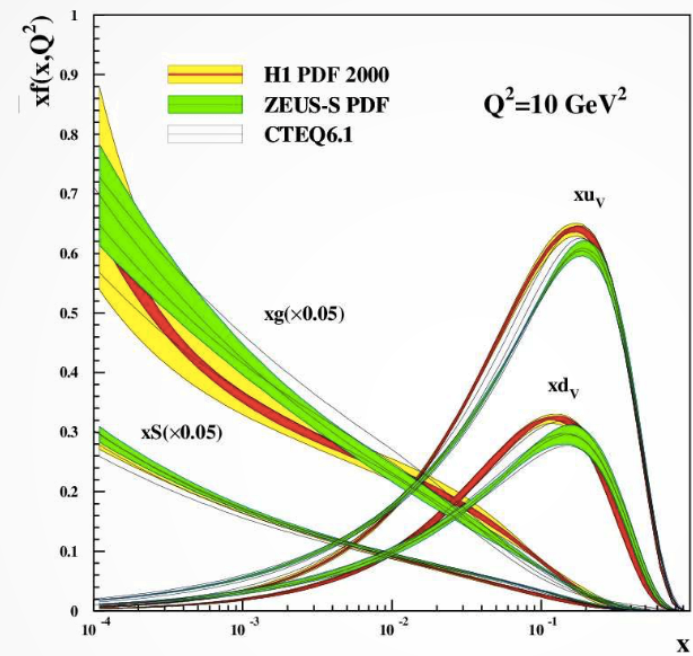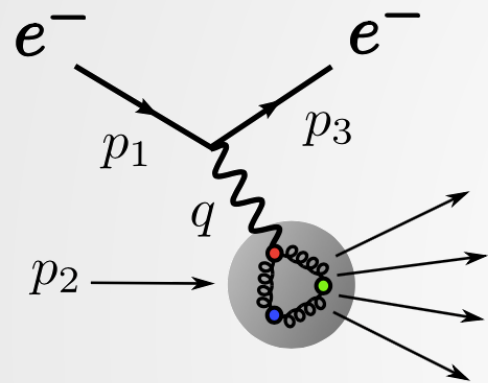leading order QCD processes for 'hard' dijet production in hadron-hadron collisions:
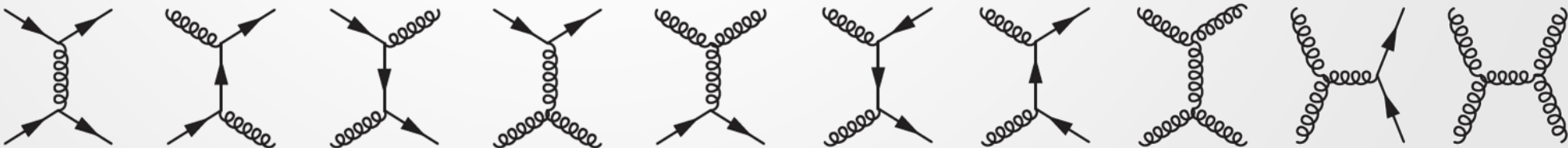
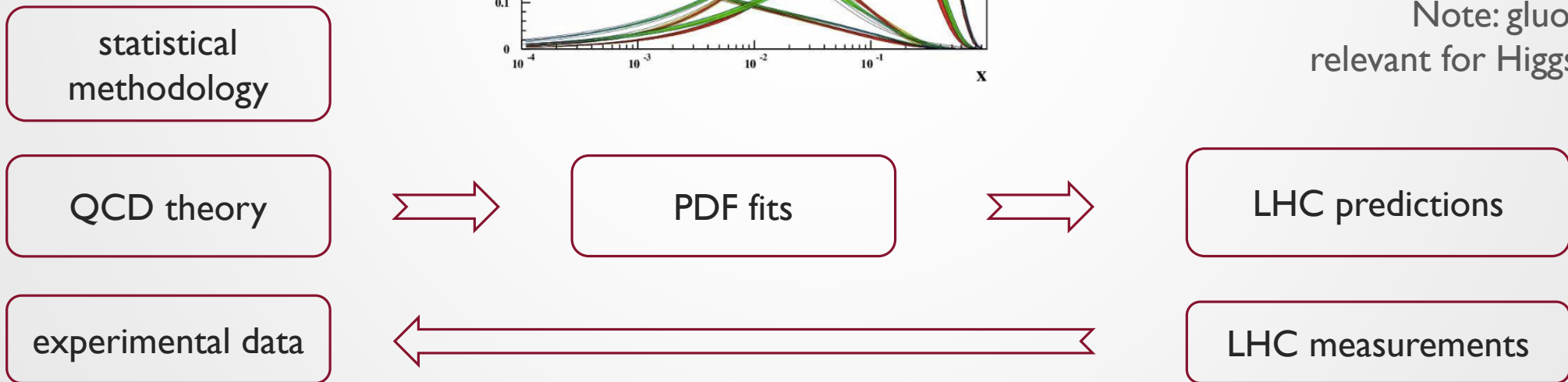# 5.2 TYPICAL INPUT DATA FOR PRE-LHC PDF FITS

- Fixed-target Deep-inelastic scattering

- Neutral current and charged current

- Collider Deep-inelastic scattering Jet

- Drell-Yan data from hadron colliders

Process	Subprocess	Partons	x range
$\ell^{\pm}\{p, n\} \to \ell^{\pm} X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^{\pm} n/p \to \ell^{\pm} X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+ \mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+ \mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^- \mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+ \mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^{\pm} p \to e^{\pm} X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+ \{d, s\} \to \{u, c\}$	d, s	$x \gtrsim 0.01$
$e^{\pm} p \to e^{\pm} c\bar{c} X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^{\pm} p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^{\pm} \to \ell^{\pm}\nu) X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+ \ell^-) X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

statistical
methodology

QCD theory ⟹ PDF fits ⟹ LHC predictions

experimental data ⟸ LHC measurements

Note: gluon PDF
relevant for Higgs production!

5.4 JET FORMATION

- Consider quark pair production in e.g. e$^{\pm}$ collisions.

- For high momentum transfer, the amplitudes are calculable. The process is described by the 1) matrix element.

- Subsequent radiation of the strongly interacting particles, however, lowers the energy scale, increasing α_s (asymptotic freedom).

- The strongly interacting parton thus 2) 'showers'. Parton showering is described by effective models, that are tuned to reproduce data.

- If the momenta falls below the QCD scale parameter ($\Lambda_{QCD} \approx 200$ MeV, where $\alpha_s \approx 1$) colorless hadrons are formed. This 3) hadronization is also described by effective models in e.g. the software package PYTHIA.

- The unstable hadrons subsequently 4) decay to (meta-)stable particles and are collectively observed as jets.

- At a lepton collider parton showering, hadronisation and hadron decay (2-4) can be measured with high accuracy. These measurements are crucial for hadron collider predictions.
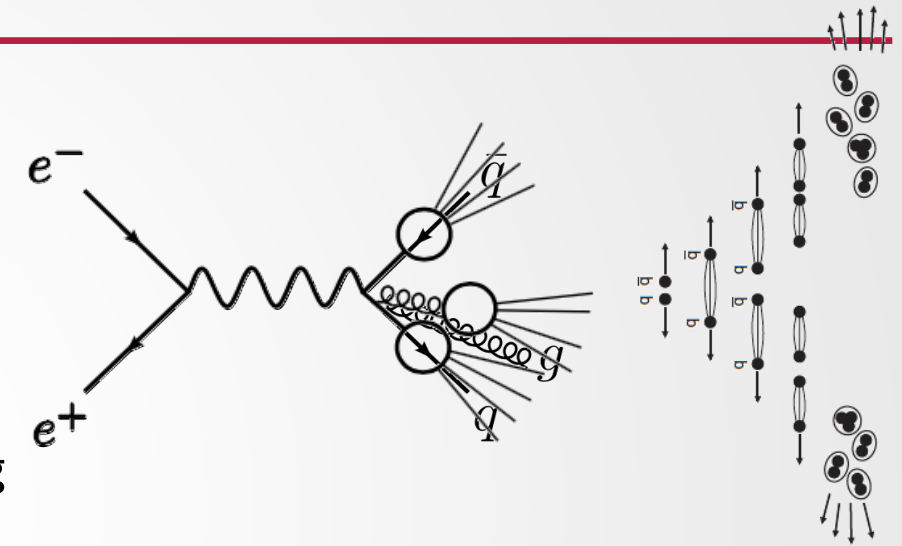
DELPHI (LEP) 4-jet event from 205 GeV run in 2000.
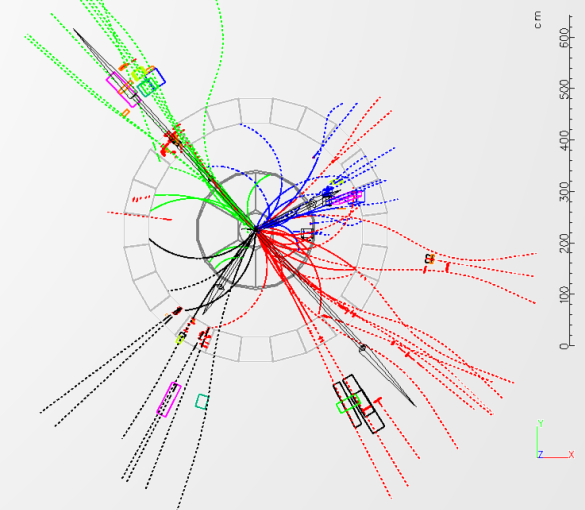
5.4 JET FORMATION

- Jet production at hadron colliders, both Tevatron and LHC, are an enormous success for QCD and the parton model.

- Inclusive spectra of jets are very well described.

Simulation of a 'typical' hard scatter event and the various stages of our current understanding.

CMS jet differential cross-section from first 7 TeV data
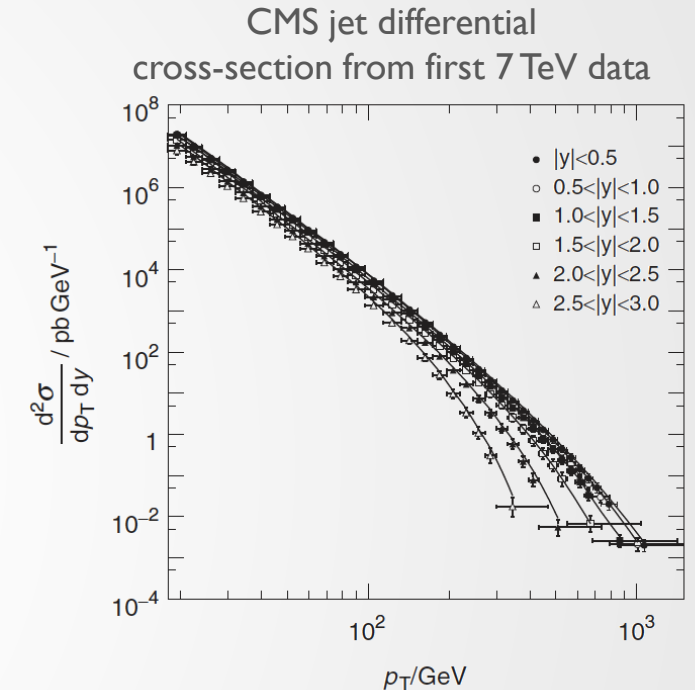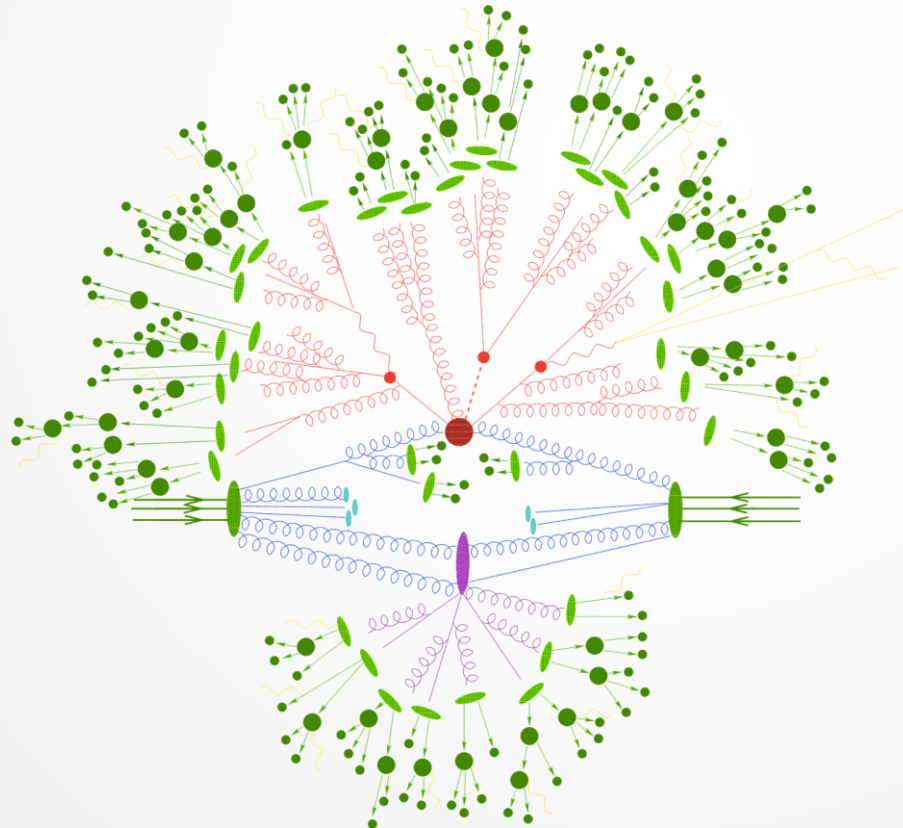
- (1) hard parton collision (calculable)
- decays (calculable) and bremsstrahlung , (2) "parton showers"
- secondary hard scattering

- (3) Parton-to-hadron transitions

- (4) hadron decays
  yellow lines signal soft photon

5.5 CROSS-SECTIONS AT HADRON COLLIDERS

- unit of cross section

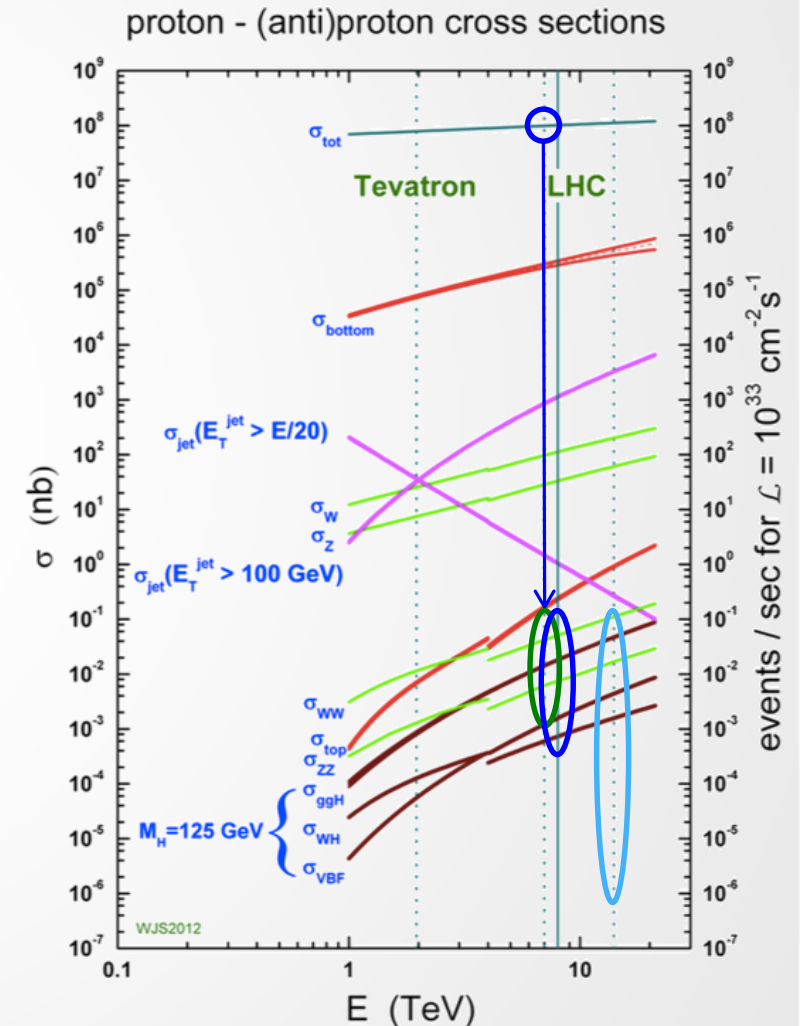 - 1b = 100 fm^2 = 10^{-24} cm^2. Typical values in nb-fb regime.

- unit of instantaneous luminosity 10^{34} – 10^{35} cm^{-2}s^{-1}

$$\frac{dN}{dt} = \mathcal{L}\sigma$$

$$N = \int_{[s]}^{[s]} dt\, \mathcal{L}(t) \cdot \sigma \cdot \epsilon(t)$$

cross-section
[nb]

number of produced events   Luminosity   efficiency
[1]   [nb^{-1} s^{-1}]   [1]
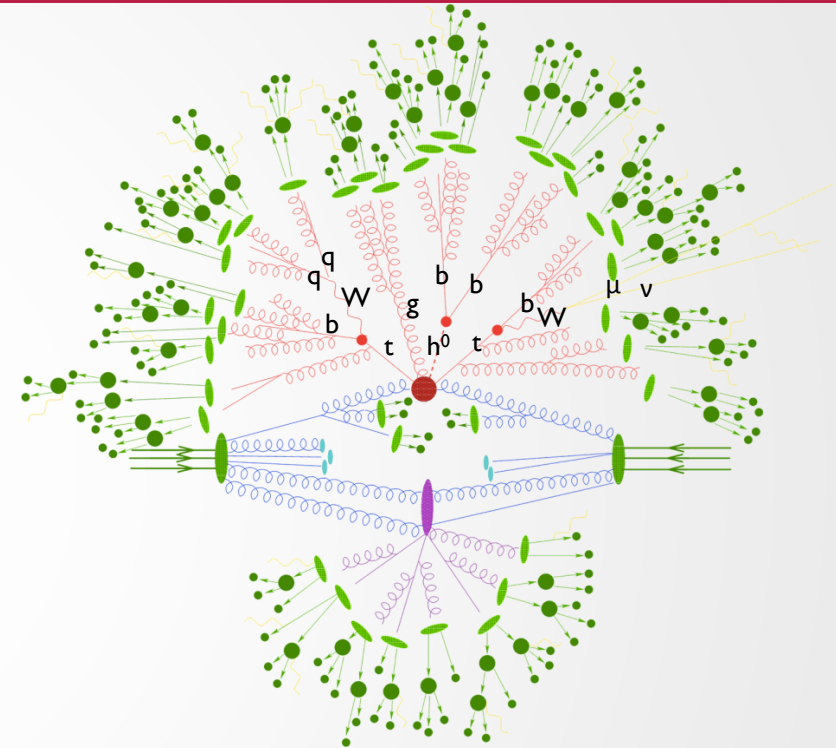
- $\sigma(hh)=\sum_{a,b}\iint dx_1 dx_2\, PDF_{a/h}(x_1)\, PDF_{b/h}(x_2)\sigma^{a,b}(x_1 P_1, x_2 P_2)$

  → All accessible parton reactions happen simultaneously!

- LHC rates for 13 TeV: ~ O(1/s) tt-pairs and O(10^2/s) Z bosons

proton - (anti)proton cross sections

5.6 A FULL HADRON-HADRON EVENT

- High energetic collisions occur at the interaction vertex
  - Heavy SM (or beyond-the-SM) particles cascade-decay
  - The interaction of secondary particles with the detector material is governed by the SM at low energies.
- At the 'energy frontier' t/W/Z/H serve as sensitive probes to the SM. These particles can be most easily identified in their leptonic decay modes.
  - BR(t → Wb) ≈ 100%
  - BR(W→lν) ≈ 11% x 3, BR(W→qq) ≈ 67%
  - BR(Z→ll) ≈ 3.4% x 3, BR(Z→$\nu\,\nu$)≈20%, BR(Z→qq)≈ 70%
  - BR(H→bb)=58%, BR(H→ZZ)=2.6%, BR(H→$\gamma\gamma$)=0.2%
- Backgrounds (of course depending on the chosen signal)
  - QCD dijet production σ(QCD)/σ(tt) ≈ 10^{10},
  - electroweak production of Z/W bosons
  - top quark pairs, …

- What are rare signatures in the SM?
  1. high missing energy
  2. same charge dileptons
  3. high jet or lepton multiplicity
  4. signatures with displaced vertices
  5. …

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THE END!

* HEAVY-ION COLLISIONS

- In Heavy Ion collisions (LHC: PbPb, RHIC: AuAu), the ions in collision partially overlap.

- For these non-central collisions with finite impact parameter, the shape of the interaction region is elliptical.

- Collisions of proton pairs, however, individually produce symmetric distributions in the transverse plane.

- But if the reacting particles undergo multiple interactions, the momentum distributions become non-symmetric, expressed by non-vanishing Fourier coefficients w.r.t the reaction plane:

$$v_n(p_t, y) = \langle \cos(n(\varphi - \Psi_R)) \rangle_p$$

- The 2nd harmonic, v_2, is called elliptical flow is sensitive to the mean free path and indicates whether multiple collisions are present.
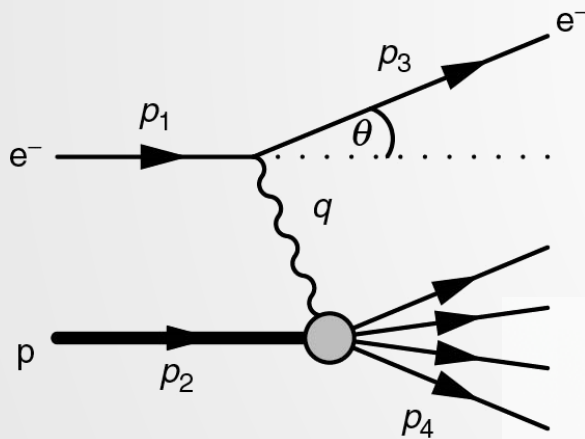
- Prominent signatures of a Quark Gluon Plasma are found in a) dilepton spectra (unaffected probes of a medium), b) jet quenching (the suppression of one of the jets of a pair which travels through the medium), and c) the suppression of J/Ψ, interpreted as the dissolution of the c$\bar{c}$ bound state in the medium.
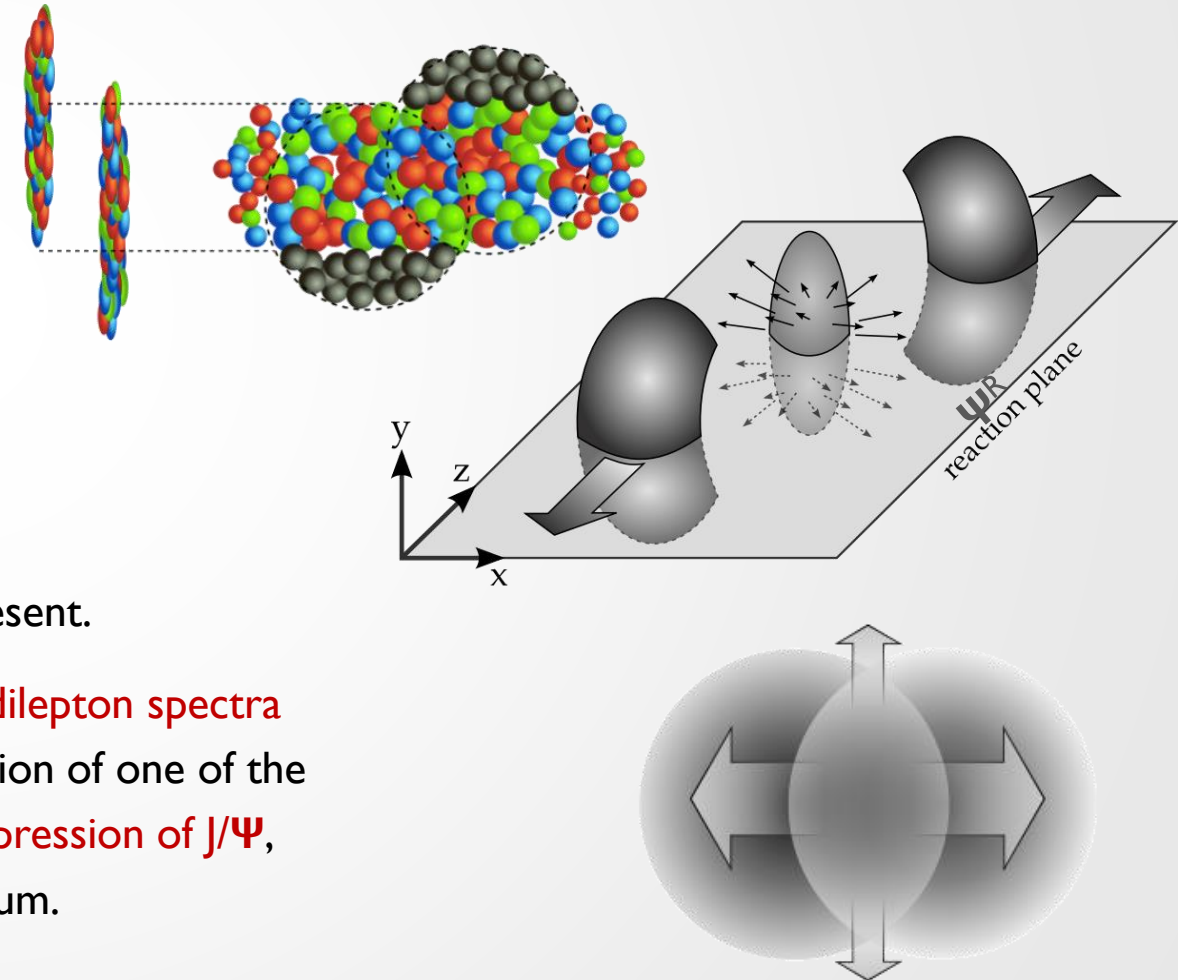
DEEP INELASTIC SCATTERING
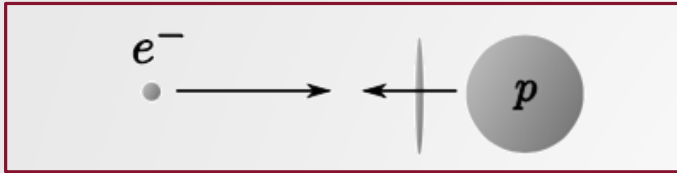
kinematics of deep inelastic scattering

Note: Bjorken-x is a kinematic degree of freedom not present in elastic scattering!

Two more L.I. variables, y and ν, appear in the literature:

- The L.I. invariant definition $y \equiv \dfrac{p_2 \cdot q}{p_2 \cdot p_1}$ can be easily calculated in the proton rest-frame as $y = \dfrac{m_p(E_1 - E_3)}{m_p E_1} = 1 - \dfrac{E_3}{E_1}$ and thus describes the fractional energy loss of the electron. Sometimes, the absolute energy loss ν of the electron is used which is defined as $\nu \equiv \dfrac{p_2 \cdot q}{m_p} = E_1 - E_3$.

- For a given $\sqrt{s}$, two quantities out of Q^2, x, y and ν are independent.

- Indeed, from $s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = 2p_1 \cdot p_2 + m_p^2 + m_e^2$ we get $2p_1 \cdot p_2 \simeq s - m_p^2$ and thus

$$y = \left(\frac{2m_p}{s - m_p^2} \right) \nu \quad \text{and} \quad Q^2 = (s - m_p^2)xy$$