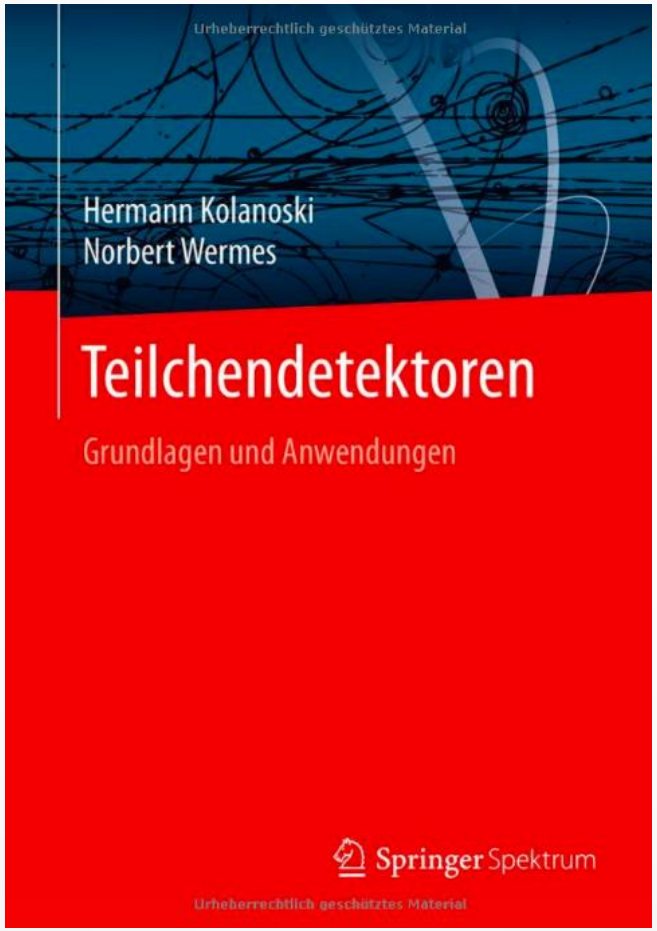
INTERACTION OF PARTICLES WITH MATTER

LECTURE 3

REFERENCES

- Vorlesung M. Krammer "Teilchendetektoren" TU Wien, SS 2015
- Kolanoski, Wermes "Teilchendetektoren", 2015
- Kleinknecht, "Detektoren für Teilchenstrahlung"
- Erika Garutti, Lecture notes, "The Physics of Particle Detectors", DESY, SS 2012



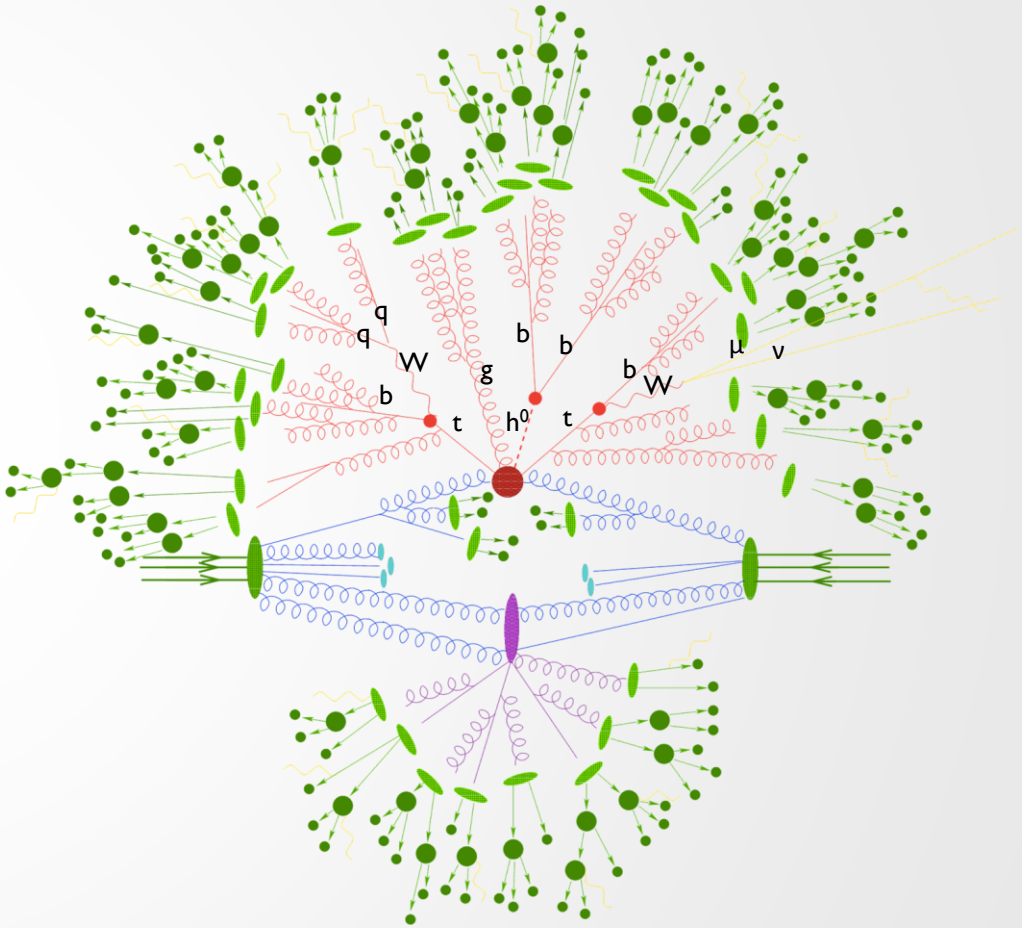
CONTENT

1. INTRODUCTION

1.1 HADRON COLLIDER ENVIRONMENT

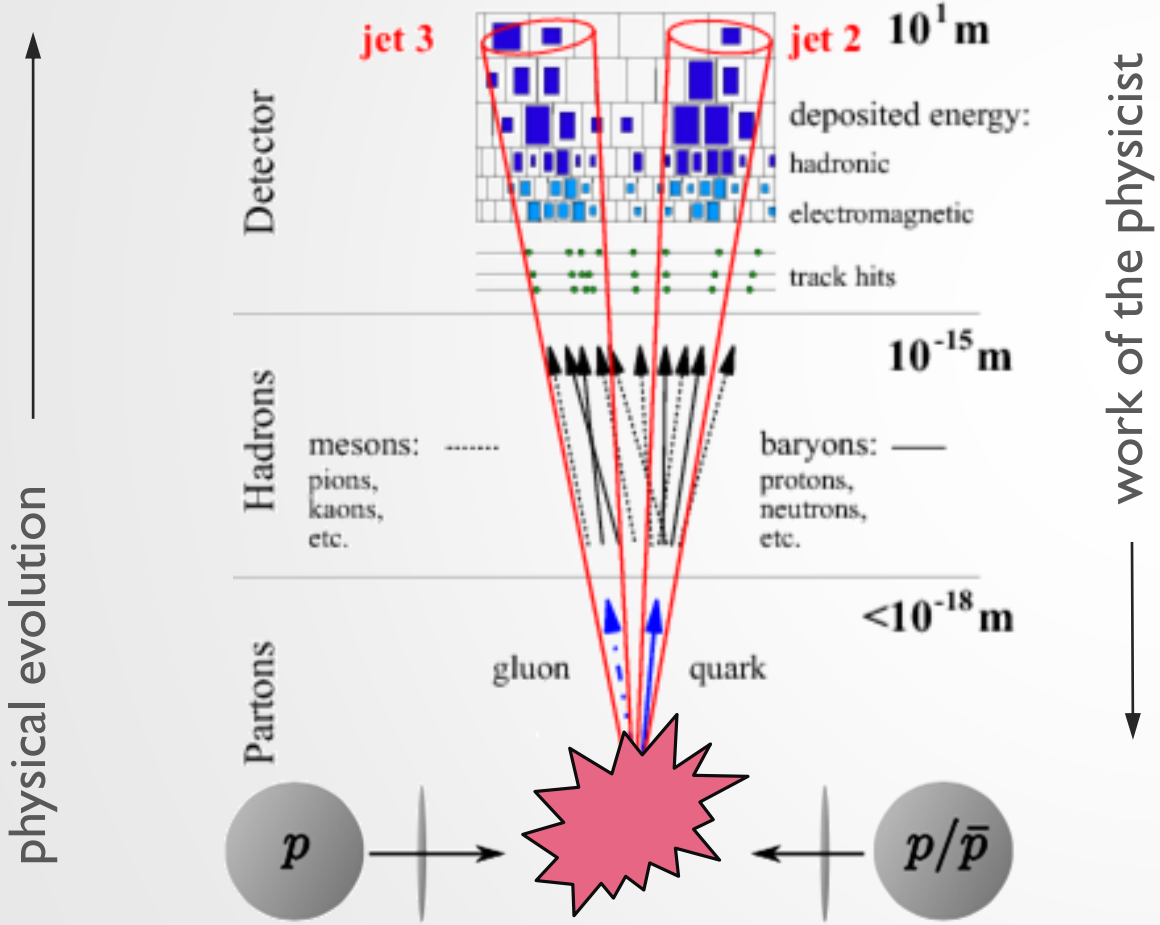
http://cds.cern.ch/record/2194545/files/MultiJetEventDisplay.png

- High energetic collisions occur at the interaction vertex
  - Heavy SM (or beyond-the-SM) particles cascade-decay
  - The interaction of secondary particles with the detector material is governed by the SM at low energies.
- Many aspects of detector technology and conceptual design are governed by the need to isolate leptons (e/μ) in the dense environment of the rest of the event.
  - E.g. leptons from e.g. the decay of W/Z bosons can be isolated and very energetic (~100 GeV)
- QCD multijet production and hadronic decay modes of t/W/Z/h produce highly energetic strongly interacting particles (quarks, gluons) that shower and 'hadronize', producing jets (σ(QCD)/σ(tt) $\approx 10^{10}$) of relatively low energetic particles.
  - At hadron colliders, simultaneous collisions happening at the same time ('pile-up) are always significant. LHC in 2016: <PU>≈15-40!
- Need to find the isolated, energetic particles in a very dense environment of low energetic background

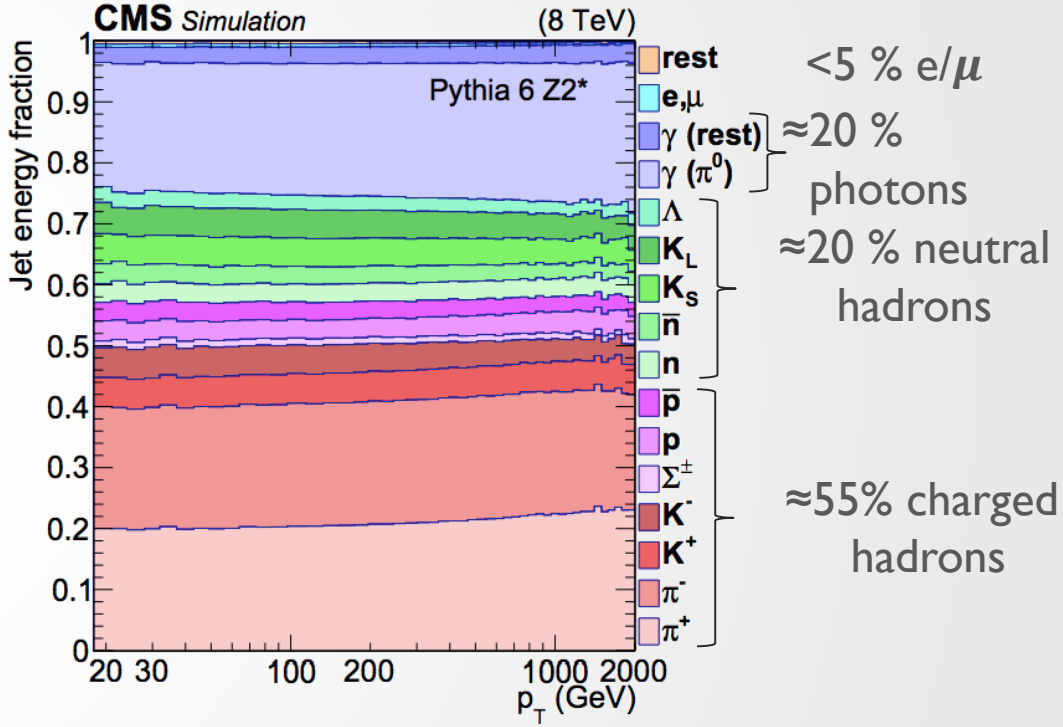


1.2 JET CONSTITUENTS

arXiv: 1506.03239, arXiv:1607.03663



simulated particle content of a hadronic jet



1. The average jet composition does not depend on E
2. The interaction of the decay products with the detector is governed by Q, E (p), m and τ

⟶ enormous simplification for data analysis

1.3 A FEW OF THE KNOWN SM PARTICLES

- Many hundreds of particles are known
- Good experimental understanding of decay branching ratios

Baryon Summary Table

This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the baryons with 3- or 4-star status are included in the Baryon Summary Table. Due to insufficient data or uncertain interpretation, the other entries in the table are not established baryons. The names with masses are of baryons that decay strongly. The spin-parity J^P (when known) is given with each particle. For the strongly decaying particles, the J^P values are considered to be part of the names.

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****	Λ_c^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$		*	$\Lambda_c(2765)^+$		*
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$		***	$\Lambda_c(2880)^+$	$5/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2^-$	***	$\Lambda_c(2940)^+$		***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$		***	$\Sigma_c(2455)$	$1/2^+$	****
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}?$	***	$\Sigma_c(2520)$	$3/2^+$	***
$N(1685)$		*	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$		*	$\Sigma_c(2800)$		***
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$		**	Ξ_c^+	$1/2^+$	***
$N(1710)$	$1/2^+$	***	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$		**	Ξ_c^0	$1/2^+$	***
$N(1720)$	$3/2^+$	****	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1750)$	$1/2^-$	***	$\Xi(2500)$		*	$\Xi_c^{\prime +}$	$1/2^+$	***
$N(1860)$	$5/2^+$	**	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1770)$	$1/2^+$	*				$\Xi_c^{\prime 0}$	$1/2^+$	***
$N(1875)$	$3/2^-$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1775)$	$5/2^-$	****	Ω^-	$3/2^+$	****	$\Xi_c(2645)$	$3/2^+$	***
$N(1880)$	$1/2^+$	**	$\Delta(2200)$	$7/2^-$	*	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2250)^-$		***	$\Xi_c(2790)$	$1/2^-$	***
$N(1895)$	$1/2^-$	**	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2380)^-$		**	$\Xi_c(2815)$	$3/2^-$	***
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1915)$	$5/2^+$	****	$\Omega(2470)^-$		**	$\Xi_c(2930)$		*
$N(1990)$	$7/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(1940)$	$3/2^-$	***				$\Xi_c(2980)$		***
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(2000)$	$1/2^-$	*				$\Xi_c(3055)$		**
$N(2040)$	$3/2^+$	*	$\Delta(2420)$	$11/2^+$	****	$\Sigma(2030)$	$7/2^+$	****				$\Xi_c(3080)$		***
$N(2060)$	$5/2^-$	**	$\Delta(2750)$	$13/2^-$	**	$\Sigma(2070)$	$5/2^+$	*				$\Xi_c(3123)$		*
$N(2100)$	$1/2^+$	*	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2080)$	$3/2^+$	**				Ω_c^0	$1/2^+$	***
$N(2120)$	$3/2^-$	**				$\Sigma(2100)$	$7/2^-$	*				$\Omega_c(2770)^0$	$3/2^+$	***
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2250)$		***				Ξ_{cc}^+		*
$N(2220)$	$9/2^+$	****	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2455)$		**						
$N(2250)$	$9/2^-$	****	$\Lambda(1520)$	$3/2^-$	****	$\Sigma(2620)$		**				Λ_b^0	$1/2^+$	***
$N(2300)$	$1/2^+$	**	$\Lambda(1600)$	$1/2^+$	***	$\Sigma(3000)$		*				$\Lambda_b(5912)^0$	$1/2^-$	***
$N(2570)$	$5/2^-$	**	$\Lambda(1670)$	$1/2^-$	****	$\Sigma(3170)$		*				$\Lambda_b(5920)^0$	$3/2^-$	***
$N(2600)$	$11/2^-$	***	$\Lambda(1690)$	$3/2^-$	****							Σ_b	$1/2^+$	***
$N(2700)$	$13/2^+$	**	$\Lambda(1800)$	$1/2^-$	***							Σ_b^*	$3/2^+$	***
			$\Lambda(1810)$	$1/2^+$	***							Ξ_b^0, Ξ_b^-	$1/2^+$	***
			$\Lambda(1820)$	$5/2^+$	****							$\Xi_b(5945)^0$	$3/2^+$	***
			$\Lambda(1830)$	$5/2^-$	****							Ω_b^-	$1/2^+$	***
			$\Lambda(1890)$	$3/2^+$	****									
			$\Lambda(2000)$		*									
			$\Lambda(2020)$	$7/2^+$	*									
			$\Lambda(2100)$	$7/2^-$	****									
			$\Lambda(2110)$	$5/2^+$	***									
			$\Lambda(2325)$	$3/2^-$	*									
			$\Lambda(2350)$	$9/2^+$	***									
			$\Lambda(2585)$		**									

Meson Summary Table

See also the table of suggested $q\overline{q}$ quark-model assignments in the Quark Model section.

• Indicates particles that appear in the preceding Meson Summary Table. We do not regard the other entries as being established.

LIGHT UNFLAVORED ($S = C = B = 0$)				STRANGE ($S = \pm1$, $C = B = 0$)		CHARMED, STRANGE ($C = S = \pm1$)		$c\overline{c}$	
	$I^G(J^{PC})$		$I^G(J^{PC})$		$I(J^P)$		$I(J^P)$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$	• $K^\pm$	$1/2(0^-)$	• $D_s^\pm$	$0(0^-)$	• $\eta_c(1S)$	$0^+(0^{-+})$
• π^0	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$	• K^0	$1/2(0^-)$	• $D_s^{*\pm}$	$0(?^?)$	• $J/\psi(1S)$	$0^-(1^{--})$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K_S^0	$1/2(0^-)$	• $D_{s0}^*(2317)^\pm$	$0(0^+)$	• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $f_0(500)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$	• K_L^0	$1/2(0^-)$	• $D_{s1}(2460)^\pm$	$0(1^+)$	• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $\rho(770)$	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$	$K_0^*(800)$	$1/2(0^+)$	• $D_{s1}(2536)^\pm$	$0(1^+)$	• $h_c(1P)$	$?^?(1^{+-})$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K^*(892)$	$1/2(1^-)$	• $D_{s2}(2573)$	$0(?^?)$	• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$	• $K_1(1270)$	$1/2(1^+)$	• $D_{s1}^*(2700)^\pm$	$0(1^-)$	• $\eta_c(2S)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1400)$	$1/2(1^+)$	$D_{sJ}^*(2860)^\pm$	$0(?^?)$	• $\psi(2S)$	$0^-(1^{--})$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K^*(1410)$	$1/2(1^-)$	$D_{sJ}(3040)^\pm$	$0(?^?)$	• $\psi(3770)$	$0^-(1^{--})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$	• $K_0^*(1430)$	$1/2(0^+)$			$X(3872)$	$0^+(1^{++})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K_2^*(1430)$	$1/2(2^+)$			• $\chi_{c0}(2P)$	$0^+(0^{++})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$	$K(1460)$	$1/2(0^-)$			• $\chi_{c2}(2P)$	$0^+(2^{++})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$	$K_2(1580)$	$1/2(2^-)$			$X(3940)$	$?^?(?^{??})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$	$K(1630)$	$1/2(?^?)$			• $\psi(4040)$	$0^-(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$	$K_1(1650)$	$1/2(1^+)$			$X(4050)^\pm$	$?(?^?)$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$	• $K^*(1680)$	$1/2(1^-)$			$X(4140)$	$0^+(?^{?+})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$	• $K_2(1770)$	$1/2(2^-)$			• $\psi(4160)$	$0^-(1^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$	• $K_3^*(1780)$	$1/2(3^-)$			$X(4160)$	$?^?(?^{??})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$	• $K_2(1820)$	$1/2(2^-)$			$X(4250)^\pm$	$?(?^?)$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$	$K(1830)$	$1/2(0^-)$			• $X(4260)$	$?^?(1^{--})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$	$K_0^*(1950)$	$1/2(0^+)$			$X(4350)$	$0^+(?^{?+})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$	$K_2^*(1980)$	$1/2(2^+)$			• $X(4360)$	$?^?(1^{--})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2100)$	$0^+(0^{++})$	• $K_4^*(2045)$	$1/2(4^+)$			• $\psi(4415)$	$0^-(1^{--})$
• $\omega(1420)$	$0^-(1^{--})$	$f_2(2150)$	$0^+(2^{++})$	$K_2(2250)$	$1/2(2^-)$			$X(4430)^\pm$	$?(?^?)$
$f_2(1430)$	$0^+(2^{++})$	$\rho(2150)$	$1^+(1^{--})$	$K_3(2320)$	$1/2(3^+)$			• $X(4660)$	$?^?(1^{--})$
• $a_0(1450)$	$1^-(0^{++})$	• $\phi(2170)$	$0^-(1^{--})$	$K_5^*(2380)$	$1/2(5^-)$				
• $\rho(1450)$	$1^+(1^{--})$	$f_0(2200)$	$0^+(0^{++})$	$K_4(2500)$	$1/2(4^-)$				
• $\eta(1475)$	$0^+(0^{-+})$	$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$	$K(3100)$	$?^?(?^{??})$				
• $f_0(1500)$	$0^+(0^{++})$	$\eta(2225)$	$0^+(0^{-+})$						
$f_1(1510)$	$0^+(1^{++})$	$\rho_3(2250)$	$1^+(3^{--})$						
• $f_2'(1525)$	$0^+(2^{++})$	• $f_2(2300)$	$0^+(2^{++})$						
$f_2(1565)$	$0^+(2^{++})$	$f_4(2300)$	$0^+(4^{++})$						
$\rho(1570)$	$1^+(1^{--})$	$f_0(2330)$	$0^+(0^{++})$						
$h_1(1595)$	$0^-(1^{+-})$	• $f_2(2340)$	$0^+(2^{++})$						
• $\pi_1(1600)$	$1^-(1^{-+})$	$\rho_5(2350)$	$1^+(5^{--})$						
$a_1(1640)$	$1^-(1^{++})$	$a_6(2450)$	$1^-(6^{++})$						
$f_2(1640)$	$0^+(2^{++})$	$f_6(2510)$	$0^+(6^{++})$						
• $\eta_2(1645)$	$0^+(2^{-+})$								
• $\omega(1650)$	$0^-(1^{--})$								
• $\omega_3(1670)$	$0^-(3^{--})$								

OTHER LIGHT	
Further States	

CHARMED ($C = \pm1$)	
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

BOTTOM ($B = \pm1$)	
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$
• $B_1(5721)^0$	$1/2(1^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)	
• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)	
• $B_c^\pm$	$0(0^-)$

$b\overline{b}$	
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_b(3P)$	$?^?(?^{?+})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$?^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

1.4 WHEN CAN WE DETECT A PARTICLE?

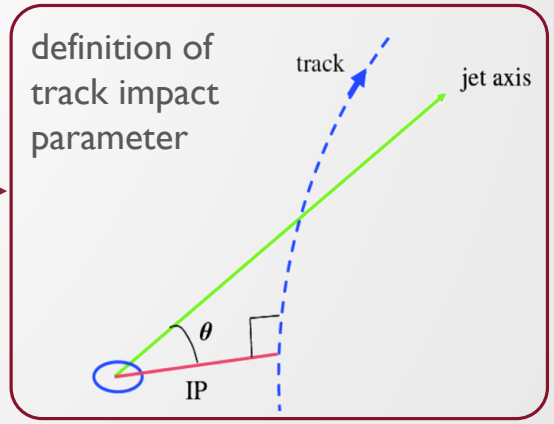
- Particles can only be measured, if they

A. live long enough after creation to reach the detector

- The majority of particle states are short lived. From the hundreds of particles only few particles (and their antiparticles) have track lengths long enough to measure them:

	γ	p	n	$e^{\pm}$	$\mu^{\pm}$	$\pi^{\pm}$	$K^{\pm}$	K_0 (K_S/K_L)
τ_0	∞	∞	∞	∞	2.2 μs	26 ns	12 ns	89 ps / 51 ns
l_{track} (p=1GeV)	∞	∞	∞	∞	6.1 km	5.5 m	6.4 m	5 cm / 27.5 m

- Track length: $L_{track} = v\tau = c\beta\gamma\tau_0$ where τ_0 being the lifetime at rest.
- small but finite life-time: B-mesons have ps lifetimes
  $\longrightarrow$ sub-mm impact parameter (IP) resolution allows tagging of b-quarks!



B. interact with the detector

- deposition of energy (dE/dx), transferred into a detector signal
- neutrinos interact only weakly: need large detector volumes (e.g. IceCube) or giant fluxes (reactor experiments)

1.5 OVERVIEW OF INTERACTIONS IN THE DETECTOR

- Different type of interactions for charged and neutral particles
- Difference "scale" of processes:
  - atomic scale for the electromagnetic and nuclear scale for the strong interactions

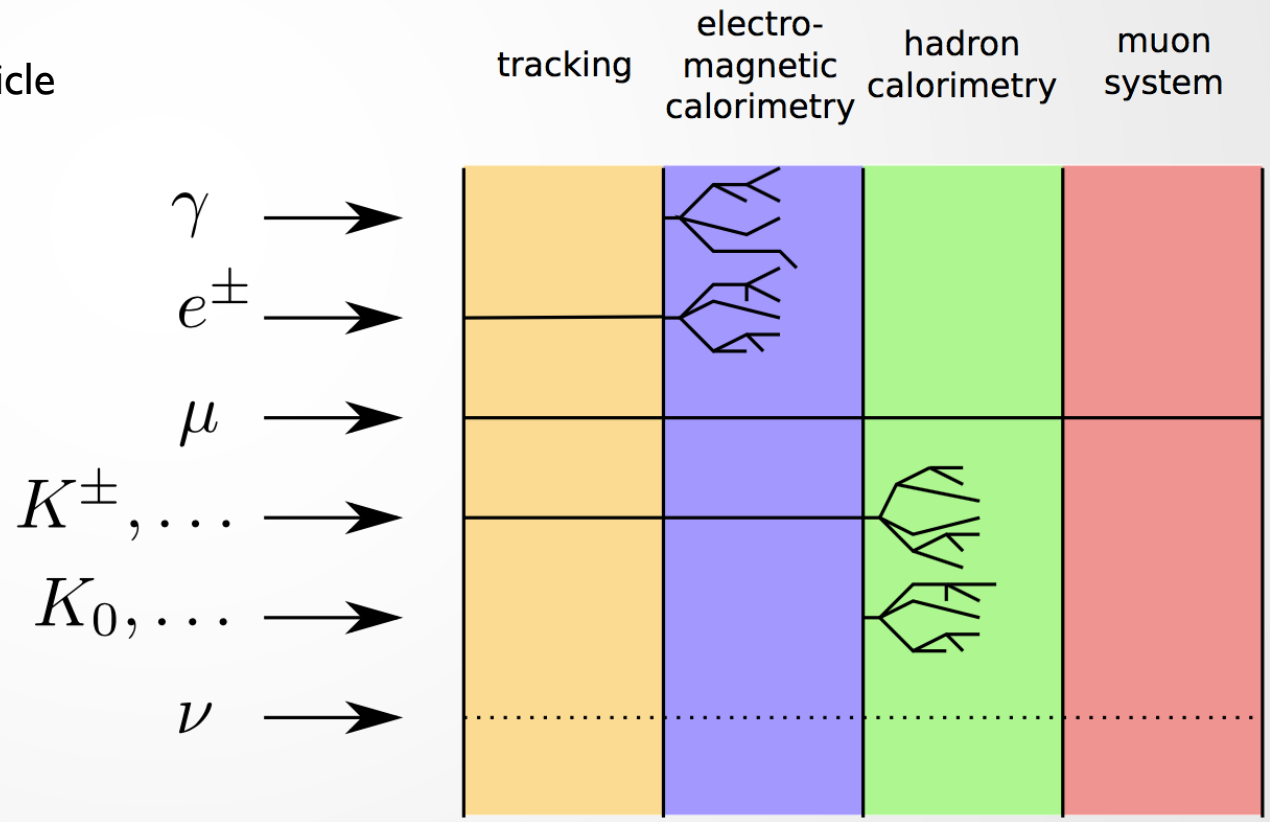
1. Detection of charged particles
   - Ionization, Bremsstrahlung, Cherenkov …
2. Detection of γ-rays
   - Photo/Compton effect, pair production
3. Detection of neutrons
   - strong interaction
4. Detection of neutrinos
   - weak interaction

- In the following, a phenomenological treatment is given with a focus on the implications for detector design!

1.6 DETECTOR DESIGN CRITERIA

1. little interaction with the measured particle
   - tracking detectors should trace the passage of a charged particle without disturbing it
2. high efficiency
   - probability of detection in case of a signal particle
3. high purity (high signal-to-noise ratio)
   - low probability of instrumental noise and unintended signal
4. high resolution
   - spatial, time, energy, momentum, angle, …
5. fast signal processing
6. simple maintenance and detector control
7. radiation hardness
8. low cost

- there is no detector that satisfies 1.-8.
- a well established compromise with existing technology:

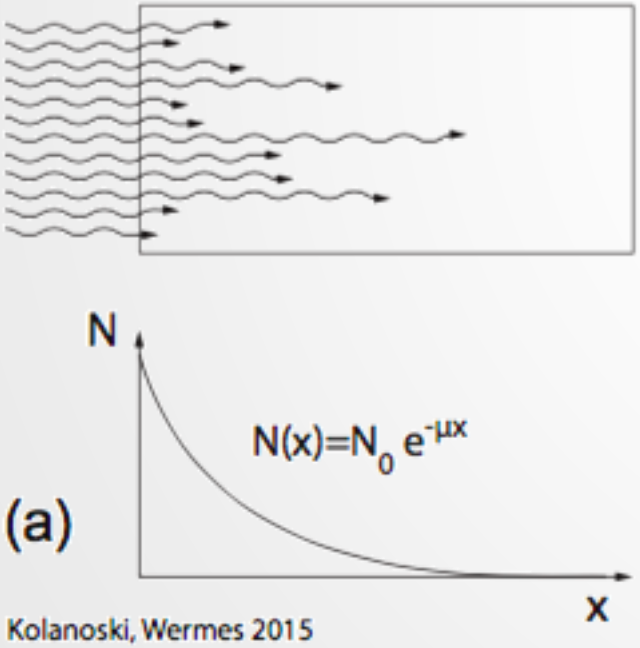


1.7 ABSORPTION OF PARTICLES AND RAYS

- Particles can either be a) consumed by a single reaction or b) continuously loose energy until stopped

a) Particle loss by absorption (dN/dx, typical for photons) leads to an exponential decay of particle intensity

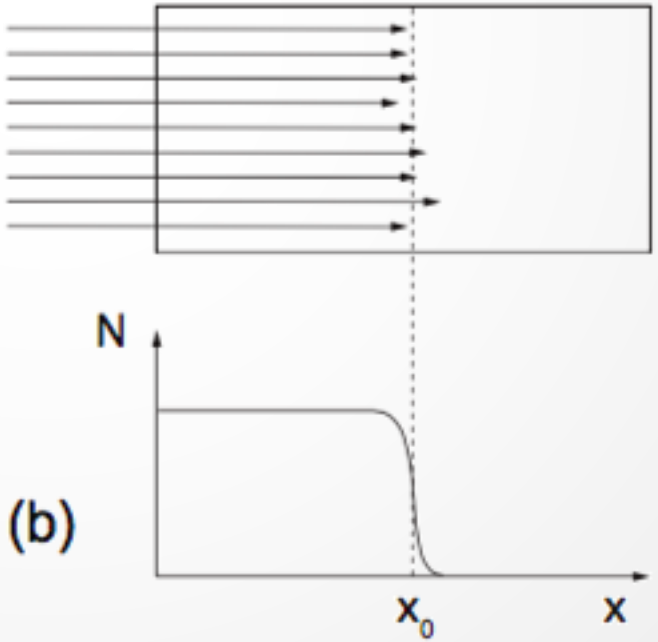
Photons



Kolanoski, Wermes 2015

b) Energy loss through matter for heavy charged particles (dE/dx) results in a limited area with a strong drop in intensity

Charged particles



Alpha particles, 1926

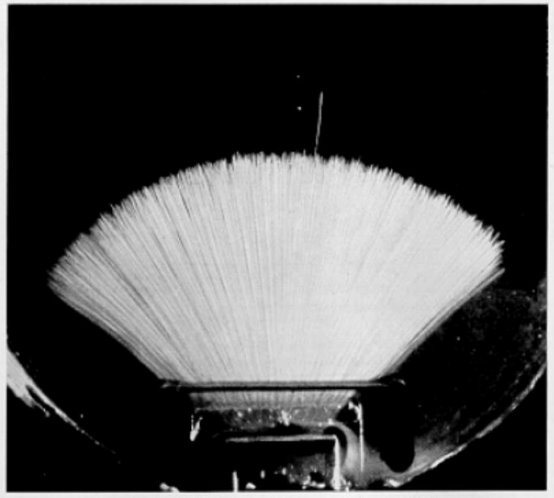


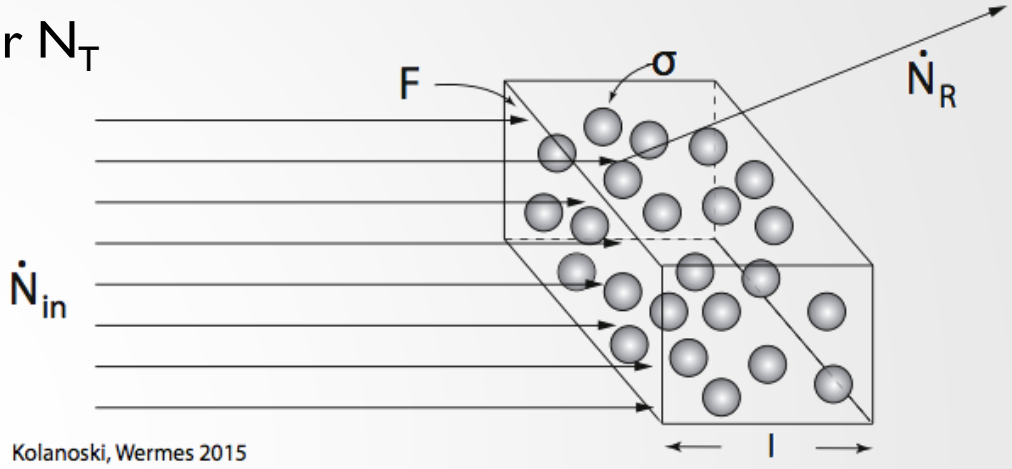
Fig. 13. K. Philipp, Naturwiss. 14, 1203 (1926).

1.7 ABSORPTION OF PARTICLES AND RAYS

- Consider a thin detector volume $V = F \cdot l$ with a number N_T particles calculated as $N_T = \frac{\rho V}{M_T} N_A$. The density **ρ** is given in g cm^{-3}, the relative atomic mass M_T in mol^{-1}, and $N_A = 6.022 \cdot 10^{23} \mathrm{mol}^{-1}$ is Avogadro's constant.
- The target density is $n = \frac{N_T}{V} = \frac{\rho}{A} N_A$ where A is the nuclear number.
- The beam sees a total cross section of $N_T\boldsymbol{\sigma}$ such that the probability of a collision becomes

$$w = \frac{\dot{N}_R}{\dot{N}_{in}} = \frac{N_T \sigma}{F} = n\sigma l$$

- The cross-section can be calculated as $\sigma = \frac{\dot{N}_R}{\dot{N}_{in}} \frac{1}{nl}$



- In a thick volume we have the differential equation $\frac{\mathrm{d}N}{N} = -n\sigma \mathrm{d}x$

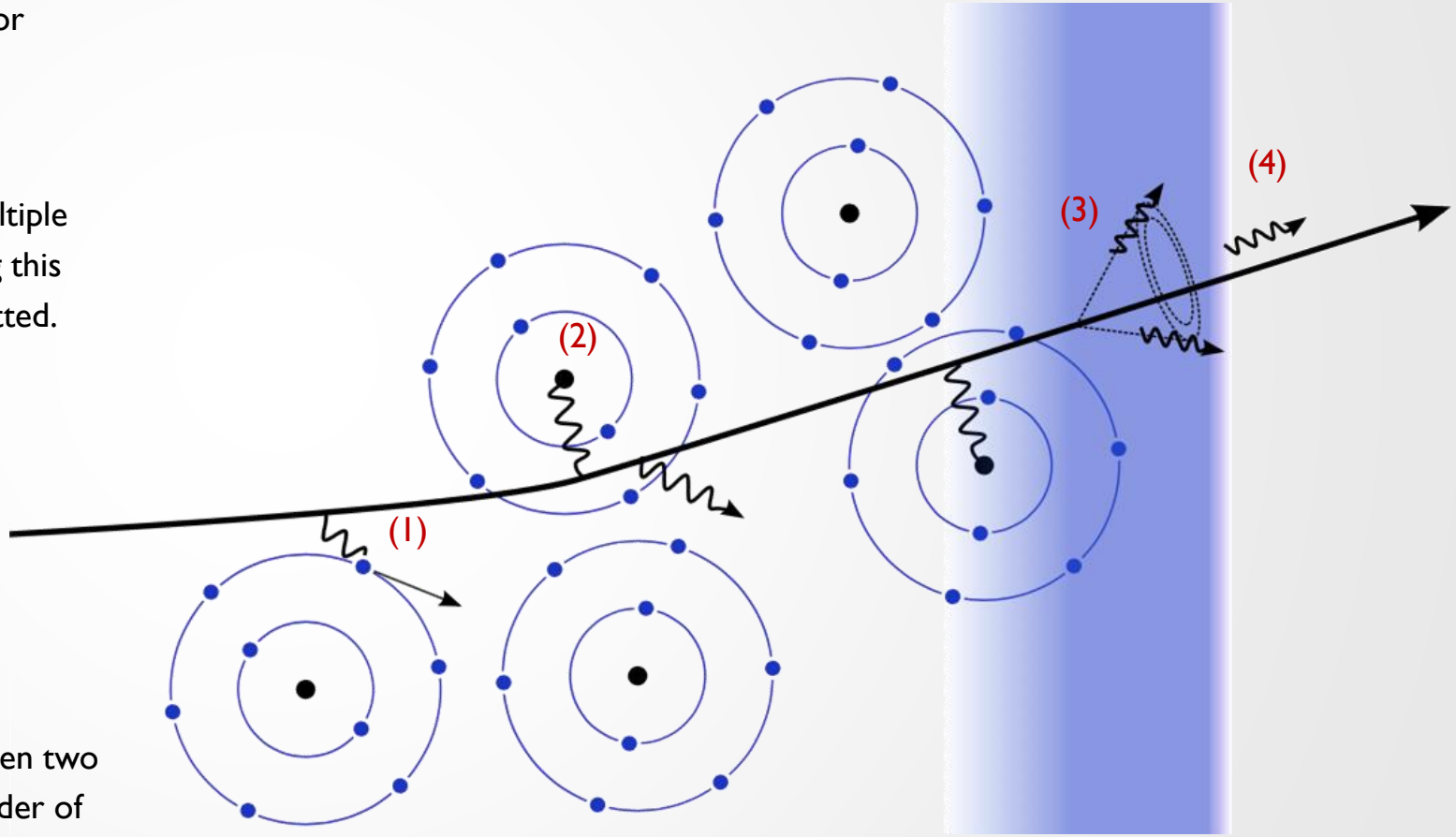
  with solution $N(x) = N_0 e^{-\mu x}$ (Beer-Lambert) and $\lambda = \mu^{-1} = (n\sigma)^{-1}$ is the mean free path.
- For cross-sections we have ($r_0 \approx 1.2$fm, $a_0 = 0.5$Å)

$$\sigma_{\mathrm{nucl}} \approx \pi r_0^2 A^{2/3} \approx 45\mathrm{mb} \cdot A^{2/3}$$
$$\sigma_{\mathrm{atom}} \approx \pi a_0^2 \approx 10^8 \mathrm{b}$$

2. INTERACTION OF CHARGED PARTICLES WITH MATTER

2.1 OVERVIEW

1. Ionization (or excitation) of detector material
   - The incoming particle loses energy by ionizing or exciting the atoms
2. Interaction with the nucleus
   - The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a bremsstrahlung photon can be emitted.
3. Emission of Cherenkov light
   - In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation.
4. Emission of transition radiation
   - When the particle crosses the boundary between two refractive media, there is a probability of the order of ≈1% to produced and X ray photon, called Transition radiation.



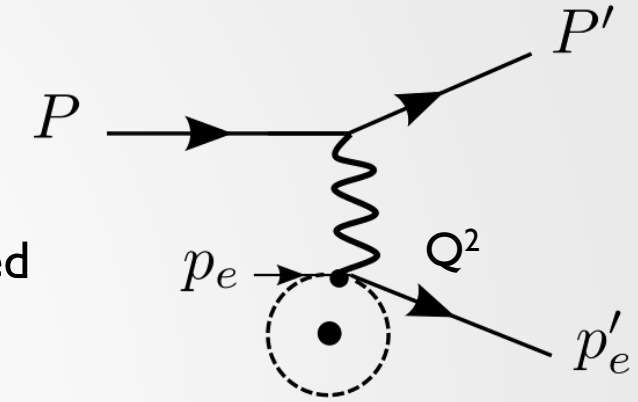
2.2 MAXIMUM ENERGY TRANSFER

- Simple rearrangements of energy momentum conservation

$$P + p_e = P' + p'_e \qquad P \cdot p'_e = P' \cdot p_e$$

$$(P - p'_e)^2 = (P' - p_e)^2 \qquad E E'_e - |\vec{P}||\vec{p}_e{}'|c^2 \cos\theta = E' m_e c^2$$

lead to $\cos\theta = \dfrac{E E'_e - E' m_e c^2}{c^2 |\vec{P}||\vec{p}_e{}'|}$ where cos $\boldsymbol{\theta}$ is the angle of the scattered electron with respect to the direction of the heavy charged particle.



- From the definition of the kinetic energy $T = E'_e - m_e c^2$ it follows that $E = E' + T$.

We express the electron momentum by using the kin. energy as

$$E_e'^{\,2} = c^2 \vec{p}_e{}'^2 + m_e^2 c^4$$

$$c^2 \vec{p}_e{}'^2 = E_e'^{\,2} - m_e^2 c^4 = T^2 + 2 m_e c^2 T$$

$$c|\vec{p}_e{}'| = \sqrt{T^2 + 2 m_e c^2 T}$$

- It follows that

$$\cos\theta = \frac{\gamma M c^2 (T + m_e c^2) - (E - T) m_e c^2}{c^2 \beta \gamma M \sqrt{T^2 + 2 m_e c^2 T}} = \frac{T\left(\gamma + \frac{m_e}{M}\right)}{\beta\gamma\sqrt{T^2 + 2 m_e c^2 T}}$$

or $T(\theta) = \dfrac{2 m_e c^2 \beta^2 \gamma^2 \cos^2\theta}{\gamma^2 (1 - \beta^2 \cos^2\theta) + 2\gamma \frac{m_e}{M} + \frac{m_e^2}{M^2}}$ with $T_{\max} = \dfrac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M} + \frac{m_e^2}{M^2}} \approx \begin{cases} 2\, m_e\, c^2\, (\beta\gamma)^2 & \gamma m_e \ll M \\ \gamma\, M\, c^2 = E & \gamma \to \infty \\ m_e c^2 (\gamma - 1) = E - m_e c^2 & M = m_e\,. \end{cases}$

Maximum energy transfer if all momenta aligned, cos $\boldsymbol{\theta}$ = 1!

2.3 ENERGY LOSS BY IONIZATION

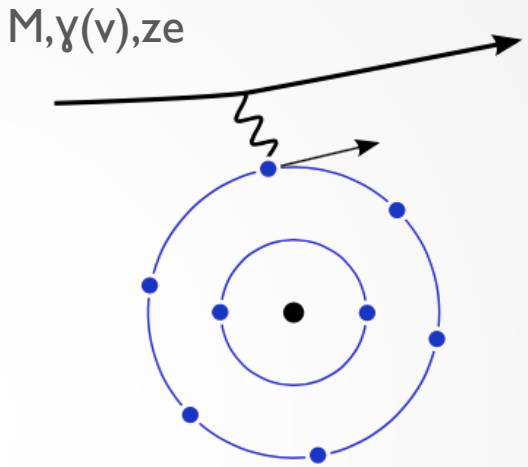
- The energy loss by ionization can be described by QM: Rutherford scattering in the electron rest frame (single photon exchange)
- In the following, however, we give Bohr's classical result
  - Particle with charge ze, mass M and velocity v moves through a medium with electron density n_e.
  - Electrons considered free and initially at rest.
  - The momentum transferred to a single electron is:

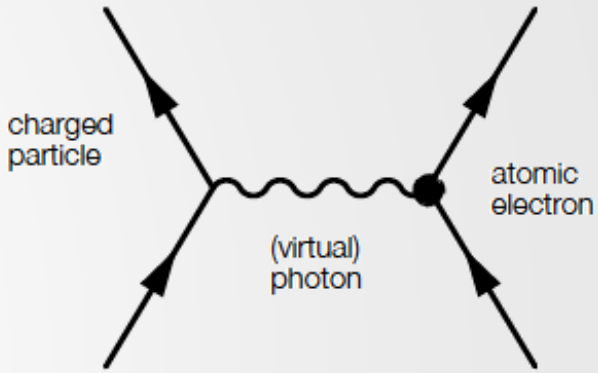
$$\Delta p_\perp = \int F_\perp \mathrm{d}t = \int F_\perp \frac{\mathrm{d}t}{\mathrm{d}x}\mathrm{d}x = \int F_\perp \frac{\mathrm{d}x}{v}\,, \qquad \Delta p_\parallel = 0$$

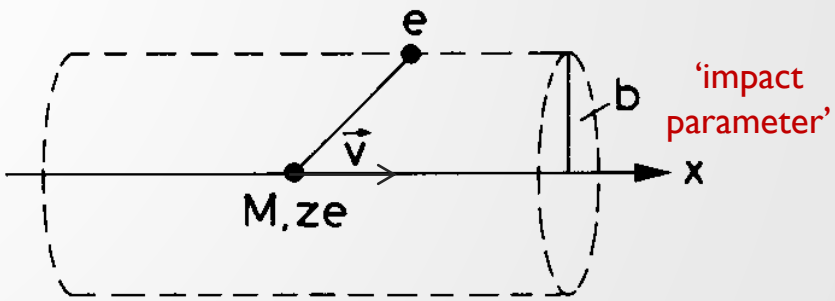
  - Furthermore $F_\perp = eE_\perp$ and we can therefore use Gauss Law (SI units)

$$\int E_\perp (2\pi b)\mathrm{d}x = \frac{ze}{\varepsilon_0} \;\rightarrow\; \int E_\perp \mathrm{d}x = \frac{ze}{2\pi\varepsilon_0 b}$$

  - transferred momentum: $\Delta p_\perp = e\int E_\perp \frac{\mathrm{d}x}{v} = \frac{ze^2}{2\pi\varepsilon_0 b v}$ and energy: $\Delta E(b) = \frac{\Delta p^2}{2m_e}$

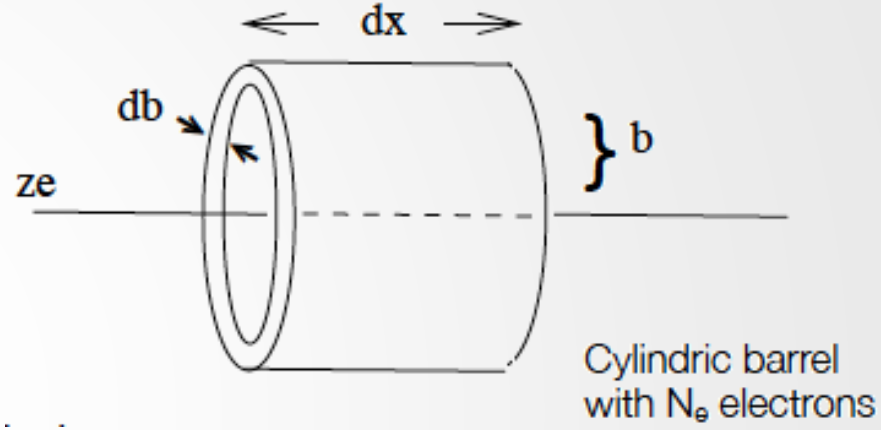






2.3 ENERGY LOSS BY IONIZATION

- Next, we integrate the loss over the electron density
- For N electrons distributed on a barrel: $N = n_e \cdot (2\pi b) \cdot db \; dx$
- Energy loss per path length dx for distance between b and b + db in medium with electron density n_e:



$$-\mathrm{d}E(b) = \frac{\Delta p^2}{2m_e} 2\pi n_e b \, \mathrm{d}b \, \mathrm{d}x = \frac{n_e z^2 e^4}{4\pi\varepsilon_0^2 m_e v^2} \frac{\mathrm{d}b}{b} \mathrm{d}x$$

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{n_e z^2 e^4}{4\pi\varepsilon_0^2 m_e v^2} \int_{b_{\min}}^{b_{\max}} \frac{\mathrm{d}b}{b} = \frac{4\pi n_e z^2 r_e^2 m_e c^2}{\beta^2} \ln \frac{b_{\max}}{b_{\min}}$$

- Here, we simplified the factors with the 'classical electron radius' $\frac{e^2}{4\pi\varepsilon_0 r_e} = m_e c^2$.
- b is the 'impact parameter' in a single-particle interaction.
- For a finite stopping power we need to determine the relevant range of interactions $[b_{min}, b_{max}]$

2.3.1 CLASSICAL RESULT

- From conservation of energy it follows (see 2.2) that the maximum energy transfer happens for a 'head-on' collision and is given by $T_{\max} = 2m_e c^2 \beta^2 \gamma^2$ (valid for $\gamma\, m_e \ll M$)
- For the maximal energy transfer: $T_{\max}(b_{\min}) = \frac{\Delta p^2}{2m_e} = \frac{(ze^2)^2}{8\pi^2 \varepsilon_0^2 m_e v^2 b_{\min}^2} = \frac{2r_e^2 m_e c^2 z^2}{\beta^2 b_{\min}^2} \propto \frac{1}{b_{\min}^2}$
- Insert in the logarithm of impact parameters: $\ln \frac{b_{\max}}{b_{\min}} = \frac{1}{2} \ln \frac{b_{\max}^2}{b_{\min}^2} = \frac{1}{2} \ln \frac{T_{\max}}{T_{\min}} = \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2}{I}$
- What about $T_{min}(b_{max})$? Classically, the energy transfer to an electron can become arbitrarily small. However, below a minimal material-dependent ionization threshold I, there are only discrete energy levels. Moreover, if the movement of the electron during the interaction is important, interference effects need to be taken into account that further reduce the interaction to negligible levels.
- Substituting also the electron density $n_e = Z \frac{\rho}{A} N_A$ we find:

A ≡ atomic mass in g/mol

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \underbrace{4\pi N_A r_e^2 m_e c^2}_{K = 0.307\,\mathrm{MeV\,cm^2\,mol^{-1}}} \rho \frac{1}{\beta^2} \frac{Z z^2}{A} \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2}{I}$$

2.3.2 BETHE-BLOCH EQUATION

- The QM treatment based on Rutherford scattering gives

$$-\left\langle \frac{dE}{dx} \right\rangle = K \frac{Z}{A} \rho \frac{z^2}{\beta^2} \left[\frac{1}{2} \ln \frac{2\, m_e\, c^2\, \beta^2\, \gamma^2\, T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(\beta\gamma,\, I)}{Z} \right]$$

- fundamental constants
  - $K = 4\pi N_A r_e^2 m_e\, c^2 = 0.307 \text{ MeV } \text{ cm}^2/\text{mol}$
  - $r_e = e^2/4\pi\epsilon_0 m_e c^2 \approx 2.8\,\text{fm}$ (the classical radius of the electron)
- properties of the incident particle
  - z, $\boldsymbol{\beta}$, γ are the charge (in units of e), the velocity, and the relativistic γ factor
- properties of the target material
  - Z atomic number, A the atomic mass in g/mol
  - I is the average energy required to ionize the medium
  - T_{max} is the maximum energy in a head-on collision
  - $\boldsymbol{\delta}$ is the density correction, relevant for high $\boldsymbol{\beta\gamma}$
  - C/Z is an additional 'shell' correction for low values of $\boldsymbol{\beta}$

Shell corrections

The properties of the EM field in the medium govern the density correction

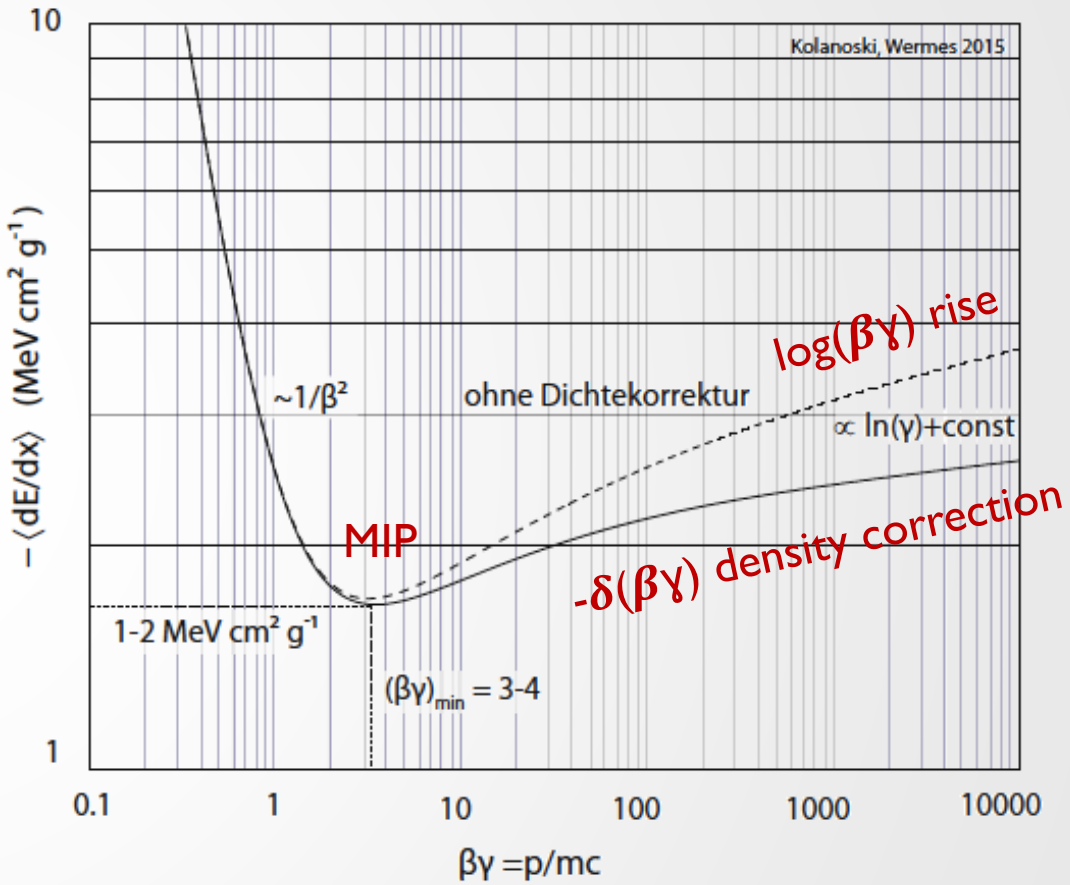
Spin-1/2 interactions (Mott scattering) instead of scalar Rutherford scattering is responsible for the $\boldsymbol{\beta}^2$ term.

These are the important dependencies:

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle \approx K \frac{Z}{\beta^2} \ln(\text{const} \cdot \beta\gamma)$$

2.3.2 BETHE-BLOCH EQUATION

- Often, dE/dx is divided by ρ and then given in units MeV cm^2 g^{-1}.
- The Bethe-Bloch equation, with its various corrections, holds in a wide range of applications for charged particles heavier than electrons.
- It describes the 'stopping power' of matter on particles.
- The plot shows $\pi^{\pm}$ on silicon. At low energies, the β^{-2} term dominates, at high energies the ln($\beta\gamma$).
- There is a broad minimum at $\beta\gamma$ = 3-3.5 or β=0.95.
  - 'Minimum Ionizing particles' (MIP)



- Ionization is the universal detection principle of semi-conducting sensors, gaseous detectors, etcetcetc!

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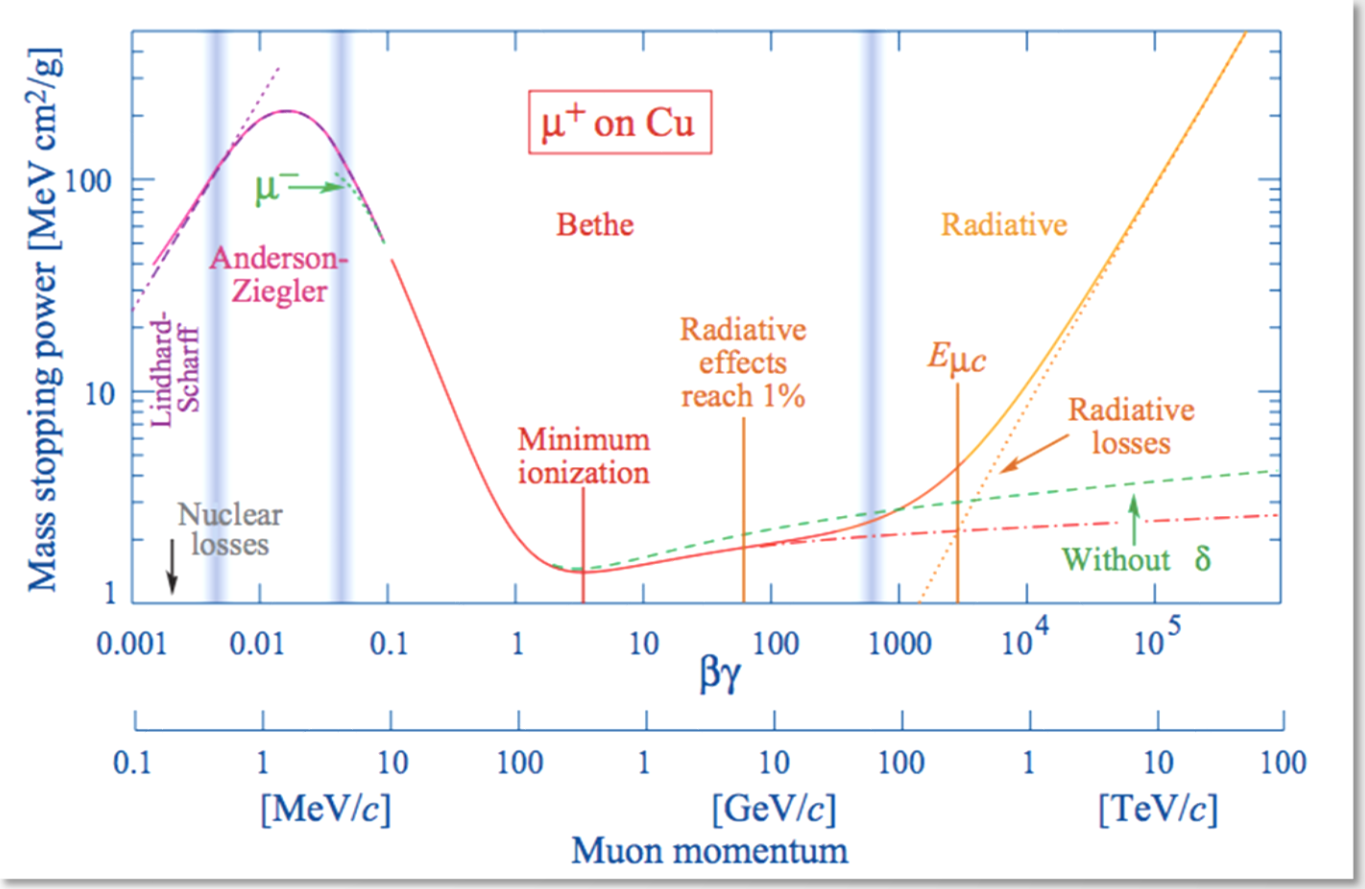
material	Z	A	ρ $\left(\frac{g}{cm^3}\right)$	I (eV)	$\frac{dE}{dx}\|_{min}$ $\left(\frac{MeV\,cm^2}{g}\right)$
C graphite	6	12.01	2.21	78	1.74
Al	13	26.98	2.70	166	1.62
Si	14	28.09	2.33	173	1.66
Fe	26	55.85	7.87	286	1.45
Pb	82	207.2	11.35	823	1.12
Ar	18	39.95	0.00166	188	1.52
CsI	108	259.8	4.51	553	1.24
polysterene	56	104.2	1.06	68.7	1.94
water	10	18	1.00	79.7	1.99
rock	11	22	2.65	136.4	1.69
nuclear emulsion			3.82	331.0	1.42

- Ionization is the universal detection principle of semi-conducting sensors, gaseous detectors, etcetcetc!
- Example: Fe 7.87 g/cm^3: A MIP looses ≈ 11.41 MeV/cm

2.3.2 BETHE-BLOCH EQUATION

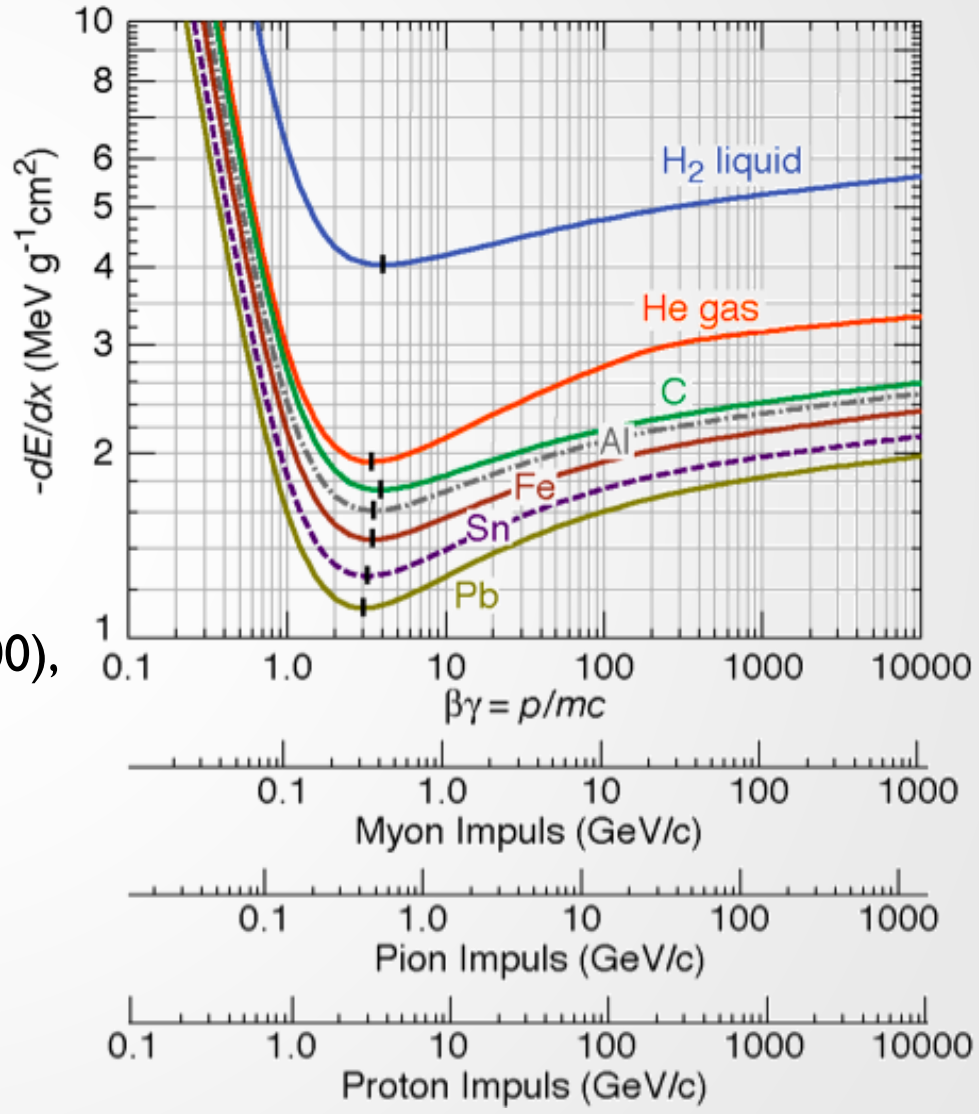
http://pdg.lbl.gov/2017//

- mass stopping power of muon over 9 orders of magnitude in momentum
- Bethe-treatment is accurate to 1% down to $\beta \approx 0.05$, below that there is no accurate theory. Even further below, non-ionizing nuclear recoils dominate.
- At ultra-relativistic energies, radiative losses become important.
- At the muon critical energy $E_{\mu c}$, this effect equals the ionization loss described by the Bethe-Bloch equation.



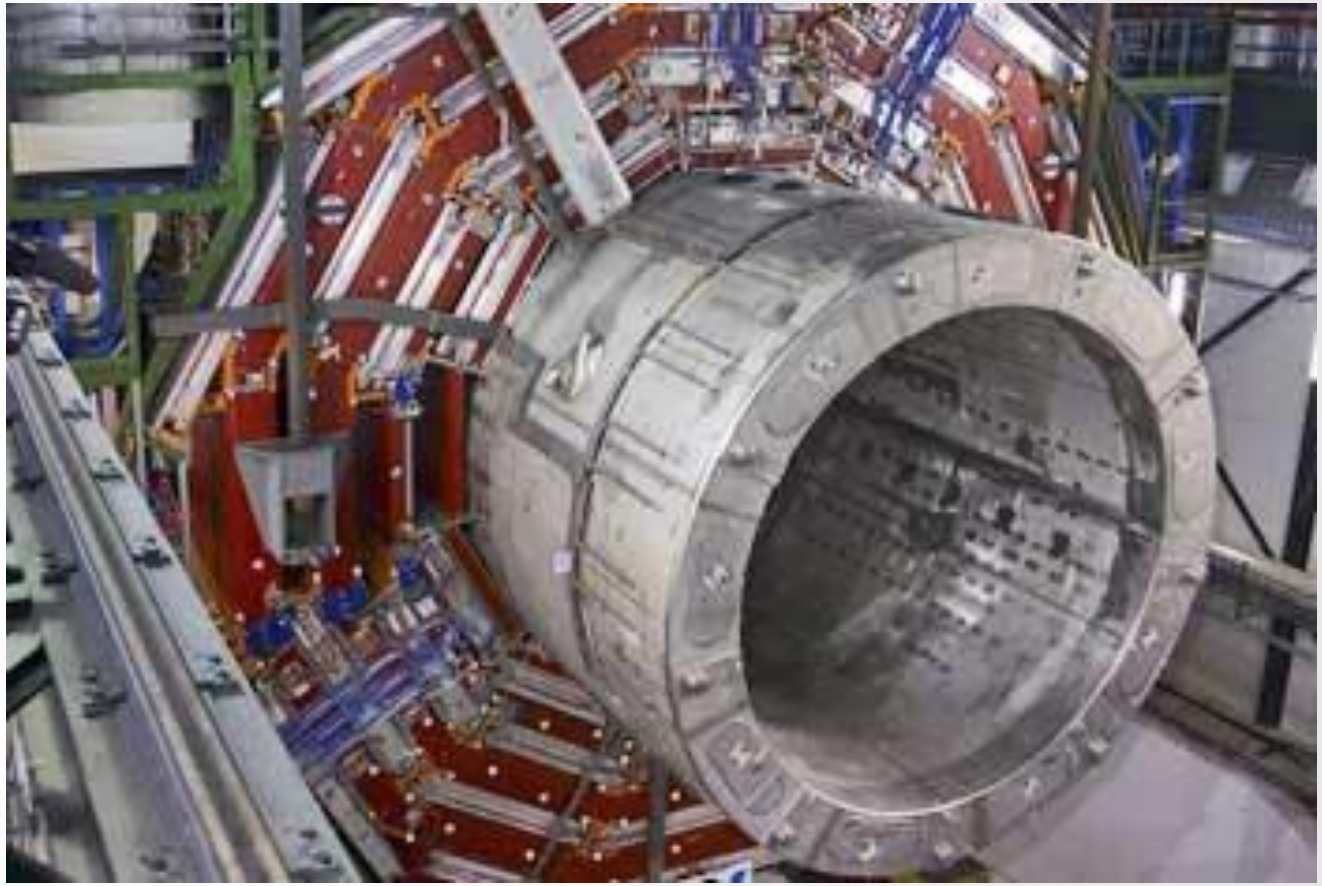
2.3.2 BETHE-BLOCH EQUATION

- The 'Bethe'-region is $0.1<\beta\gamma<100\text{-}1000$.
- The plot shows mass stopping power in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead.
- Typical value of 2 MeV cm^2/g (except H_2) for a MIP.
  - liquid H_2 has Z/A=1 which otherwise is ≈ 0.5
- Radiative effects, relevant for muons and pions, are not included (significant for muons in iron for $\beta\gamma \geq 1000$), and at lower momenta for muons in higher-Z absorbers.
- Note the qualitatively different behavior of the density correction in gaseous Helium.
- The minimum drops from $\beta\gamma\approx 3$ to 3.5 from Z=7 to 100. (or $\beta \approx 0.95$).



EXAMPLE: MUONS TRAVERSING A LARGE DETECTOR

- CMS magnet: 12.5m length, 6.3m diameter, cooled to 4.7K
- 3.8T magnetic field inside, return yoke (steel) provides ~2T
- The magnet and the return yoke are large metallic structures

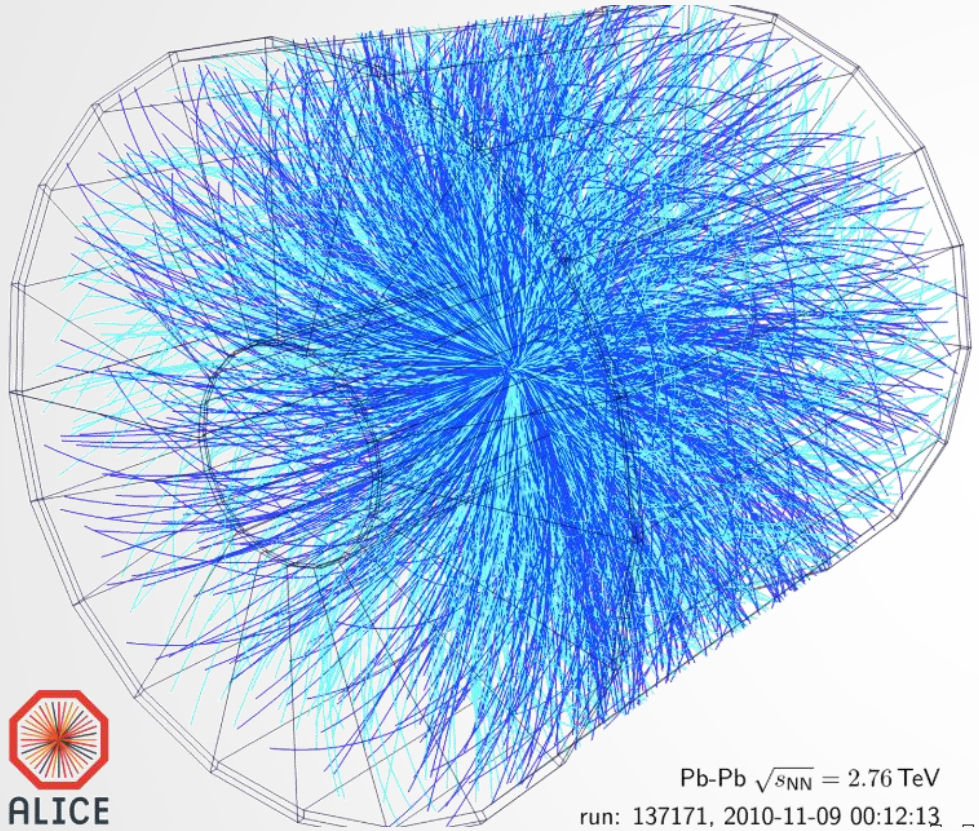


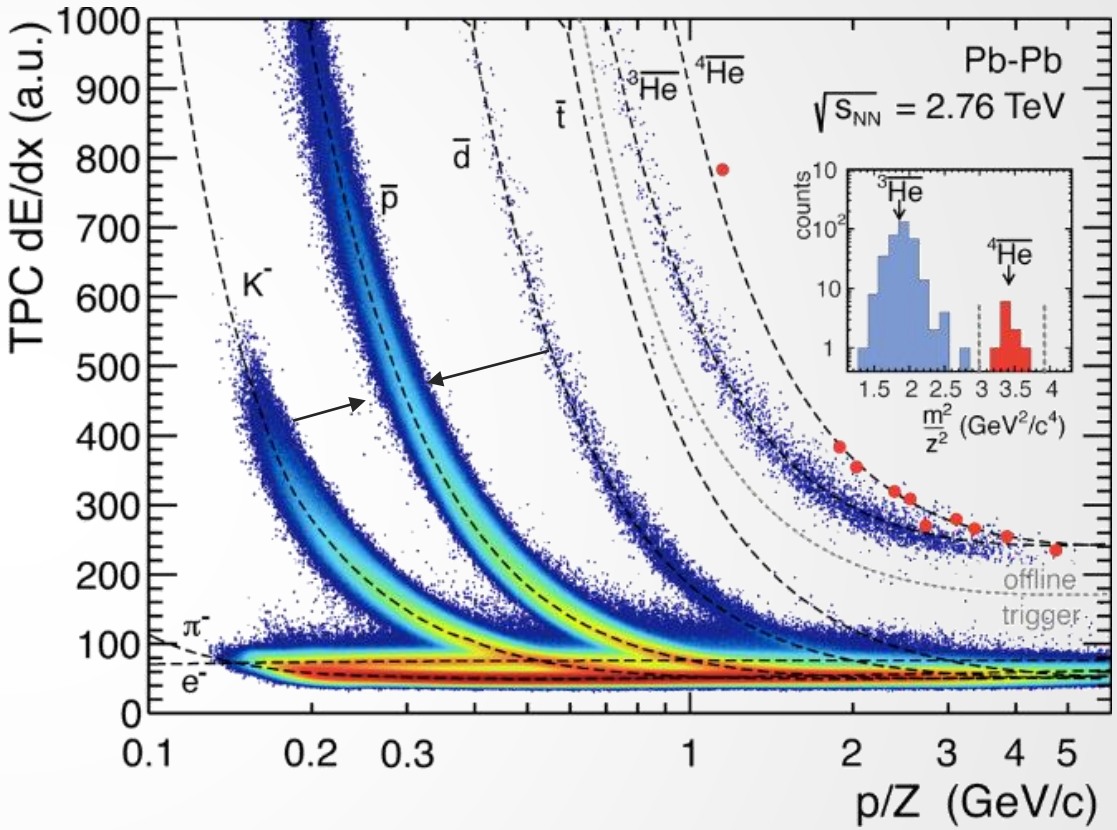
- What is the ionization loss of a 50 GeV muon in a 50cm iron layer?

$$-\Delta x \cdot \left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle \Bigg|_{p_T(\mu)=50\mathrm{GeV}} \approx 2.2 \frac{\mathrm{MeV cm}^2}{\mathrm{g}} \cdot 7.87 \frac{\mathrm{g}}{\mathrm{cm}^3} \cdot 50\,\mathrm{cm} \approx 0.86\ \mathrm{GeV}$$

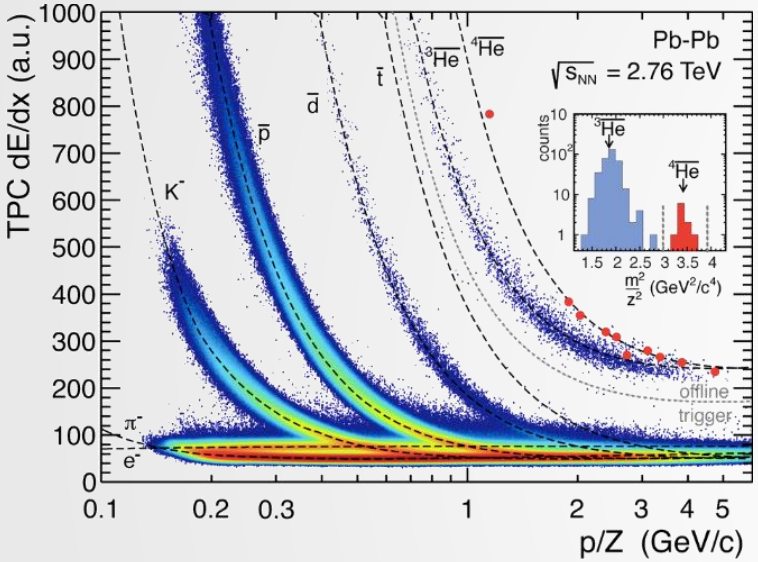
$\longrightarrow$ few % losses in the whole detector

2.3.3 PARTICLE IDENTIFICATION WITH BETHE-BLOCH





- dE/dx can be used for identifying particles.
- Left: Time Projection Chamber (ALICE detector, Ne/CO_2 gas). Right: dE/dx measurement in Pb/Pb collisions.
- Note: The spread in dE/dx limits particle Identification.


Pb-Pb
$\sqrt{s_{NN}}$ = 2.76 TeV
TPC dE/dx (a.u.)
p/Z (GeV/c)
K^-
$\bar{p}$
$\bar{d}$
$\bar{t}$
$^3\overline{He}$
$^4\overline{He}$
π^-
e^-
offline
trigger
counts
$\frac{m^2}{z^2}$ (GeV^2/c^4)

2.3.4 STOCHASTICS OF THE ENERGY LOSS

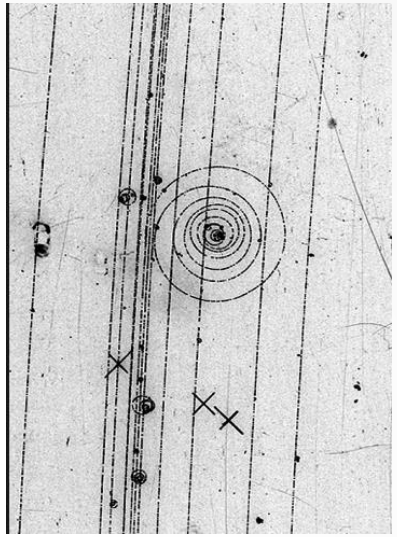
- Bethe-Bloch equation describes the mean energy loss
  - "Few concepts in high-energy physics are as misused as <dE/dx>" [Pdg Live review of particles in matter, 2017]
- δ-electrons

  For ultra-relativistic particles (2.2) gives $T(\theta)\Big|_{\gamma\to\infty} = \frac{2m_e c^2}{\tan^2\theta}$
  - rate of electrons in relatively central collisions (δ-electron) is significant and limited only by $T_{max} < \gamma Mc^2$
  - These large fluctuations are described by 'Landau' distribution:

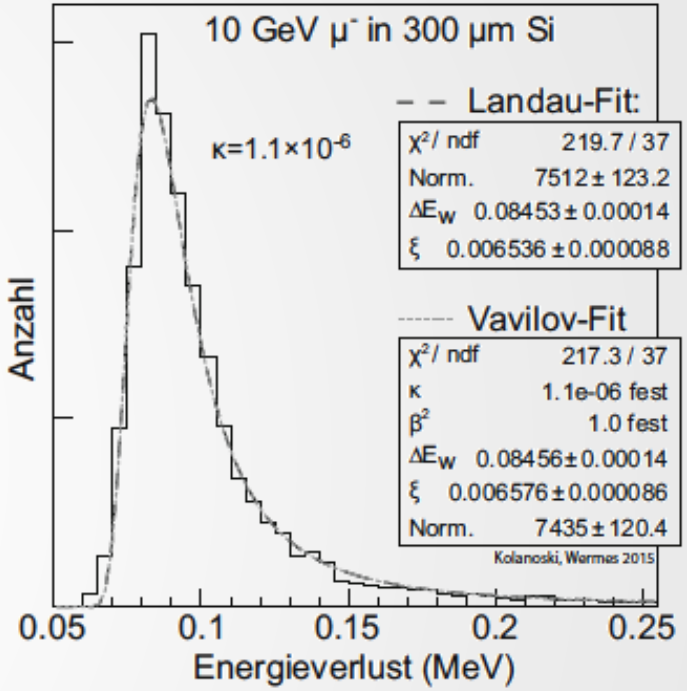
$$f_L(\lambda) = \frac{1}{\pi}\int_0^\infty e^{-t\ln t - \lambda t}\sin(\pi t)\,dt$$

$$\lambda = \lambda(\Delta E_w, \xi) = \frac{\Delta E - \Delta E_w}{\xi} - 0.22278$$

- Effects in the detector
  - limit detector resolution (can correct only the mean energy loss ΔE_w)
  - limit the separation capabilities in particle Id
  - spatial widening of secondary ionization and bremstrahlung



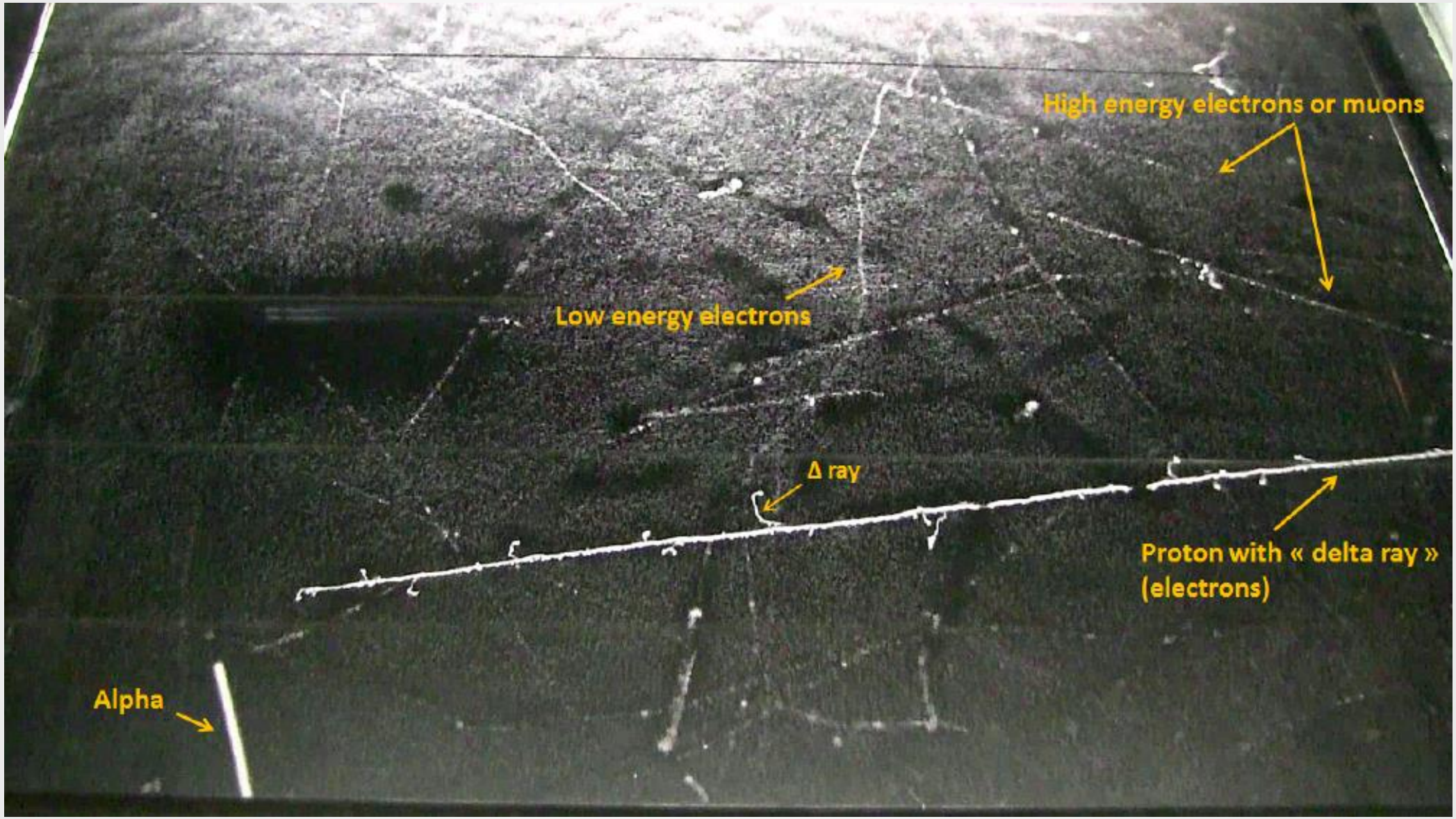
1958, 2m Bubble Chamber operated with 1150 liters of liquid hydrogen (later discovered Ω^- predicted by M. Gell-Mann)



- The shape parameter is $\kappa = \frac{\xi}{T_{max}}$ where $\xi = \frac{1}{2}K\frac{Z}{A}\rho\frac{z^2}{\beta^2}\Delta x$ is the prefactor in the Bethe-Bloch equation multiplied with the thickness of the material and divided by T_{max}.
- For thick absorbers ($\kappa \gtrsim 1$), the energy loss becomes Gaussian, for thin absorbers ($\kappa \ll 1$) it has strong tails

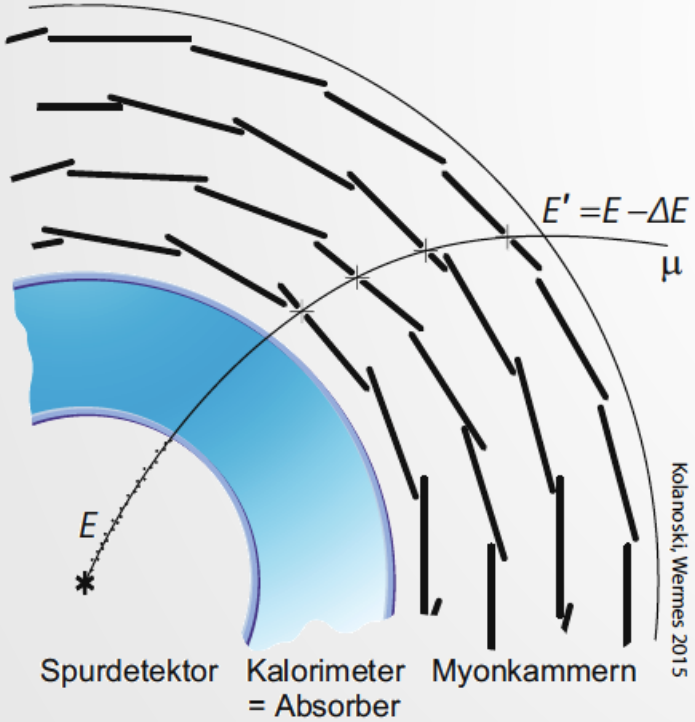
2.3.5 DELTA ELECTRONS

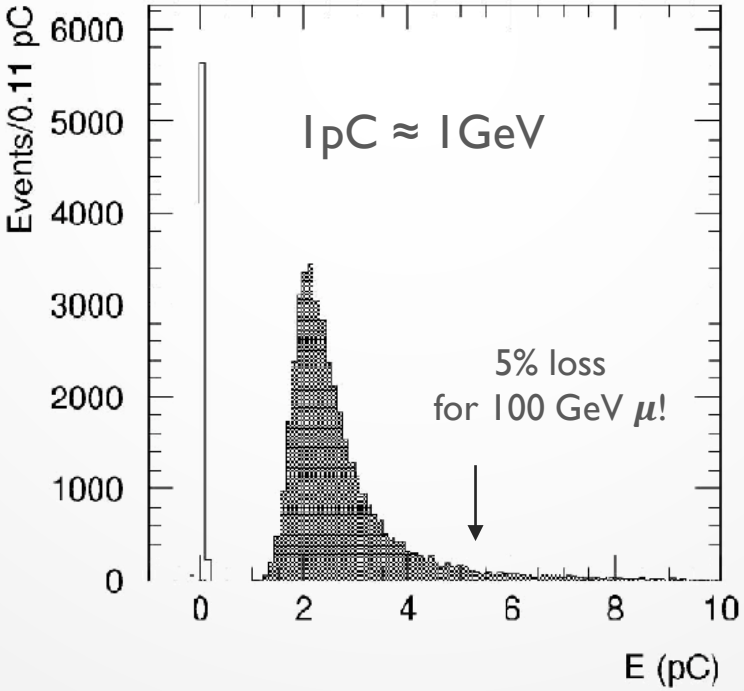
Image source: Wikipedia



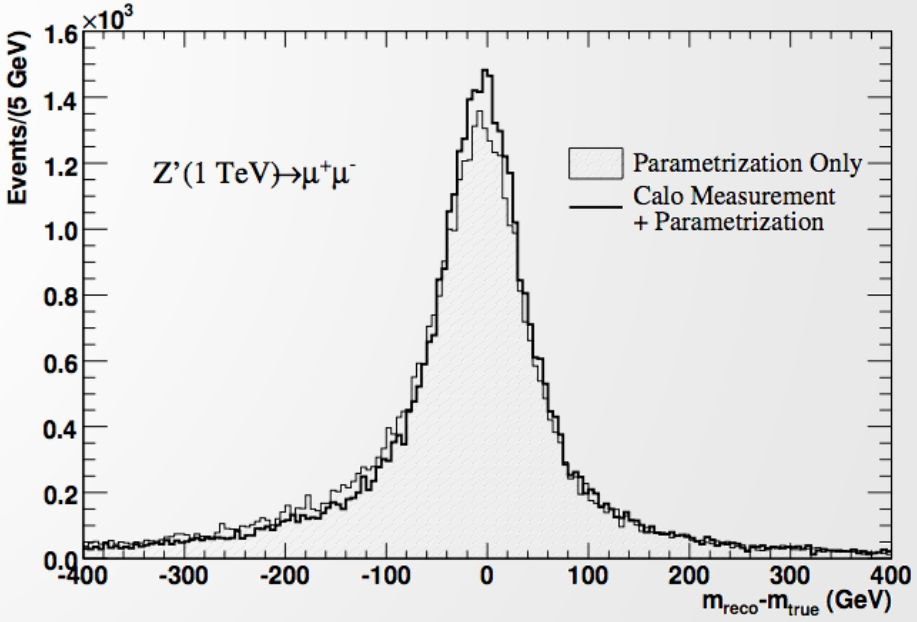
2.3.6 EXAMPLE: ATLAS MUON RECONSTRUCTION

- The ATLAS calorimetry for 100 GeV muons is equivalent to 175cm of Fe, but large T_{max}, so $\kappa \approx 0.001$.
- A small faction of high energetic muons deposits a significant energy fraction in the calorimetry.
- Because this is an instrumented region, the muon reconstruction can correct for the effect.





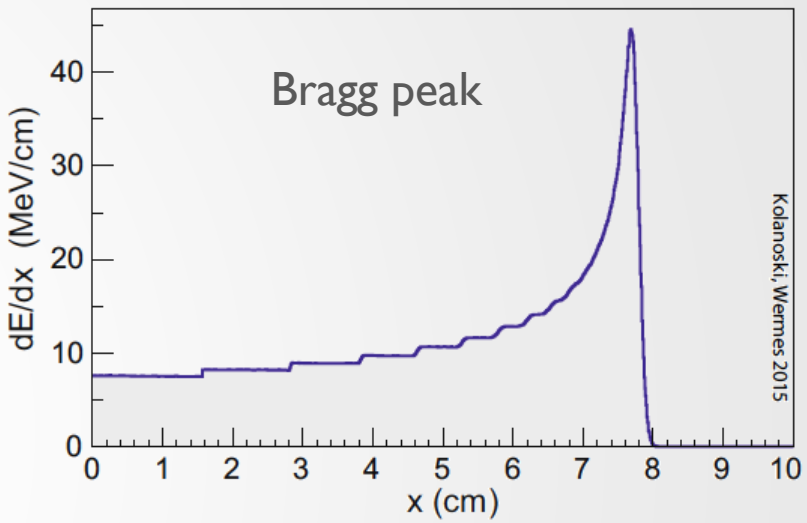
improved resolution for hypothetical signal:

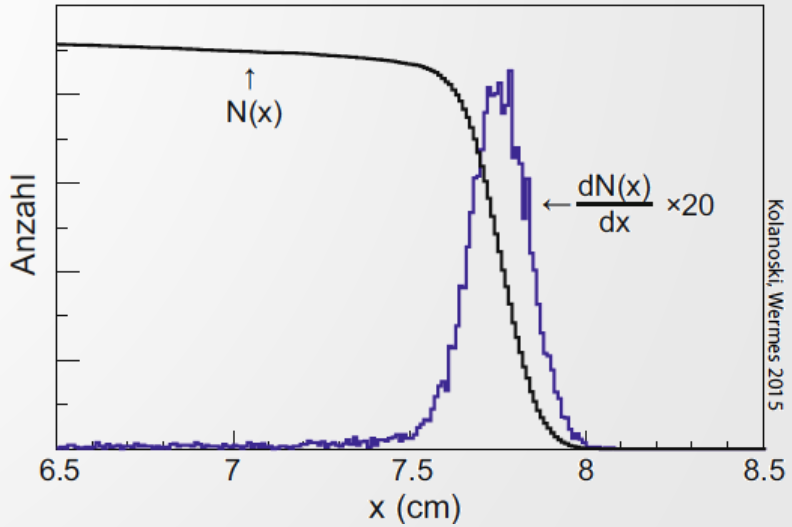


http://inspirehep.net/record/826342

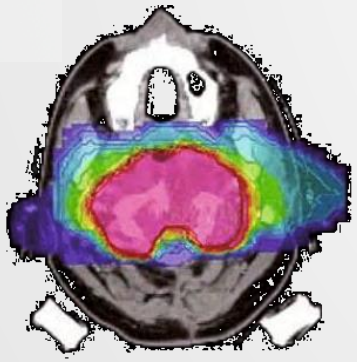
2.3.7 PENETRATION DEPTH AND BRAGG PEAK

- Because <dE/dx> grows with β^{-2} below the minimum (MIP) in the Bethe-Bloch regime, most heavy charged particles deposit their energy in a relatively narrow region.
  - Example (right): 100 MeV protons on water
    - The dislocation effect of e.g. δ-electrons is not taken into account
  - This 'Bragg-Peak' is exploited in e.g. medical cancer treatment.

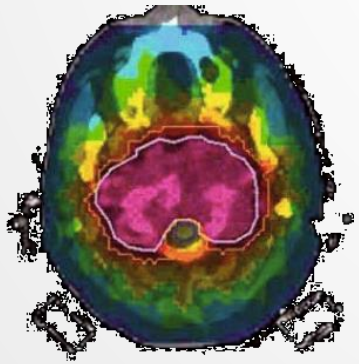




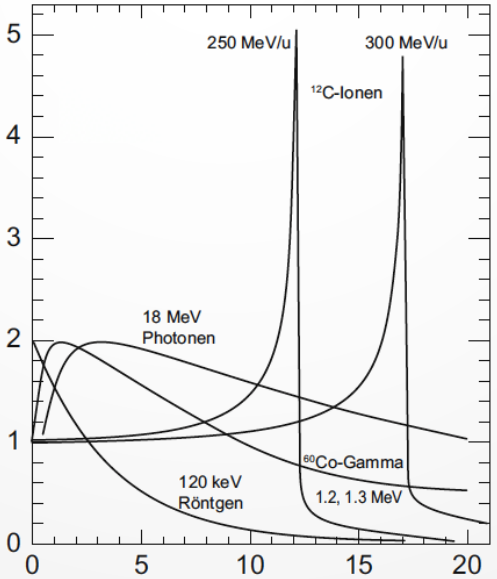
C^{12} Ions



gamma rays



energy deposits vs. depth in water

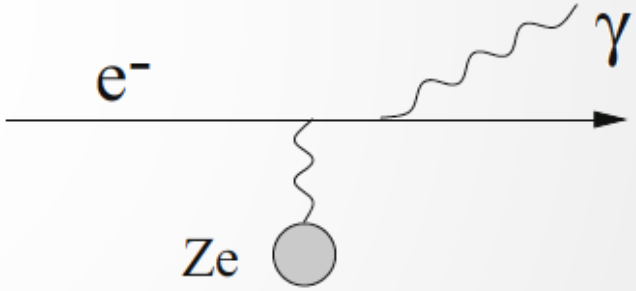


2.4 BREMSSTRAHLUNG

- So far, we have dealt with heavy charged particles where ionization of the atomic shell is the most important source of energy loss.
- Particles can also loose energy by interacting with the nucleus or the atomic shell.
  For electrons, this 'bremsstrahlung' is the dominant source of energy loss.
- In lecture 1, we calculated the energy loss of an accelerated charge as $P = \frac{q^2 \dot{v}^2}{6\pi\varepsilon_0 c^3}$.

  Because $m\dot{v} \propto Ze$ in the field of a nucleus of charge Z, we find $\boxed{P \propto \frac{Z^2 e^2}{m^2}}$ (Note: $\propto Z^2$, because interaction is coherent).

Bremsstrahlung interaction of an electron in the Coulomb field of a nucleus



- Therefore, the bremsstrahlung of the muon is suppressed by a factor $(m_\mu/m_e)^2 = 40000$.
- Careful QM calculations take into account dielectric suppression (polarization of the medium for low energies), coherent scattering on the shell, and energy dependent shielding of the nucleus by the shell.

2.4 BREMSSTRAHLUNG

- The fractional energy loss is nearly independent of energy, therefore, the loss per unit length is efficiently described by the 'radiation length' X_0 after which the energy of a highly energetic electron reduces to 1/e = 36.7%

$$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{E}{X_0}$$

- The radiation length can be approximated to within 2.5% (Helium: 5%) by the empirical expression

$$\rho X_0 = \frac{716.408\ \mathrm{g\ cm^{-2}} A}{Z(Z+1)\ln\frac{287}{\sqrt{Z}}}$$

- The critical energy E_c is defined as the energy where bremsstrahlung (dominant at high energies) and ionization losses (dominant at lower energies) agree.
  - Factor 10000 between electrons and muons!
- high-Z materials have low E_c (Ion. ∝ Z while brem. is ∝ Z^2)

material	Z	density (g/cm^3)	X_0 (g/cm^2)	X_0 (cm)	E_k (MeV)	E_k^μ (GeV)
Be	4	1.85	65.19	35.3	113.7	1328
C (Graphit)	6	2.21	42.65	19.3	81.7	1060
Al	13	2.70	24.01	8.9	42.7	612
Si	14	2.33	21.82	9.36	40.2	582
Fe	26	7.87	13.84	1.76	21.7	347
Cu	29	8.96	12.86	1.43	19.4	317
Ge	32	5.32	12.25	2.30	18.2	297
W	74	19.30	6.76	0.35	8.0	150
Pb	82	11.35	6.37	0.56	7.4	141
U	92	18.95	6.00	0.32	6.7	128
Szintillator						
NaI	11, 53	3.66	9.49	2.59	13.4	228
CsI	55, 53	4.53	8.39	1.85	11.2	198
BaF_2	56, 9	4.89	9.91	2.03	13.8	233
$PbWO_4$	82, 74, 8	8.30	7.39	0.89	9.64	170
Polystyrol	1, 6	1.06	43.79	41.3	93.1	1183
Gases (20°C, 1 bar)						
H_2	1	$0.0838 \cdot 10^{-3}$	61.28	731000	344.8	3611
He	2	$0.1249 \cdot 10^{-3}$	82.76	662610	257.1	2352
Air	≈ 7.36	$1.205 \cdot 10^{-3}$	36.66	30423	87.9	1115
Ar	18	$1.66 \cdot 10^{-3}$	19.55	11763	38.0	572
Xe	54	$5.48 \cdot 10^{-3}$	8.48	1547	12.3	232
other materials:						
water	1, 8	1.0	36.1	36.1	78.3	1031
rock	11	2.65	26.5	10.0	49.1	693
fotoemulsion		3.82	11.33	2.97	17.4	286

2.4 BREMSSTRAHLUNG

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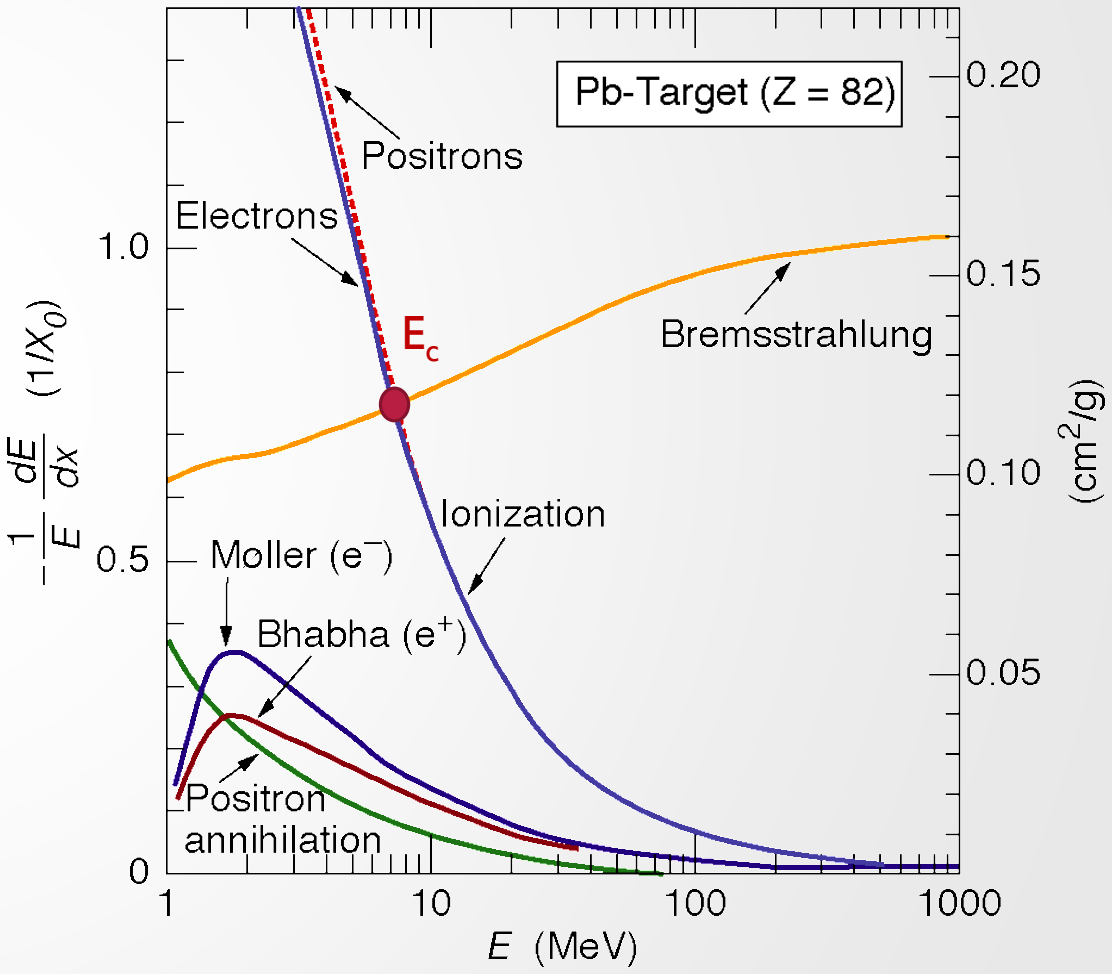
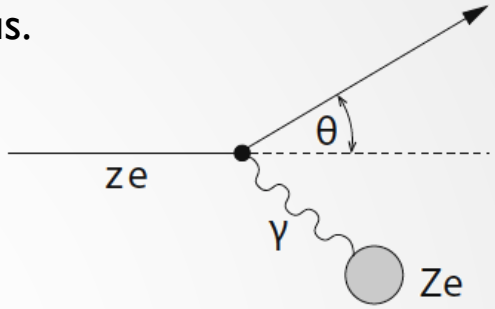


2.5 MULTIPLE SCATTERING - MOLIÈRE THEORY

- Charged particles can change their direction when interacting with the nucleus.
  Rutherford scattering formula (the two charges are z and Z in units of e)

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rutherford}} = z^2 Z^2 \alpha^2 \hbar^2 \frac{1}{\beta^2 p^2} \frac{1}{4\sin^4 \theta/2}$$

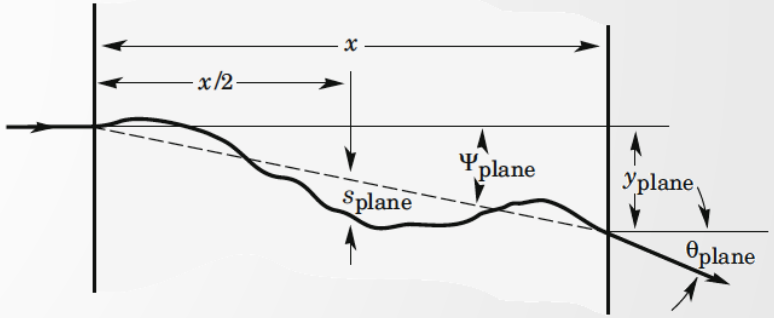


Rutherford scattering

- For many subsequent scattering events,
  Moliere-theory allows to predict the *angular spread*.
  Expect a Gaussian distribution:

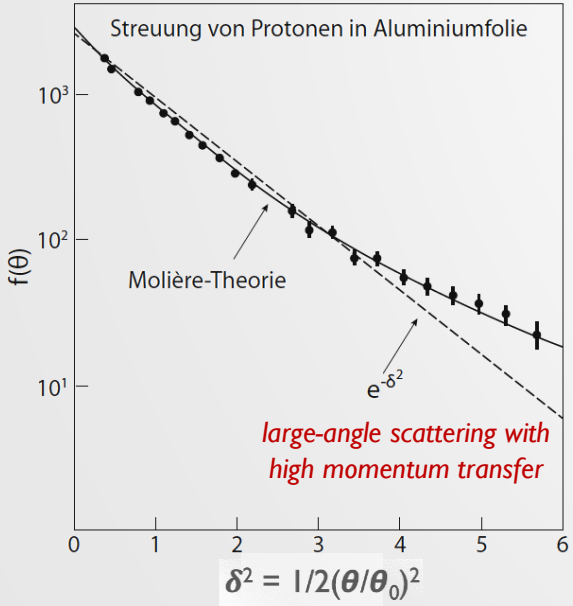
$$f(\theta_{\text{plane}}) = \frac{1}{\sqrt{2\pi}\theta_0} e^{-\theta^2_{\text{plane}}/(2\theta_0^2)}$$

$\boldsymbol{\theta}_0$ is used to model deflection:



$$\langle \psi_{plane} \rangle = \frac{1}{\sqrt{3}} \theta_0$$

$$\langle y_{plane} \rangle = \frac{1}{\sqrt{3}} x\, \theta_0 \qquad \langle s_{plane} \rangle = \frac{1}{4\sqrt{3}} x\, \theta_0$$



Left: Scattering of protons with T=2.18 MeV on an Aluminium foil of x = 13 μm. The large-angle deviations predicted by Rutherford scattering make a 'non-Gaussian tail' for δ^2>4.

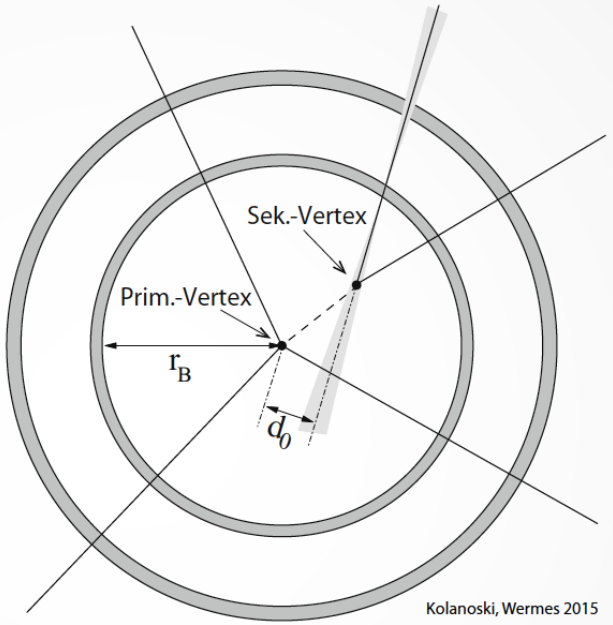
Here, $\delta^2 = 1/2(\theta/\theta_0)^2$ where θ_0 is predicted by the 'Highland formula':

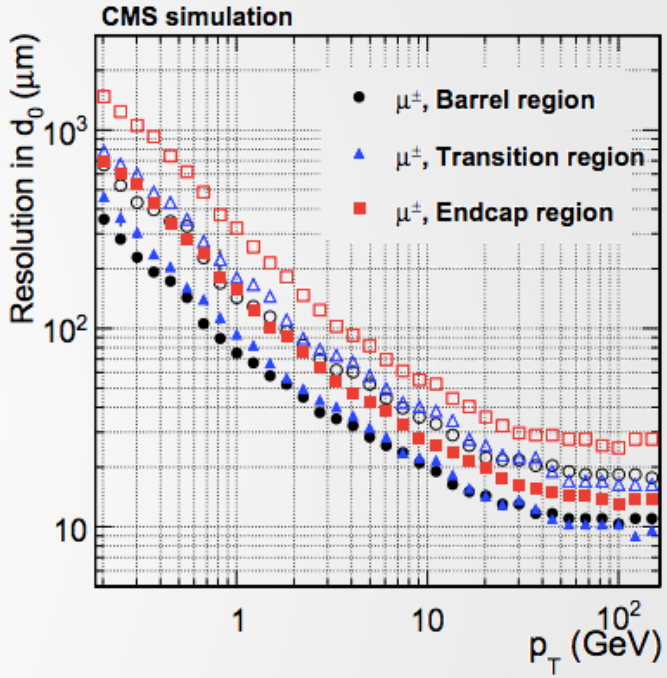
$$\theta_0 \approx \sqrt{\langle \theta^2_{plane} \rangle} \approx \frac{13.6\,\text{MeV/c}}{p\beta} z \sqrt{\frac{x}{X_0}} \left(1 + 0.038 \ln \frac{x}{X_0}\right)$$

→ x/X_0 characterizes deflection

2.6 IMPACT PARAMETER RESOLUTION

- In order to identify b-quarks, the displaced decay of B mesons inside a jet is identified by reconstructing secondary, transversally displaced, vertices.
- B lifetime 1.5ps, <l> = $\beta\gamma c\tau \approx$ 2mm for e.g. a 10 GeV particle.
- The impact parameter is the most discriminating feature of a secondary vertex. Multiple scattering in the beam-pipe limits impact parameter resolution.
- Assume r_B=50 mm and a beam-pipe of thickness 1mm and p(π) of 5 GeV.
- The resulting resolution is $\Delta d_0 = \theta_0 \, r_B$





Material	X_0 [mm]	x/X_0	θ_0 [rad]	Δd_0 [μm]
Al	89	0.011	0.0003	15.0
Be	353	0.003	0.00015	7.5

Choose Be instead of Al for the beampipe
to gain a factor 2 in impact parameter resolution!

2.7 ELECTROMAGNETIC FIELD IN DIELECTRIC MEDIUM

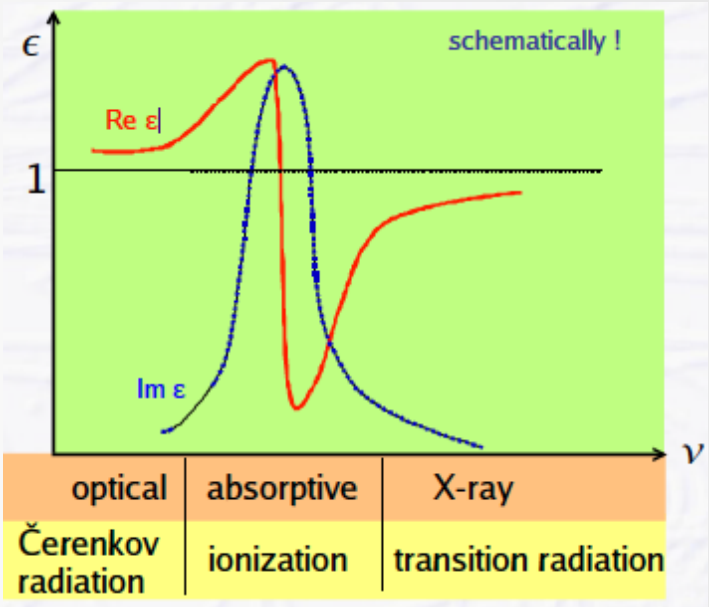
- Energy can be lost to photons in a medium. Consider a dielectric medium.
- The medium is modelled as N_i damped harmonic oscillators (ω_i, Γ_i) per unit volume. The relative permittivity ε is complex:

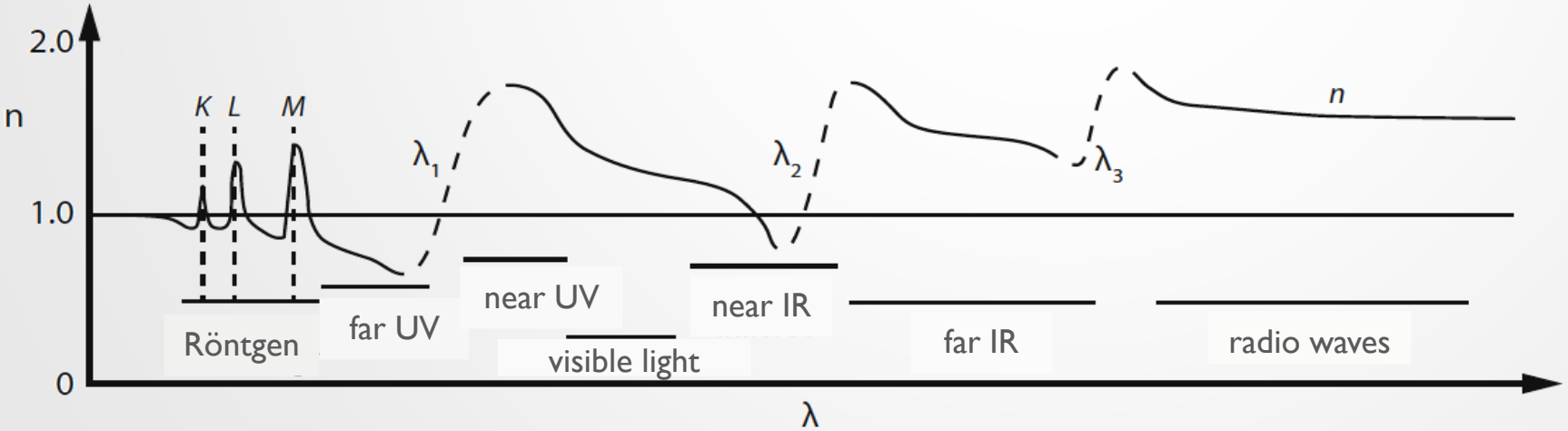
$$\varepsilon = 1 + \frac{e^2}{m\varepsilon_0}\sum_i \frac{N_i}{(\omega_i^2 - \omega^2) + i\omega\Gamma_i} \qquad \sqrt{\varepsilon} = n - i\kappa$$

$$\omega^2 = \frac{|\vec{k}|^2 c^2}{\varepsilon}$$

Dispersion relation

- Behavior as a function of the frequency of an incident wave:
  - high frequencies, above resonance: $n \lesssim 1$ but reaching 1 quickly (X-Ray regime)
  - low frequencies, below resonance $n>1$ (optical regime)
  - at a resonance frequency: absorption/ionization (anormal dispersion)





typical dispersion relation for a transparent medium. Dashed lines show regions of anomalous dispersion

http://pcwww.liv.ac.uk/~awolski/Teaching/Liverpool/PHYS370/AdvancedElectromagnetism-Part2.pdf

2.7 ELECTROMAGNETIC FIELD IN DIELECTRIC MEDIUM

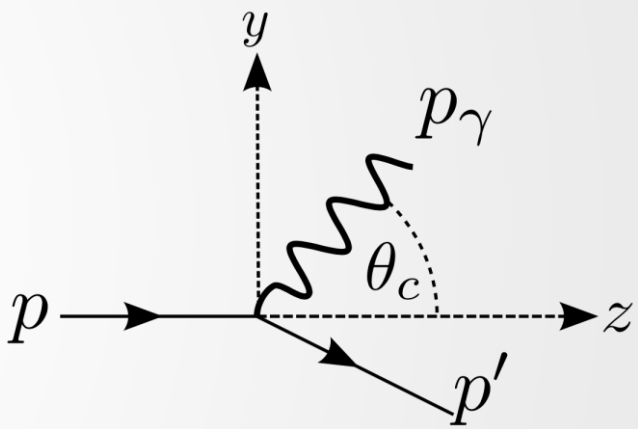
- Let's write the 4-momentum of the incident charged particle:

$$p^\mu = M\gamma \begin{pmatrix} c \\ \vec{v} \end{pmatrix} = Mc\gamma \begin{pmatrix} 1 \\ \vec{\beta} \end{pmatrix} = \begin{pmatrix} \frac{E}{c} \\ \vec{p} \end{pmatrix}$$ (of course it satisfies $\frac{1}{c^2}\left(E^2 - c^2\vec{p}^{\,2}\right) = M^2c^2$)

- For the photon we make the ansatz $p_\gamma = \begin{pmatrix} \frac{\hbar\omega}{c} \\ \hbar\vec{k} \end{pmatrix}$.

  Then we can use $p' = p - p_\gamma$ to find

$$M^2 = M^2 - \frac{2}{c^2} p \cdot p_\gamma \quad \text{or} \quad 0 = p \cdot p_\gamma = \frac{E\hbar\omega}{c^2} - \hbar\vec{k}\cdot\vec{p} \ .$$



- We can express the angular frequency as

$$\omega = \frac{\vec{k}\cdot\vec{p}}{E/c^2} = \vec{k}\cdot\vec{v} = |\vec{k}||\vec{v}|\cos\theta_c = kv\cos\theta_c$$

  and use the dispersion relation $\omega^2 = \frac{|\vec{k}|^2c^2}{\varepsilon} \rightarrow |\vec{k}| = \sqrt{\varepsilon}\frac{\omega}{c}$ to find

$$\boxed{\begin{aligned} 1 &= \sqrt{\varepsilon}\frac{|\vec{v}|}{c}\cos\theta_c \\ \cos\theta_c &= (\beta n)^{-1} \end{aligned}}$$

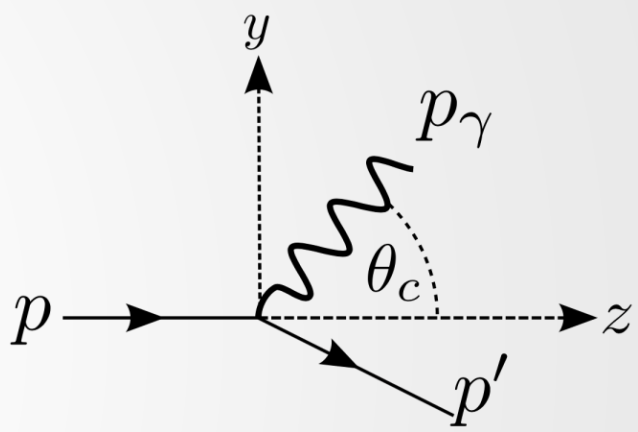
- IF(!) we are in the optical regime, there is is a real solution
  - real photon emission for v > c / n (Čerenkov)
  - Close to resonance (ionisation), or in the hard UV, there is no real solution.

2.8 TRANSVERSE REACH OF THE EM INTERACTION

- Next, we assume k_x=0 (no loss of generality) and use $\omega = vk_z = vk\cos\theta_c$ and $\frac{\omega^2\varepsilon}{c^2} = k_y^2 + k_z^2$ to express k_y as

$$k_y^2 = \left(\frac{\omega}{v}\right)^2\left(\frac{v^2}{c^2}\varepsilon - 1\right) = \left(\frac{\omega}{v}\right)^2\left(\frac{v^2}{(c/\sqrt{\varepsilon})^2} - 1\right).$$

- The phase velocity in the medium is $c_m = c/\sqrt{\varepsilon},\ \beta' = v/c_m$ and therefore $k_y = \frac{\omega}{v}\sqrt{\beta'^2 - 1}$.



- For β'>1 (optical!), the plane wave solution is real and the field extends infinitely in the y direction: $e^{i(\omega t - \vec{k}\cdot\vec{r})}$

- For β'<1, however, we have $e^{-i\frac{\omega}{v}(z-vt)}e^{-y/y_0}$ with

$$y_0 = \frac{v}{\omega}\underbrace{\frac{1}{\sqrt{1-\beta'^2}}}_{\gamma'} = \frac{v\sqrt{\varepsilon}}{\omega_0}\gamma' = \frac{c}{\omega_0}\beta'\gamma' = \frac{\beta'\gamma'}{k}$$

where ω_0 and k_0 are the vacuum angular frequency and wave vector:

$$\omega = ck/\sqrt{\varepsilon} = \omega_0/\sqrt{\varepsilon} \qquad k_0 = \omega/c = k/\sqrt{\varepsilon}$$

Take home message (1):
The transverse reach of the electromagnetic interaction is governed by $y_0 = \beta'\gamma'/k$ of the incident particle! This explains the dependence of the density effect on $\beta\gamma$.

2.9 ČERENKOV RADIATION

- Below the excitation energies of the medium (optical regime) ε is real and $\varepsilon > 1$ so that $\cos\theta_c = (\beta n)^{-1}$ has a real solution if $\boldsymbol{\beta} > n = \sqrt{\varepsilon}$. In this case, 'Čerenkov'-radiation is emitted. For $\boldsymbol{\beta} \longrightarrow 1$: $\theta_{\max} = \cos^{-1} n^{-1}$
- The minimum energy of the particle can be calculated from

$$\frac{E_{\text{th}}}{mc^2} = \gamma_{\text{th}} = \frac{1}{\sqrt{1-\beta_{\text{th}}^2}} = \frac{1}{\sqrt{1-\varepsilon^{-1}}} = \frac{n}{\sqrt{n^2-1}} \ .$$

- The maximum angle is attained for $\boldsymbol{\beta}=1$ and satisfies

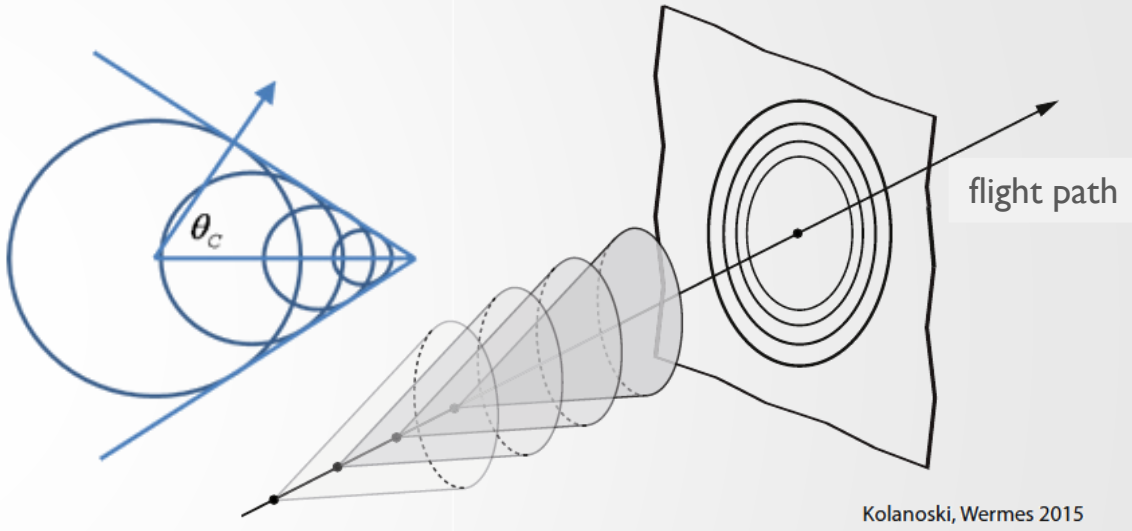
$$\sin\theta_{max} = \sqrt{1-\cos^2\theta_{max}} = \sqrt{1-\frac{1}{n^2}} = \frac{1}{\gamma_{th}} \ .$$

- The angle of radiation for a given $\boldsymbol{\beta}$ as a function of $\boldsymbol{\theta}_{max}$ is

$$\sin^2\theta_c = 1-\cos^2\theta_c = 1-\frac{1}{\beta^2 n^2} = 1-\frac{\beta_{th}^2}{\beta^2} = \frac{1}{\gamma_{th}^2}\frac{\gamma^2-\gamma_{th}^2}{\gamma^2-1}$$

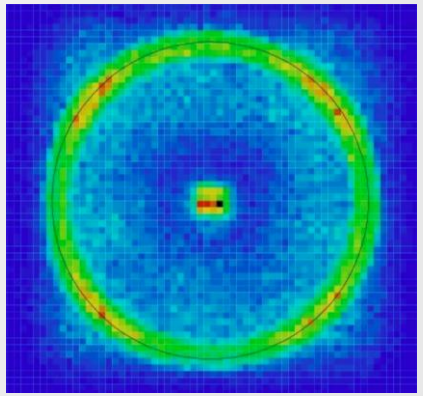
$$= \sin^2\theta_{max}\frac{\gamma^2-\gamma_{th}^2}{\gamma^2-1} \quad \text{and therefore} \quad \frac{\theta_c}{\theta_{max}} = \frac{R_c}{R_{max}} \approx \sqrt{1-\frac{\gamma_{th}^2}{\gamma^2}} \ .$$

- For $\gamma \gg \gamma_{th}$, Čerenkov light is emitted in narrow rings. It is very weak: ≈0.1-1% of Ionization.
- It also emitted promptly, in contrast to scintillation and ionization signals.



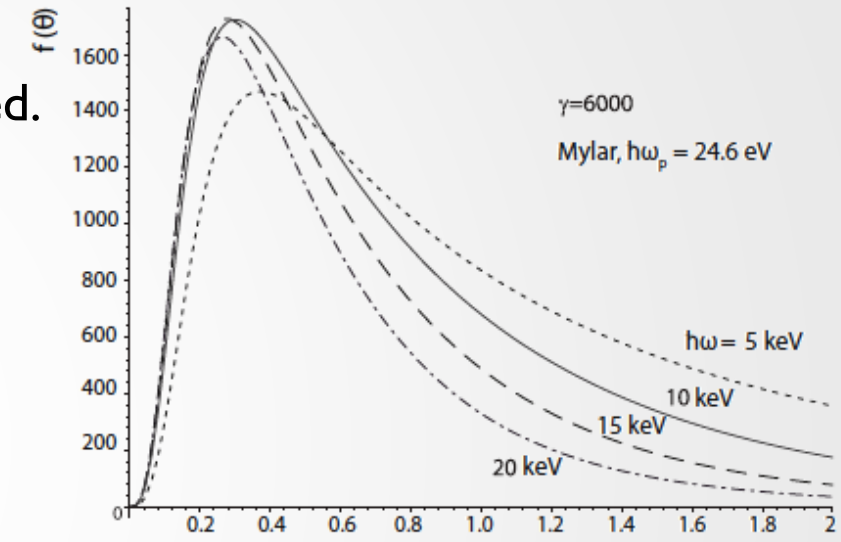
Kolanoski, Wermes 2015

Čerenkov ring:

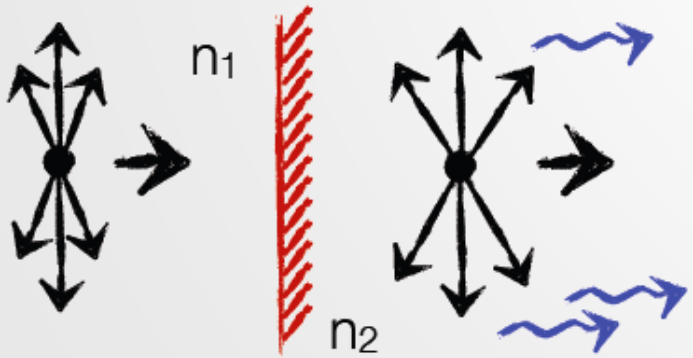


2.10 TRANSITION RADIATION

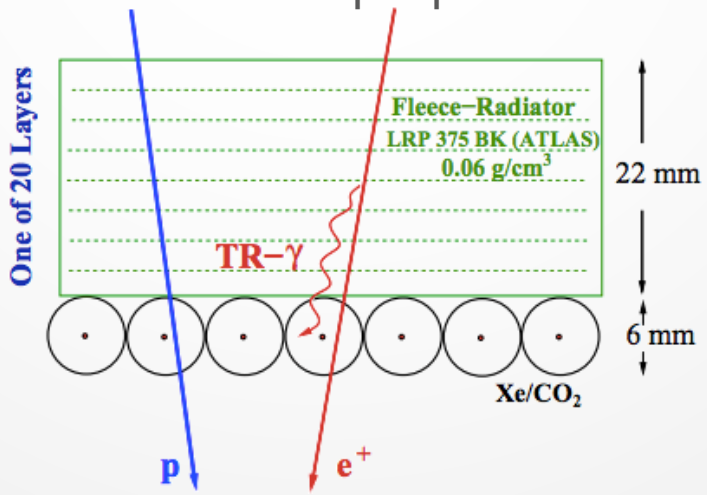
- If a particle traverses a surface separating two materials with different refraction index $n_1 \neq n_2$, transition radiation is emitted.
- The most probable emission angle is $\theta_{mpv} \sim \gamma^{-1}$, and Intensity $\sim \gamma$.
- For ultra-relativistic particles (γ>1000), hard UV radiation is emitted, closely collimated with the particle.
- Measuring the intensity (γ) allows to identify charged particles when their momentum is known.

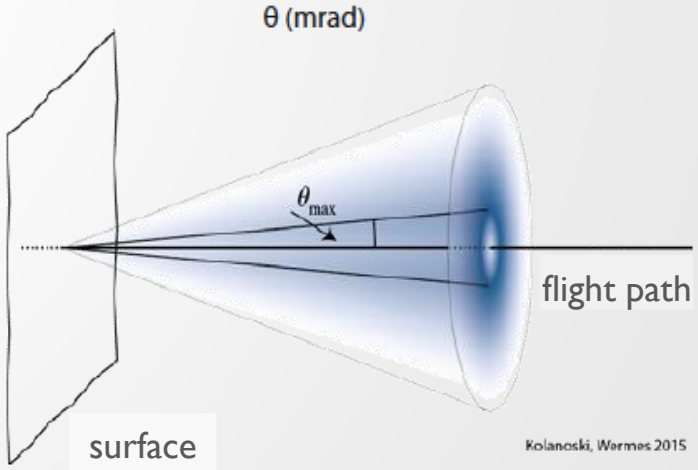


principle:



AMS TRD for e^+/p separation:



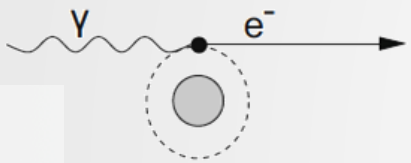


3. INTERACTION OF PHOTONS WITH MATTER

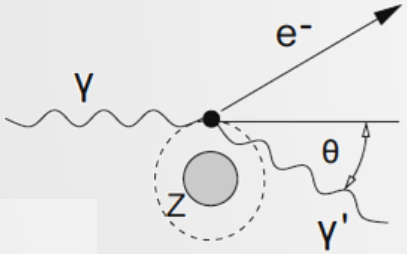
3.1 OVERVIEW

- High energetic photons typically interact by absorption. Ansatz: $N(x) = N_0\, e^{-\mu x}$

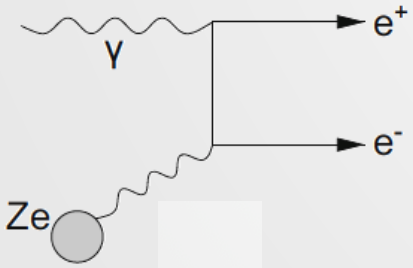
Photo effect ($\propto Z^5/E_\gamma^{3.5}$)

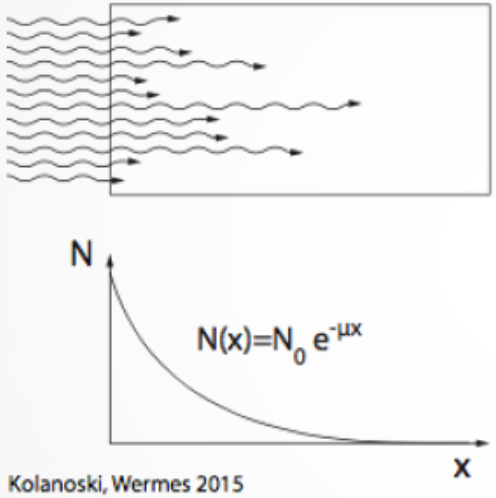


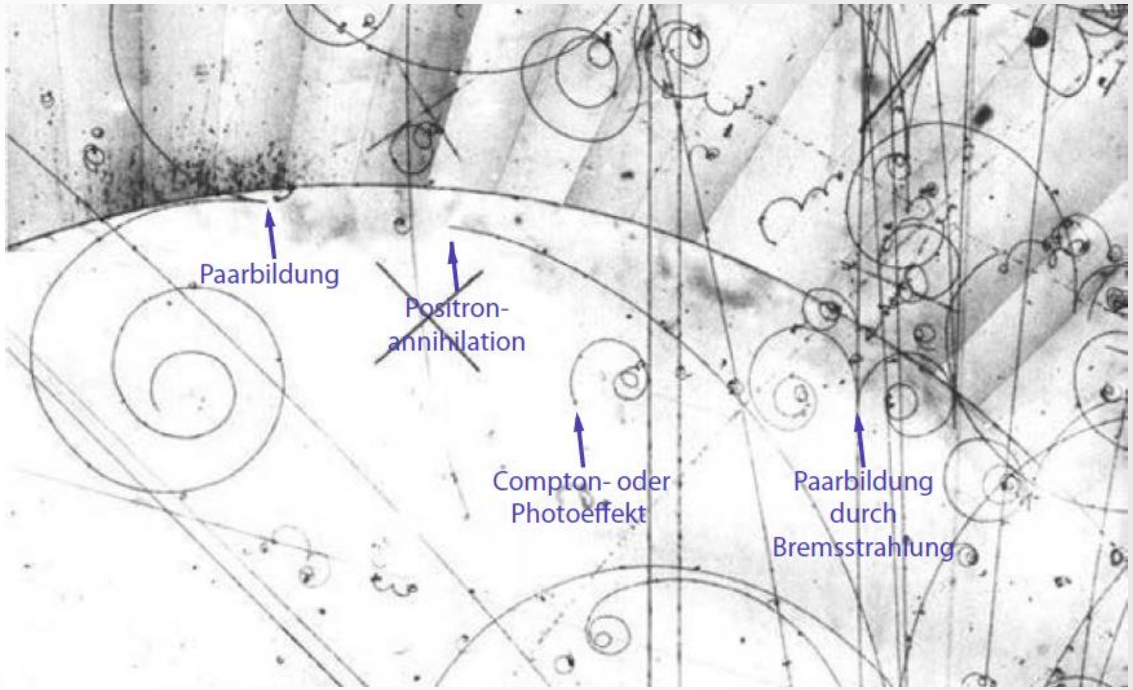
Compton effect ($\propto Z$)



Pair creation ($\propto Z^2$)







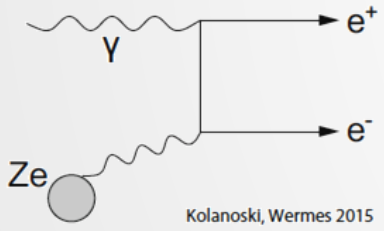
electromagnetic shower (bottom-up) in the 15-foot Bubble Chamber at FNAL

- Photo effect: the photon looses its energy to an atom which emits an electron
  - secondary emission of characteristic X-rays and Auger electrons when the holes are re-filled
- Compton effect: (quasi-) elastic scattering on an electron in the atomic shell ($\gg E_{bind}$)
- Pair creation: the photon converts to an e^+e^- pair in the electric field of the nucleus

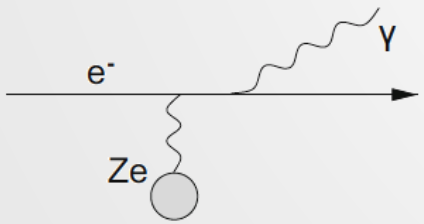
3.2 MEAN FREE PATH OF PHOTONS

https://physics.nist.gov/ (Photon cross section database)

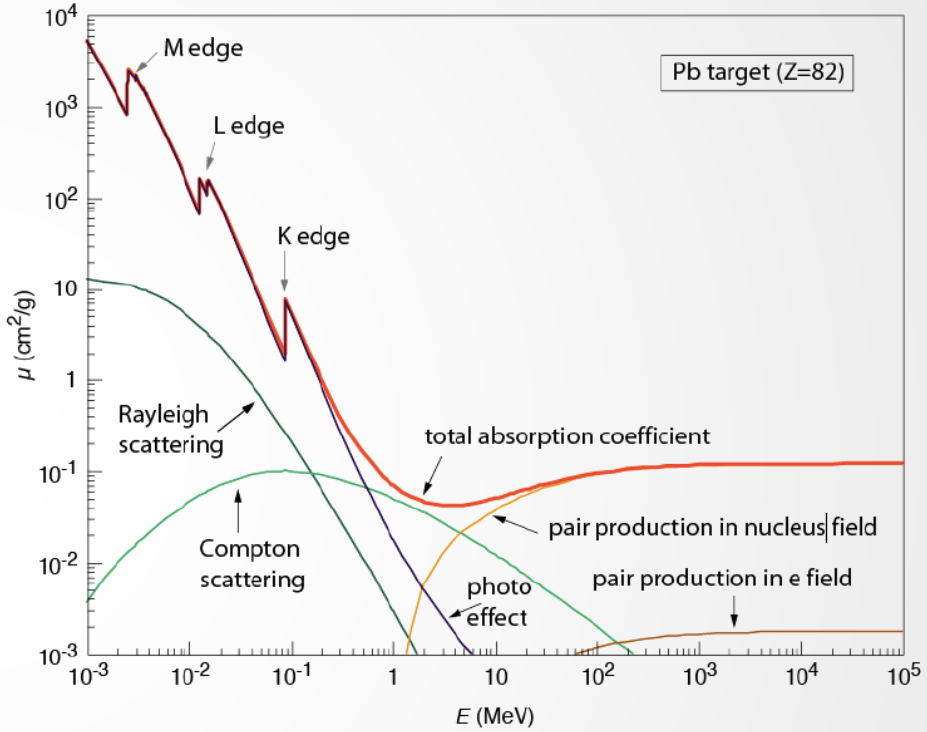
- Photo-effect increases when the energy passes the energy thresholds of nuclear shells (K,L,M,..) (there is always a smaller contribution from elastic coherent Rayleigh scattering on the shell)
- The Compton effect and pair production dominate ≥ 1MeV ≈ $2m_ec^2$



pair creation

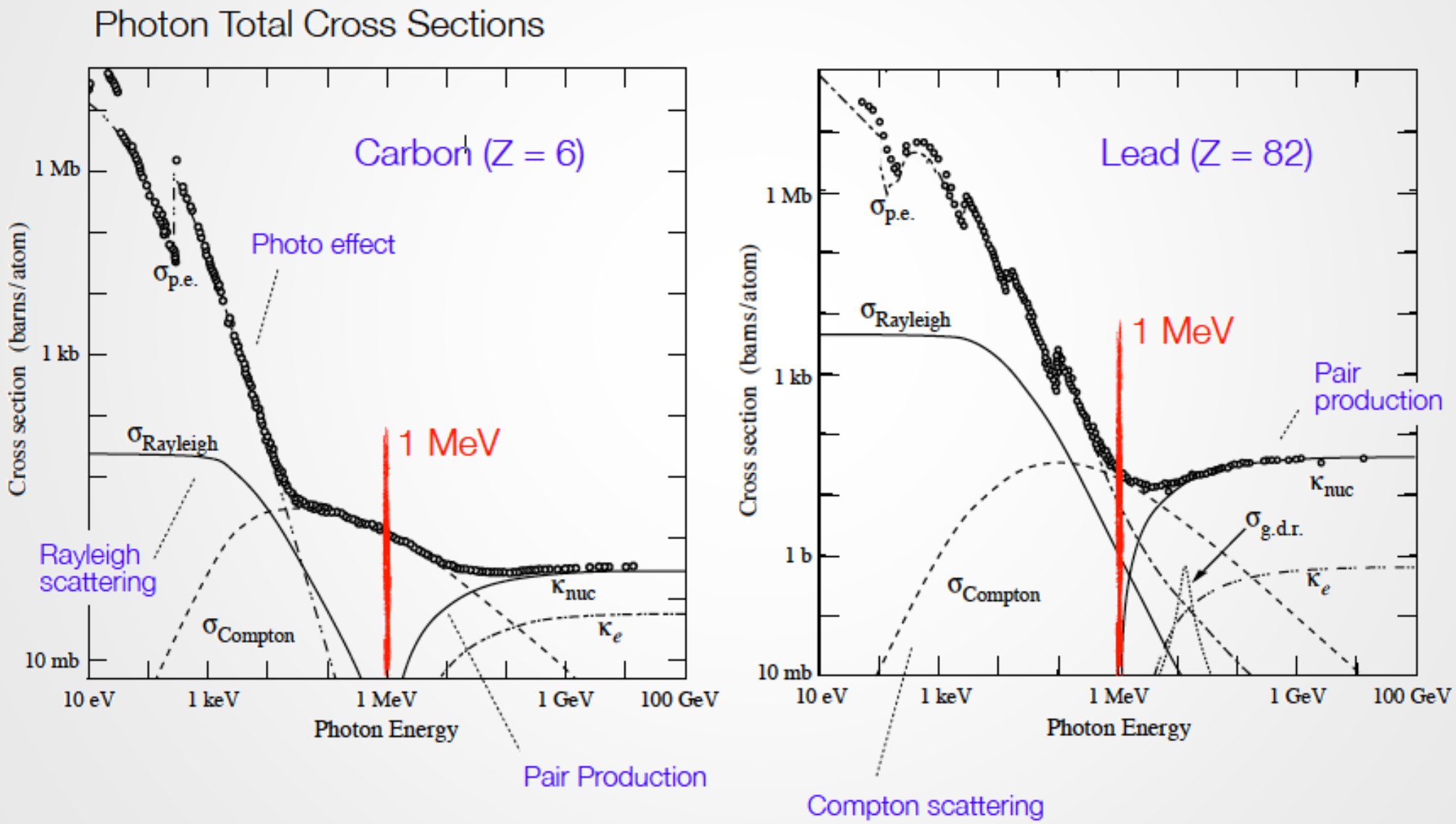


electron bremstrahlung



- Pair creation(E>1.022 MeV) and electron bremsstrahlung, the two dominant processes at high energies, have the same (leading-order) matrix element and are thus related by $\lambda_\gamma = \frac{9}{7} X_0$. When $e^\pm$ brem 1/e= 63%, γ loose 54%.

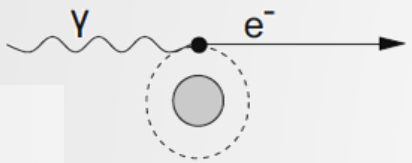
3.3 MATERIAL DEPENDENCE



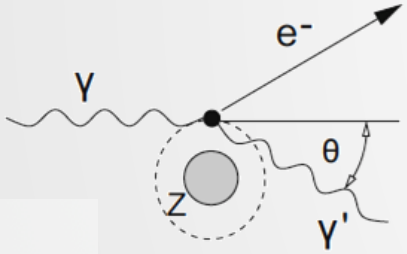
3.4 OVERVIEW OF ELECTROMAGNETIC INTERACTIONS

Photons

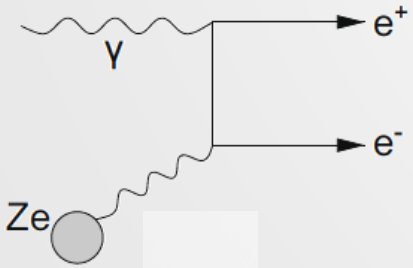
Photo effect ($\propto Z^5/E_\gamma^{3.5}$)

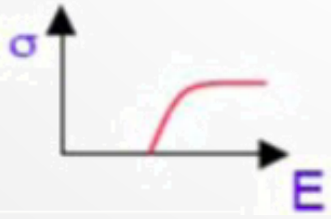


Compton effect ($\propto Z$)



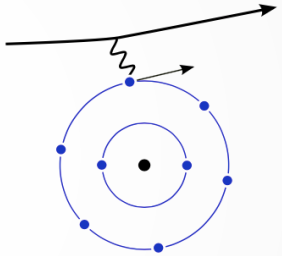
Pair creation ($\propto Z^2$)





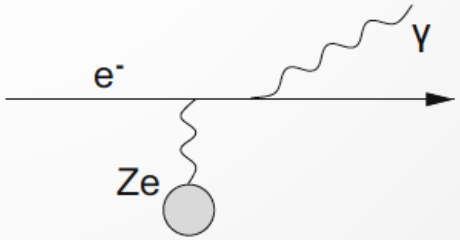
Electrons

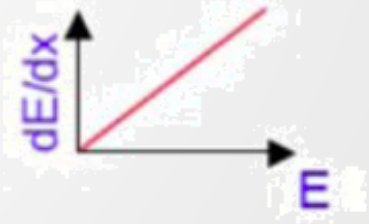
Ionization ($\propto Z$)



Bremsstrahlung ($\propto Z^2$)

small for all heavier charged particles

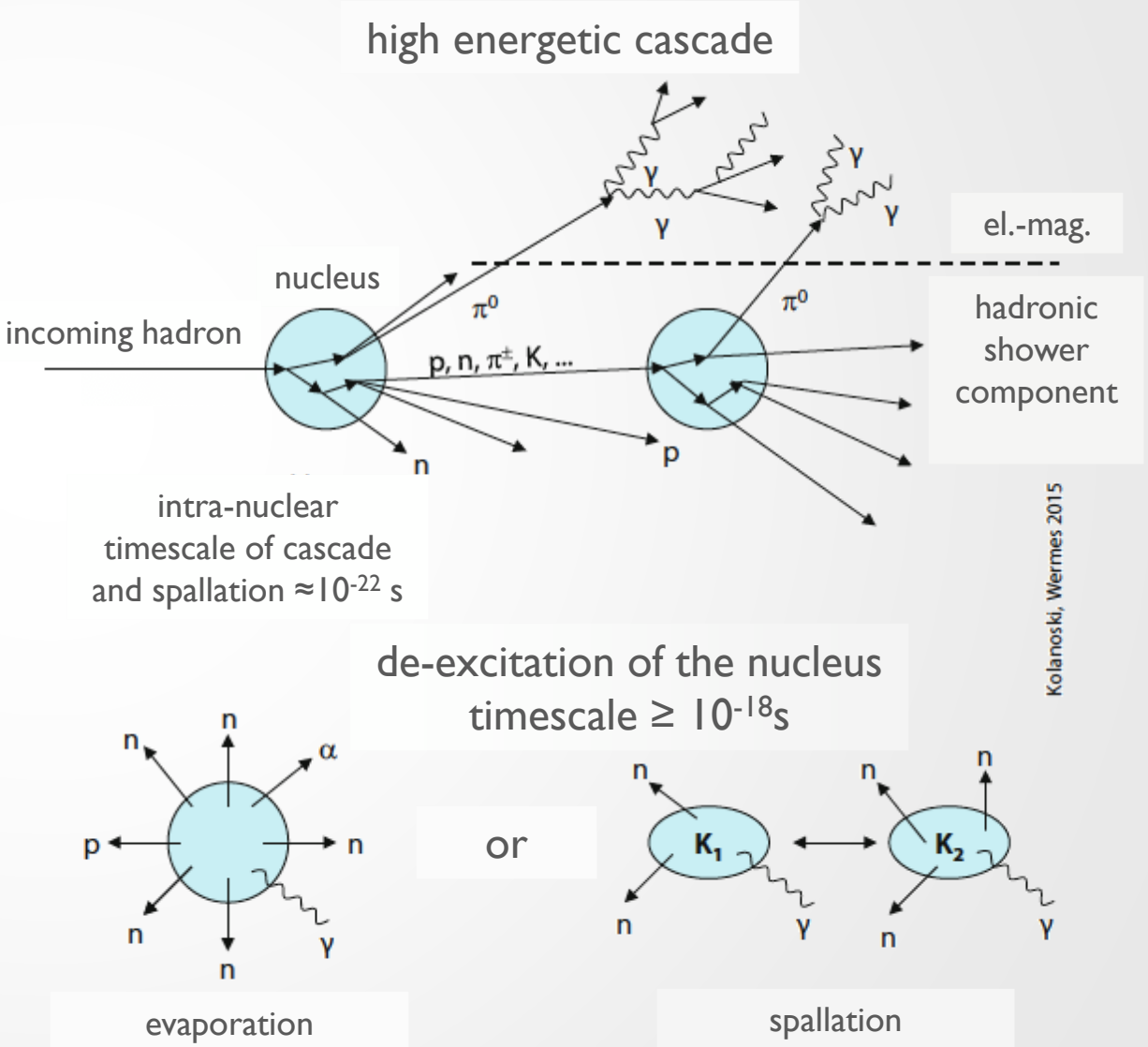




4. HADRONIC INTERACTIONS

4.1 OVERVIEW

- Hadrons interact inelastically with a single nuclei in the nucleus
- Secondary hadronic decay products interact with other nuclei, leading to shower formation.
- If neutral pions are produced, they decay to photons with subsequent electromagnetic showers.
- Between hadronic interactions, charged shower particle also ionize the material, leading to an EM cascade (also, nuclei can de-excite emitting γ)
- The fluctuation of the shower components is large and, depending on the charge and the momenta of the particles, each has a different reconstruction efficiency.
- This limits the resolution in reconstructing the energy of the incident particle.



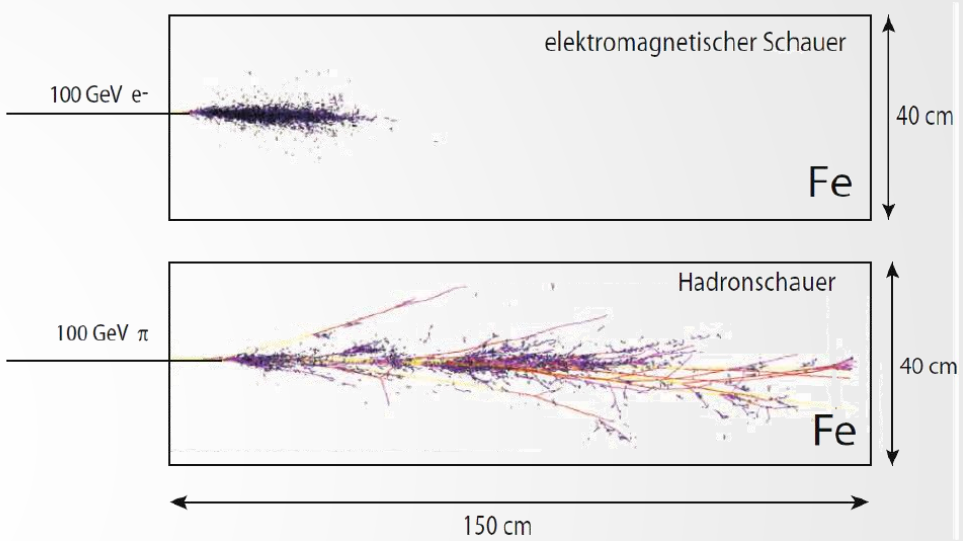
4.2 HADRONIC AND ELECTROMAGNETIC SHOWERS

- The hadronic interaction is stronger, but has a short range.
- Again, an exponential law is an excellent description

$$N(x) = N_0\, e^{-x/\lambda_a}$$ with the absorption length λ_a

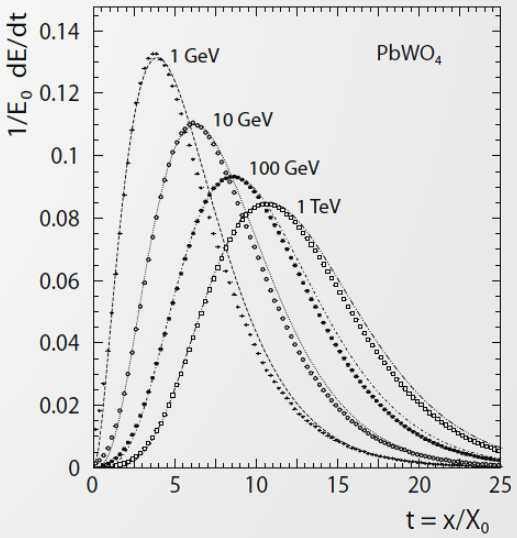
$$\lambda_a = \frac{A}{N_A\, \rho\, \sigma_{inel}} \approx 35\, \frac{\mathrm{g}}{\mathrm{cm}^2}\, \frac{A^{\frac{1}{3}}}{\rho} \propto A^{-\frac{2}{3}}$$

(assuming $\rho \propto A$)



- The processes for energy loss of photons and electrons (bremsstrahlung, pair production) lead to electromagnetic shower development, however, at a shorter length scale.
- The shower depth (electromagnetic and hadronic) grows logarithmically with energy.
- Neutral hadrons (e.g. neutrons) therefore penetrate deeply.

electromagnetic shower in P_bWO_4



4.3 MATERIAL DEPENDENCE

- It is beneficial to have similar response to EM and hadronic showers ('compensating')
- Low-Z materials have a lower (benefitial) ratio λ_a/X_0.

Material	Z	ρ (g/cm^3)	X_0 (cm)	A	λ_a (cm)	λ_a/X_0	$dE/dx\|_{min}$ (MeV/cm)
H_2O	1,8	1.00	36.1	18	83.3	2.3	1.99
Luft	7,8	$1.205 \cdot 10^{-3}$	$3.0 \cdot 10^4$	14.3	$7.5 \cdot 10^4$	2.5	$2.19 \cdot 10^{-3}$
Be	4	1.85	35.3	9	42.1	1.2	2.95
C	6	2.21	19.3	12	38.8	2.0	3.85
Al	13	2.70	8.9	27	39.7	4.5	4.36
Fe	26	7.87	1.76	56	16.8	9.5	11.42
Cu	29	8.96	1.43	64	15.3	10.7	12.57
W	74	19.30	0.35	184	9.9	28.3	22.10
Pb	82	11.35	0.56	207	17.6	31.4	12.73
U	92	18.95	0.32	238	11.0	34.4	20.48

THE END!