Numerical Optimization of Electromagnet Current Distribution in Superconducting Linear Acceleration System

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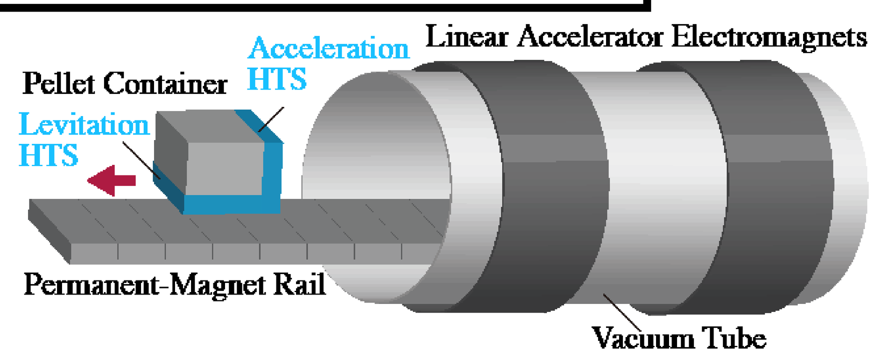
I. Introduction

Background

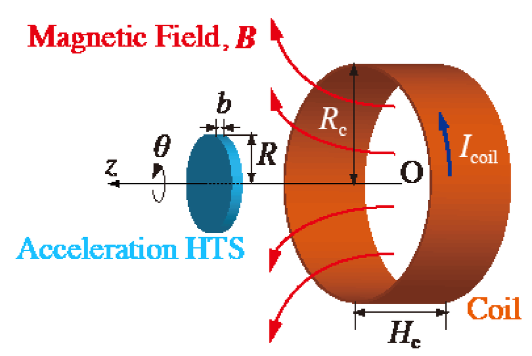
Superconducting Linear Acceleration (SLA) System for Pellet Injection

Conventional systems, a pneumatic pellet and a centrifuge pellet injections, inject ice pellets of frozen hydrogen gas into the fusion reactor at the velocity of 1-1.5 km/s.



In order to inject the ice pellets into the plasma core, Yanagi et al. recently propose a novel pellet injection system. This system electromagnetically accelerates the ice pellets on the magnetic levitation train. They adopt two types of high-temperature superconducting (HTS) film for acceleration and levitation.

⇒ They estimate 5-10 km/s as the velocity of the pellet injection.



Purpose

(1) We implement the NSGA-II(non-dominated sorting genetic algorithm II) of the multi-objective optimization in the FEM code for analyzing the shielding current density in an HTS film.

(2) We improve the pellet speed by using the NGnet (normalized Gaussian network) method as the shape optimization of current distribution in the electromagnet.

II. Governing Equation and Equation of Motion

Shielding Current Density in HTS Sample

$\boldsymbol{j} = (2/b)\nabla S \times \boldsymbol{e}_z$ (1) Here, $S(r,t)$: scalar function, b : thickness.

Integro-Differential Equations

$$\mu_0 \partial_t \int_0^R Q(r,r')S(r',t)r'dr' + \frac{2}{b}S = -\partial_t \langle B_z \rangle - \frac{1}{r}\partial_t (rE_\theta). \quad (2)$$

Here, $\boldsymbol{B}$: applied magnetic field by an electromagnet, $\langle\ \rangle$: average operator over the HTS thickness, $\boldsymbol{E}$: electric field.

J-E Constitutive Equation (Power Law)

$$\boldsymbol{E} = E(|\boldsymbol{j}|)\frac{\boldsymbol{j}}{|\boldsymbol{j}|}, \quad (3a) \qquad E(j) = E_C\left(\frac{j}{j_C}\right)^N. \quad (3a)$$

Here, j_C : critical current density, E_C : critical electric field, N : index.

Newton's law of motion

$$\frac{d^2 Z}{dt^2} = \frac{4\pi}{m}\int_0^R \frac{\partial S}{\partial r}\langle B_r \rangle r dr. \quad (4)$$

Here, m : total mass of the pellet container, $B_r(r,z)$: r-component of an applied magnetic flux density $\boldsymbol{B}$.

Initial and Boundary Conditions

$$S(r,0) = v = 0 \quad \text{at} \quad t = 0 \quad (5a)$$
$$Z = Z_0 \quad \text{at} \quad t = 0 \quad (5b)$$
$$S(R,t) = 0 \quad (6)$$

Integro-Differential Equations

$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{S} \\ v \\ Z \end{bmatrix} = \begin{bmatrix} -W^{-1}U[\boldsymbol{e}(\boldsymbol{S}) + v\boldsymbol{c}(Z) + \boldsymbol{h}(Z)] \\ \frac{4\pi}{m}\boldsymbol{a}^T(Z)\boldsymbol{S} \\ v \end{bmatrix} \quad (7)$$

Here, $W \equiv UW^*U + F$ (8)

W^*: symmetric matrix by function Q and FEM's shape functions,

U, F : matrix determined from the boundary condition,

$\boldsymbol{S}$: nodal vector corresponding to the scalar function $S(r,t)$,

$\boldsymbol{e}(\boldsymbol{S})$: nodal vector corresponding the electric field $\boldsymbol{E}$,

$\boldsymbol{a}(Z), \boldsymbol{c}(Z), \boldsymbol{h}(Z)$: nodal vector corresponding to the applied magnetic flux density $\boldsymbol{B}$.

ODEs (7) are solved by the Runge-Kutta method with the adaptive stepsize control.

Parameters

R = 5 cm, b = 1 mm, m = 10 g, Z_0 = 1 mm, N = 20, R_1 = 5 cm, R_2 = 7 cm, L = 10 cm, E_C = 1 mV/m, j_C = 1 MA/cm², n = 101.

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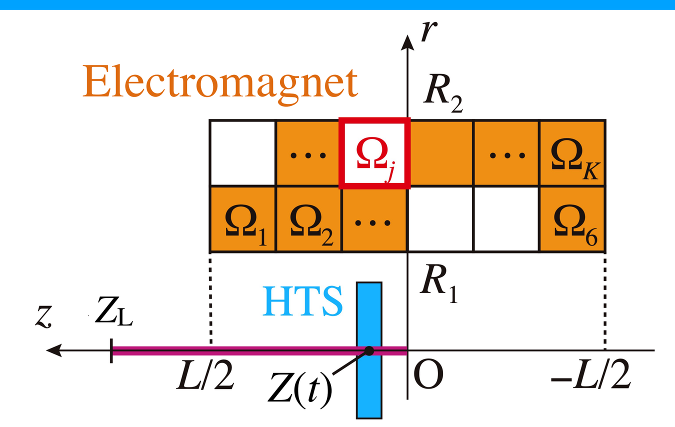
III. Simulation of SLA System

Filaments



Schematic view of the electromagnet cross-section. Here, ■: on-state, □: off-state, In addition, the on/off states in the number K of the filaments:

$$x_j \equiv \begin{cases} 1 & (\Omega_j : \text{on} - \text{state}) \\ 0 & (\Omega_j : \text{off} - \text{state}) \end{cases} \quad (j = 1, 2, \cdots, K) \quad (10)$$

Currents

$$I_j(t,Z) \equiv \begin{cases} (\alpha t/K)x_j & (0 \le Z \le Z_L) \\ 0 & (\text{otherwise}) \end{cases} \quad (j = 1, 2, \cdots, K) \quad (11)$$

Here, α: the increasing rate of the current, Z_L: the limit of acceleration range.

NGnet Method

$$\Omega_j \leftarrow \begin{cases} \text{on} & (g(\boldsymbol{c}_j) \ge 0) \\ \text{off} & (g(\boldsymbol{c}_j) < 0) \end{cases} \quad (j = 1, 2, \cdots, K) \quad (12)$$

$$g(\boldsymbol{y}) = \sum_{j=1}^{M} w_j b_j(\boldsymbol{y}) \quad (13)$$

$$b_j(\boldsymbol{y}) = G_j(\boldsymbol{y}) / \sum_{k=1}^{M} G_k(\boldsymbol{y}) \quad (14)$$

$$G_j(\boldsymbol{y}) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{|\boldsymbol{y} - \boldsymbol{\mu}_j|^2}{2\sigma^2}\right) \quad (15)$$

Here, σ : the positive constant value,

$\boldsymbol{c}_j$: the center position of the jth filament,

$\boldsymbol{\mu}_j$: the center position of the function G_j.

M : the number of the 2D Gaussian function G_j.

Objective functions

$$\begin{cases} \text{Maximize} : f_v(\boldsymbol{x}) \equiv v_o(\boldsymbol{x})/v_h \\ \text{Maximize} : f_u(\boldsymbol{x}) \equiv U_{out}(\boldsymbol{x})/U_{in}(\boldsymbol{x}) \end{cases} \quad (16)$$

Here, v_o : the final velocity for the optimization,

v_h : the final velocity for the homogeneous current distribution,

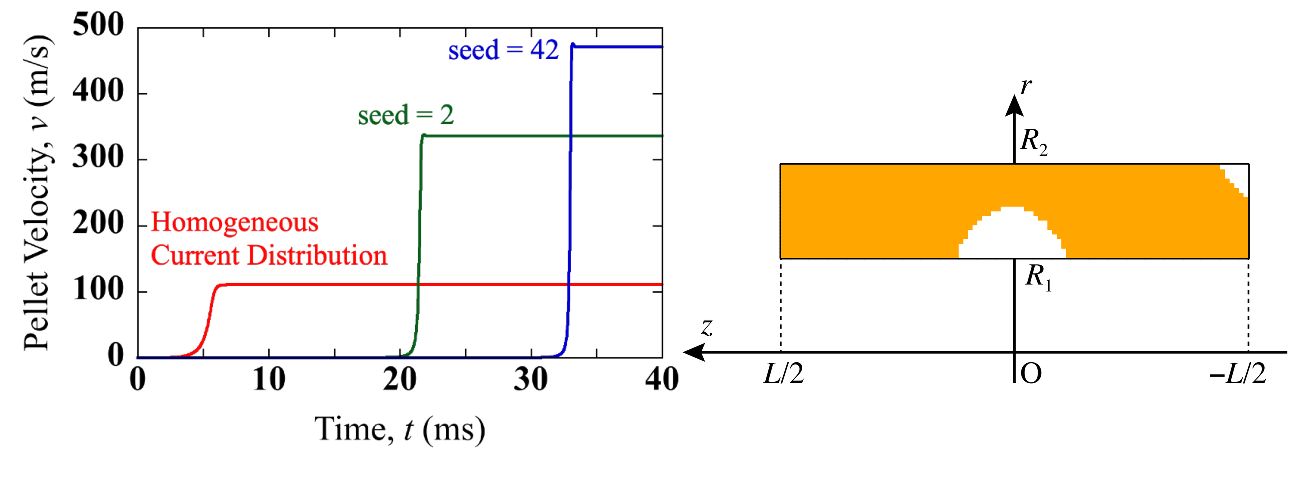
U_{out} : the output energy for the SLA system,

U_{in} : the input energy for the SLA system.

Parameters for multi-objective optimization problem

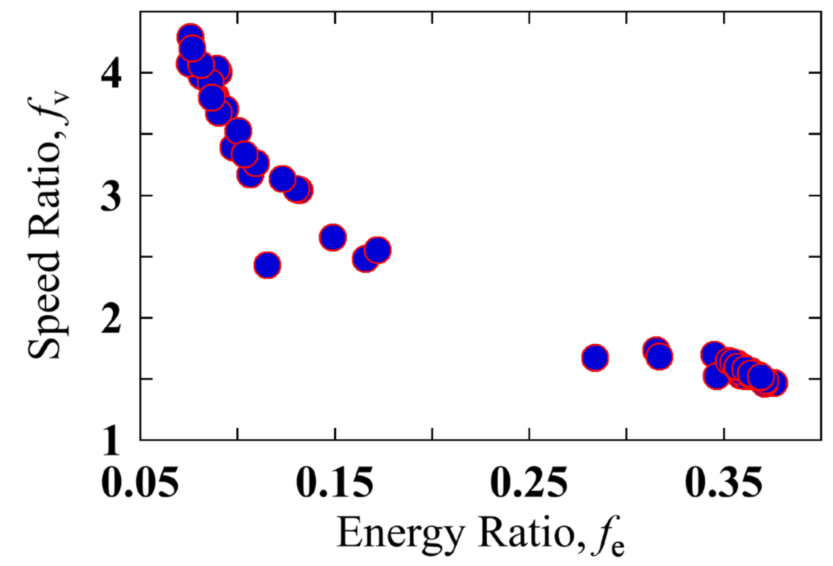
NGnet method	Number of division for r-direction	N_r	20
	Number of division for z-directon	N_z	100
	Number of filaments	K	2000
	Number of Gaussian functions	M	180
	Constant value	σ	m
NSGA-II	Number of populations	N_P	50
	Number of generations	N_G	500
	Value of seed		0-100

Pellet velocity and shape of electromagnet



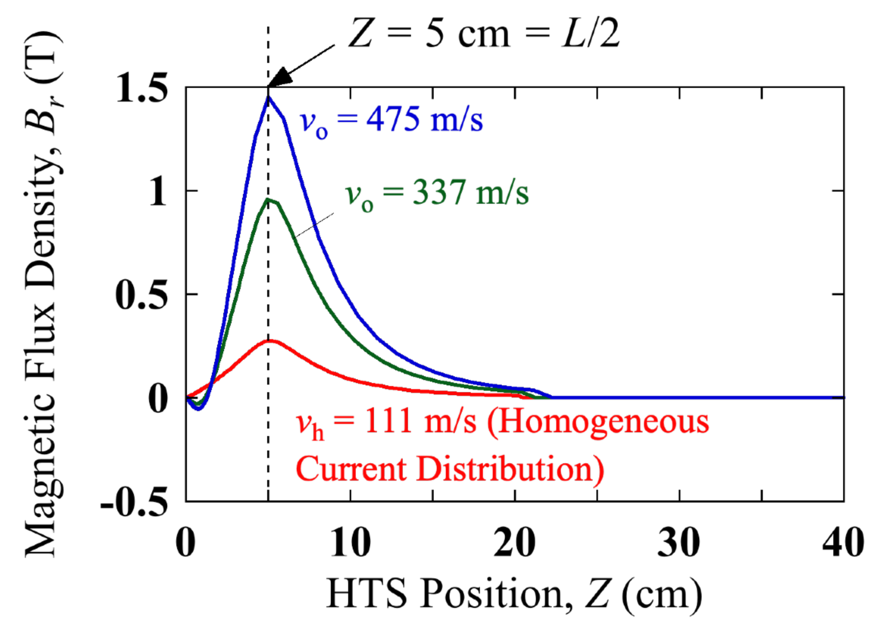
(a) Time dependences of the pellet velocity for the homogeneous current and optimized current distributions. (b) the current distribution in the electromagnet for the case where the velocity is v_o = 475 m/s. Here, ■: on-state, □: off-state.

Relationship between pellet speed and energy efficiency



Dependence of the speed ratio f_v on the energy ratio f_e

Applied magnetic flux density



Dependence of the electromagnetic flux density on the HTS

IV. Conclusion

By using the NGnet method and the NSGA-II, the shape optimization of the electromagnet can be obtained.

(1) The pellet speed increases significantly compared with the use of the homogeneous current distribution in the magnet. The pellet velocity is increased by about 4.8 times.

(2) The applied magnetic flux density becomes the maximum at the edge ($z = L/2$) of the magnet without depending on the shape of the magnet.

(3) The pellet speed and the energy efficiency are a trade-off relation.