Abstract
The J-PARC muon g-2/EDM experiment (E34) will use an unprecedented technique, the Three-Dimensional Spiral Injection Scheme, to inject the beam into a 66 cm diameter storage orbit, which is about 1/20th of the previous experiment (BNL-E821). In order to establish the injection scheme, we are currently demonstrating the 3-D spiral injection on an electron gun test bench. In this scheme, the beam is injected at an angle into the solenoidal storage magnet. The vertical beam motion along the solenoid axis is controlled by a pulse kicker of a few 100 ns time duration. Once the beam is guided near the storage plane, a weak focusing magnetic field holds the beam in betatron oscillations and stores it in an almost two-dimensional plane without using an electric field. Therefore, a stable and accurate control of the pulse kicker and the weak focusing magnetic field is one of the major technical challenges to realize this experiment. The tuning of the parameters of the weak focusing magnetic field (static field) and the kicker magnetic field (dynamic field) is very coupled, and the mutual adjustment of these parameters is essential for the beam storage.
In this presentation, we discuss the results and consideration of our study of the best solution for the time range of kicker timing by changing the parameters of the weak focusing magnetic field and the kicker magnetic field. In addition, we will discuss the difference in the pitch angle acceptance when changing the weak focusing field flux.
The J-PARC muon g-2/EDM experiment (E34) will use an unprecedented technique, the Three-Dimensional Spiral Injection Scheme, to inject the beam into a 66 cm diameter storage orbit, which is about 1/20th of the previous experiment (BNL-E821).

**How do we store the beam?**

1. The beam is injected into the storage solenoid magnet moves in a spiral motion using the fringe field of the solenoid magnet.
2. Create a radial kicker magnetic field to give a vertical kick (Vertical force according to Fleming’s left-hand rule).
3. This vertical kick guides the beam near the storage plane.
4. A weak focusing magnetic field holds the beam in Betatron oscillation and stores it in an almost two-dimensional plane without using an electric field.

> For the success of this injection, the generation of the kicker field that gives this vertical kick is essential and giving the accurate kick and holding it in the weak focusing magnetic field is directly related to the high injection efficiency of the beam.
In order to establish the injection method, we will demonstrate the 3-D spiral injection on an electron gun test bench which is a scaled-down version of the production experiment.

In the future, we will install the pulse magnetic field kicker device in the storage magnet.

I am mainly studying this kicker device and storage magnet.

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**Schematic diagram of the electron gun test bench at KEK**

- **Electron gun**
- **80 keV electron beam**
- **Collimator**
- **Quadrupole magnet**
- **Bending magnet**
- **Kicker coil**
- **Storage magnet**
- **Screen (Copper plate)**
- **CMOS camera**

**Prototype pulse magnetic field kicker device for our test bench**

**Table: Comparison between J-PARC(E34) and Electron gun test bench**

<table>
<thead>
<tr>
<th></th>
<th>J-PARC(E34)</th>
<th>Electron gun test bench</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Particle</strong></td>
<td>$\mu^+$</td>
<td>$e^-$</td>
</tr>
<tr>
<td><strong>Diameter of the storage orbit</strong></td>
<td>66 cm</td>
<td>24 cm</td>
</tr>
<tr>
<td><strong>Storage field flux</strong></td>
<td>3 T</td>
<td>82.5 Gauss</td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td>300 MeV/c</td>
<td>296.9 keV/c</td>
</tr>
<tr>
<td><strong>Cyclotron period</strong></td>
<td>7.4 ns</td>
<td>5.0 ns</td>
</tr>
</tbody>
</table>
Pulse magnetic field kicker device

- **Principle of Operation**
  - Create a radial magnetic field by applying a pulse current in the reverse direction to one turn coils symmetrical above and below the storage plane.

- **Advantages**
  - The pulse magnetic field generation time can span multiple cyclotron cycles.
  - The solenoid axial magnetic field is a canceling magnetic field distribution.
  - Vertical kick can be given without disturbing the orbital center because the kick field is axisymmetric.

![Image of the kicker coil creating the kicker magnetic field](image-url)
Generation and distribution of weak focusing magnetic field

- Our storage solenoid magnet can create the weak focusing magnetic field around the storage plane by changing the current balance between the main coil and the auxiliary coil.
  - The beam can be held by betatron oscillation near the storage plane without using an electric field.
  - In this study, we consider three different models of weak focusing magnetic field: "none", "medium", and "strong".

<table>
<thead>
<tr>
<th>Model</th>
<th>Main Current (A)</th>
<th>Auxiliary Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>8.14</td>
<td>4.40</td>
</tr>
<tr>
<td>Medium</td>
<td>8.76</td>
<td>6.84</td>
</tr>
<tr>
<td>Strong</td>
<td>9.35</td>
<td>9.16</td>
</tr>
</tbody>
</table>

Photo of the solenoid coil inside the storage magnet
Change the weak focusing magnetic field flux

- **Effect of different weak focusing magnetic field distributions on the pitch angle acceptance**

  - The stronger the weak focusing magnetic field is, the larger the pitch angle acceptance that can be stored.
  - For the multi-particle model, the pitch angle alignment of the stored particles can be changed by tuning the weak focusing magnetic field.

  → The balance of injection efficiency needs to be considered.

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**Examples of pitch angles for each weak focusing magnetic field model**

- None
- Medium
- Strong

Range of weak focus (medium)
Range of weak focus (Strong)

Examples of pitch angles for each weak focusing magnetic field model:
Based on the specifications of the pulse power supply, the actual kicker device's current waveform signal was acquired by CT.

- The pulse power supply alone → **Confirmed to meet the specification.**
- The pulse power supply + kicker coil → **The fall time is 100 ns, which is different from the specification.**

### Pulse current source

**Made by PPJ**

<table>
<thead>
<tr>
<th>Item</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current rise time</td>
<td>25 ns</td>
</tr>
<tr>
<td>Current fall time</td>
<td>50 ns</td>
</tr>
<tr>
<td>Peak current</td>
<td>40 A (20 A + 20 A)</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>5 kV</td>
</tr>
<tr>
<td>Load inductance</td>
<td>1.5 μH</td>
</tr>
<tr>
<td>Repetition</td>
<td>50 Hz</td>
</tr>
</tbody>
</table>

Duration time is long due to the pulse power supply limitations.

→ How much does this change the kicker timing?
→ Conduct beam orbit calculations with a waveform close to the actual current waveform.
Assume a triangular wave and “base” for beam orbit calculation

- As shown in the figure below, I defined a triangular wave that is close to the real waveform.
- Use the parameters in the figure as a "base" for discussion.
- The time at the injection point is set to $t_0 = 0$ ns.
- Parameters that can be changed: weak focusing magnetic field flux ("Medium" ⇔ "Strong"), pitch angle, peak current, kicker coil position

The triangular wave is defined as:

$$f(t) = \begin{cases} 
\frac{1}{T_1} t & (0 \leq t < T_1) \\
-\frac{1}{T_2 - T_1} (t - T_1)(T_1 \leq t < T_2) 
\end{cases}$$

Rise time : 20 [ns] \hspace{1cm} Fall time : 100 [ns]

Duration time : 120 [ns]

Kicker coil
- Pulse current : $I(t) = I_0 f(t)$
- Peak current : $I_0 = \pm 40$ [A]
- Coil radius : 10.25 cm

Injection point
- $(x_0 [\text{cm}], y_0 [\text{cm}], z_0 [\text{cm}]) = (-10.88, 11.71, -35.20)$
- Pitch [mrad] = 621.17

Kicker coil position $z = \pm 15$ cm

Storage plane
Study of appropriate kicker timing

While changing the parameters from the "base", I checked the time range of kicker timing that could accumulate one electron with the assumed triangle wave.

- **Change the weak focusing magnetic field flux**
  
  If we change the model of the weak focusing magnetic field, it could not be stored well even if it was incident at the same pitch angle. ("base" : pitch = 621.17 [mrad])

  → While changing the pitch angle, I checked the time range of kicker timing that could be stored.

- **Results**
  
  - Kick in flying (t₀<0) and use the falling time part of the waveform.
  - Changing the model of the weak focusing magnetic field requires changing the kick start timing and pitch angle accordingly, but **we were able to find the best solution with a bunch width acceptance of about 80 ns in each case. (The current expected bunch width of the electron beam is 60 ns.)**
• **Summary**

  At the test bench of KEK, we are conducting a demonstration experiment to establish the three-dimensional spiral injection scheme, which is one of the key technologies for the J-PARC Muon g-2/EDM Experiment.

  In our storage magnet, we can apply a weak focusing field near the storage plane. We studied how the pitch angle changes when the weak focusing magnetic field flux changes. As a result, it is found that **the stronger the weak focusing magnetic field is the larger the pitch angle acceptance that can be stored**.

  We assumed a triangular wave that is closer to the real waveform. Based on the assumption that the triangle wave and the actual data would not be significantly different, we explored how the kicker timing would change by changing each parameter.

  ➢ **In order to increase the time range of the kicker timing, it is better to use the falling time part of the waveform by kicking in flying.**

  ➢ **When the weak focusing magnetic field flux changes, the best pitch angle changes too. However, the best acceptance of the kicker timing was found to be about 80 ns in each of the weak focusing magnetic field models.**

• **Future study**

  We used peak current and kicker coil position as the "base" parameters, but we will also change these parameters to determine the best kicker parameters.

  It will be discussed in the multi-particle model, and **the balance between the weak focusing magnetic field flux and the injection efficiency** will be considered.

  **Install the kicker device on the electron gun test bench and run it as a system.**