Formulation for the reluctance of the wide air gap

No magnetic flux spread in the air gap, ➔ the reluctance can be calculated using air gap space volume.

Magnetic flux will spread in the wide air gap, ➔ the reluctance calculation becomes difficult.
A simple equivalent model of the electromagnet

Electromagnet with a wide air gap

A infinite length solenoid with a wide air gap

A infinite length solenoid with a reverse direction finite length solenoid
Magnetic flux distribution of the finite solenoid

Self-inductance of the finite solenoid can be calculated using the Nagaoka coefficient $K_n$

$$L = K_n \frac{\mu_0 S}{d} N^2 = K_n \frac{1}{\mathcal{R}_{in}} N^2 = \frac{N^2}{\mathcal{R}_{in} + \mathcal{R}_{out}}$$

Assuming the leakage flux can be neglected:

$$L = \frac{N^2}{\mathcal{R}_{in} + \mathcal{R}_{out}}$$

$$\mathcal{R}_{out} = \frac{1 - K_n}{K_n} \mathcal{R}_{in}$$
Magnetic Flux distribution in a Wide air gap

Wide air gap

Magnetic flux

Outside space of solenoid

Part II

Part I

Inside space of solenoid

Equivalent magnetic circuit

\[ \phi_m \]

\[ \phi_{in} \]

\[ \phi_{out} \]

\[ R_{in} \]

\[ R_{out} \]

The magnetic flux are the same direction in part I and part II

Parallel connection

\[ \frac{1}{R_{air}} = \frac{1}{R_{in}} + \frac{1}{R_{out}} \]

The reluctance of the wide air gap

\[ R_{air} = (1 - K_n) R_{in} = (1 - K_n) \frac{d}{\mu_0 S} \]
**Self-inductance of electromagnet**

**Electromagnet with a wide air gap**

**Equivalent magnetic circuit**

\[
\mathcal{R}_{\text{air}} = (1 - K_n) \frac{d}{\mu_0 S} \quad (5)
\]

\[
L = \frac{N^2}{\mathcal{R}_{\text{core}} + \mathcal{R}_{\text{air}}} \quad (6)
\]
Cooling system design for real-scale electromagnet

Base on the thermal equilibrium equation:

\[ U = (P_C + P_m) \cdot t_{\text{op}} = U_c + U_m + U_r \]

\[ = \int_{T_0}^{T_{\text{max}}} m_c C_c + m_m C_m \, dT + \int_{T_0}^{T_r} Q t_{\text{op}} \rho_r C_r \, dT \]

- \( U_c \): specific heat capacity of coil windings
- \( U_m \): specific heat capacity of magnetic core
- \( U_r \): endothermic energy of coolant

- \( m_c C_c \): mass and specific heat capacity of coil winding
- \( m_m C_m \): mass and specific heat capacity of magnetic core
- \( T_{\text{max}} - T_0 \): temp. rise of electromagnet
- \( Q \): coolant volumetric flow rate
- \( \rho_r C_r \): liquid density and specific heat capacity of coolant
- \( T_r - T_0 \): temp. rise of coolant

\[ Q = \frac{(P_C + P_m)t_{\text{op}} - (m_c C_c + m_m C_m)(T_{\text{max}} - T_0)}{t_{\text{op}} \rho C_r (T_r - T_0)} \]
3M Fluorinert Electronic Liquid FC-40

It is necessary that use a low dielectric constant coolant to reduce the effect on the self-inductance of the electromagnet because the electromagnet is operated at high-frequency.

<table>
<thead>
<tr>
<th></th>
<th>FC-40 Fluorinert</th>
<th>Pure water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiling point</td>
<td>165 °C</td>
<td>100 °C</td>
</tr>
<tr>
<td>Liquid density</td>
<td>1870 kg/m³</td>
<td>1000 kg/m³</td>
</tr>
<tr>
<td>Liquid specific heat</td>
<td>1050 J/kg · K</td>
<td>4217 J/kg · K</td>
</tr>
<tr>
<td>Dielectric constant</td>
<td>1.9</td>
<td>80</td>
</tr>
</tbody>
</table>

URL: https://www.3mae.ae/3M/en_AE/p/d/v000586117/#variation1
Cooling system design for real-scale electromagnet

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\text{op}}$</td>
<td>360 s</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>1855 kg/m³</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>8960 kg/m³</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>4800 kg/m³</td>
</tr>
<tr>
<td>$C_r$</td>
<td>1100 J/kg·K</td>
</tr>
<tr>
<td>$C_C$</td>
<td>386 J/kg·K</td>
</tr>
<tr>
<td>$C_m$</td>
<td>600 J/kg·K</td>
</tr>
<tr>
<td>$T_0$</td>
<td>25°C</td>
</tr>
<tr>
<td>$T_{\text{max}}$</td>
<td>75°C</td>
</tr>
<tr>
<td>$T_r$</td>
<td>50°C</td>
</tr>
<tr>
<td>$Q$</td>
<td>21.5 L/s</td>
</tr>
<tr>
<td>$Q \cdot t_{\text{op}}$</td>
<td>7731 L</td>
</tr>
</tbody>
</table>

Symbols:
- $t_{\text{op}}$: Operation time
- $\rho$: Density
- $C$: Heat capacity
- $T$: Temperature
- $Q$: Flow rate