The benefits of the analytical approach for the optimization of superconducting magnets from the early design stage

Young Scientist Plenary MT27 Conference, November 17th 2021

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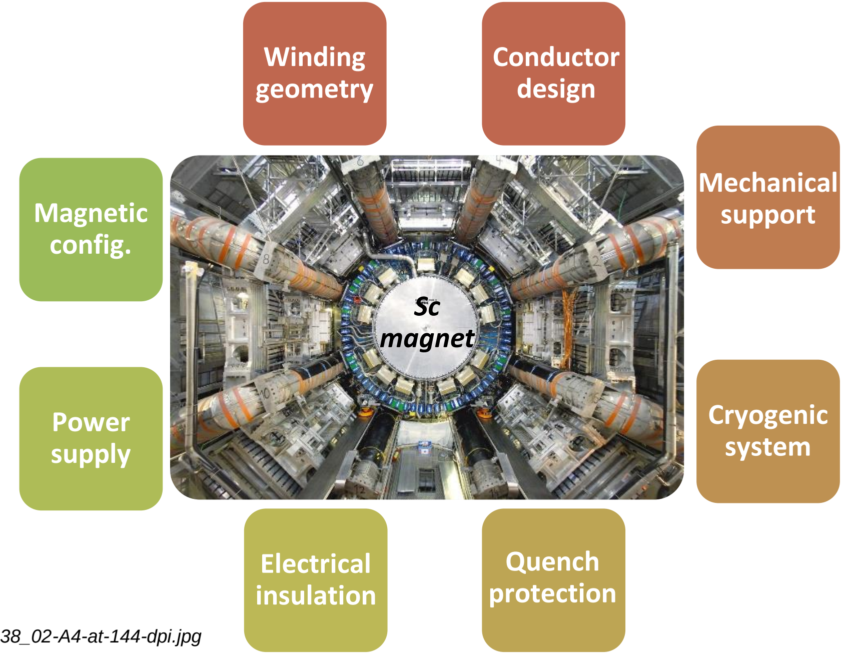
Superconducting magnet design method

- **Superconducting magnet**
  - Multiphysics
  - Multifunction
  - Different sizes & environments

complex object

- **Difficult to design since it requires different**
  - experts
  - technologies
  - design tools

- **To reduce complexity of design process**
  - Predesign phase is often limited to winding geometry definition from magnetic configuration requirements
  - The other systems are then usually treated more sequentially in the conceptual design phase



Winding geometry

Conductor design

Magnetic config.

Sc magnet

Mechanical support

Power supply

Cryogenic system

Electrical insulation

Quench protection

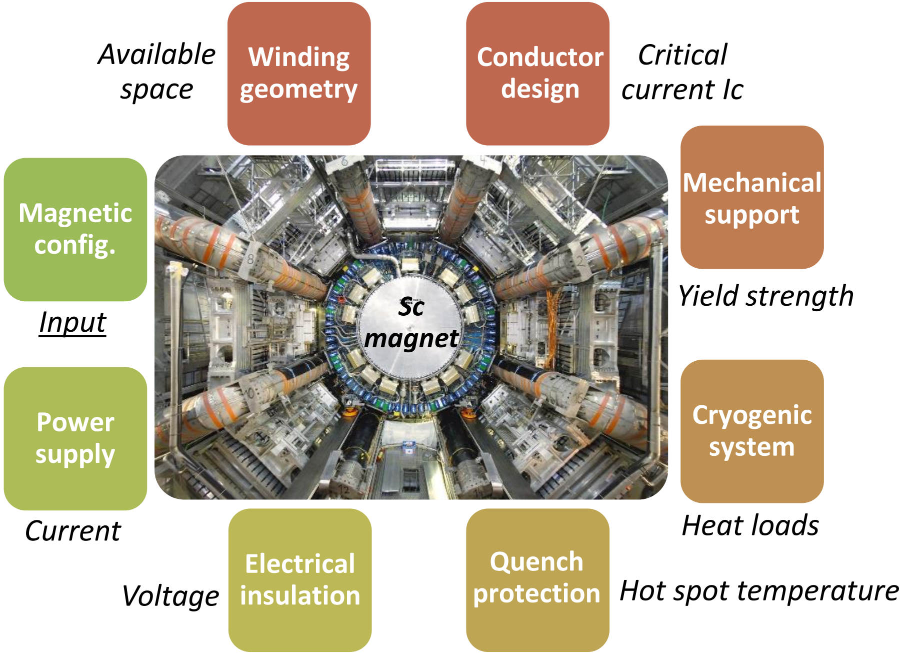
ATLAS BT picture source: https://mediaarchive.cern.ch/MediaArchive/Photo/Public/2007/0706038/0706038_02/0706038_02-A4-at-144-dpi.jpg

Superconducting magnet design method

- **During predesign**, a **more global approach including** different **systems** will lead to a **more optimized magnet** in terms of cost and performance

- Such an approach is **difficult** to **formulate and to interface** between experts/tools

- **An analytical approach at this stage allows to**
  - account for the *limiting effects* of different **systems in parallel** in a simplified yet **realistic/conservative** and **rapid** way
  - derive scaling laws of the magnet properties with respect to its parameters (e.g. peak field vs winding thickness)



Available space

Winding geometry

Conductor design

Critical current Ic

Mechanical support

Yield strength

Cryogenic system

Heat loads

Quench protection

Hot spot temperature

Electrical insulation

Voltage

Power supply

Current

Magnetic config.

Input

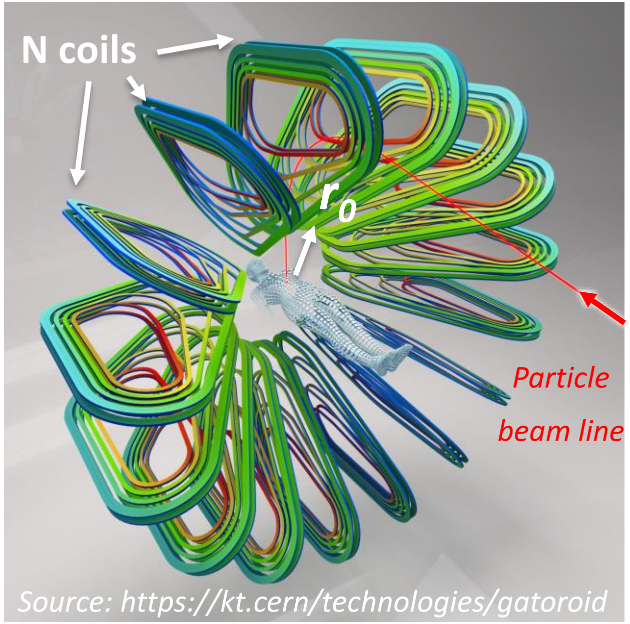
Sc magnet

Sc magnet design example : GaToroid

- **GaToroid[1,2] = Toroidal gantry for hadrontherapy**
  - $\boldsymbol{N}$ coils → number of treatment angles
  - Aperture $\boldsymbol{r_0}$ → free bore for patient

- **Stored energy $\boldsymbol{E}$** is a good indicator of magnet **cost and complexity** as it contains the information of forces and needs to be extracted by the quench protection system
  - Computed through $\iiint_{space} \frac{B^2}{2\mu_0} dV$ (magnetic configuration) or $\frac{1}{2} L I^2$ (winding geometry)
  - Needs to be **minimized** to reduce cost & complexity

- **Question** : how does $\boldsymbol{E}$ scale with $\boldsymbol{N}$ and $\boldsymbol{r_0}$?



Source: https://kt.cern/technologies/gatoroid

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[1] L. Bottura, *A Gantry and Apparatus for Focusing Beams of Charged Particles, European Patent Application EP 18173426.0, May 2018*

[2] L. Bottura, E. Felcini, G. De Rijk, B. Dutoit, *"GaToroid: A Novel Toroidal Gantry for Hadron Therapy", Nucl. Instrum. Methods Phys. Res. A, 2020*

Sc magnet design example : GaToroid

- **3D** geometry **too complex**, so stored energy $\boldsymbol{E}$ **analytical** formula derived from **simplified** 2D approach in toroidal plane

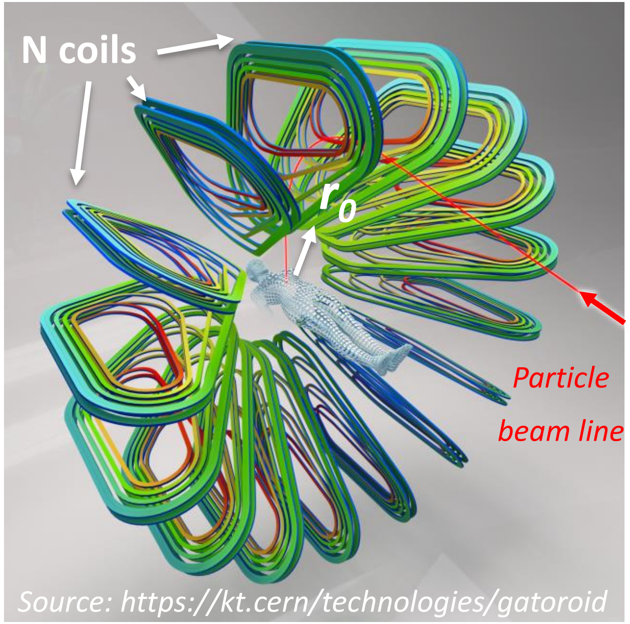
$$\boldsymbol{E} = \frac{\mu_0 (2\boldsymbol{N}I_0)^2 h}{4\pi}\left[ln\left(\frac{\boldsymbol{r_2}}{\boldsymbol{r_1}}\right) - \frac{2ln(2\boldsymbol{N})}{2\boldsymbol{N}} + \frac{1}{2\boldsymbol{N}} ln\left(\frac{\boldsymbol{r_1 r_2}}{r_c{}^2}\right) + \frac{2ln(2) - ln\left(1-\cos\left(\frac{2\boldsymbol{N}d}{\boldsymbol{r_1}}\right)\right) - ln\left(1-\cos\left(\frac{2\boldsymbol{N}d}{\boldsymbol{r_2}}\right)\right)}{4\boldsymbol{N}}\right]$$

$$with\ \boldsymbol{r_1} = \boldsymbol{r_0} + r_c + 0.15\ m, \boldsymbol{r_2} = \boldsymbol{r_1} - 2r_c + 2.296\ m, d = 0.137\ m, r_c = 0.128\ m, h = 1.75\ m$$

- **Answer to E scaling with $\boldsymbol{N}$ and $\boldsymbol{r_0}$:**

	Analytical formula				Numerical code			
		$r_0 = 1.5m$	$r_0 = 1.65m$	$r_0 = 2m$		$r_0 = 1.5m$	$r_0 = 1.65m$	$r_0 = 2m$
$\boldsymbol{E}$ scaling	$N = 16$	**0,92**	**1,03**	**1,27**	$N = 16$	**0,90**	**1,01**	**1,26**
	$N = 20$	**0,90**	**1,00**	**1,22**	$N = 20$	**0,89**	**1,00**	**1,24**
	$N = 24$	**0,90**	**0,99**	**1,20**	$N = 24$	**0,89**	**0,99**	**1,23**

→ $\boldsymbol{E}$ scaling law vs $\boldsymbol{N}$ and $\boldsymbol{r_0}$ derived from the simplified analytical formula agrees well with numerical results !



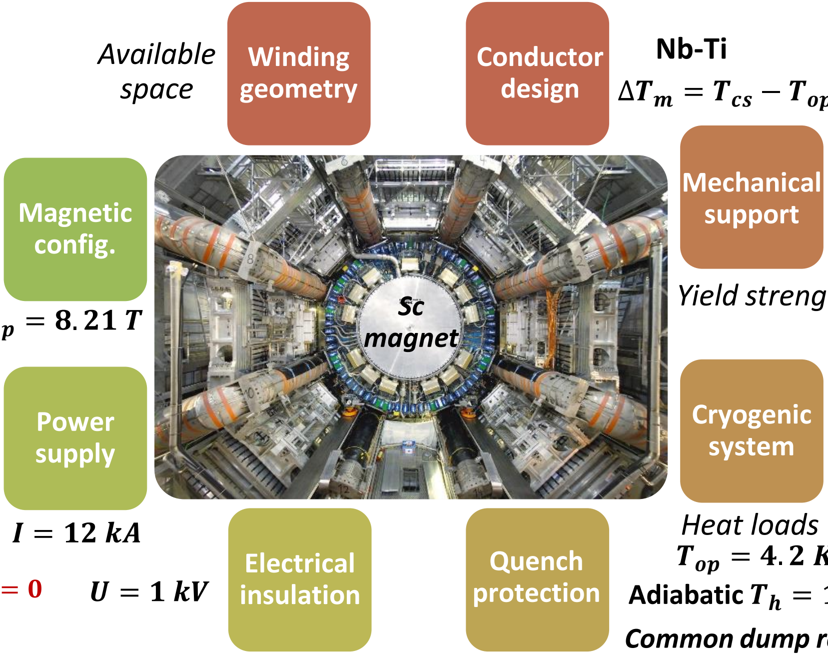
Source: https://kt.cern/technologies/gatoroid

Sc magnet design example : GaToroid

- **Global analytical approach** to account for ***limiting effects*** of **each system** and **optimize the magnet design**
  - **Winding geometry**
    - Stored magnetic energy: $E = \frac{1}{2}LI^2$
  - **Conductor design**
    - Stability: $J_{sc} = J_c(B_p, T_{op} + \Delta T_m)$
    - Geometry: $CuSc = \frac{A_{cu}}{A_{sc}}, f = \frac{A_{cu}+A_{sc}}{A_{cond}} = \frac{3}{4}$
    - Current: $I = J_0 A_{cond} = J_{sc}A_{sc} = J_{cu}A_{cu}$
    - Physical properties: $\mu C_p = \frac{f}{1+CuSc}(CuSc\mu_{cu}C_{p,cu} + \mu_{sc}C_{p,sc}), \rho = \frac{1+CuSc}{fCuSc}\rho_{cu}$
  - **Quench protection system**
    - Discharge on dump R_d time constant: $\tau = L/R_d$
    - Max voltage: $U = R_d I$
    - MIITs: $\int {J_0}^2 dt = \frac{\tau}{2}{J_0}^2 = \int_{T_{op}}^{T_h} \frac{\mu C_p}{\rho} dT : (2)$

$$\left.\begin{array}{l} \text{Stability} \\ \text{Geometry} \\ \text{Current} \end{array}\right\} \boldsymbol{J_0 = \frac{fJ_c(B_p, T_{op} + \Delta T_m)}{1 + CuSc}} : (1)$$

$$(1)\ \&\ (2): \boldsymbol{CuSc^2 \int_{T_{op}}^{T_h} \frac{\mu_{cu}C_{p,cu}}{\rho_{cu}} dT + CuSc \int_{T_{op}}^{T_h} \frac{\mu_{sc}C_{p,sc}}{\rho_{cu}} dT - \frac{E}{U_d I}{J_c}^2(B_p, T_{op} + \Delta T_m) = 0}$$



Available space

Winding geometry

Conductor design

Nb-Ti

$\boldsymbol{\Delta T_m = T_{cs} - T_{op} = 1K}$

Magnetic config.

Mechanical support

Yield strength

Sc magnet

$\boldsymbol{E = 540\ MJ, B_p = 8.21\ T}$

Power supply

Cryogenic system

$\boldsymbol{I = 12\ kA}$

$\boldsymbol{U = 1\ kV}$

Electrical insulation

Quench protection

Heat loads

$\boldsymbol{T_{op} = 4.2\ K}$

Adiabatic $\boldsymbol{T_h = 150\ K}$

Common dump resistor

Sc magnet design example : GaToroid

- **Global analytical approach** to account for **limiting effects** of **each system** and **optimize magnet**

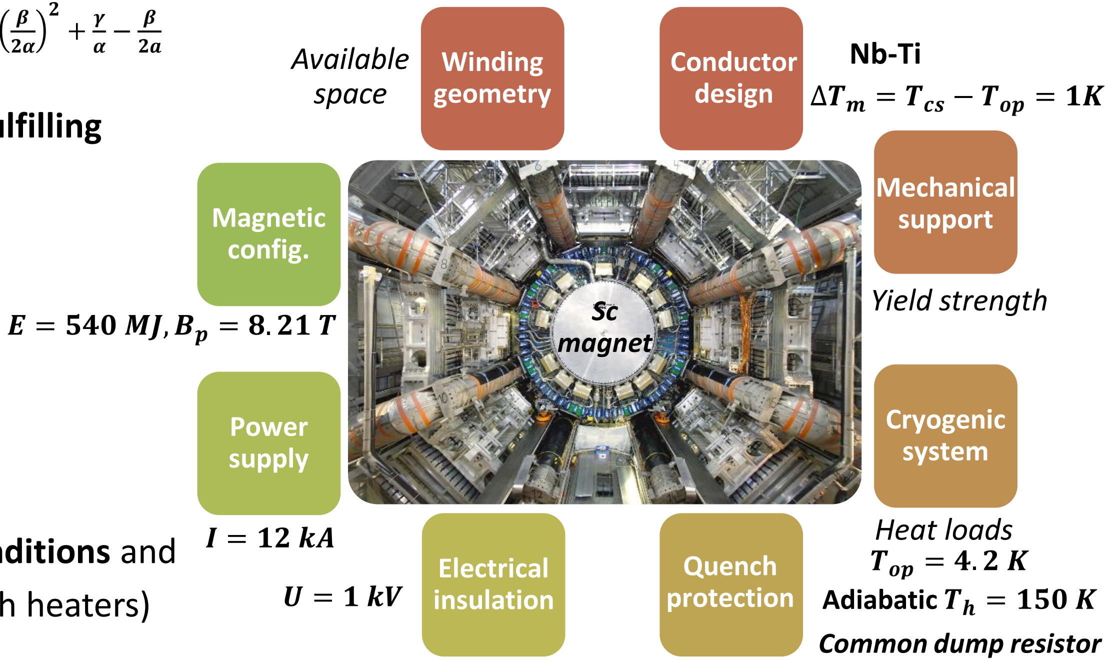
$$CuSc^2\int_{T_{op}}^{T_h}\frac{\mu_{cu}C_{p,cu}}{\rho_{cu}}dT+CuSc\int_{T_{op}}^{T_h}\frac{\mu_{sc}C_{p,sc}}{\rho_{cu}}dT-\frac{E}{U_dI}J_c{}^2(B_p,T_{op}+\Delta T_m)=0$$

Solution: $\alpha CuSc^2+\beta CuScb-\gamma=0$ → $CuSc=\sqrt{\left(\frac{\beta}{2\alpha}\right)^2+\frac{\gamma}{\alpha}}-\frac{\beta}{2a}$

- **Parameters** ensuring that all **systems** are **fulfilling** the defined **criteria**:
  - Copper to superconducting ratio $CuSc=7.87$
  - Engineering current density $J_0=29\ A/mm^2$
  - Conductor area $A_{cond}=414\ mm^2$
  - Discharge time constant $\tau=90\ s$

→ **Magnet is optimized at predesign stage**

- Exercise **can be repeated** with **different conditions** and **different quench protection systems** (e.g. quench heaters)



Available space

Winding geometry

Conductor design

Nb-Ti

$\Delta T_m=T_{cs}-T_{op}=1K$

Magnetic config.

Mechanical support

Yield strength

$E=540\ MJ, B_p=8.21\ T$

Sc magnet

Power supply

Cryogenic system

$I=12\ kA$

Heat loads

$T_{op}=4.2\ K$

Adiabatic $T_h=150\ K$

$U=1\ kV$

Electrical insulation

Quench protection

Common dump resistor

Perspectives

- This global **analytical** approach allows to **optimize** the **magnet from the predesign** stage → the optimization can be further refined locally with numerical/FEM models during the conceptual design phase

- This **analytical baseline** is interesting to **formulate** and to **integrate from** the **predesign** phase in a sc **magnet project** as it can also be the **embryo** of:
  - **System codes** such as SYCOMORE or PROCESS if combined with a **numerical platform**
  - **A tool to rapidly control** the impact of deviations from design specifications on the magnet future operation if combined with an **updated parameters database along magnet manufacturing and quality controls** (e.g. Jc fit impact on Tcs, RRR impact on hot spot, etc.)
  - **A fast and light modeling tool** during **magnet commissioning/operation** if combined with statistical datatabase from quality controls and **commissioning data**

- It also allows **better learning and understanding** of the different **systems** of a **sc magnet**, and of their **physical and technological issues** and could also enable the different **experts** to **communicate** with a **clearer** baseline

Thank you for your attention

Special thanks to A. Torre, J.L. Duchateau, L. Zani, R. Riccioli, Q. Gorit, Q. Le Coz & I. Meziane, for fruitful discussions and support