Black holes and quantum gravity

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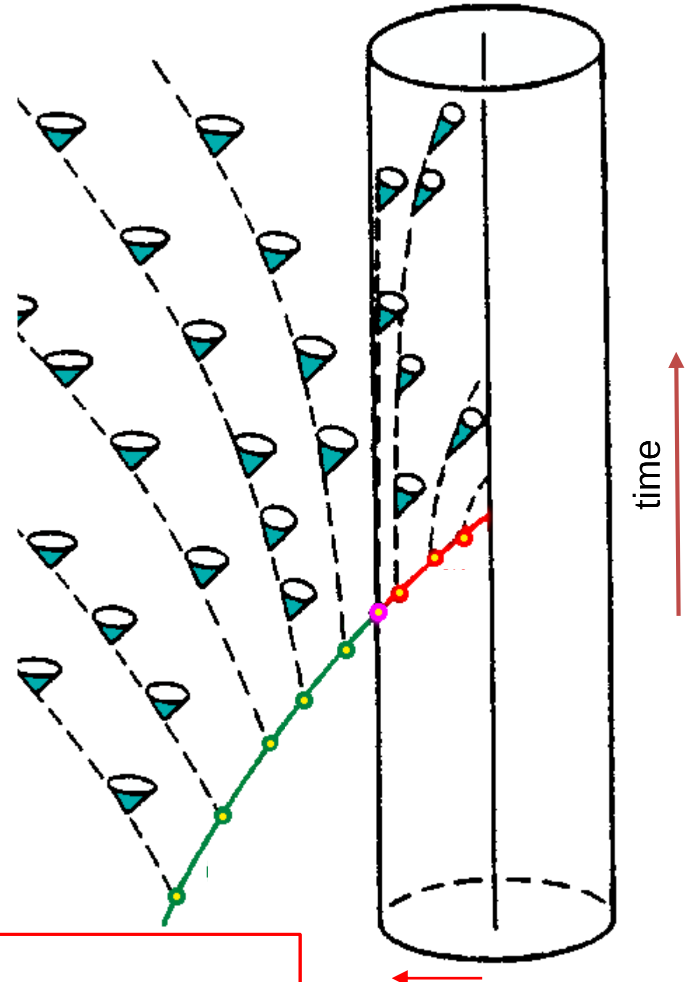
Black holes

- Black holes have two very odd properties:
  - Event horizon
  - Singularity cloaked inside the event horizon

- Implications of the singularity are not studied well.
  - Hard (if not impossible) to see.
  - Even theoretically, it is difficult to study.
  - Einstein's General Relativity (GR) is an effective theory.
    Supposed to break down near singularities.



$$r_S = \frac{2\,GM}{c^2}$$

- Event horizon:
  - Matters behind it are (classically) inaccessible to the observer outside.
  - Creates various puzzling situations concerning this "information loss."

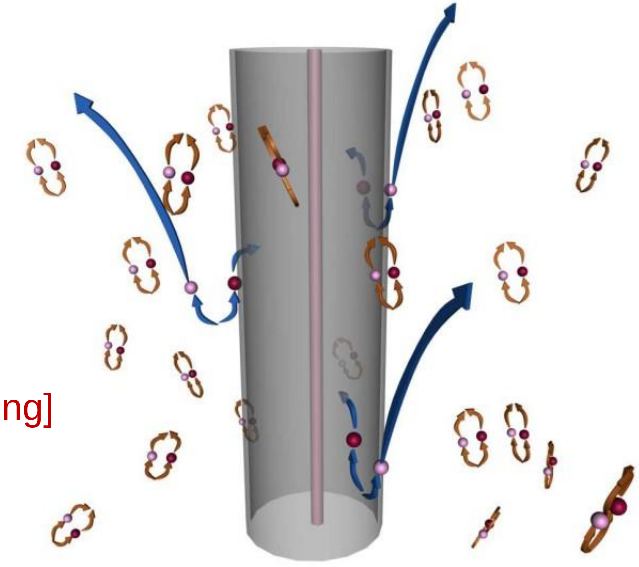
Today, we shall only discuss a classic consequence of the event horizon: black hole thermodynamics

Black hole thermodynamics

- Entropy = "missing information": For black holes, geometrically realized.
  - Bekenstein-Hawking entropy: S = (area of event horizon) / 4G …… "**area law**"
  - Hawking temperature: T = "surface gravity" ~ tidal force at the event horizon

- Satisfy "thermodynamic" laws, derived from GR:
  - 1st law: "heat" exchange $dM = \mu\, dQ + \Omega\, dJ + \boldsymbol{T\, dS}$
  - 2nd law: Entropy non-decreases. $dS(t)/dt \geq 0$

- Real thermodynamics of matters inside black holes:
  - Hawking radiations: Establishes $S = \frac{A}{4G} \sim A\, m_{Planck}^2$ [Hawking]
  - Statistical account for this entropy [Strominger, Vafa] (1996)

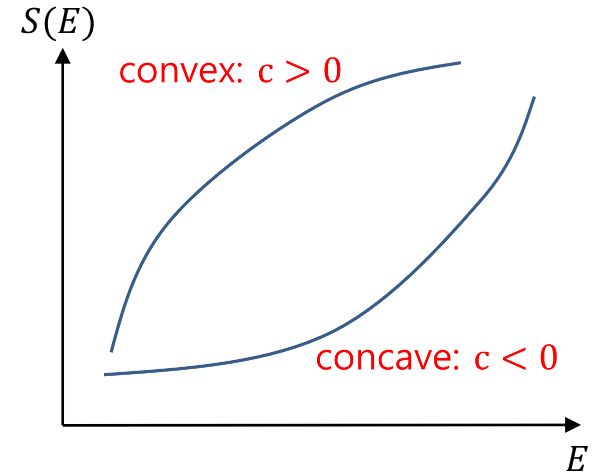


- Fundamental implication:
  - Imprint of quantum gravity at macroscopic scale (cf. black body radiation & Planck's QM)

Exotic entropy of black holes

- Black holes have negative specific heat.

$$T_{\rm H} = \frac{\hbar c^3}{8\pi G M k_{\rm B}} \approx 6.169 \times 10^{-8}\ {\rm K} \times \frac{M_\odot}{M}$$



- Why? Entropy grows too fast: $\frac{dS}{dE} > 0$ and $\frac{d^2S}{dE^2} > 0$.

- Note: $\frac{1}{\mathrm{T}} = \frac{dS}{dE} \quad \rightarrow \quad \frac{d^2S}{dE^2} = \frac{d(1/T)}{dE}$

- Very apparently, a bit similar to the evolution of stars.
  - E lowered by radiation $\rightarrow$ V decreases & K increases $\rightarrow$ T increases.
  - After radiation & gravitational shrink, one probes d.o.f. at larger kinetic energy.
  - After the collapse to BH, black hole entropy probes d.o.f. at the Planck scale.

$S = \frac{A}{4G} \sim GM^2 \sim (M/m_P)^2$ with $m_P \sim 10^{19}\ GeV \sim 10^{-8} kg$

- Black holes demand unusual structures of the Hilbert space of quantum gravity.
  - Very fast growing entropy at high energy.

Exotic entropy of quantum gravity

- Fast-growing entropy at high E is in fact familiar in quantum gravity.
- E.g. string theory has many high E degrees of freedom:
  - Elementary strings: $S(E) \sim E/T_H$, where $(T_H)^2 \propto$ string tension.
  - ∞ tower of oscillations: "Hagedorn growth" [Hagedorn] (1965) [Sundborg] [Atick, Witten]
  - Hagedorn's original work was on hadron towers. (→ Similar to string spectrum)

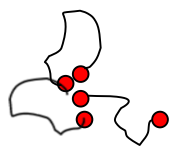
- One usually motivates string theory's huge extra d.o.f. from the cancelation of UV divergence. But they are needed just to account for black holes.
  - Quantum gravity must be a drastic completion of GR, no matter if it is string theory or not.

- Note that observed black holes also exhibit such fast-growing entropies.
  - May have observable implication, though entropy itself is (almost) impossible to measure.

- The point I'd like to highlight here is that, even the entropy of perturbative strings <u>doesn't grow as fast</u> as $S_{BH} \propto M^2$: Needs non-perturbative d.o.f.…?

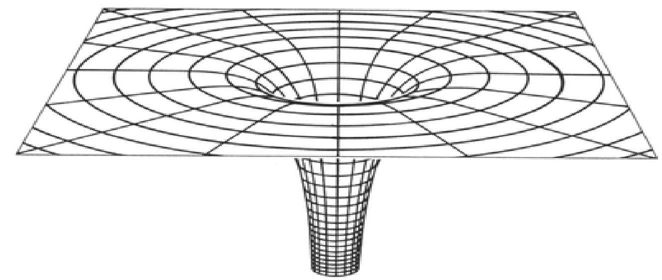
Statistical entropy of black holes

- To form such black holes, non-perturbative “solitonic” objects are needed.
  - New solitons (called “D-branes”) [Polchinski] (1995) & extra bound states → faster growth.



Do honest statistical mechanics of
the soliton quantum mechanics

strong coupling



Agrees w/ emergent gravitational picture?

- The statistical picture of area law was confirmed with “extremal” black holes:
  - Carry electric charge & saturate the bound $M \geq \# \, Q$.
  - In $D = 3 + 1$ dimensions: $S = 2\pi Q^2$

  [Strominger, Vafa] (1996) [Maldacena, Strominger, Witten] (1997) [Vafa] (1997) ……

- Since Q is M, similar growth as Schwarzschild BH: $S_{BH} \propto M^2$.
  - Very fast growth from non-perturbative bound states.
  - Fast growth → negative susceptibility (like negative specific heat).

High temperature?

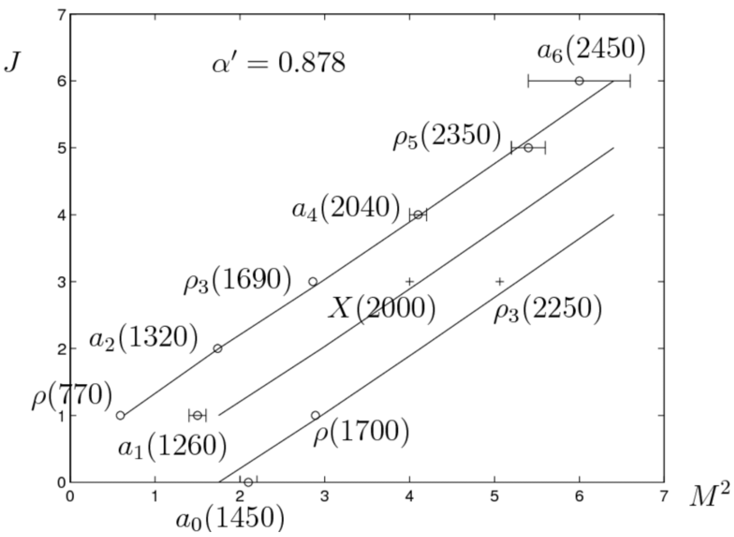
- As black holes illustrate, too many d.o.f. at high E yields strange physics.
- Similar issue discussed by Hagedorn for hadrons in 1960's.

  - Now we know QCD: hadron tower vs. quark-gluon

    → QCD at high T is in "quark-gluon plasma phase"

  - Similar "high T" phase transition for gravity?

  - Relevant for, say, studying very early universe?



- Hard to even set up such a question for gravity.
  - For thermodynamics at fixed T, needs "finite volume" "IR regulator". ("E ∝ volume")
  - Universe may have its own "size cutoff." Hard to directly study at the moment.

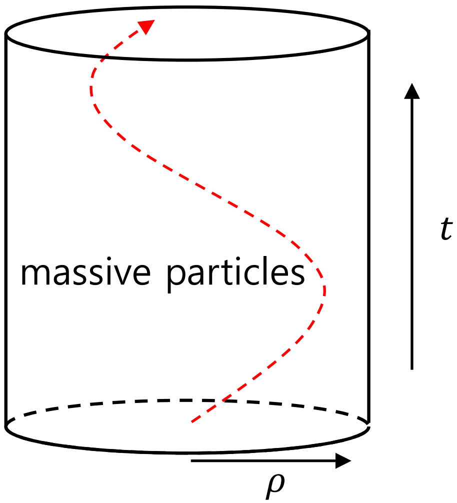
- Can't put gravity in an artificial box: Everything subject to equivalence principle.
  - But there is a simple model to put gravity in a "box" & one can draw general lessons.

Anti de Sitter (AdS) spacetime

- We need a theoretically consistent setup for the "finite box" ~ IR regulator
- Put gravity in AdS: [Hawking, Page] (1983)

$$ds^2_{D+1} = d\rho^2 - \cosh^2\frac{\rho}{\ell}\, dt^2 + \ell^2 \sinh^2\frac{\rho}{\ell}\, ds^2(S^{D-1})$$

  - Confining force to $\rho = 0$: $\Phi \approx -g_{tt}(\rho)/2$: "gravitational box"
  - Renders many thermal questions of gravity well defined.



- Incidentally, we know a microscopic description of such quantum gravity.
  - We call it "AdS/CFT correspondence" [Maldacena] (1997)
  - QFT at the boundary $S^{D-1} \times R$ 'holographically' describes the QG inside.
  - SU(N) gauge theory at $N \gg 1$: large number of "gluons"
  - Newton constant related to the # of degrees of freedom: $\hbar^{-1} \sim \ell^{D-1}/G_{D+1} \propto N^2$.

- Surprisingly, in this setup, quantum gravity at high T is again represented by the thermodynamics of new black hole solutions.

Black holes in AdS

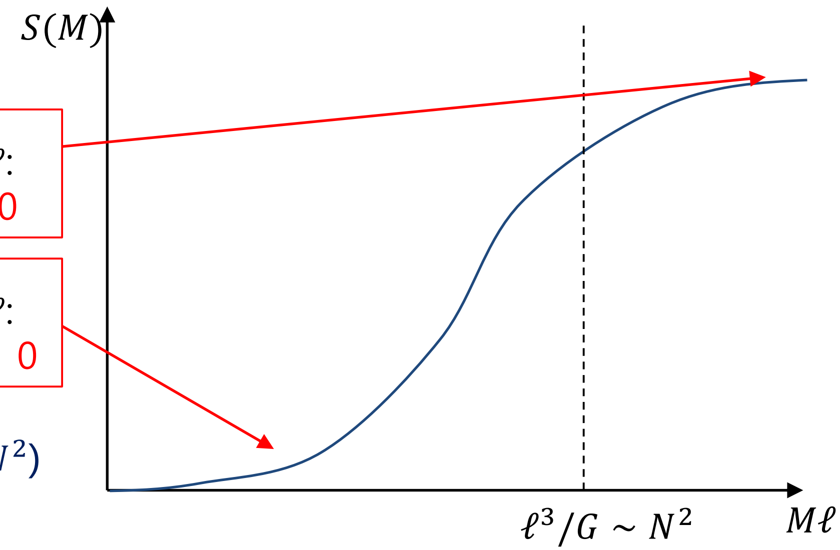
- AdS$_5$ BH's were best studied from 4d QFTs, but general lesson is universal.
- Schwarzschild black holes in AdS$_5$:

[Hawking, Page]

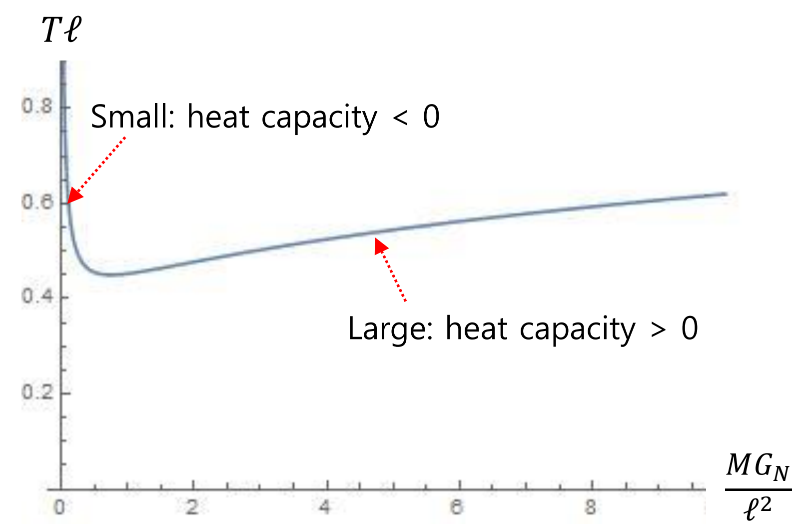
$\propto E^{\frac{d-1}{d}}$ at $E\ell \gg N^2 \leftrightarrow R_{Sch} \gg \ell$:
Convex curve. Specific heat > 0

$\propto E^{\frac{d-2}{d-3}}$ at $E\ell \ll N^2 \leftrightarrow R_{Sch} \ll \ell$:
Concave curve. Specific heat < 0

$(\ell^3/G \propto N^2)$



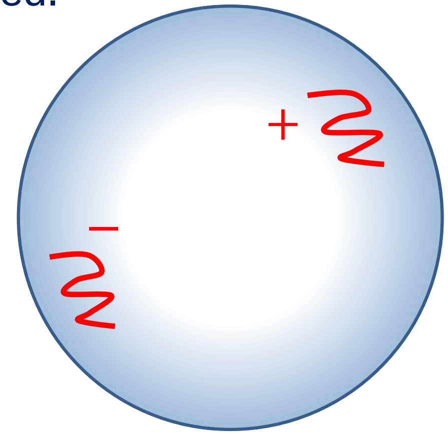
- T-E curve: $$T = \frac{r_+}{\pi\ell^2} + \frac{1}{2\pi r_+}$$ $$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



- Key features:
  - Small BH: IR regulated BH's in flat spacetime
  - Large BH: Represent the thermodynamics of full QG (at high T)

Phase transition & QFT

- Gauge theory at finite T can be in different phases.
- QFT on $S_r^3 \times R$, i.e. "compact" space:
  - low T: System confines. "Net charge 0" → forbids individual gluon excitations.
  - high T: System deconfines. Locally, individual gluons virtually liberated.



- (wavelength of locally liberated gluon) $<$ (size r of space)
- Expect a large N phase transition at $T_c \sim 1/r$:

confinement-deconfinement phase transition

- Hawking & Page (1983) found a similar transition for quantum gravity in AdS:
  - low T: A phase dictated by thermal gas of graviton, without black holes
  - high T: The large black hole is thermally nucleated.

- One expects the two phase structures to be dual to each other. [Witten] (1998)

deconfined plasma of gluons ↔ "large black hole ~ high T phase of QG"

Challenges & recent advances

- QFT is at strong coupling, so is difficult to study. Some solutions are:
  - Use computer. Numerically see BH. [Hanada, Hyakutake, Ishiki, Nishimura] (2013)
  - Analytically solvable "supersymmetric" model → charged rotating black holes. Some physics very similar to Schwarzschild or Kerr black holes.

- Strong-coupling studies using "SUSY non-renormalization" [Seiberg], etc.
  - Black holes carry two spins J_1, J_2 & electric charge R.
  - Only very recently, at large spins/charge, we successfully enumerated the entropy of these systems, which precisely accounts for black hole's area law:

$$S=\sqrt{3}\pi\left[2\underline{(3c-2a)}\left(J_1+\frac{R}{2}\right)\left(J_2+\frac{R}{2}\right)\right]^{\frac{1}{3}}$$

[Sunjin Choi, Joonho Kim, SK, June Nahmgoong] (2018), [Joonho Kim, SK, Jaewon Song] (2019), ……

c and a are two "central charges" of the QFT $\propto N^2$

  - Also showed that this is the entropy of deconfined quark-gluon plasma, establishing the microscopic picture of quantum black holes.

Conclusion & future directions

- "Exotic" thermodynamics of "normal" black holes:
  - Reflects high energy d.o.f. of quantum gravity.
  - To me, this almost forces quantum gravity to be "something like" string theory.

- "Normal" thermodynamics of "novel" black holes (say in AdS):
  - Quantum gravity undergoes a (QCD-like) phase transition. Less d.o.f.
  - Generally suggests a new phase of gravity at high T. For instance, early Universe…?

- Folklore: "Black hole thermodynamics is irrelevant at macroscopic ones."
  - Correct only in the sense that the Hawking temperature is too small.
  - Still the entropy is huge, probing the Planck scale degrees of freedom.

- Entropy is a subtle theoretical quantity, and isn't directly measurable.
  - But the 1st or 2nd laws etc. might have nontrivial observable implications in a not-so-far future, especially in the modern golden age of black hole observations these days.