

# On the space of quantum field theories

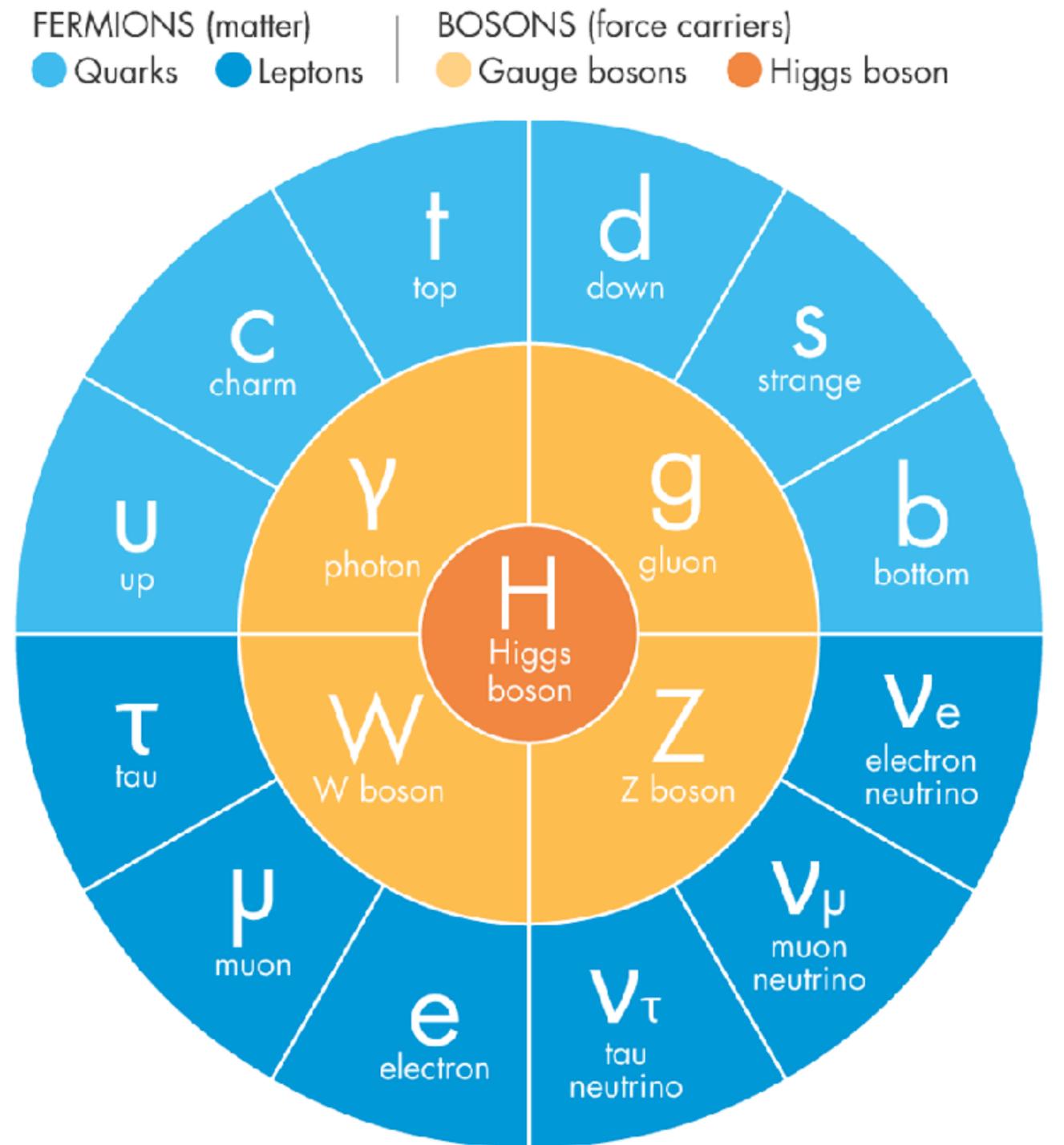
2020 Meeting of the Division of Particles and Fields of the KPS

Dec. 3<sup>rd</sup> 2020

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**(KAIST)**

# Why quantum field theory?

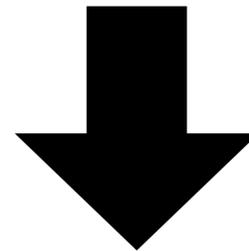
- Nature is described in terms of quantum field theory.
- Standard Model of Particle physics is spectacularly successful.
- (Almost) No experimental signature that violates the Standard Model.
- QFT arises naturally when we consider a relativistic quantum system.



Particle Physics  
Inflationary Cosmology

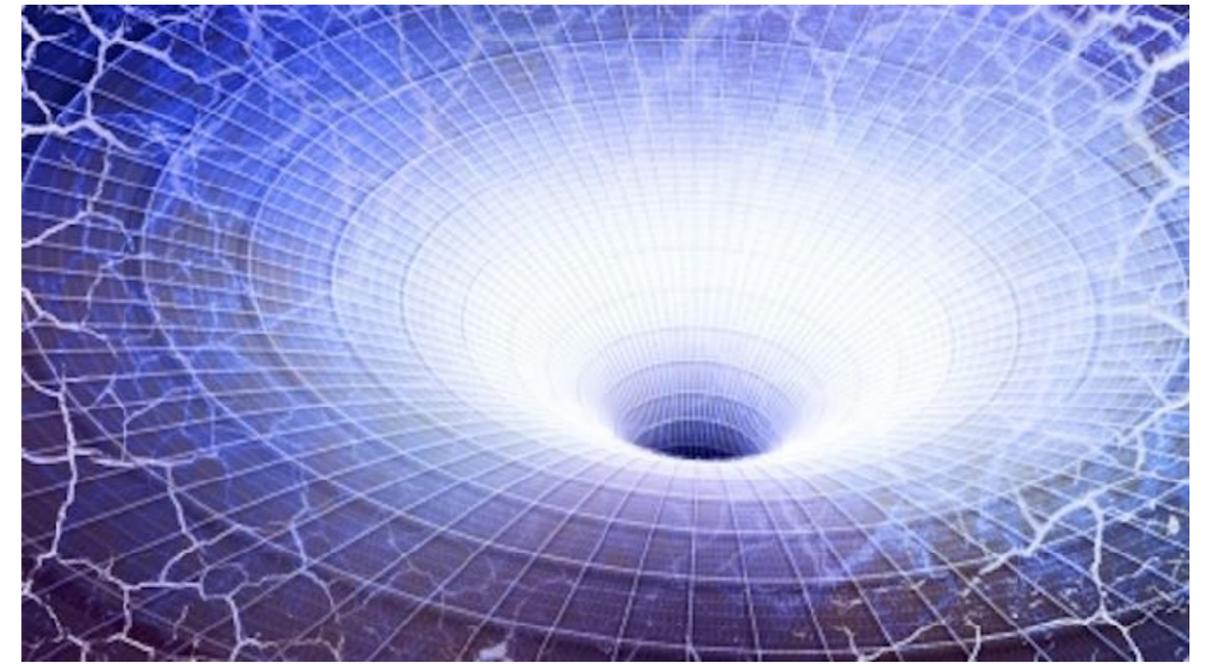
Condensed Matter Physics  
Quantum Hall system, Non-Fermi liquids,  
Topological phases of matter

World-sheet String Theory  
Quantum Gravity

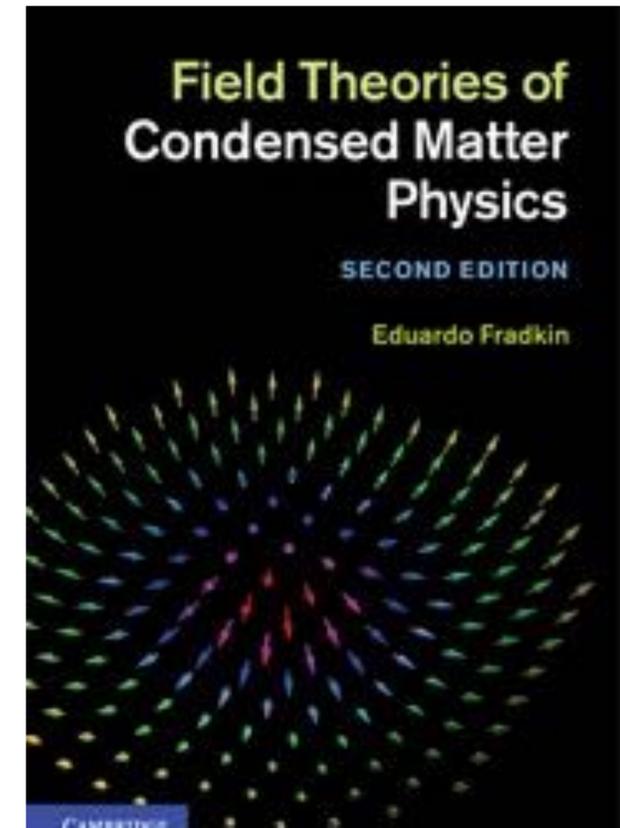
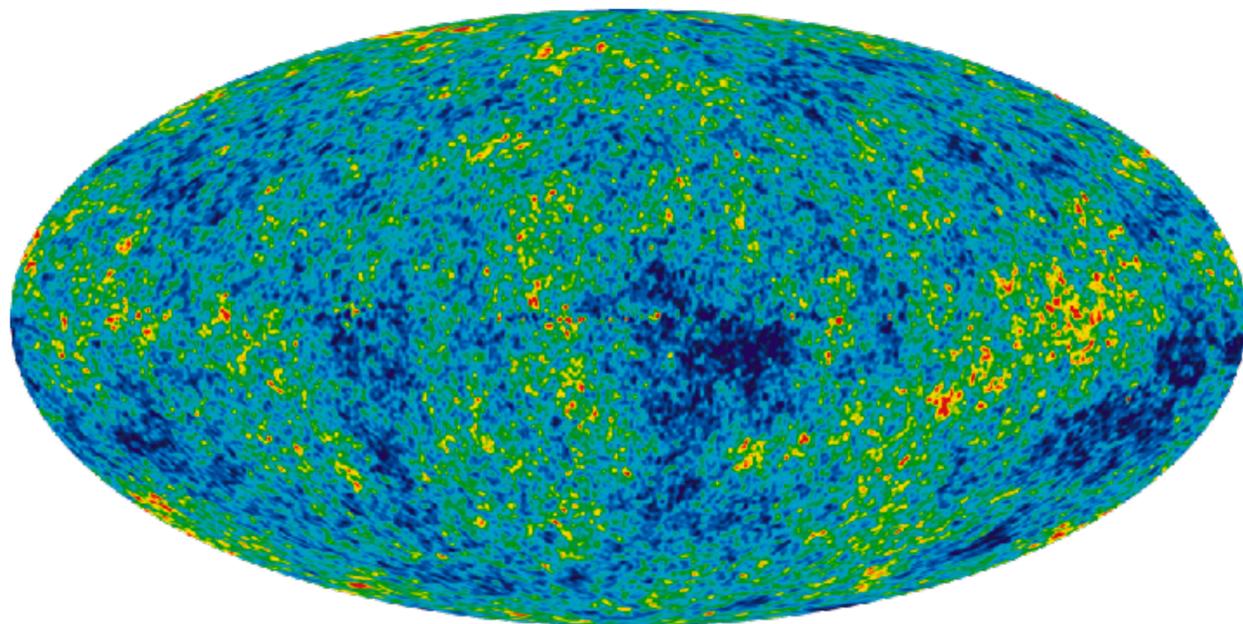


Quantum Mechanics  
Many Body / Relativity

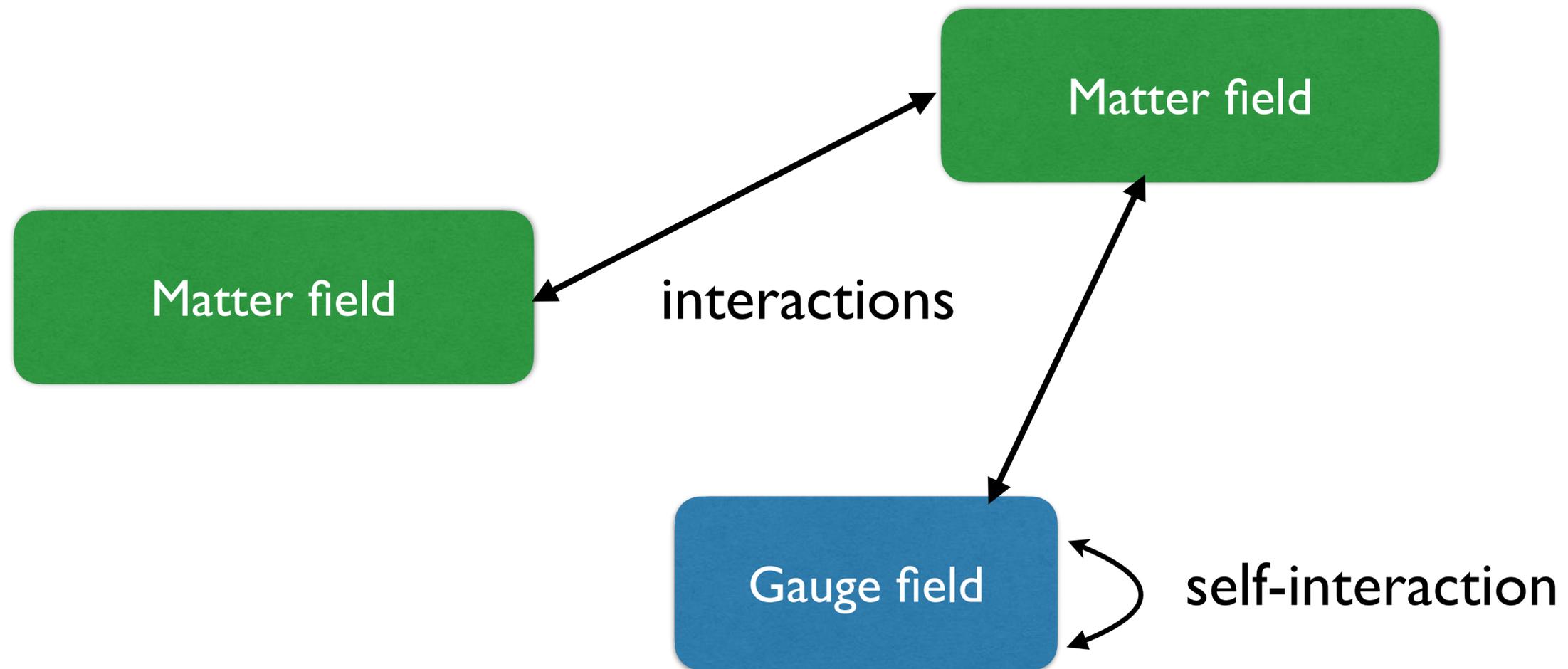
Quantum Field Theory



**Quantum field theory is a universal language of theoretical physics**



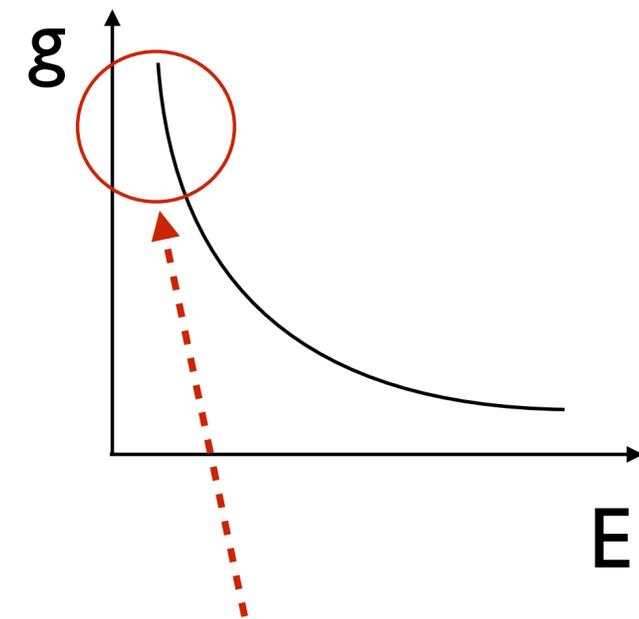
# Typical structure of a QFT



When the interactions are **weak**, one can use perturbation theory.

# Challenge: strong coupling

- **Strong-coupling dynamics** underlie the most challenging questions in physics.
  - Color confinement
  - Black Holes
  - High-temperature superconductivity
  - Fractional quantum Hall effect



Perturbation theory  
not applicable here.

# Challenge: Quantum systems with no path-integral description

- Conventionally, QFT is defined in terms of a **path-integral** over the field configurations.

$$Z = \int [D\phi] e^{iS[\phi]} \quad \langle \mathcal{O} \rangle = \int [D\phi] \mathcal{O} e^{iS[\phi]}$$

- Recently, it became evident that there is a vast landscape of QFTs with no path-integral formulation!
- Sometimes called “**non-Lagrangian**” theories.

# Examples of “non-Lagrangian” QFTs

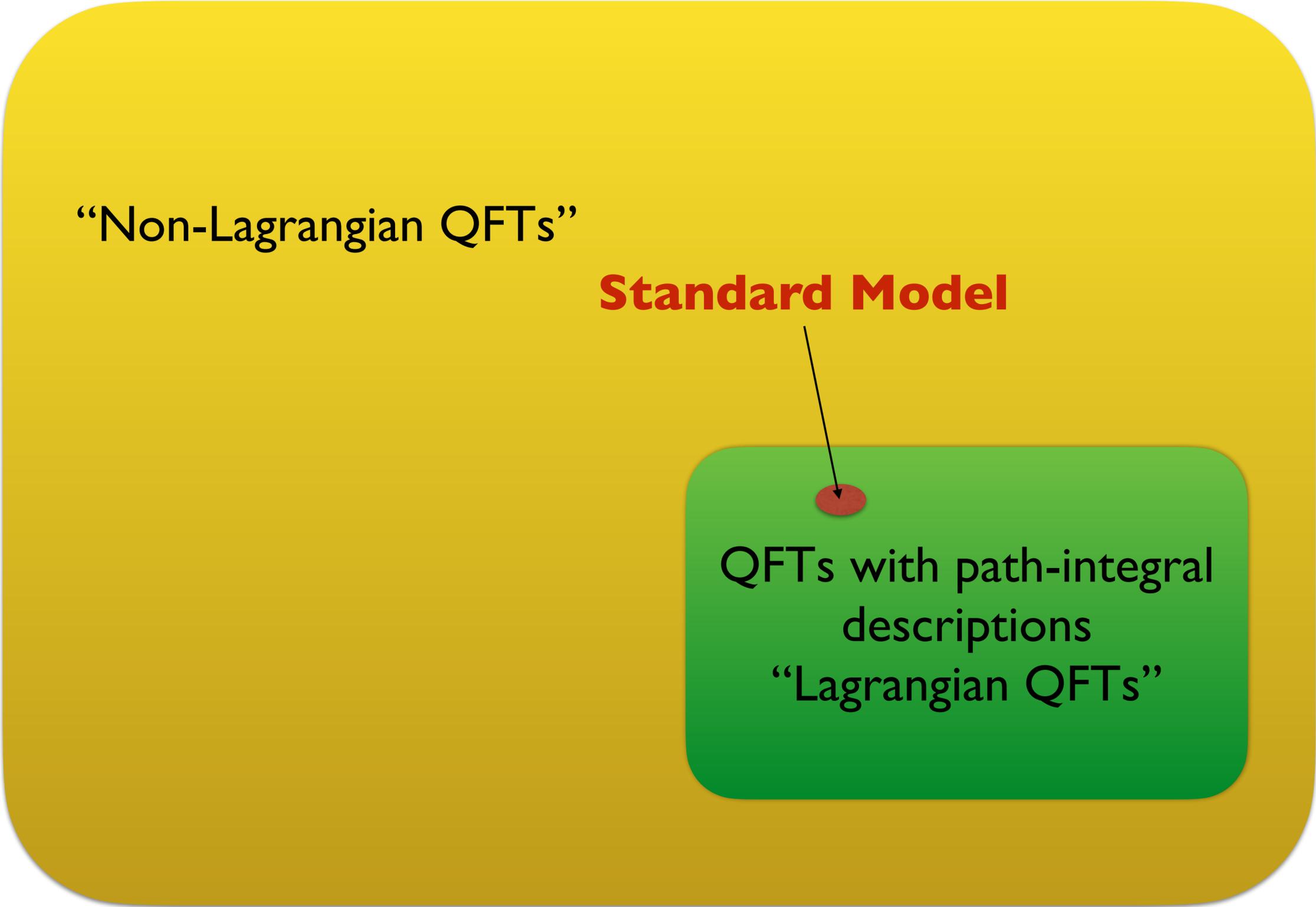
## - partial list of such theories

- 4d Argyres-Douglas theory [Argyres-Douglas '95]...
- Limits of string/M-theory compactifications at strong coupling.  
eg) 6d  $\mathcal{N}=(2, 0)$  theories [Witten '95],  $E_{6, 7, 8}$  theories [Minahan-Nemeschansky '97]
- Certain QFTs at infinite coupling limits. [Argyres-Seiberg '07][Gaiotto '09]
- CFT duals of numerous supergravity solutions with AdS geometry.
- 4d  $\mathcal{N}=3$  SCFTs [Garcia-Etxebarria & Regalado '15]
- the list goes on and on... and it is **growing very rapidly!**

# The space of QFTs

“Non-Lagrangian QFTs”

**Standard Model**



QFTs with path-integral  
descriptions  
“Lagrangian QFTs”

- There is no reason to believe that every physical system can be written in terms of **free fields** + **interactions**.
- There may be a strongly-coupled quantum system that exists **in nature** that does not allow a path-integral description.
- We have to consider such QFTs and search for a way to understand them.
- Let me illustrate this point by focusing on a simple quantum system, which exemplifies the **limitations of the conventional paradigm** for the QFT.

# Electric-Magnetic duality

$$\begin{array}{ccc} \nabla \cdot E = \rho & & \nabla \cdot E = \rho_e \\ \nabla \cdot B = 0 & & \nabla \cdot B = \rho_m \\ \nabla \times E = -\frac{\partial B}{\partial t} & \longrightarrow & \nabla \times E = -J_m - \frac{\partial B}{\partial t} \\ \nabla \times B = J + \frac{\partial E}{\partial t} & & \nabla \times B = J_e + \frac{\partial E}{\partial t} \end{array}$$

If there is a **magnetic monopole**, the Maxwell's equations are modified.

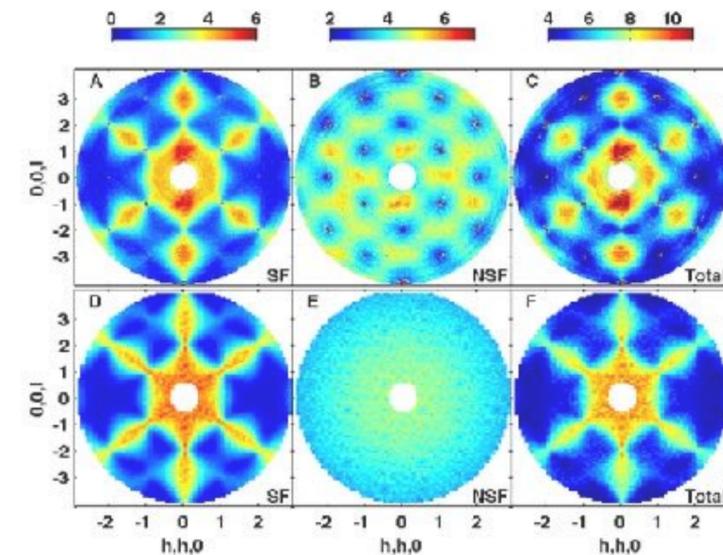
It is invariant under the **duality** transformation

$$E \rightarrow -B \quad B \rightarrow E$$

# Magnetic monopoles

- Magnetic monopole can be realized theoretically (and experimentally!).
- **Grand Unified Theory**: Created when the universe undergoes a phase transition from Grand Unification to Electroweak epoch.
- It **exists in a condensed-matter** system as a **collective-excitation**. (experimentally realized!)
- They are indispensable in **string theory**.

[t' Hooft-Polyakov]

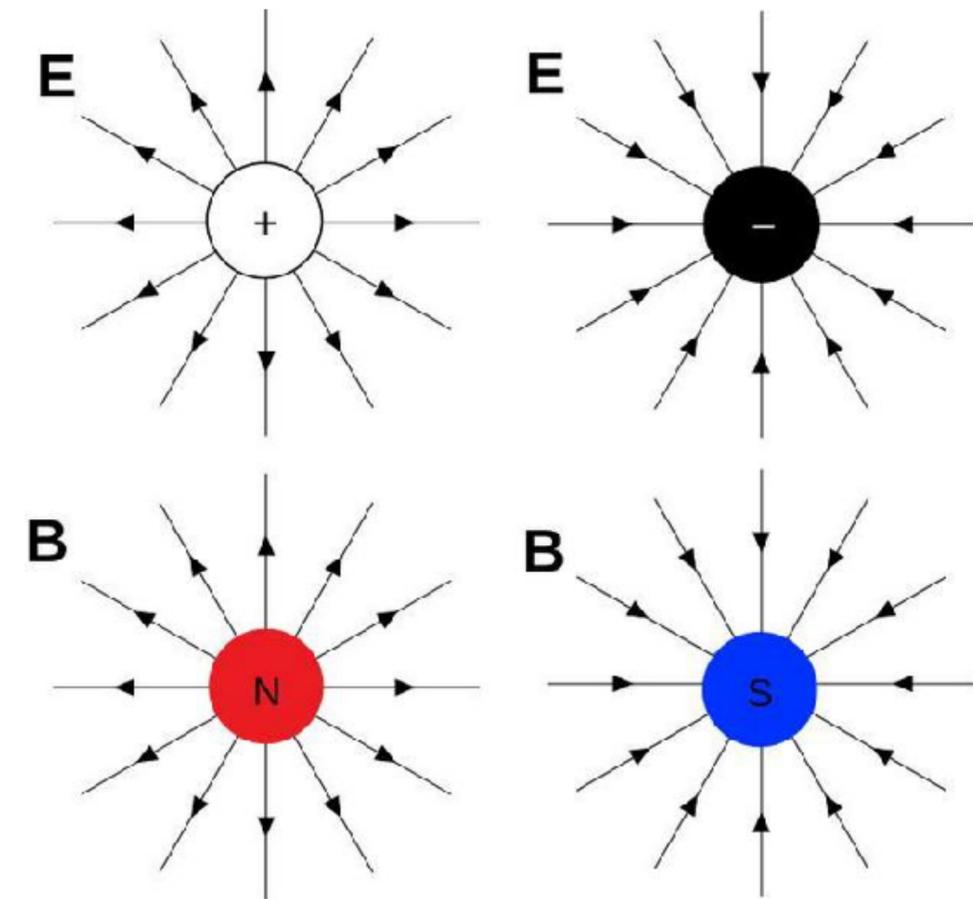


[Fennel et al, Science '09]

# Electron-Monopole system

## A simple toy model QFT?

- Consider a field theory with massless **electron** and **monopole** interacting via **photon**.
- Possibly the simplest interacting QFT.  
(no non-abelian gauge bosons)
- It is *very difficult to construct such a QFT* having (light) **electrons** and **monopoles** simultaneously.  
[Dirac '30][Schwinger '60][Zwanziger '60]
- Is it a **fundamental limitation** of **Nature** or just a **short-coming** of the gauge theory **description**?



# The electron-monopole QFT is strongly-coupled

- Such a system is **inevitably strongly-coupled**, due to the Dirac-quantization condition:

$$eg = 2\pi\hbar n, \quad n \in \mathbb{Z}$$

- This QFT (if it exists) **does not have a classical limit!**  
No Lagrangian. No path-integral formulation.
- [Argyres-Douglas '95] showed for the first time that such a QFT (with extended supersymmetry) **indeed exists**.
- This type of 'non-Lagrangian' QFTs play a prominent role in understanding strong-coupling phase of string/M-theory.

**So, what should we do to  
understand this theory?**

**One way: Find a Lagrangian!**  
(hey, didn't you just say you can't?)

**It was recently found that there exists  
a Lagrangian theory that flows to the  
enigmatic Electron-Monopole theory!**

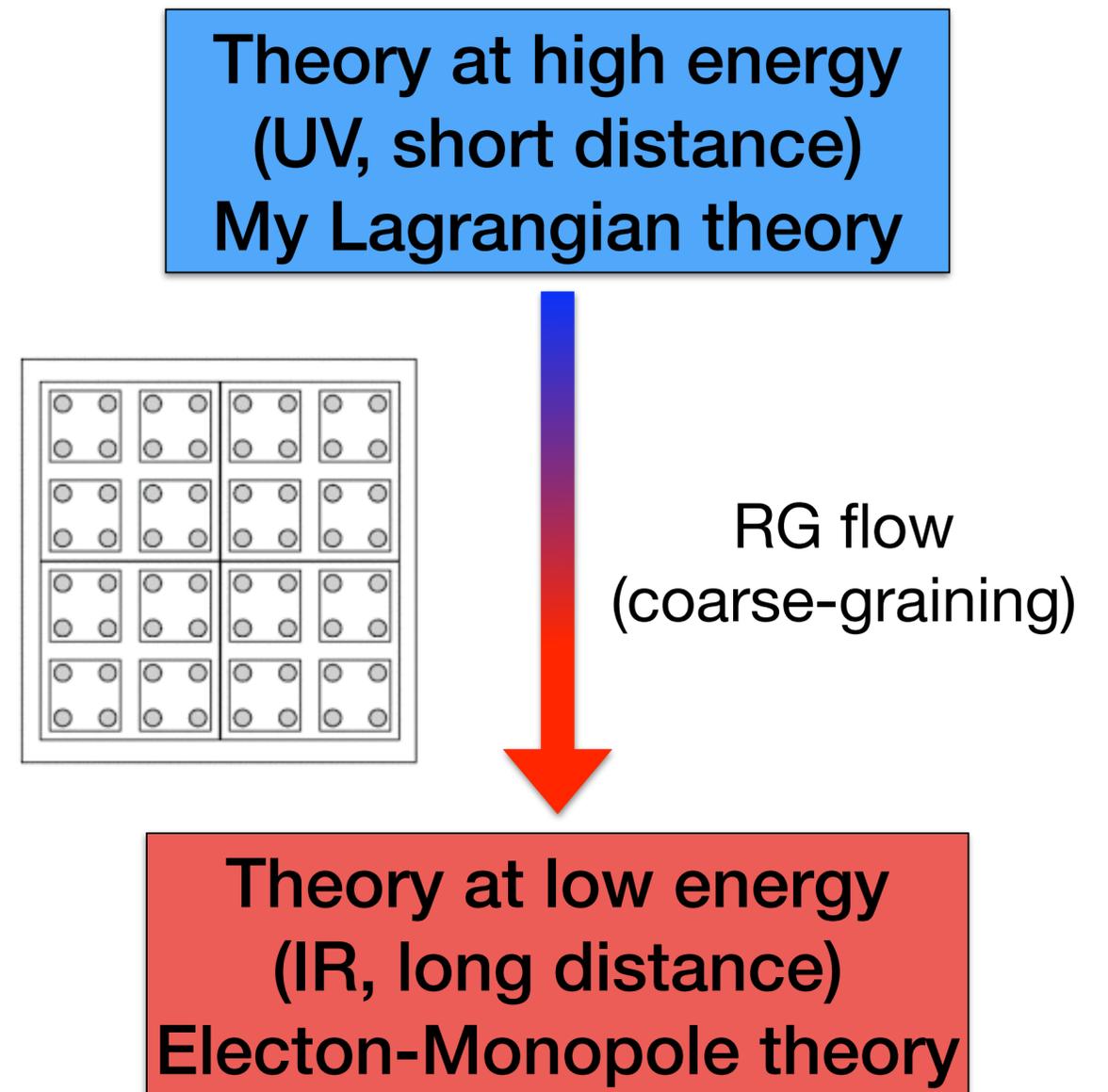
[Maruyoshi-JS '17]

A “Lagrangian” for a “non-Lagrangian” theory

What does it mean?

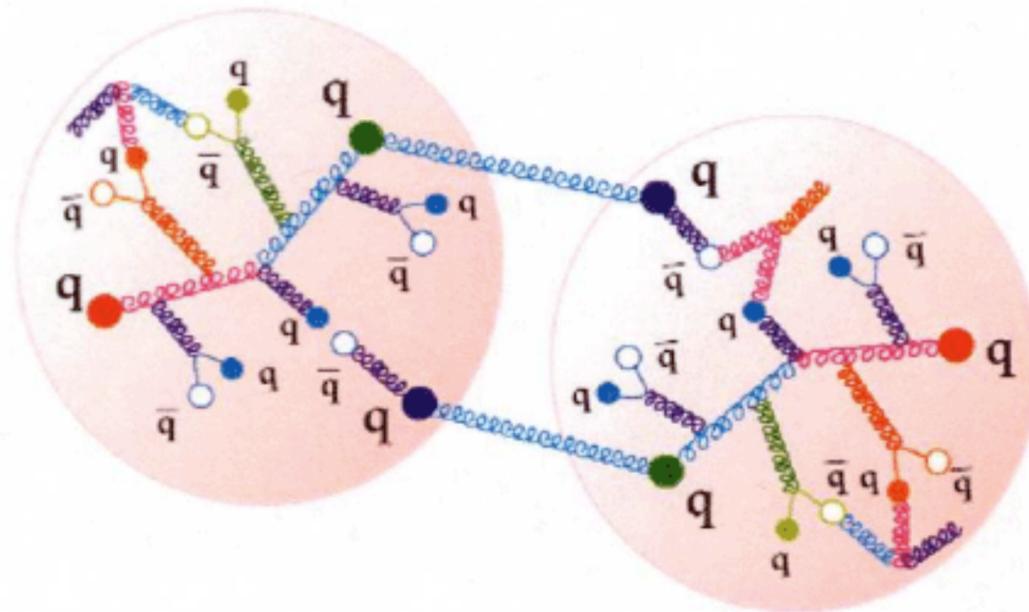
# Wilsonian Renormalization Group (RG) Flow

- Start with a theory defined at short-distance. (UV theory)
- As one integrate-out the high-energy modes, the **effective Hamiltonian/Lagrangian change**. “RG flow”
- There is a fixed point where the couplings no longer change. “**IR fixed point**”



# It is analogous to QCD

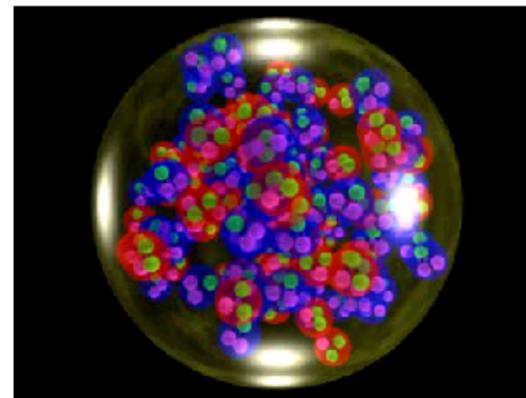
- Our theory provides a **microscopic** description for the “**macroscopic**” theory of monopoles and electrons.
- Analogous to the QCD (**quark** and **gluon**) as a microscopic description for the macroscopic **proton** and **neutron**.



The microscopic theory contains gauge bosons ('**gluons**') and matters ('**quarks**') similar to QCD.

No photon/electron/monopoles to begin with.

Photon/electron/monopole **emerge at strong coupling!**



(+ Emergent SUSY enhancement)

Monopole  
Electron  
Photon



Composite of Quarks  
& Gluons

Such phenomenon is quite ubiquitous!

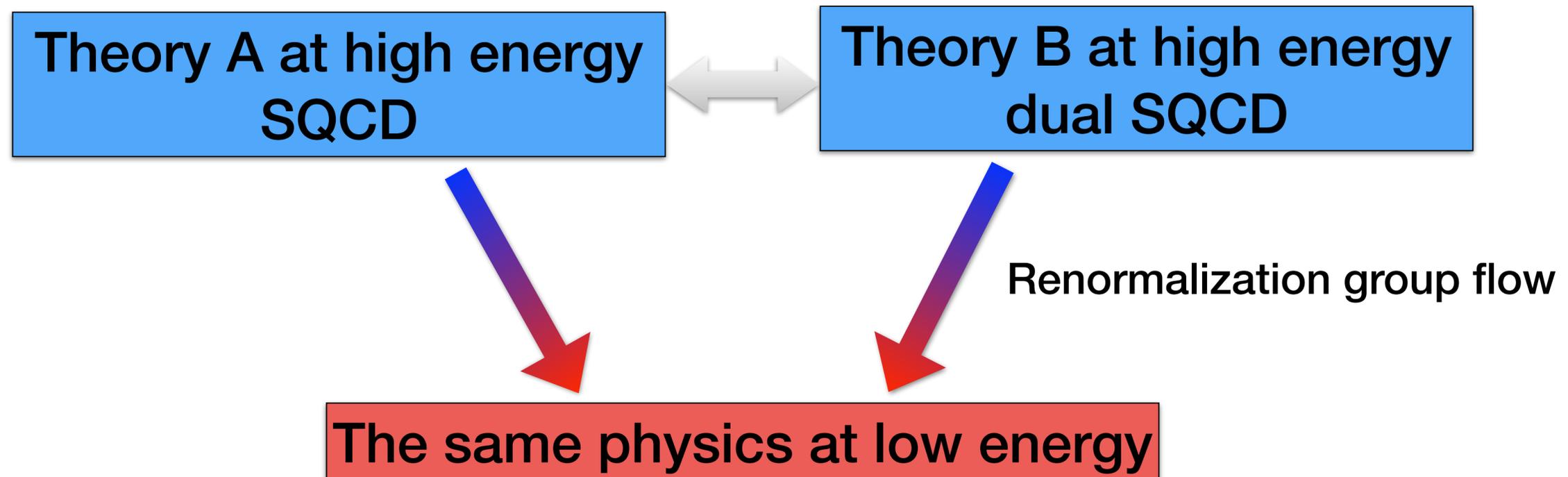
[Agarwal-Maruyoshi-JS][Benvenuti-Giacomelli]...  
[Maruyoshi-Nardoni-JS '19]  
[Razamat-Zafirir][Agarwal-KH Lee-JS '20]

**What do they all mean?**

# Duality

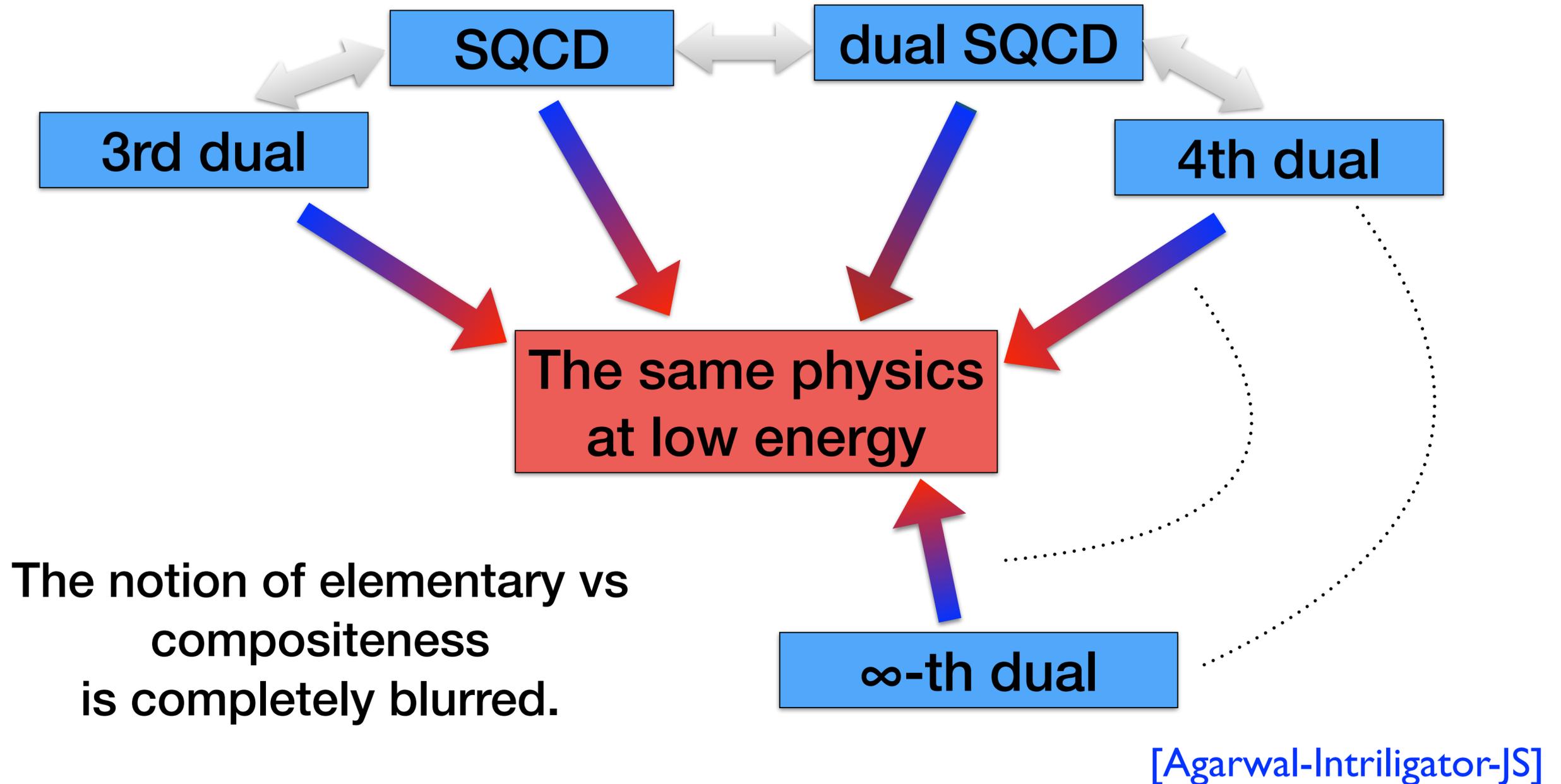
Equivalence of two seemingly different systems

[Seiberg]+...



Weak coupling  $\longleftrightarrow$  Strong coupling

# Infinitely many dualities



# Some implications

## - What is the correct framework of quantum fields?

- No uniform way to describe a quantum system (QFT).
- The notion of **elementary** and **composite** is **arbitrary**.
- Conventional way (path-integral) does not fully capture the **power of QFT**.
- *We may* need an **entirely new framework** for the **quantum field theories**!
- It is **not Nature's limitation**, but our **lack of understanding**.
- Why should we care? Opportunity for **new physics**!

# The space of QFTs

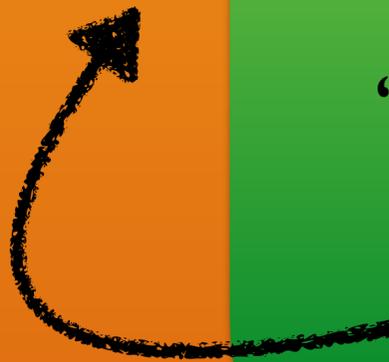
“Non-Lagrangian/Non-conventional QFTs”

**Standard Model**

“Non-Lagrangians” connected  
to Lagrangian QFTs via RG

**Are we here?**

“Lagrangian QFTs”



**Conclusion**

# Conclusion

- We know that quantum field theory works very well.  
(It is a universal language of physics)
- There exists **inherently quantum**, **strongly-coupled** QFT that has **no classical/weak-coupling limit**.
- Such “non-Lagrangian” QFTs are **ubiquitous** in the space of QFTs. We have to explore them. Chance for new physics?
- Need to develop new methods to study such theories.
- Quantum field theory is extremely versatile and rich. Exciting discoveries await us in the uncharted territory!