Why are theorists so interested in Black holes?
Information paradox

Sang-Heon Yi (CQUeST)

based on the works

블랙홀 정보손실문제의 과거와 현재, 그리고 미래
물리학과 첨단기술 with 염 동한

[arXiv:2006.11717] with D. Bak, C. Kim and J. Yoon

3 Dec. 2020

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- Entanglement in QM and QFT
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  $\Longrightarrow$ Hawking, Penrose, Unruh et al vs 't Hooft, Susskind, Page et al
- Reincarnation by Almheiri, Marolf, Polchinski and Sully (AMPS)
- Various suggestions for resolutions

- Recent Devopments: AdS/CFT Correspondence
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- Generalized EE formula and Quantum extremal surface
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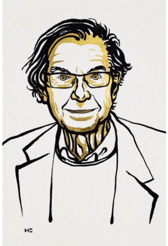
2020 Nobel Prize in Physics

The Nobel Prize in Physics 2020



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Roger Penrose

Prize share: 1/2

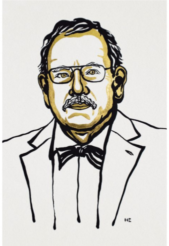

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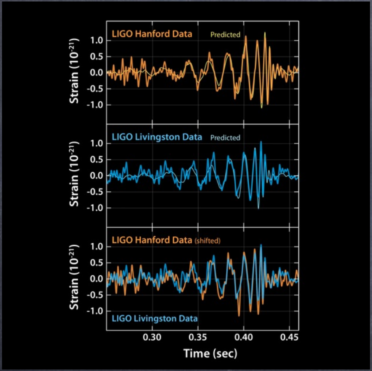
그림 www.nobelprize.org

Roger Penrose "for the discovery that
black hole formation is a robust prediction of the general theory of relativity"

Experiments

Sound and Image of black holes



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Black Holes

Spacetime region from which even light cannot escape because of strong gravity

The boundary of this region is named as the event horizon

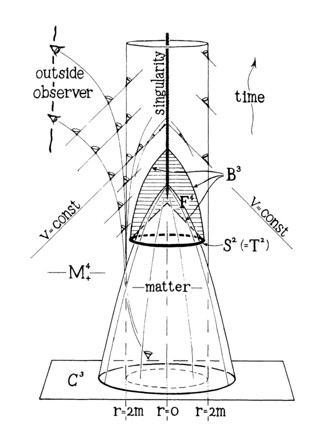


Figure from Penrose's 1964 PRL paper

Light cones are tilted by gravity

Characteristics of black holes are well summarized by the so-called Penrose diagram

Penrose Diagram = Conformal Diagram

To explore a causal structure of an infinitely extended spacetime, it is very useful to consider a map from a physical spacetime $\mathcal{M}$ to another space $\mathcal{N}$ as

$$\mathcal{M} \quad \longrightarrow \quad \widetilde{\mathcal{M}} = \mathcal{M} \cup \mathscr{I} \quad \subset \mathcal{N}$$

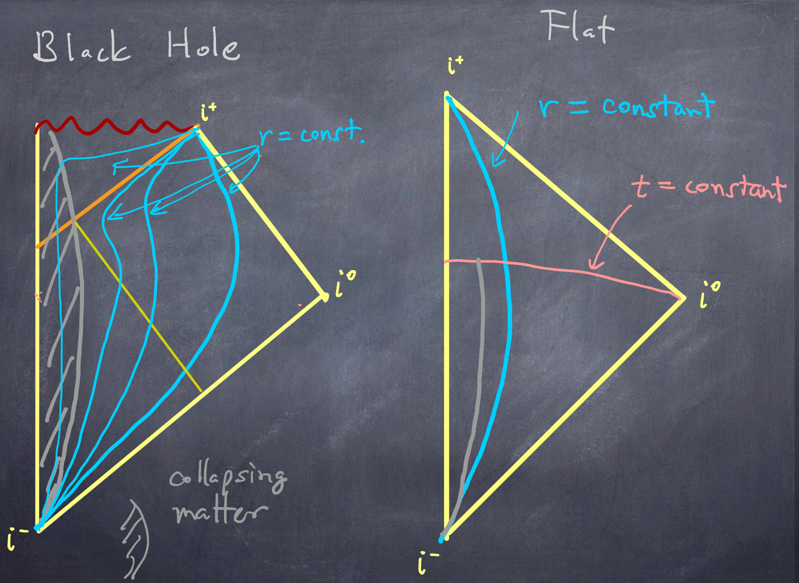
in such a way that

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x) \,, \qquad \mathscr{I} \text{ is a boundary to } \mathcal{M}$$

$\tilde{g}$: unphysical metric $\qquad$ g: physical metric

One can draw a two-dimensional diagram (time + radial directions) by using $\tilde{g}$: Penrose diagram

Penrose digram of collapsing black holes



Black Holes

Penrose and Hawking showed that

General Relativity predicts the formation of black holes under a gravitational collapse and a singularity is unavoidable inside black holes

- No hair theorem
- Black hole thermodynamics and Area law (Bekenstein) → black hole entropy = horizon area
- Hawking radiation and black hole evaporation

Black holes behave as a thermal object

In fact as a black body with a Planck distribution radiation

If black holes evaporate, what happens in the information of matters forming black holes? $\Longrightarrow$ Information loss problem

-continued

Why are we interested in black hole information loss problem?

This issue tells us that

there is a conflict between the well-established physical principles!!

- Equivalence principle (general relativity)
- Superposition principle (quantum mechanics)
- Locality and etc

Special Relativity + QM

$\Longrightarrow$ Quantum Field Theory

Our framework: (compared to Planck scale)

local effective field theory for a low energy physics

Information Paradox

► Thermal Hawking radiation conflicts with unitary scattering matrix approach for black holes

- Black Hole evaporation and Information loss ['70 Hawking]
- Unitary evolution and Complementarity [85∼ 93 't Hooft + Susskind]
- Page's approach to this issue [93 Page]

- AdS/CFT Era [97 Maldacena et al]

  Entanglement Entropy(EE) and its application to BH

-continued

Initial two groups on the problem

• Hawking, Penrose, Unruh, Wald etc: Information loss side

The evolution from a pure state to a mixed state is OK!

If we accept that a distant observer cover only the partial $\overbrace{\text{Cauchy surface}}^{\text{observables}}$

we should conclude that the evolution from 'pure' to 'mixed' is allowed

(see next slides) [Unruh + Wald 1703.02140]

• 't Hooft, Susskind, Many string theorists: Unitary evolution side

From the viewpoint of a distant observer

Black hole may be regarded as a (complicated) quantum state

Then, ingoing and outgoing particle scattering matrix should preserve the unitarity

$\Longrightarrow$ "Central Dogma" in recent developments

Conflict

There is a modern version of explaining this conflict: AMPS

Reincarnation of information loss problem in 2012 !!

[Almheiri + Marolf + Polchinski + Sully]

To explain this version in some detail,

we need to introduce the concept of entanglement entropy (EE)

(and Page curve)

Entanglement in QM

Entanglement is a quantum mechanical correlation

Example: Entangled pairs (EPR pair) of spin up/down state $|\pm\rangle$

$$|0\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle_{\mathsf{L}}|-\rangle_{\mathsf{R}} + |-\rangle_{\mathsf{L}}|+\rangle_{\mathsf{R}}\Big]\,, \qquad |\pm\rangle_{\mathsf{L/R}} \in \mathcal{H}_{\mathsf{L/R}}$$

Partial tracing out the Hilbert space $\mathcal{H}_{\mathsf{R}}$

$$\rho_{\mathsf{L}} = \mathrm{Tr}_{\mathsf{R}}\, \rho_{\mathsf{tot}}\,, \qquad \rho_{\mathsf{tot}} = |0\rangle\langle 0|$$

gives us a <u>mixed state</u> ρ_{L} as

$$\rho_{\mathsf{L}} = \mathrm{Tr}_{\mathsf{R}}|0\rangle\langle 0| = \frac{1}{2}\Big(|+\rangle_{\mathsf{L}\,\mathsf{L}}\langle +| + |-\rangle_{\mathsf{L}\,\mathsf{L}}\langle -|\Big) \neq |\psi\rangle\langle\psi|$$

c.f. Pure state density matrix $\quad \rho = |\psi\rangle\langle\psi|$

In general, partial tracing out a entangled state gives us a mixed state

-continued

An entangled state $|0\rangle$ may be characterized by

$$\langle 0|\mathcal{O}|0\rangle \neq {}_{\mathsf{L}}\langle +|\mathcal{O}|+\rangle_{\mathsf{L}}\ {}_{\mathsf{R}}\langle -|\mathcal{O}|-\rangle_{\mathsf{R}}\,, \quad {}_{\mathsf{L}}\langle +|\mathcal{O}|-\rangle_{\mathsf{L}}\ {}_{\mathsf{R}}\langle -|\mathcal{O}|+\rangle_{\mathsf{R}}\,, \qquad \text{etc}$$

On the contrary,

Non-entangled state $|\varphi\rangle = |+\rangle_{\mathsf{L}}|+\rangle_{\mathsf{R}}$, $|+\rangle_{\mathsf{L}}|-\rangle$, etc satisfies

$$\langle \varphi|\mathcal{O}|\varphi\rangle = {}_{\mathsf{L}}\langle +|\mathcal{O}|+\rangle_{\mathsf{L}}\ {}_{\mathsf{R}}\langle -|\mathcal{O}|-\rangle_{\mathsf{R}}\,, \quad {}_{\mathsf{L}}\langle +|\mathcal{O}|-\rangle_{\mathsf{L}}\ {}_{\mathsf{R}}\langle -|\mathcal{O}|+\rangle_{\mathsf{R}}\,, \qquad \text{etc}$$

What about QFT?

QFT and Entanglement

Claim: Any state in QFT is highly entangled state!

Let us consider two disjoint regions Σ_L and Σ_R with their common boundary C (corridor) on a Cauchy surface Σ as

(Σ_L and Σ_R are spacelike separated by construction)

$$\Sigma = \Sigma_L \cup \Sigma_R \cup C$$

Basic facts in QFT:

Feld operators in the region $D(\Sigma_L)$ are entangled with the operators in the region $D(\Sigma_R)$ (though they are spacelike separated)

$$\lim_{x_1, x_2 \to x} \langle \Psi | \phi(x_1) \phi(x_2) | \Psi \rangle \neq \langle \Psi | \phi(x) | \Psi \rangle \langle \Psi | \phi(x) | \Psi \rangle \qquad x \in C$$

simply because LHS diverges while RHS is finite

For a collapsing black hole, field operators inside and outside of horizon

(Σ_{in} and Σ_{out}) are entangled!!

-continued

In QFT

Causality

$$[\phi(x),\phi(y)]_{\mp}=0 \qquad \text{for} \quad |x-y|^2>0 \quad \text{(spacelike separation)}$$

Cluster decomposition theorem: Any n-point function for any k

$$\langle\Omega|\phi(x_1)\cdots\phi(x_k)\phi(x_{k+1}+s)\cdots\phi(x_n+s)|\Omega\rangle \underset{|s|^2\to\infty}{\longrightarrow} \langle\Omega|\phi(x_1)\cdots\phi(x_k)|\Omega\rangle\,\langle\Omega|\phi(x_{k+1})\cdots\phi(x_n)|\Omega\rangle$$

- Comments:

Usually, QFT satisfying these conditions is called as a local theory

even though $|\Omega\rangle$ is a highly entangled state!

EE is a non-local but we don't say that QFT is non-local in this case

-continued

Evaporating Black Hole

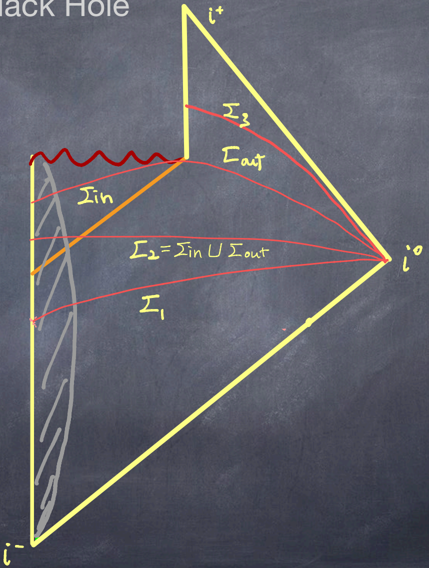
Final stage

$\Rrightarrow$ Flat Spacetime

Let us assume that an initial state is pure at Cauchy surface Σ_1

$|\psi(\Sigma_1)\rangle$

$\rho = |\psi(\Sigma_1)\rangle\langle\psi(\Sigma_1)|$



-continued

Since Σ_{out} is evolved to Σ_3

[Σ_3: Cauchy surface after the complete evaporation]

any operator(state) on Σ_3 would be entangled with the inside degrees

and so a state on Σ_3 should be a mixed state!

Recall that a Cauchy surface $\Sigma_{\text{in}} \cup \Sigma_{\text{out}}$ defines a single Hilbert space $\mathcal{H}$ with a global vacuum $|\Omega\rangle$

$\Longrightarrow$ Tracing out Σ_{in} leads to a mixed state on Σ_{out}!!! (Information loss)

In fact,

Hawking's computation tells us that the radiation becomes a thermal system at the so-called Hawking temperature

$$T_H = \frac{\kappa}{2\pi} \sim \frac{1}{M}$$

Schwarzschild BH case

Page's Approach (Page curve) ['93]

If we believe that unitarity is preserved in the evaporation process, one may ask how the information leaks out from the black hole

$\Longrightarrow$ This is a really difficult and important question

Or more mildly, one may ask whether one may see a characteristic for a information leak: When does the information begin to leak out?

Simplifying assumption: Black Hole + Radiation = Pure state

Assuming that the pure state is typical, Page argued that a smaller part of the radiation is very nearly maximally mixed and the total information is mostly encoded in the correlation among all the parts

At initial state

$$\rho_{\mathbf{R}} \simeq \frac{1_{\mathbf{R}}}{\dim \mathrm{R}} : \quad \text{(nearly max. ent.)} \quad \dim \mathrm{R} << \dim \mathrm{BH}$$

Conversely, at a final stage

$$\rho_{\mathbf{BH}} \simeq \frac{1_{\mathbf{BH}}}{\dim \mathrm{BH}} : \quad \text{(nearly max. ent.)} \quad \dim \mathrm{BH} << \dim \mathrm{R}$$

Entanglement Entropy (EE)

Let us denote the density matrix of the whole system as ρ

Density matrix of the system A after tracing out its complement A^c:

$$\rho_A \equiv \mathrm{Tr}_{A^c}\, \rho$$

S(A): entanglement entropy of a region A with its complement A^c

is defined by a von Neumann entropy of ρ_A

$$S(A) = -\mathrm{Tr}_A(\rho_A \ln \rho_A)$$

S(A) measures how much the system A is entangled with A^c

Bipartite system: $\boxed{S(A) = S(B)}$ $\qquad B = A^c$

c.f. Mutual information

$$I(A, B) \equiv S(A) + S(B) - S(A \cup B)$$

which measures how much A and B are entangled.

-continued

One can see that coarse graining process loses some information and gives us larger entropy

If we forget everything, the final information would be just a dimension of the Hilbert space $\mathcal{H}$ and so

$$\text{Boltzmann entropy}(= \ln \dim \mathcal{H}) \;\geq\; \text{Entanglement Entropy (EE)}$$

If we can define/obtain a temperature of the system

$$\text{Thermal entropy} \geq \text{EE}$$

Bekenstein-Hawking Area law $S_{BH} = \frac{A_H}{4G}$ = Thermal entropy

1st/2nd law of black hole thermodynamics OK

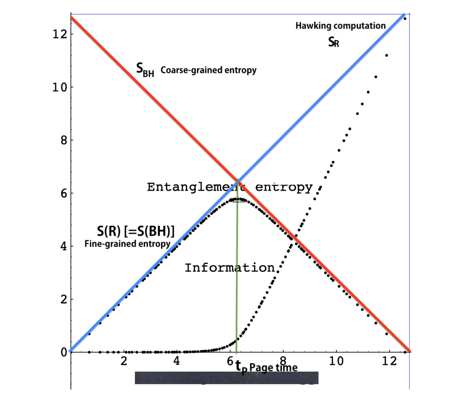
Therefore,

$$\text{Thermal E} = \boxed{S_{BH}} \geq \boxed{S(BH)} = \text{EE}$$

where $S(BH)$ is the entanglement entropy of black hole with radiation

-continued

Entropy: $S_{BH} \sim A_H$ - Bekenstein Hawking vs S(BH) - Entanglement



[Note

Pure total state $\Longrightarrow$

$S(R) = S(BH)$]

c.f. Page's argument contrasts

with our picture

for Hawking thermal radiation

From Page's paper

Consensus:

If anyone tries to resolve the information paradox, (s)he should explain this behavior of the entanglement

Simplified version of conflict

Pair production in vacuum (just outside of the horizon)

$\Longrightarrow$ maximally entangled pairs $\left[\mathrm{Tr}_{\mathbf{R}}|0\rangle\langle 0| = \frac{1}{2}\left(|+\rangle\langle +| + |-\rangle\langle -|\right) \quad \Longrightarrow \quad \text{max. mixed}\right]$

$$|0\rangle = \frac{1}{\sqrt{2}}\Big[|+\rangle_{\mathrm{L}}|-\rangle_{\mathrm{R}} + |-\rangle_{\mathrm{L}}|+\rangle_{\mathrm{R}}\Big]$$

This Hawking pair production process adds up the entanglement in the radiation, since the pair production does not change the background significantly (*i.e.* still black hole background)

Conflict with Page's result: small part of the radiation is nearly maximally entangled $\Longrightarrow$ Paradox

AMPS version of the information paradox:

Are the collection of $|+\rangle$'s entangled with the radiation degrees or with the behind horizon degrees?

c.f. Monogamy property of entanglement $\Longrightarrow$ Paradox

AMPS arguments

One may think that the conflict occurs near the singularity or the very late time of the black hole evaporation process

However, this is not the case!

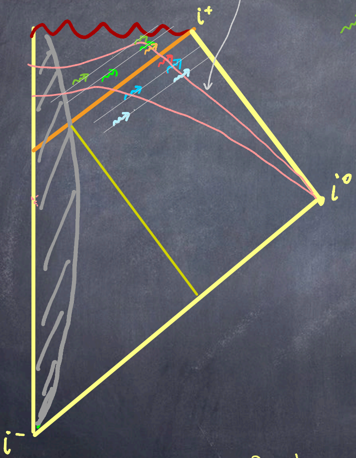
And the conflict occurs just after the Page time near the horizon

which corresponds to the low curvature regime for a big black hole

$\Longrightarrow$ AMPS arguments

AMPS arguments

nice time slice : Single Hilbert space



- : Early radiation E
- : Inside partners of early rad.
- : Late rad. b
- : Inside partner of $\tilde{b}$

Distant observer sees

Thermal state (mixed)

$$\rho = \frac{1}{Z}\sum_i e^{-\beta H}|\psi_i\rangle\langle\psi_i|$$

$\Rightarrow$ b is entangled with E

-continued

If we accept the Page's argument, the entanglement entropy of the radiation $S(R) = S(BH)$ should be reduced after the Page time

Then, after the Page time a newly generated late Hawking particle b should be entangled with early radiations E

This aspect of bE system implies a non-vanishing mutual information

$$I(b, E) \neq 0$$

(Recall that mutual information measures the entanglement between b and E)

On the contrary,

Hawking pair production picture b should be maximally entangled with $\tilde{b}$

This means that $I(b, E) = 0$

$\Longrightarrow$ Contradiction!!!

-continued

An (single) infalling observer at late time is sufficient to claim both results for $\tilde{b}bE$ system: Violation of Monogamy property of the entanglement

Non-vanishing Ent. between b and E vs Maximal Ent. between b and $\tilde{b}$

Complementarity is not sufficient to overcome this difficulty

c.f. Black hole complementarity: Infalling observer and distant observer cannot see the whole system

This AMPS argument suggests strongly that the conflict occurs even on the near horizon not in the deep black hole interior

Usually, the small curvature and low energy could be assumed on the near horizon of large black holes (Hawking's approximation should be valid)

Which one is wrong among our assumptions or principles???

Suggested Resolutions

- Not a paradox (information loss) (Hawking, Unruh, Wald et al)
- Not a complete thermal Radiation (Page et al)
- Remnants
- FuzzBall (Mathur et al)
- Firewalls (no concrete microscopic models) (AMPS)
- ER=EPR (AdS/CFT defenders Maldacena, Susskind et al)
- Computational Complexity (Preskill et al)
- etc

Unitary evolution supporters have not provided an explanation for the Page curve before last year

Page's original paper: a bit model not a gravity theory

-continued

Firewall : Imagine a high energy curtain to break an entanglement with the behind horizon degrees

Fuzzball: horizonless micro geometries(coarse grained microstates)

These are big departures from our understanding of near horizon since the semiclassical approximation breaks down even near the horizon

ER=EPR : Einstein-Rosen bridge (wormhole) = Einstein-Rosen-Podolski pair (entanglement)

This is a favorite for AdS/CFT practitioners

Others : various advantage vs disadvantages

-continued

AdS/CFT people believe that unitarity could be realized in gravity!

Question:

How to explain the Page's curve in gravity beyond a bit model?

[How to implement $S(R) = S(BH)$ in gravity?]

Specifically, ER=EPR has an explanation for the Page's curve

$\Longrightarrow$ We need more ingredients

- AdS/CFT
- Holographic construction for entanglement entropy (Ryu-Takyanagi)
- Quantum Extremal surface (QES) and Entanglement Wedge(EW)
- Island picture in the EW

-continued

► Area law $S_{BH} = A_H/4G$ is a thermal entropy

Generalized entropy in black holes

$$S(BH) = \frac{A_H}{4G} + S(\text{matters})$$

► Ryu-Takayanagi surface and HEE [AdS Boundary $= A \cup A_c$]

bulk extremal surface $\mathcal{A}$ homologous to the boundary region A

$$S(A) = \underset{\mathcal{A}}{\text{ext}} \frac{\mathcal{A}}{4G}$$

What should we do if there are additional matters in the bulk?

$$S_{gen}(\mathcal{A}) = \min \underset{\mathcal{A}}{\text{ext}} \Big[\frac{\mathcal{A}}{4G} + S_{bulk}(\mathcal{B})\Big] \qquad \partial\mathcal{B} = \mathcal{A} \cup A$$

$S_{gen}(\mathcal{A}) = S(A)$ $\quad \mathcal{A}$: QES

Generalized Entanglement Entropy

Last year progress [Almheiri et al]

GEE formula for radiation with islands

$$S_{EE}(\mathrm{Rad}) = \underset{X}{\mathrm{mim}} \left[\mathrm{ext} \left\{ \frac{A(X)}{4G} + S_{\text{semi-cl.}}(\mathcal{B} \cup \mathrm{Is}) \right\} \right]$$

Is denotes the so-called the island of the entanglement wedge

The appearance of this structure seems to be an unique feature of semi-classical gravity theory

See the next slide for a picture for Ent. Islands

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Entanglement Island for collapsing BH case in the asymptotically flat space

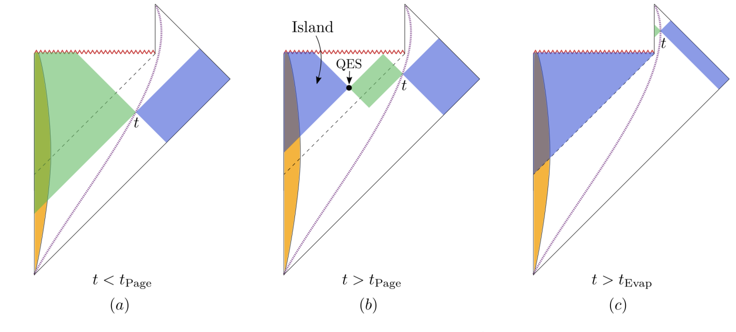

Fig. from [arXive:2006.06872]

Purple dotted curve: distant observer and cutoff surface

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Double holography $\Longrightarrow$ Is is natural in 2 dim. higher embedding

QM + CFT_2 $\Longrightarrow$ 2d gravity + + CFT_2

$\Longrightarrow$ 3d gravity + end of world brane (EOW)

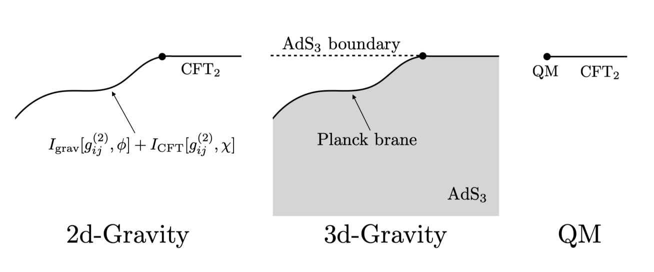
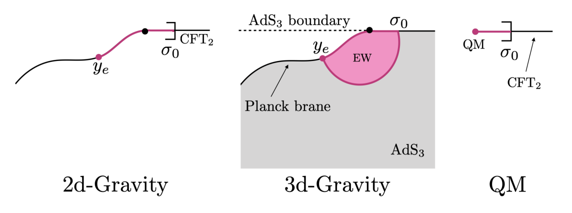


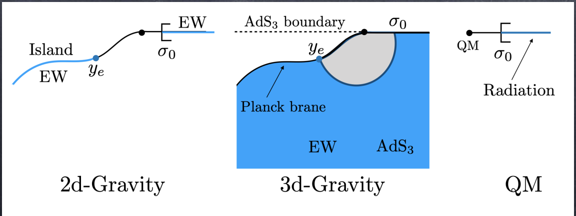
Fig. from Almheiri et al [arXiv:1908.10996]

-continued : Fig. from Almheiri et al

EW at late times (time slice of the geometry)



QES and EW for black hole



Replica wormholes and Petz map

The previous GEE may be derived by using the semi-classical path integral in gravity theory with the well-known formula for EE

($n \to 1$ limit of Renyi entropy)

$$S_{EE} = \lim_{n \to 1} \left[\frac{1}{1-n} \ln \mathrm{Tr}\, \rho^n \right]$$

If we try to compute $\mathrm{Tr}\, \rho^n$ in the Euclidean path integral, there is another saddle point known as replica wormhole geometry:

"the black hole interior can be connected in various ways

among the n copies" Almheiri et al [2006.06872]

Assisted by the path integral,

behind horizon reconstruction is performed by the Petz map

Penington et al [1911.11977]

My work: Double holography and EW island in Janus black holes

Circle fibration of AdS_D over AdS_{D-1}

$$ds_D^2 = dy^2 + \cosh^2 y \; ds_{D-1}^2$$

Let us consider action

$$I = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left[R + \frac{2}{\ell^2} - \frac{1}{2}(\partial\phi)^2\right]$$

Janus black hole solution ($D = 3$ case) : dual to interface CFT (ICFT)

$$ds_3^2 = dy^2 + f(y)ds_2^2 \,, \qquad \phi(y) = \frac{1}{\sqrt{2}}\ln\frac{1+\sqrt{1-2\gamma^2}+\sqrt{2}\gamma\tanh y}{1+\sqrt{1-2\gamma^2}-\sqrt{2}\gamma\tanh y}$$

where γ denotes the deformation parameter and

$$f(y) = \frac{1}{2}\left[1+\sqrt{1-2\gamma^2}\cosh 2y\right], \qquad 0 \leq \gamma < \frac{1}{\sqrt{2}}$$

Note that $\gamma = 0$ corresponds to AdS_3 and $T_H = \frac{1}{\beta} = \frac{r_H}{2\pi\ell}$

Excellent Review Papers

- Entanglement in QFT

"APS Medal for Exceptional Achievement in Research: Invited article on entanglement properties of quantum field theory"

Witten

Rev.Mod.Phys. 90 (2018) 4, 045003 arXive:1803.04993 [hep-th]

- Information loss paradox

"The entropy of Hawking Radiation"

[arXive:2006.06872]

Almheiri et al

Even for novices, the essential points of the recent developments are very well presented in this review paper

Conclusion

Reviews on basic stuffs on black hole information and Entanglement entropy

- Information paradox and Page curve
- QES / EW and ER=EPR
- 2d JT gravity and double holography And island picture
- Some open questions:
  In QFT we assume that $[\psi(x), \psi(y)]_{\pm} = 0$ (usual causal fields)
  bet. spacelike separated x, y: locality in AdS/CFT??

My work

- Janus-deformation of AdS black holes (Janus/ICFT)

In this setup,

- Double holography, Page curve, EW Island, effective 2d gravity,
  Phase transition before Page time, etc