Longitudinal beam dynamics examination (corrections)

(1h30 – Free access to lecture notes and paper documents)

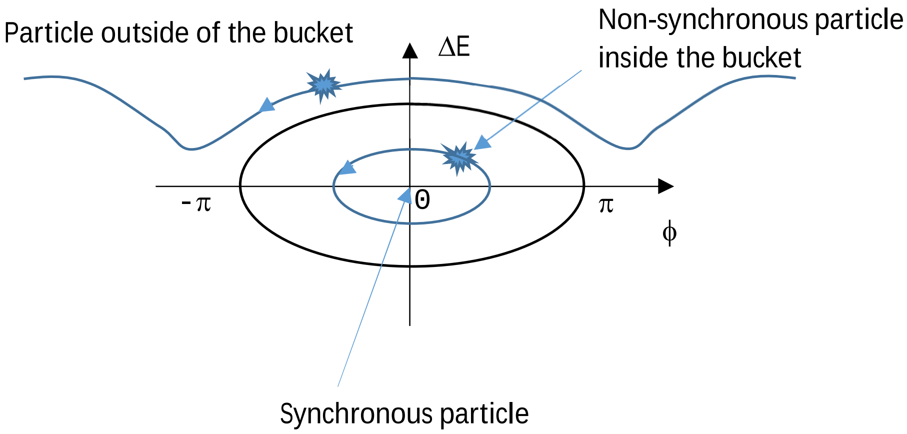
Exercise A:

1) Explain why the bending radius is not equal to the machine radius in a synchrotron.

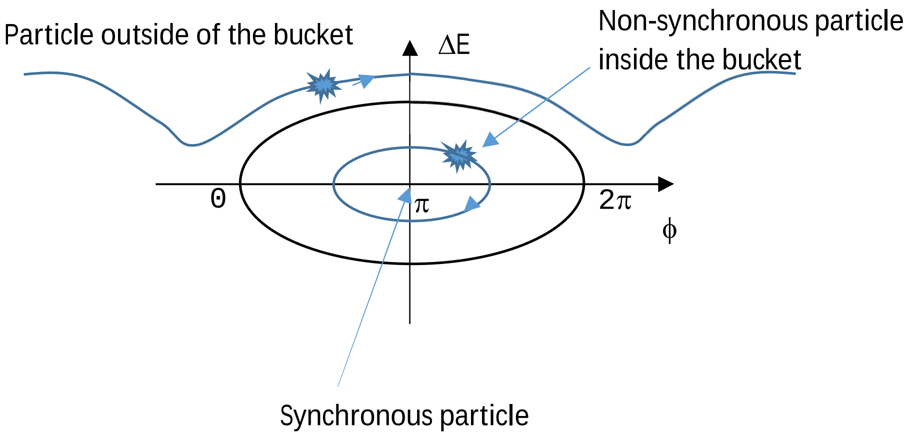
The bending radius would be equal to the machine radius if the circumference of the machine were fully occupied by dipoles. However, there are other elements required for the functioning of the accelerator that are taking space (kickers to inject and extract the beam, RF cavities, quadrupoles, sextupoles, instrumentation, etc.). Therefore, the bending radius is always lower than the machine radius.

2)
   a. Draw a sketch in phase space with coordinates of your choice of a stationary bucket below transition and a sketch of a stationary bucket above transition.
   b. In both sketches, draw a synchronous particle, a non-synchronous particle inside the bucket, and a particle outside the bucket. Indicate their trajectory and direction of motion.
   c. Assuming the RF voltage and harmonic number is the same below and above transition, can we conclude on whether the height of the bucket is larger below or above transition?

Below transition:



Above transition:



The height of the stationary bucket is given by (equation of max energy spread slide 84)

$$\Delta E_{\max}^{sep}(\phi_s) = \sqrt{\frac{2\,\beta_s^2\,E_s\,e\,\hat{V}_{RF}}{\pi\,h\,|\eta|}}\;G(\phi_s)$$

$G(\phi_s)=1$ as the bucket is stationary. Therefore the size of the bucket depends on the energy of the synchronous particle E_s, but also on the absolute value of the slippage factor $|\eta|$. Therefore we cannot conclude.

3) Do all particles inside a full bucket take the same time to perform one full revolution in phase space? Explain qualitatively why. How is this time to perform a revolution called?

No, particles with large synchrotron amplitude in phase space take longer to perform a turn in phase space than particles with small amplitude. This is due to the non-linear restoring force from the RF cavity, which decreases with amplitude. For small amplitudes, the restoring force is linear and particles rotate with the same synchrotron frequency in phase space. The time to perform a revolution in phase space is defined as the synchrotron period.

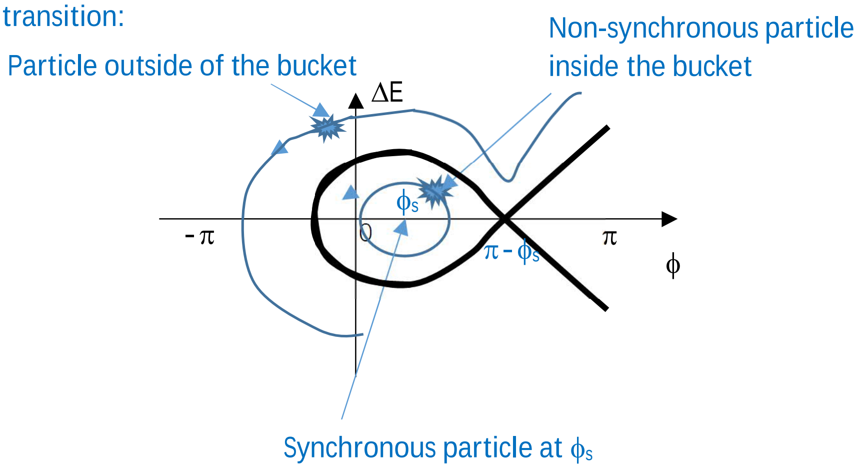
4) For a given machine with fixed harmonic number,
   a. Draw a sketch of a stationary bucket below transition.
   b. Explain what machine parameters need to be changed to initiate acceleration.
   c. Draw a sketch of the accelerating bucket, right after the start of acceleration and highlight the differences with a bucket at constant energy in terms of synchronous phase, bucket length, bucket shape and bucket height for the case of a constant RF voltage. In both sketches, draw a synchronous particle inside the bucket, a non-synchronous particle inside the bucket, and a particle outside the bucket. Indicate their trajectory and direction of motion.

a) Stationary bucket (same as question 2 below transition).

b) To initiate acceleration, one needs to change the RF cavity phase and to synchronize the magnetic field rate of change to the resulting energy gained by the synchronous particle every turn and the RF voltage. The RF voltage may need to be increased to keep a stable area in phase space.

c) Accelerating bucket:

Below transition:



The differences between the bucket right after the start of acceleration is:

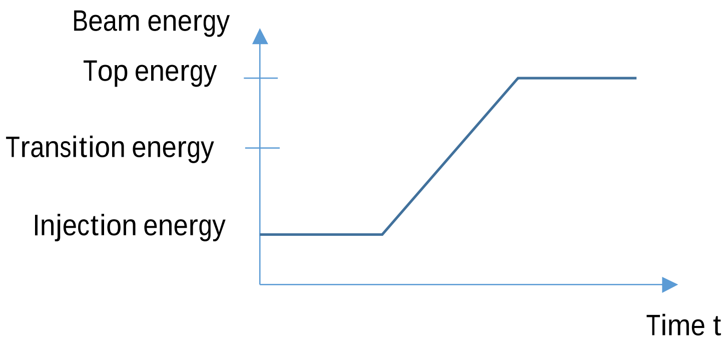
- ϕ_s is different from 0
- The bucket length is no longer 2π , but smaller
- The bucket is not symmetric
- The bucket height is

$$\Delta E^{sep}_{max}(\phi_s) = \sqrt{\frac{2\,\beta_s^2\,E_s\,e\,\hat{V}_{RF}}{\pi\,h\,|\eta|}}\;G(\phi_s)$$

(equation of max energy spread slide 84)

Right after the start of acceleration, the energy of the synchronous particle E_s is still close to the energy of that particle before the acceleration starts, therefore also the slippage factor $|\eta|$. The harmonic number h and RF voltage V_{RF} are constant, but G decreases. Therefore, the bucket height decreases right after acceleration starts.

5) If the harmonic number, RF voltage and magnetic field ramp rate are kept constant along acceleration, draw qualitatively how the bucket height changes from injection energy to top energy (starting from a stationary bucket and finishing with a stationary bucket), and explain how it should change well below transition, close to transition and well above transition energy.



When starting acceleration, the bucket height drops (see explanation in question 4c)

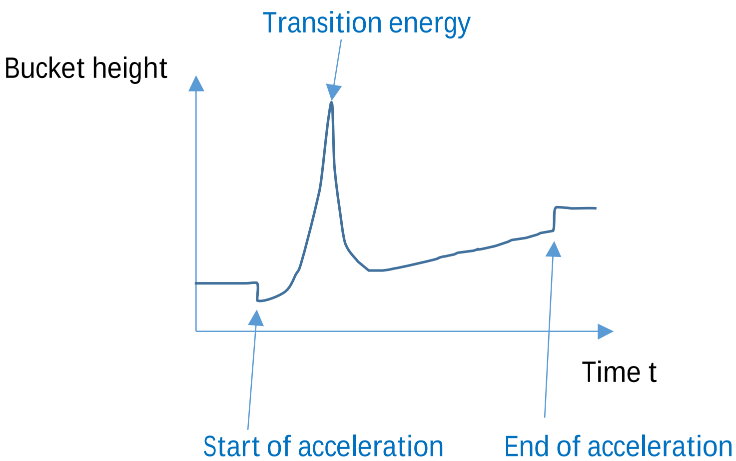
For a beam well below transition, the slippage factor decreases slightly with increasing energy, and the energy of the synchronous particle increases with increasing energy. Therefore, the size of the bucket increases.

For a beam well above transition, the slippage factor (in absolute value) increases with increasing energy, and the energy of the synchronous particle increases with increasing energy. However, far away from transition, the slippage factor can be considered almost constant. Therefore, we can assume that the bucket height increases.

Around transition: the slippage factor approaches zero and therefore the bucket height diverges.

When acceleration stops, the bucket height increases as the synchronous phase is pi, and the function G becomes 1.

(See also slide 90)



6) What are the benefits and the limits of using a synchronous phase close to $\pi/2$ during acceleration?

Using a synchronous phase during acceleration that is getting close to $\pi/2$ allows reducing the time of ramp (provided the magnetic field rate and RF voltage can be increased), but it significantly reduces the stable area in phase space and could lead to significant beam losses.

7) In what circumstances is there no stable RF bucket?

- When the RF voltage is zero, there is no RF bucket.
- If the magnetic field rate is 0 and the RF voltage is nonzero, there is always a bucket.
- If the magnetic field rate is nonzero, there is no RF bucket when the RF voltage is too small compared to the value needed for the following equation to have a solution:

$$\sin\phi_s = 2\pi\rho R \frac{\dot{B}}{\hat{V}_{RF}}$$

8) Can the momentum compaction factor of a machine be negative?

Yes, it can. In that case the transition gamma is imaginary.

9) What are the main differences between lepton and hadron synchrotrons in terms of longitudinal beam dynamics?

Velocity and revolution frequency are almost constant for leptons. Therefore electron beams are always above transition energy. For current synchrotrons, leptons are much more subject to synchrotron radiation than protons, and the RF system should be designed to compensate for the energy loss.
One must care about transition energy for proton synchrotrons.

Exercise B: the CERN SPS

The parameters for the CERN SPS (Super Proton Synchrotron) are provided in Table 1.

Table 1: SPS parameters

Circumference	6911 m
Momentum compaction factor	$1.92\ 10^{-3}$
Bending radius	741.3 m

Two types of beams are accelerated in the SPS and their parameters are provided in Table 2.

Table 2: parameters of the "LHC beam" and the "Fixed Target (FT) beam"

	LHC beam	FT beam
Injection momentum (GeV/c)	26	14
Extraction momentum (GeV/c)	450	400
Momentum compaction factor	$1.92\ 10^{-3}$	$1.92\ 10^{-3}$
Harmonic number	4620	4620
Magnetic field rate in T/s	0.35	0.76

1) Transition energy
   a. Define qualitatively the transition energy in one sentence.

   See slide 42: special energy for which the revolution frequency is constant with momentum, since the velocity variation is exactly compensated by the trajectory variation.

   b. Compute the transition energy for both beams.

   $\gamma_{tr} = \sqrt{\frac{1}{\alpha_p}} = 22.8$ for both beams

   c. Could transition energy be different for the LHC beam and the Fixed Target beam? What machine optics parameter would need to be changed?

   Yes, one could change the distribution of dispersion in the horizontal plane and therefore the momentum compaction factor.

   d. Compute the relativistic factor gamma and slippage factor for both beams at injection and top energy.

$$\eta = \frac{1}{\gamma^2} - \alpha_p$$

$$\gamma = \frac{E}{E_0} = \frac{\sqrt{p^2c^2 + {E_0}^2}}{E_0} = \sqrt{\frac{p^2c^2}{{E_0}^2} + 1}$$

Therefore,
- for the LHC beam at injection, $\gamma = 27.7$ and η= -6.18e-4<0 → above transition
- for the LHC beam at extraction, $\gamma = 479$ and η= -1.92e-3<0 → above transition
- for the FT beam at injection, $\gamma = 14.9$ and η= 2.57e-3>0 → below transition
- for the FT beam at injection, $\gamma = 426$ and η= -1.91e-3<0 → above transition

e. Does the LHC beam cross transition?

No, as slippage factor does not change sign.

f. Does the FT beam cross transition?

Yes, as the slippage factor changes sign.

g. What needs to be done when transition energy is crossed by a beam?

One needs to change the phase of the RF system from ϕ_s to $\pi - \phi_s$ to keep the bunch inside the bucket after transition crossing.

2) Bucket for the LHC beam

a. Define in one sentence the synchrotron tune.

It is the number of synchrotron oscillations per machine turn in the longitudinal phase space.

b. Compute the synchrotron tune at – constant – injection energy for the LHC beam with an RF voltage of 2 MV.

$$Q_{sync} = \frac{\Omega_{sync}}{\omega_s} = \sqrt{\frac{e\hat{V}_{RF}\,h}{2\pi\beta^2 E_s}\eta\cos\phi_s}$$

(definition of Q_s slide 76)

The energy is constant so $\cos(\phi_s)=1$

We find Q_s = 6e-3

c. With an RF voltage of 2 MV, what is the maximum energy spread of the LHC beam injected into the SPS that would be allowed without losses?

To avoid losses, all the particles injected into the SPS should be inside the bucket, which means that – assuming no phase or voltage offset and similar longitudinal bunch shapes – the difference of energy between the injected particles and the synchronous particle should not be larger in absolute value than the difference of energy between the synchronous particle and the separatrix at the phase of the synchronous particle (referred to as ΔE^{sep}_{max} in the course, p.84).

Assuming here that the energy spread is defined as the difference between the maximum energy and the minimum energy of particles inside the bunch (other definitions could be envisaged), the maximum energy spread allowed is +/- ΔE^{sep}_{max}. Using

$$\Delta E^{sep}_{max}(\phi_s) = \sqrt{\frac{2\,\beta_s^2\,E_s\,e\,\hat{V}_{RF}}{\pi\,h\,|\eta|}}\;G(\phi_s)$$

(equation of separatrix slide 84),
Since the bucket is stationary, $\cos(\phi_s)=1$ and the maximum energy spread of an incoming bunch is +/-108 MeV.

d. By how much would the allowed energy spread of the injected LHC beam change if the RF voltage were set to its maximum of 10 MV?

From the equation above, the allowed energy spread would increase by a factor $\sqrt{10\ MV/2\ MV}$ = 2.23

3) Acceleration

a. Assuming the magnetic field rate is constant along beam acceleration, compute the time it takes to accelerate the LHC and the FT beam from injection to top energy.

- LHC beam at injection: $B = \frac{p}{e\rho} = \frac{26}{0.3*741.3} = 0.11T$
- LHC beam at extraction: $B = \frac{p}{e\rho} = \frac{26}{0.3*741.3} = 2.02T$
- FT beam at injection: $B = \frac{p}{e\rho} = \frac{26}{0.3*741.3} = 0.06T$
- FT beam at injection: $B = \frac{p}{e\rho} = \frac{26}{0.3*741.3} = 1.8T$

Therefore the ramping time is $t = \frac{B_{extr} - B_{inj}}{\dot{B}}$
And is 5.4 s for the LHC beam and 2.28 s for the FT beam

b. Check that the energy ramp rate is 78 GeV/s for the LHC beam and 169 GeV/s for the FT beam.

Energy ramp rate is (E_{extr}-E_{inj})/ramp time

c. Compute the synchronous phase along acceleration for these two beams, assuming the RF voltage during acceleration is 7 MV.

$$\phi_s = \arcsin\left(2\pi \rho R \; \frac{\dot{B}}{\hat{V}_{RF}} \right)$$

For the LHC beam, the synchronous phase is 165 degrees (as it is above transition).

For the FT beam, the synchronous phase is 33.8 degrees below transition and 146.2 degrees above transition.

Physical constants:

- Elementary charge: $e = 1.60\ 10^{-19}$ C
- Electron mass: $m_e = 9.11\ 10^{-31}$ kg
- Proton mass: $m_p = 1.67\ 10^{-27}$ kg
- Speed of light: $c = 3.00\ 10^{8}$ m/s
- Vacuum permittivity: $\varepsilon_0 = 8.85\ 10^{-12}$ F/m