Joint Universities Accelerator School

Longitudinal beam dynamics examination

(1h30 – Free access to lecture notes and paper documents)

This exam is composed of two independent exercises totalling 108 points. The marks will be normalized to 20.

Exercise A: (58 pts)

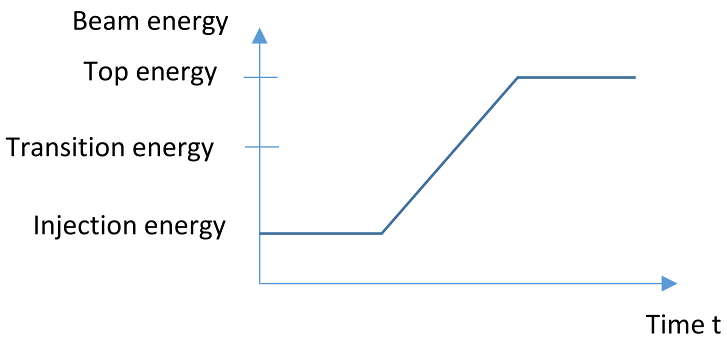
1) Explain why the bending radius is not equal to the machine radius in a synchrotron. **(3 pts)**

2)
   a. Draw a sketch in phase space with coordinates of your choice of a stationary bucket below transition and a sketch of a stationary bucket above transition. **(5 pts)**
   b. In both sketches, draw a synchronous particle, a non-synchronous particle inside the bucket, and a particle outside the bucket. Indicate their trajectory and direction of motion. **(5 pts)**
   c. Assuming the RF voltage and harmonic number are the same below and above transition, can we conclude whether the height of the bucket is larger below or above transition? **(4 pts)**

3) Do all particles inside a full bucket take the same time to perform one full revolution in phase space? Explain qualitatively why. What is the name given to this time taken to perform a revolution? **(4 pts)**

4) For a proton synchrotron with fixed harmonic number,
   a. Draw a sketch of a stationary bucket below transition (as in question 2 above). **(2 pts)**
   b. Explain which machine parameters need to be changed to initiate acceleration. **(3 pts)**
   c. Draw a sketch of the accelerating bucket, just after the start of acceleration and highlight the differences with a bucket at constant energy in terms of synchronous phase, bucket length, bucket shape and bucket height for the case of a constant RF voltage **(5 pts)**. In both sketches, draw a synchronous particle inside the bucket, a non-synchronous particle inside the bucket, and a particle outside the bucket. Indicate their trajectory and direction of motion. **(5 pts)**

5) If the harmonic number, RF voltage and magnetic field ramp rate are kept constant during acceleration, draw qualitatively how the bucket height changes from injection energy to top energy (starting from a stationary bucket and finishing with a stationary bucket, see figure below), and explain how it should change well below transition, close to transition and well above transition energy. **(10 pts)**



6) What are the benefits and the limits of using a synchronous phase close to $\pi/2$ during acceleration? **(3 pts)**

7) In what circumstances is there no stable RF bucket? **(3 pts)**

8) Can the momentum compaction factor of a machine be negative? **(3 pts)**

9) What are the main differences between lepton and hadron synchrotrons in terms of longitudinal beam dynamics? **(3 pts)**

Exercise B: the CERN SPS **(50 pts)**

The parameters for the CERN SPS (Super Proton Synchrotron) are provided in Table 1.

Table 1: SPS parameters

Circumference	6911 m
Momentum compaction factor	1.92×10^{-3}
Bending radius	741.3 m

Two types of beams are accelerated in the SPS and their parameters are provided in Table 2.

Table 2: parameters of the “LHC beam” and the “Fixed Target (FT) beam”

	LHC beam	**FT beam**
Injection momentum (GeV/c)	26	14
Extraction momentum (GeV/c)	450	400
Momentum compaction factor	1.92×10^{-3}	1.92×10^{-3}
Harmonic number	4620	4620
Magnetic field rate in T/s	0.35	0.76

1) Transition energy
   a. Define qualitatively the transition energy in one sentence. **(3 pts)**

   b. Compute the value of the Lorentz γ-factor at transition energy for both beams. **(4 pts)**

   c. Could transition energy be different for the LHC beam and the Fixed Target beam? What machine optics parameter would need to be changed? **(3 pts)**

   d. Compute the relativistic factor gamma and slippage factor for both beams at injection and top energy. **(5 pts)**

   e. Does the LHC beam cross transition? **(2 pts)**

   f. Does the FT beam cross transition? **(2 pts)**

   g. What needs to be done when transition energy is crossed by a beam? **(3 pts)**

2) Bucket for the LHC beam
   a. Define in one sentence the synchrotron tune. **(3 pts)**

   b. Compute the synchrotron tune at – constant – injection energy for the LHC beam with an RF voltage of 2 MV. **(4 pts)**

   c. With an RF voltage of 2 MV, what is the maximum energy spread of the LHC beam injected into the SPS that would be allowed without losses? **(4 pts)**

   d. By how much would the allowed energy spread of the injected LHC beam change if the RF voltage were set to its maximum of 10 MV? **(4 pts)**

3) Acceleration
   a. Assuming the magnetic field rate is constant along beam acceleration, compute the time it takes to accelerate the LHC and the FT beam from injection to top energy in the SPS. **(6 pts)**

   b. Check that the energy ramp rate is 78 GeV/s for the LHC beam and 169 GeV/s for the FT beam. **(3 pts)**

   c. Compute the synchronous phase along acceleration for these two beams, assuming the RF voltage during acceleration is 7 MV. **(4 pts)**

Physical constants:

- Elementary charge: $e = 1.60 \times 10^{-19}$ C
- Electron mass: $m_e = 9.11 \times 10^{-31}$ kg
- Proton mass: $m_p = 1.67 \times 10^{-27}$ kg
- Speed of light: $c = 3.00 \times 10^{8}$ m/s
- Vacuum permittivity: $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m