

# Recent progress in the calculation of the Higgs trilinear coupling in models with extended scalar sectors

Based on JB, Kanemura, PLB 796 (2019) 38-46 & EPJC 80 (2020) 3, 227  
and JB, Kanemura, Shimoda, arXiv:2011.07580 (accepted in JHEP)

**Johannes Braathen**

*5th International Workshop "Higgs as a Probe of New Physics" Special Edition 2021  
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# Why investigate $\lambda_{hhh}$ ?

# Probing the shape of the Higgs potential

- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:

→ the location of the EW minimum:

$$v = 246 \text{ GeV}$$

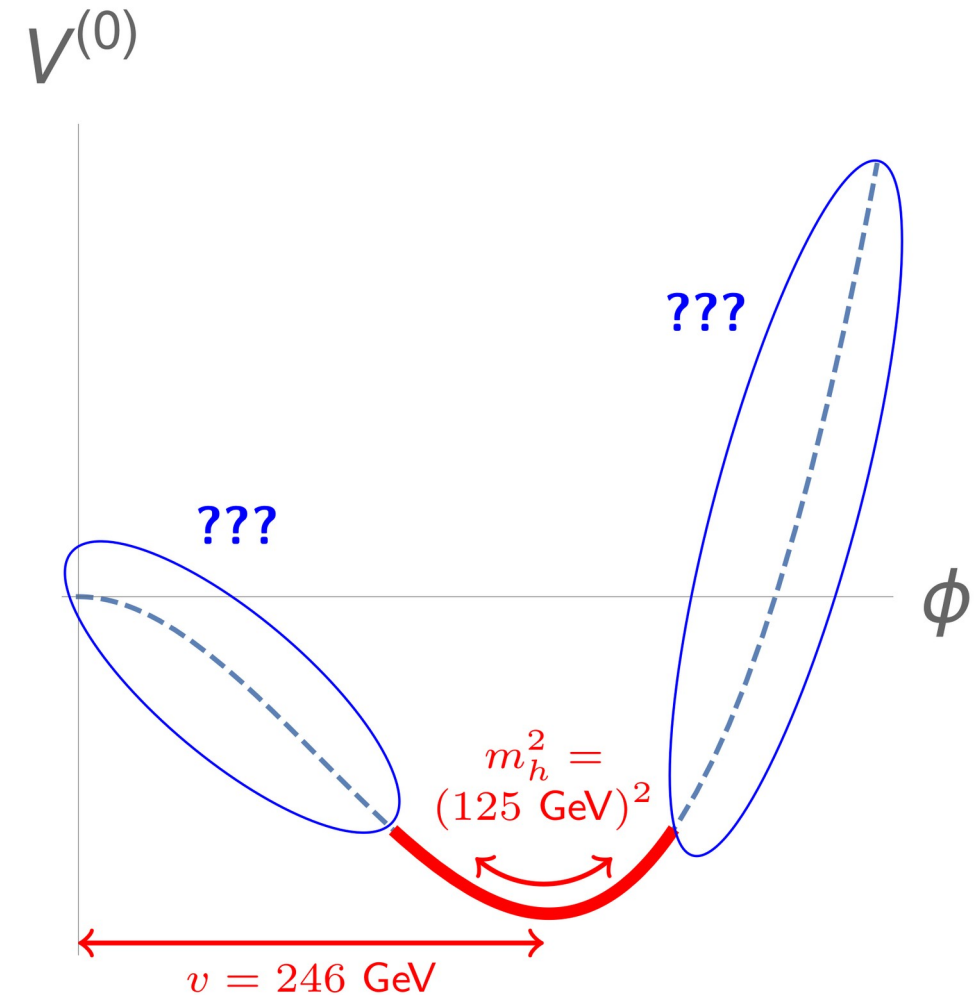
→ the curvature of the potential around the EW minimum:

$$m_h = 125 \text{ GeV}$$

However we still don't know the **shape** of the potential, away from EW minimum → depends on  $\lambda_{hhh}$

- $\lambda_{hhh}$  determines the nature of the EWPT!

⇒ O(20%) deviation of  $\lambda_{hhh}$  from its SM prediction needed to have a strongly first-order EWPT → necessary for EWBG [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]



# Distinguish aligned scenarios with or without decoupling

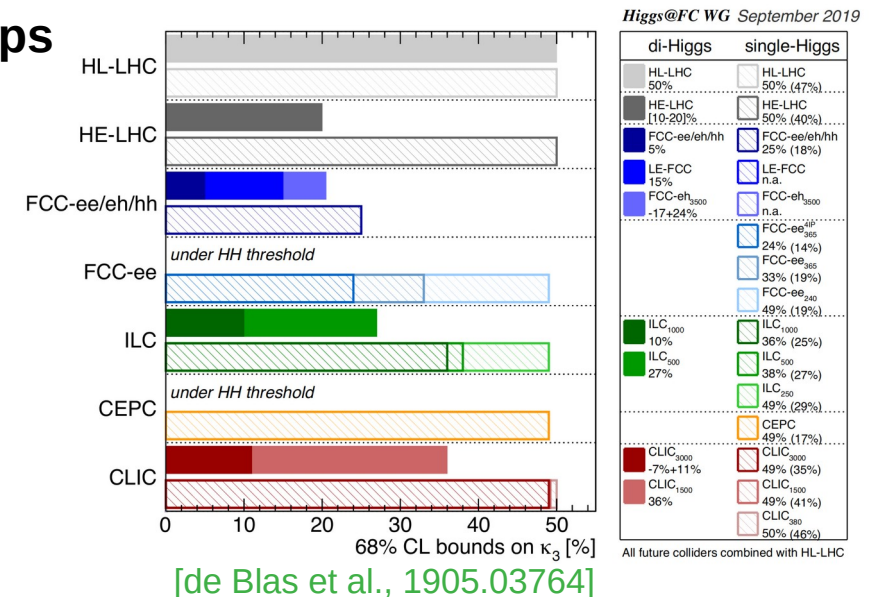
- Aligned scenarios already seem to be favoured → Higgs couplings are SM-like at tree-level
- Non-aligned scenarios (e.g. in 2HDMs) could be almost entirely excluded in the close future using **synergy** of **HL-LHC** and **ILC**!
  - **Alignment through decoupling?** or **alignment without decoupling?**

- If alignment without decoupling, Higgs couplings like  $\lambda_{hhh}$  can still exhibit large deviations from SM predictions because of **non-decoupling effects from BSM loops**

- Current best limit (at 95% CL):  

$$-3.7 < \lambda_{hhh} / (\lambda_{hhh})^{SM} < 11.5$$
 [ATLAS-CONF-2019-049]

- Improvement at future colliders:
  - **HL-LHC:**  $\lambda_{hhh} / (\lambda_{hhh})^{SM}$  within ~ 50-100%;
  - At lepton colliders – **ILC, CLIC** – within some tens of %;
  - At a **100-TeV hadron collider**, down to 5-7%



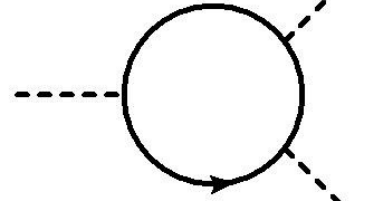
see also backup and [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], etc.

# Non-decoupling effects at one loop

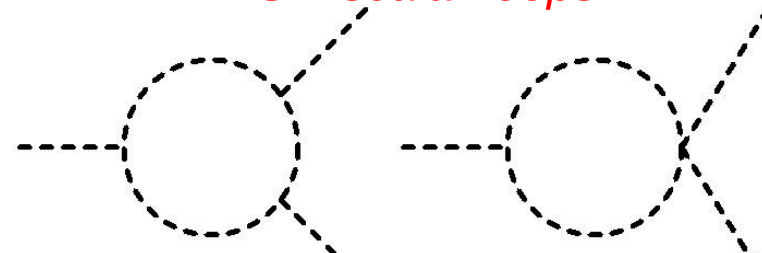
# One-loop non-decoupling effects

- Leading one-loop corrections to  $\lambda_{hhh}$  in models with extended sectors (e.g. 2HDM):

SM top quark loop



BSM scalar loops



$$\delta^{(1)} \lambda_{hhh} \supset \frac{1}{16\pi^2} \left[ -\frac{48m_t^4}{v^3} + \sum_{\Phi} \frac{4n_{\Phi} m_{\Phi}^4}{v^3} \left( 1 - \frac{\mathcal{M}^2}{m_{\Phi}^2} \right)^3 \right]$$

First found in 2HDM:  
[Kanemura, Kiyoura,  
Okada, Senaha, Yuan '02]

$\mathcal{M}$  : BSM mass scale, e.g. soft breaking scale  $M$  of  $Z_2$  symmetry in 2HDM

$n_{\Phi}$  : # of d.o.f of field  $\Phi$

- Size of new effects depends on how the BSM scalars acquire their mass:  $m_{\Phi}^2 \sim \mathcal{M}^2 + \tilde{\lambda}v^2$

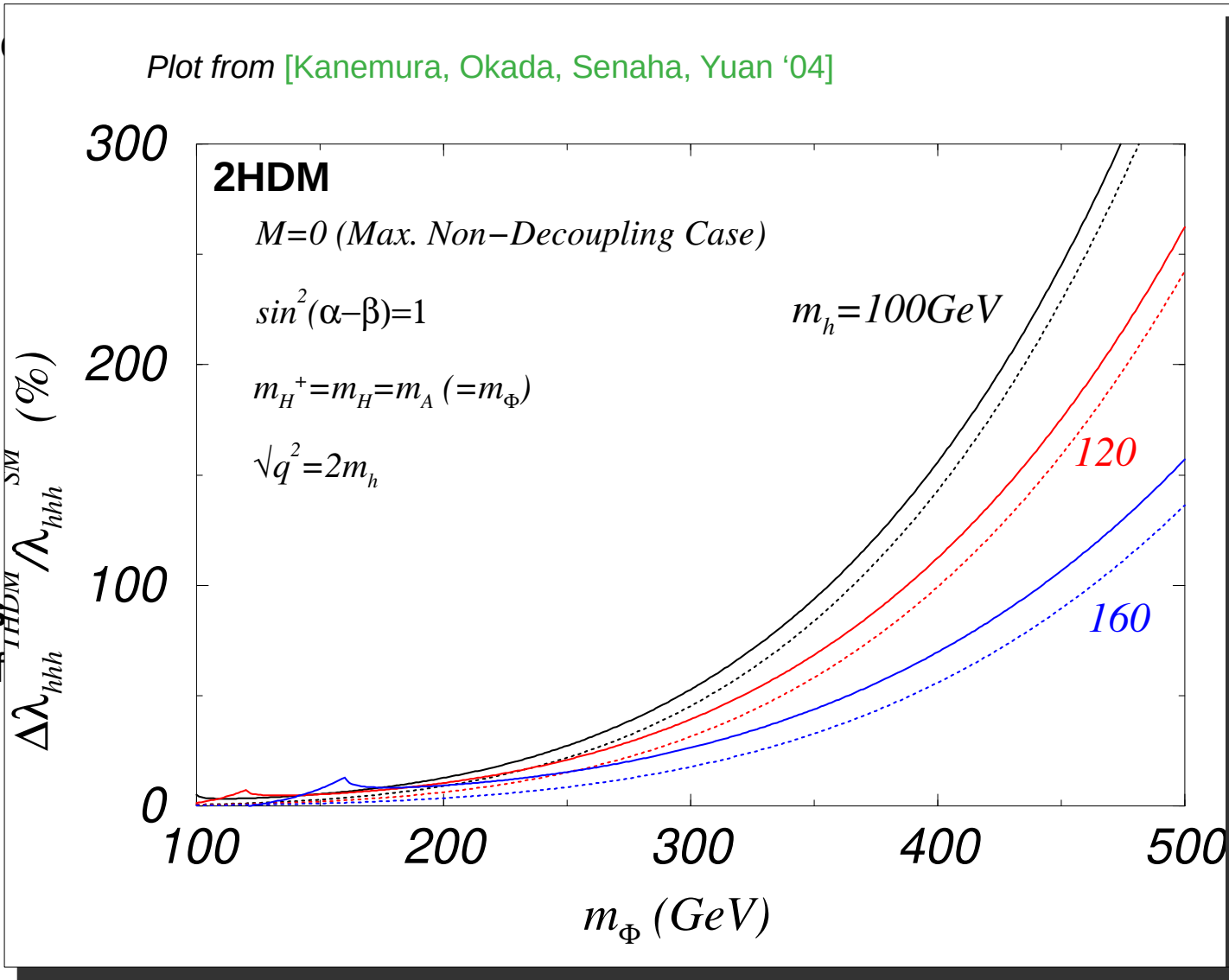
$$\left( 1 - \frac{\mathcal{M}^2}{m_{\Phi}^2} \right)^3 \longrightarrow \begin{cases} 0, & \text{for } \mathcal{M}^2 \gg \tilde{\lambda}v^2 \\ 1, & \text{for } \mathcal{M}^2 \ll \tilde{\lambda}v^2 \end{cases} \longrightarrow \text{Huge BSM effects possible!}$$



# One-loop non-decoupling effects

➤ Leading one-loop c

Plot from [Kanemura, Okada, Senaha, Yuan '04]



$$\delta^{(1)} \lambda_{hhh} \supset$$

$\mathcal{M}$  : BSM mass  
 $n_\Phi$  : # of d.o.f of

➤ Size of new effects

First found in 2HDM:  
 [Kanemura, Kiyoura,  
 Okada, Senaha, Yuan '02]

$$\mathcal{M}^2 + \tilde{\lambda} v^2$$

**Huge BSM effects possible!**

# One-loop results for $\lambda_{hhh}$

- Deviations of  $\lambda_{hhh}$  by **several hundred percent** w.r.t SM prediction can occur at one loop in wide range of BSM theories – 2HDM, IDM, singlet models, triplet model – **without violating unitarity!**

- *Classical scale invariant models*: additional symmetry entirely fixes the one-loop Higgs potential

⇒ **Universal one-loop result in CSI theories!**

*(detailed derivation in backup)*

$$(\lambda_{hhh}^{\text{CSI}})^{\text{1-loop}} = \frac{5[M_h^2]v_{\text{eff}}}{v} = \frac{5}{3}(\lambda_{hhh}^{\text{SM}})^{\text{tree}}$$

(Higgs effective potential mass ~ pole mass)

(Higgs VEV)

- **What happens at two loops??**



# Our results

# Our calculations

- We want to know **how large** the two-loop corrections to  $\lambda_{hhh}$  can become:

- ➔ *Effective Higgs trilinear coupling*

- (i.e. neglect subleading effects from ext. momentum)

$$\lambda_{hhh} \equiv \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min.}}$$

- ➔ *Dominant two-loop corrections to  $V_{\text{eff}}$*  = diagrams involving **heavy BSM scalars and top quark**

- ➔ *Aligned scenarios* → no mixing + compatible with experimental results

- Several models investigated:

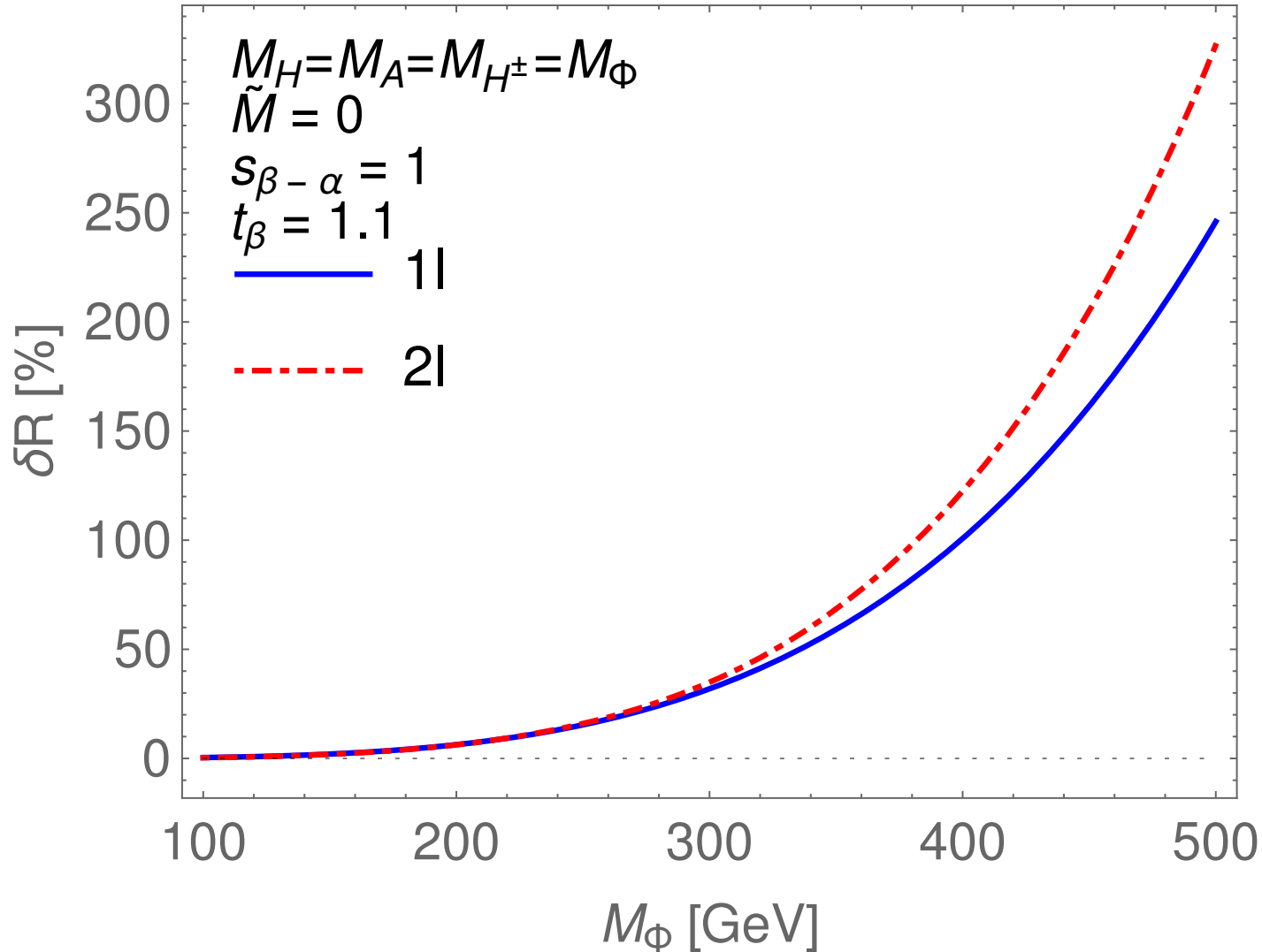
- (aligned) 2HDM**, IDM, singlet extension, **CSI-2HDM**, CSI N-scalar model

- Numerical results in the following:

$$\delta R \equiv \frac{\hat{\lambda}_{hhh}^{\text{BSM}} - \hat{\lambda}_{hhh}^{\text{SM}}}{\hat{\lambda}_{hhh}^{\text{SM}}}$$

# BSM deviation of $\lambda_{hhh}$ in an aligned 2HDM

Taking degenerate BSM scalar masses:  $M_\phi$

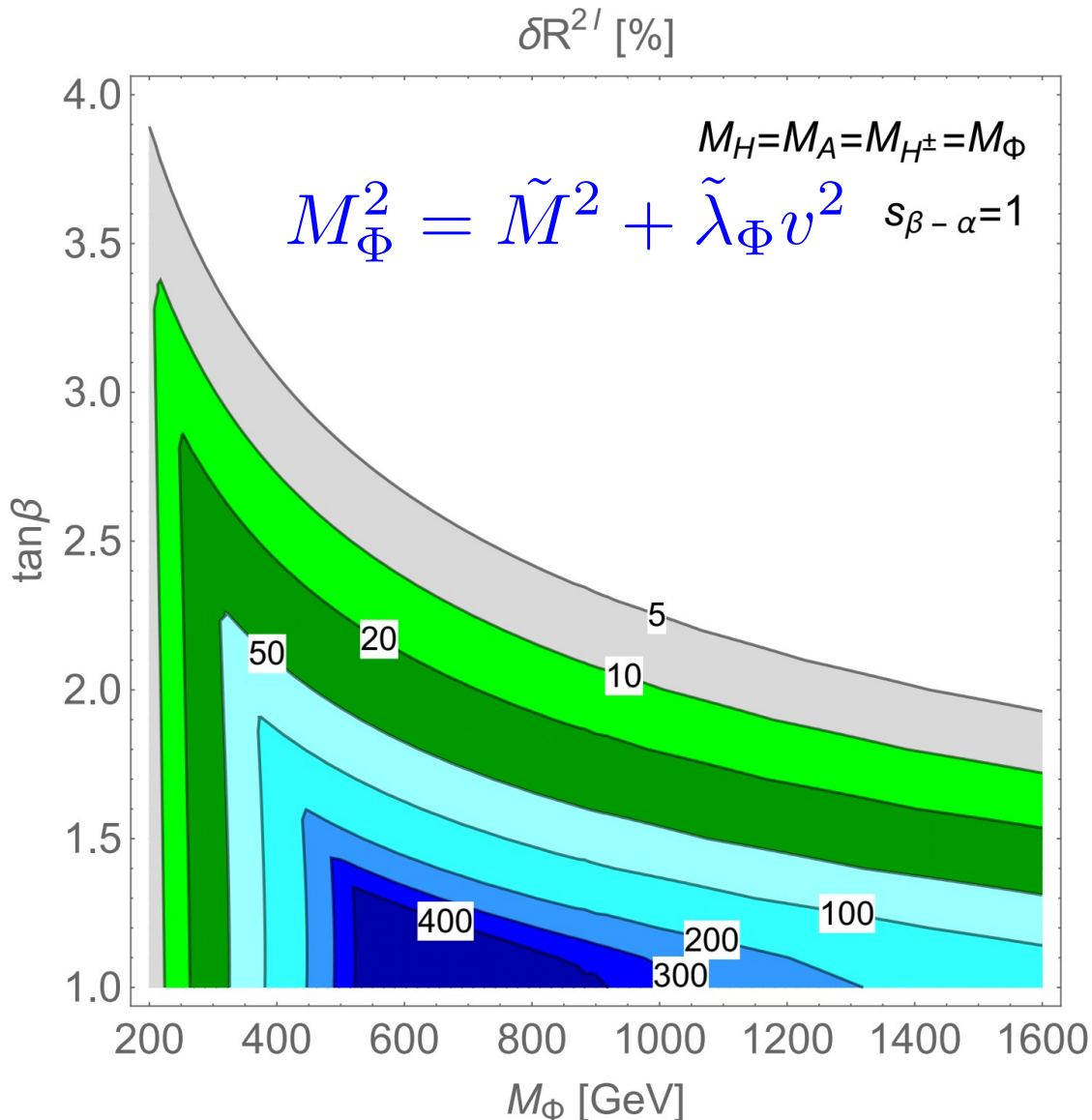


[JB, Kanemura 1903.05417]

- $\tilde{M} = 0 \rightarrow$  maximal non-decoupling effects
- $\delta^{(2)}\lambda_{hhh}$  typically 10-20% of  $\delta^{(1)}\lambda_{hhh}$  for most of mass range, at most 30%

# Maximal BSM deviation in an aligned 2HDM scenario

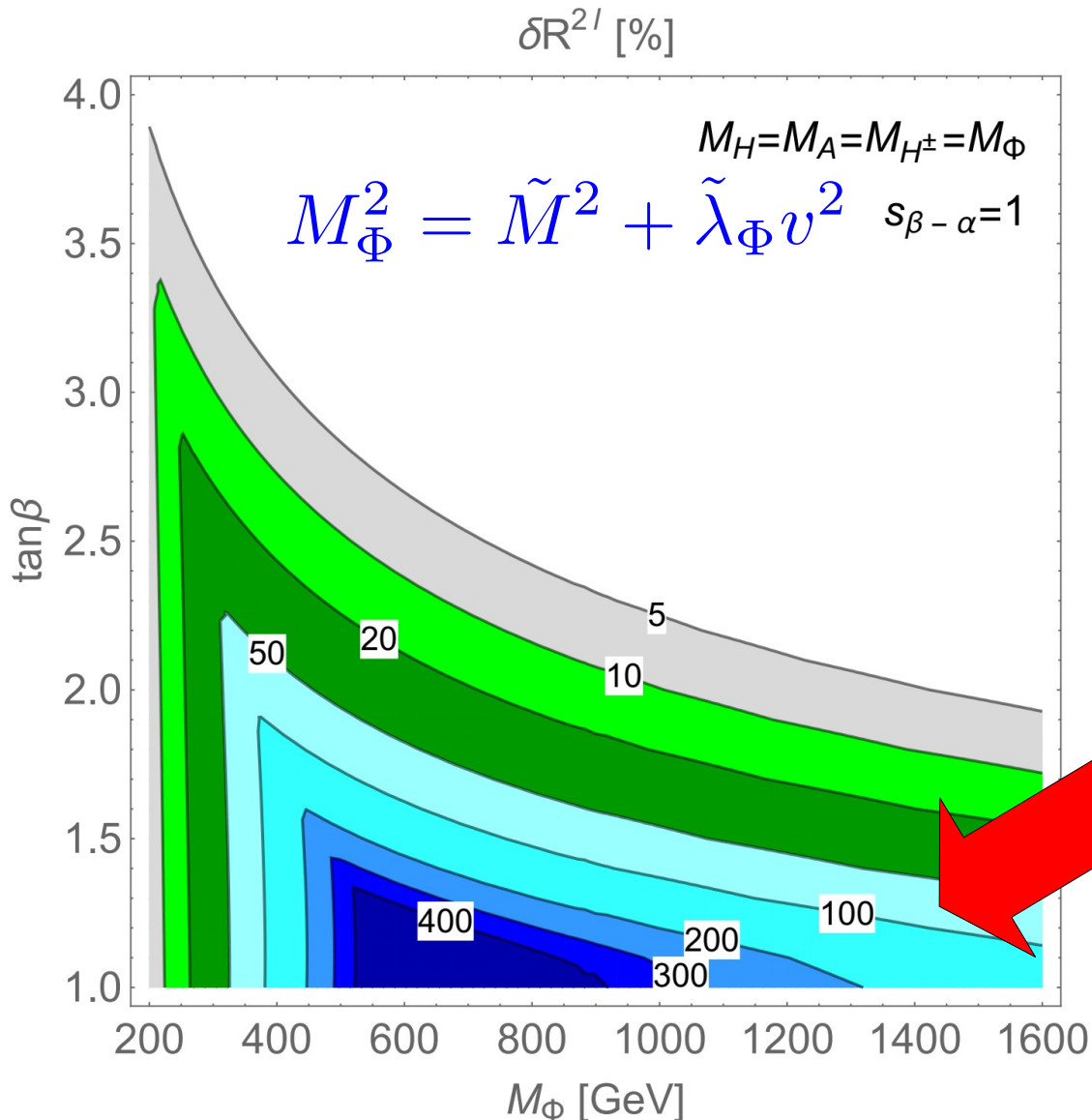
[JB, Kanemura 1911.11507]



- Maximal  $\delta R$  (1l+2l) allowed while fulfilling perturbative unitarity [Kanemura, Kubota, Takasugi '93]
- Max. deviations for low  $\tan\beta$  and  $M_\Phi \sim 600-800$  GeV  $\rightarrow$  heavy BSM scalars acquiring their mass from Higgs VEV **only**
  - 1 loop: up to  $\sim 300\%$  deviation at most
  - 2 loops: additional  $100\%$  (for same points)
- For increasing  $\tan\beta$ , unitarity constraints become more stringent  $\rightarrow$  smaller  $\delta R$
- **Blue region:** probed at **HL-LHC** (50% accuracy on  $\lambda_{hhh}$ )
- **Green region:** probed at lepton colliders, e.g. **ILC** (50% accuracy at 250 GeV; 27% at 500 GeV; 10% at 1 TeV)

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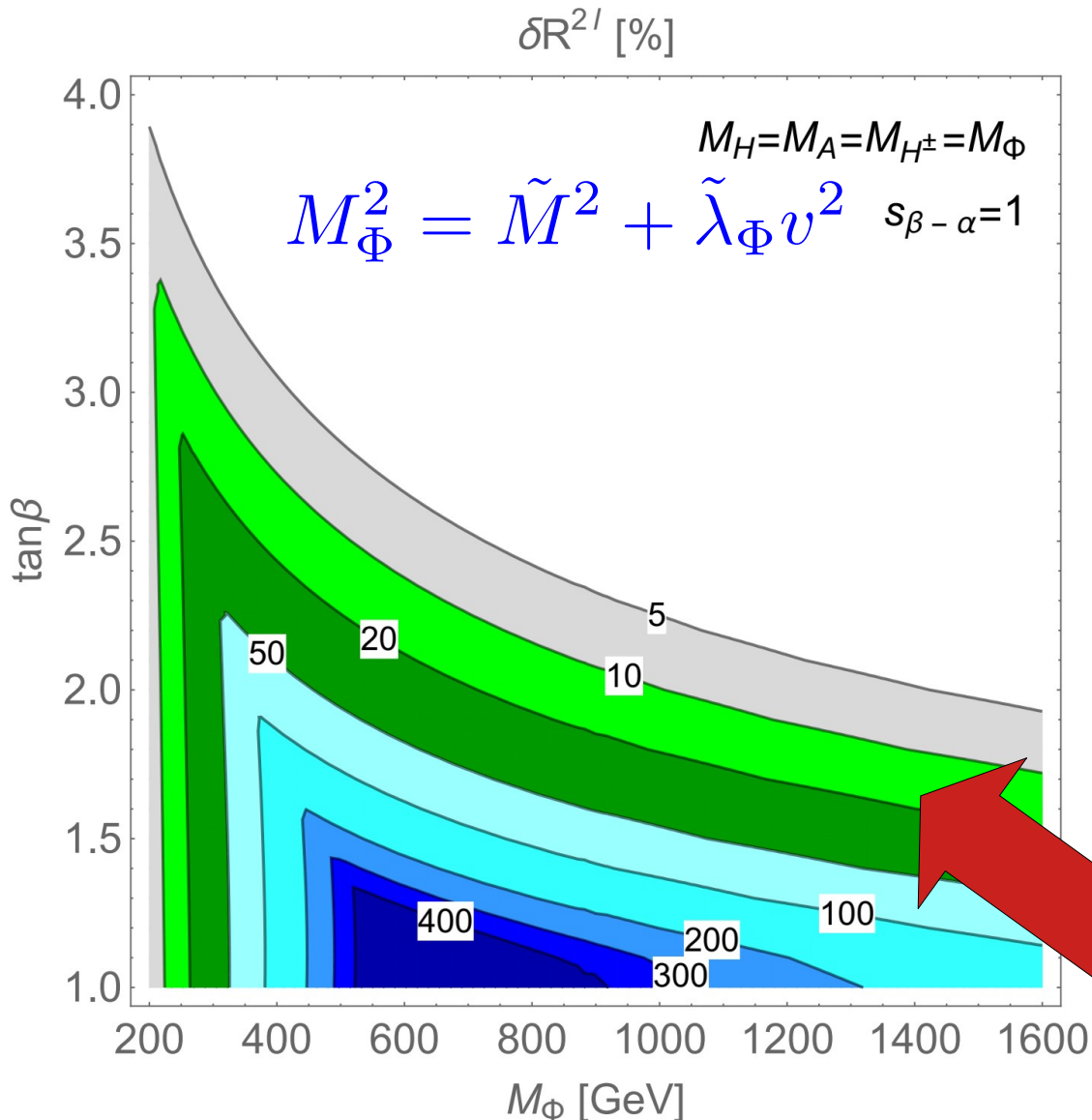
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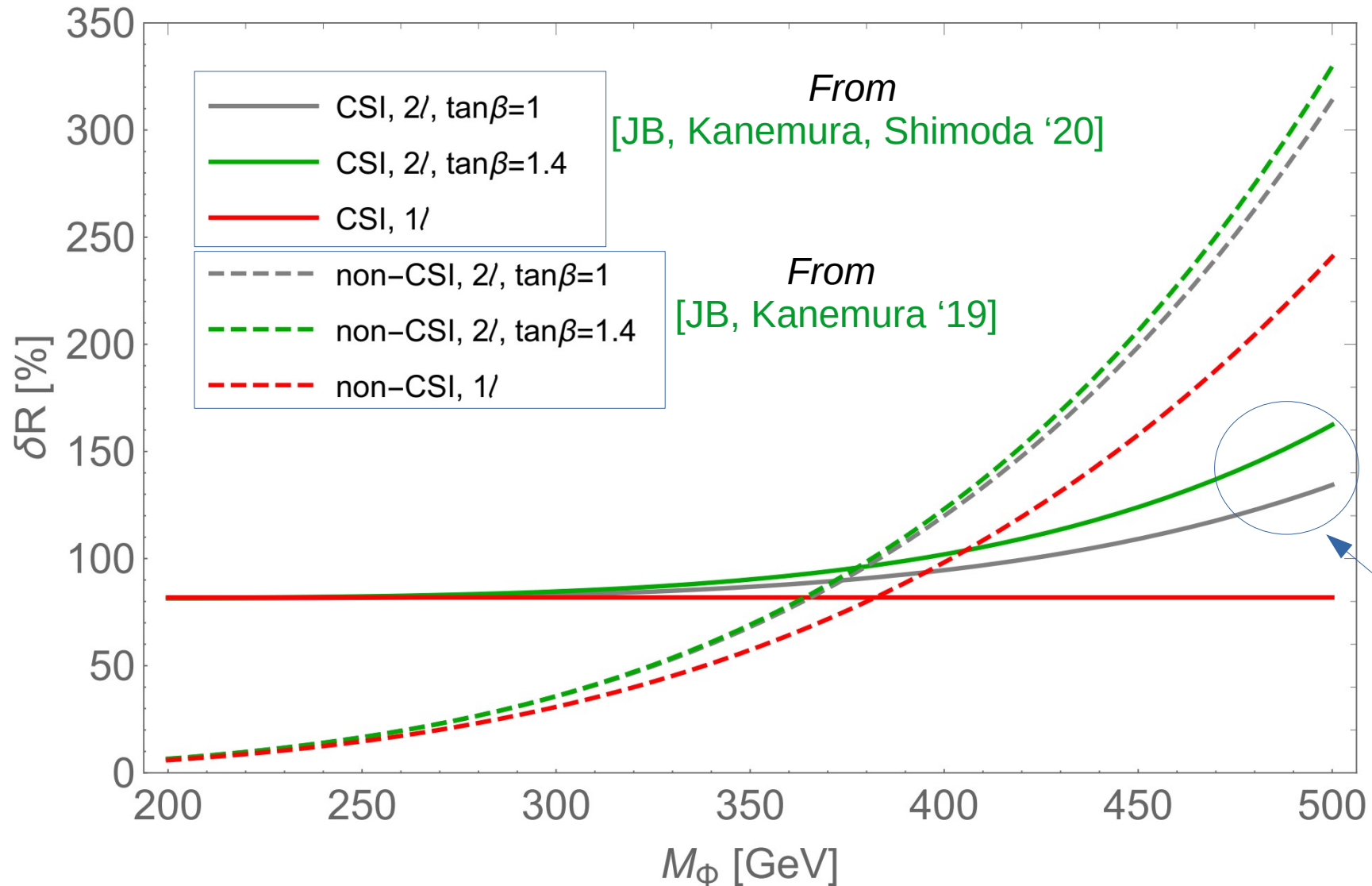
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# Comparing $\lambda_{hhh}$ in 2HDM scenarios with or without CSI

Taking degenerate BSM masses:  $M_\phi = M_H = M_A = M_{H^\pm}$

[JB, Kanemura, Shimoda '20]

see also Makoto Shimoda's poster (this morning JST)



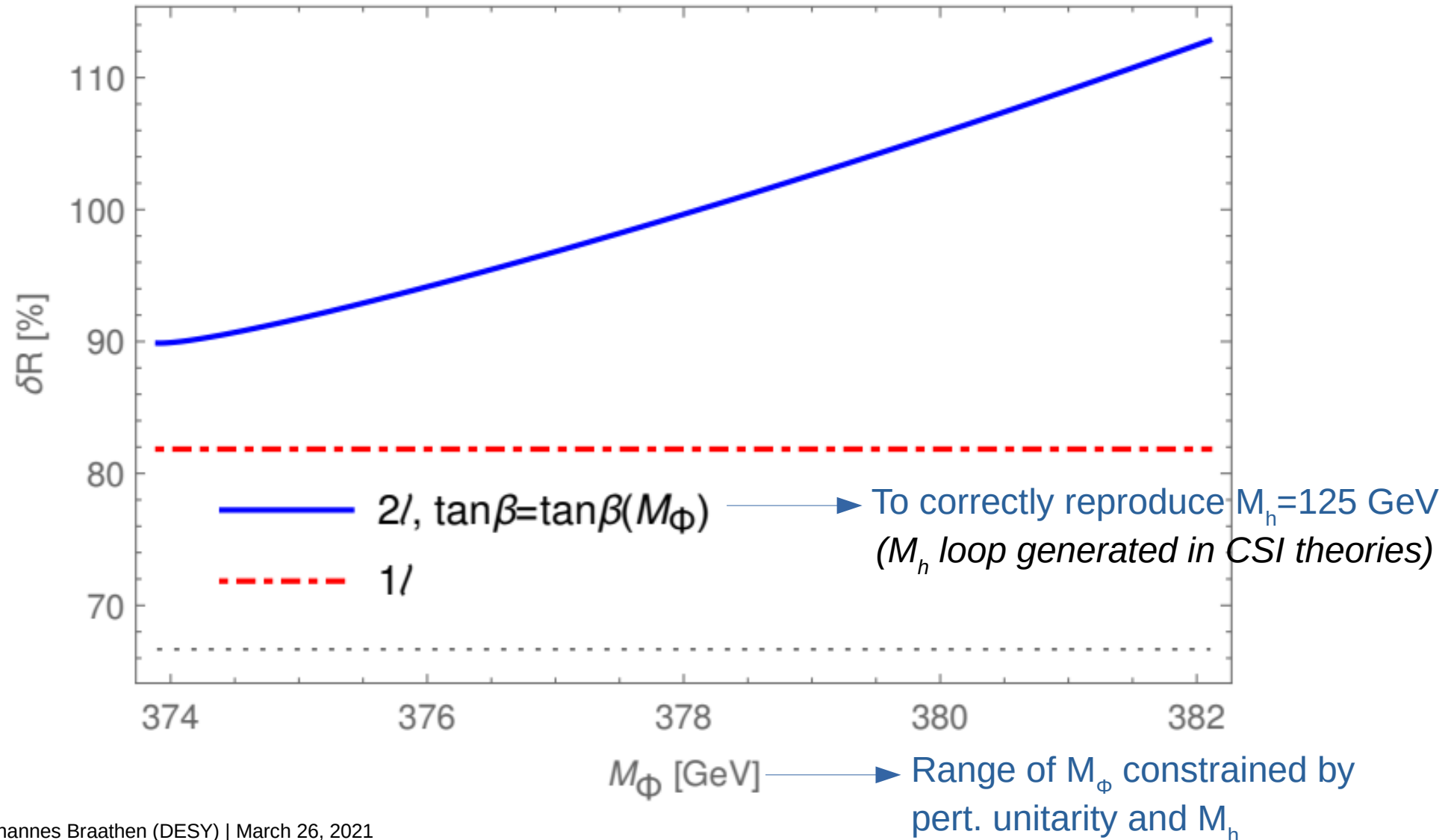
We can now distinguish CSI scenarios with different values of  $\tan\beta$ !



# Allowed range of BSM deviations in a CSI-2HDM

Perturbative unitarity and  $M_h$  strongly constrain the allowed range of BSM parameters!

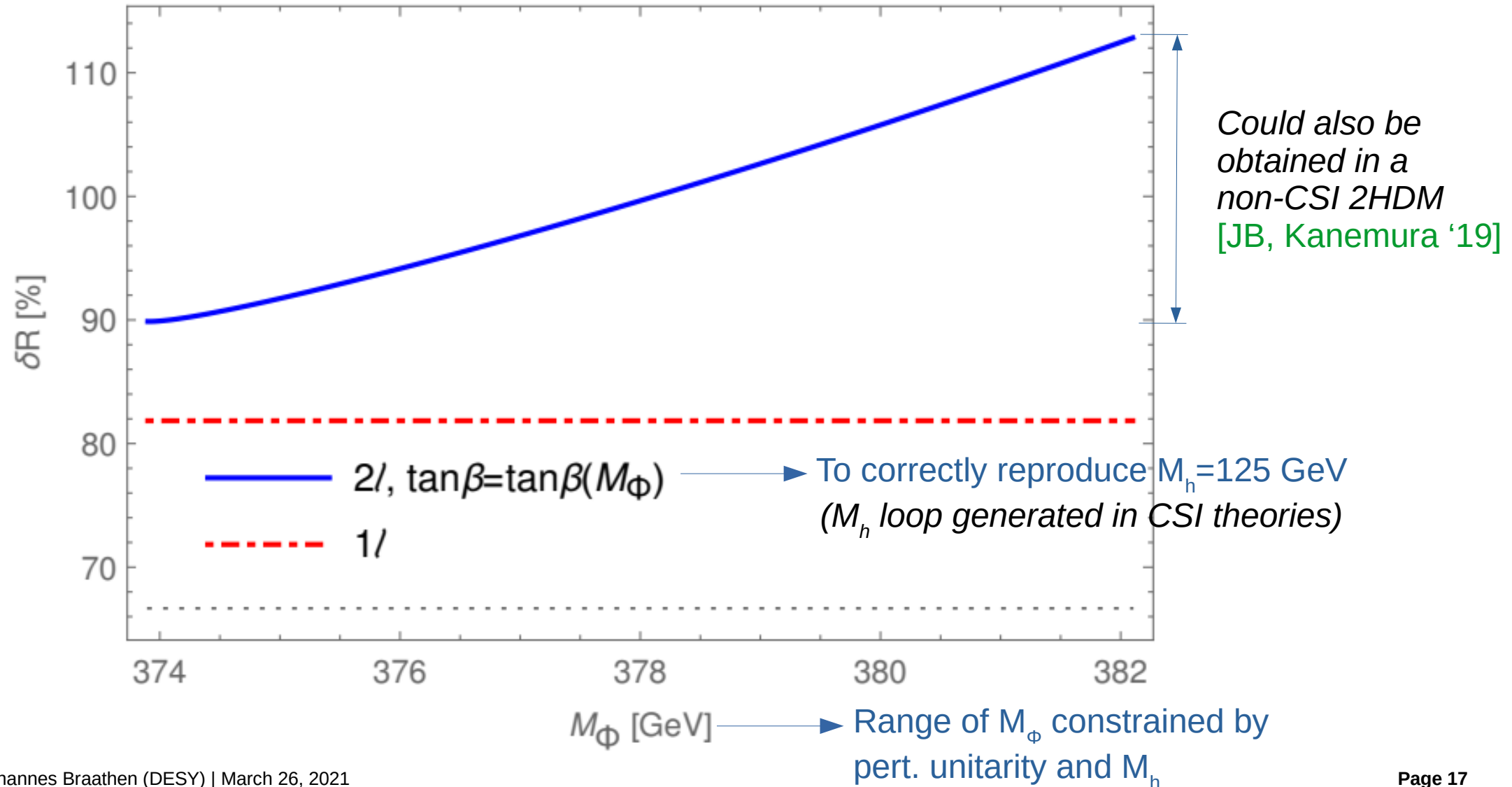
[JB, Kanemura, Shimoda '20]



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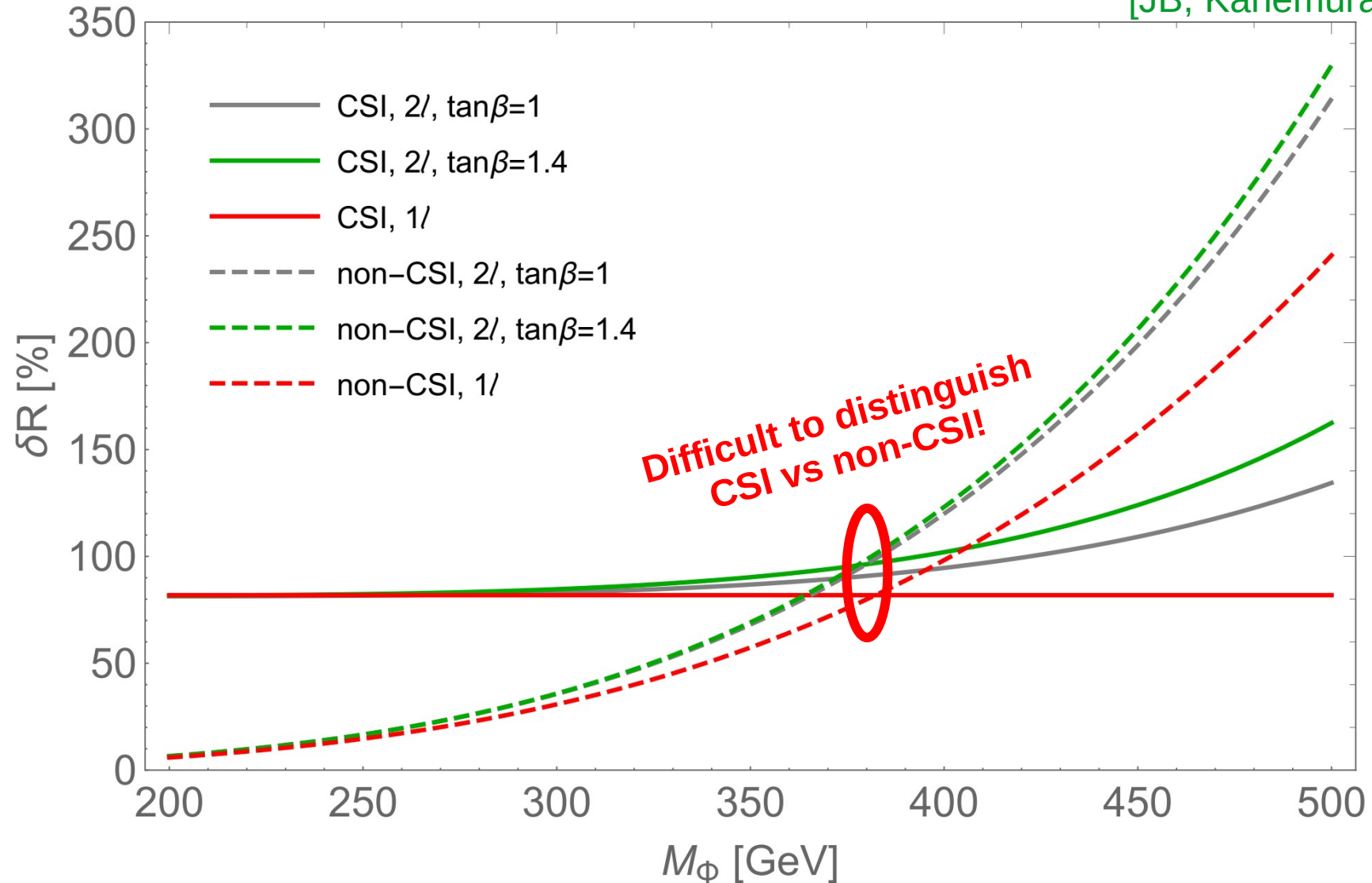
[JB, Kanemura, Shimoda '20]



# Comparing $\lambda_{hhh}$ in 2HDM scenarios with or without CSI

Taking once again degenerate BSM masses:  $M_\phi = M_H = M_A = M_{H^\pm}$

[JB, Kanemura, Shimoda '20]



# Summary

## Explicit two-loop calculation of $\lambda_{hhh}$ in theories with extended scalar sectors

- ▷ Two-loop corrections amount **typically to 10-20% of one-loop contributions** (max.  $\sim 30\%$ )
  - ⇒ Non-decoupling effects found at one loop are **not drastically changed**
  - ⇒ Computations beyond one loop will be **necessary** given the expected accuracy of the measurement of  $\lambda_{hhh}$  at future colliders – **HL-LHC** (50% acc.), **ILC** (27% at 500 GeV, down to 10% at 1 TeV), etc.
  - ⇒ Precise calculation of Higgs couplings ( $\lambda_{hhh}$ , etc.) can **allow distinguishing aligned scenarios with or without decoupling**, by accessing **non-decoupling effects!**
- ▷ New results for CSI theories in **[JB, Kanemura, Shimoda '20]** (see also **Makoto Shimoda's poster presentation!**)
  - ⇒ Two-loop corrections **allow distinguishing different scenarios with CSI**
  - ⇒ Separate models with or without CSI difficult with only  $\lambda_{hhh}$ , but possible with **synergy** of  $\lambda_{hhh}$  and either collider or GW signals (see e.g. **[Hashino, Kakizaki, Kanemura, Matsui '16]**)

# Thank you for your attention!

## Contact

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# Backup slides

# Future determination of $\lambda_{hhh}$

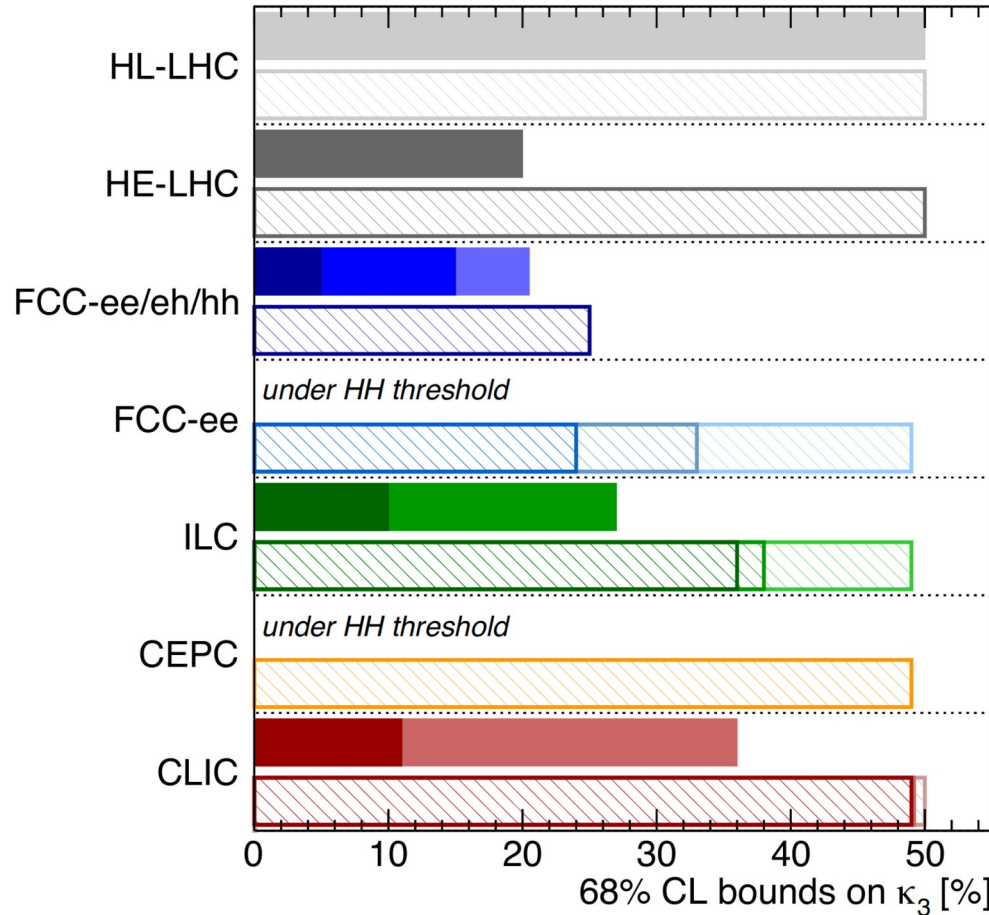
Expected sensitivities in literature, assuming  $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

di-Higgs exclusive result

Higgs@FC WG September 2019

di-Higgs	single-Higgs
HL-LHC 50%	HL-LHC 50% (47%)
HE-LHC [10-20]%	HE-LHC 50% (40%)
FCC-ee/eh/hh 5%	FCC-ee/eh/hh 25% (18%)
LE-FCC 15%	LE-FCC n.a.
FCC-eh <sub>3500</sub> -17+24%	FCC-eh <sub>3500</sub> n.a.
	FCC-ee <sup>4IP</sup> <sub>365</sub> 24% (14%)
	FCC-ee <sub>365</sub> 33% (19%)
	FCC-ee <sub>240</sub> 49% (19%)
ILC <sub>1000</sub> 10%	ILC <sub>1000</sub> 36% (25%)
ILC <sub>500</sub> 27%	ILC <sub>500</sub> 38% (27%)
	ILC <sub>250</sub> 49% (29%)
	CEPC 49% (17%)
CLIC <sub>3000</sub> -7+11%	CLIC <sub>3000</sub> 49% (35%)
36%	CLIC <sub>1500</sub> 49% (41%)
	CLIC <sub>380</sub> 50% (46%)

All future colliders combined with HL-LHC



single-Higgs exclusive

single-Higgs global

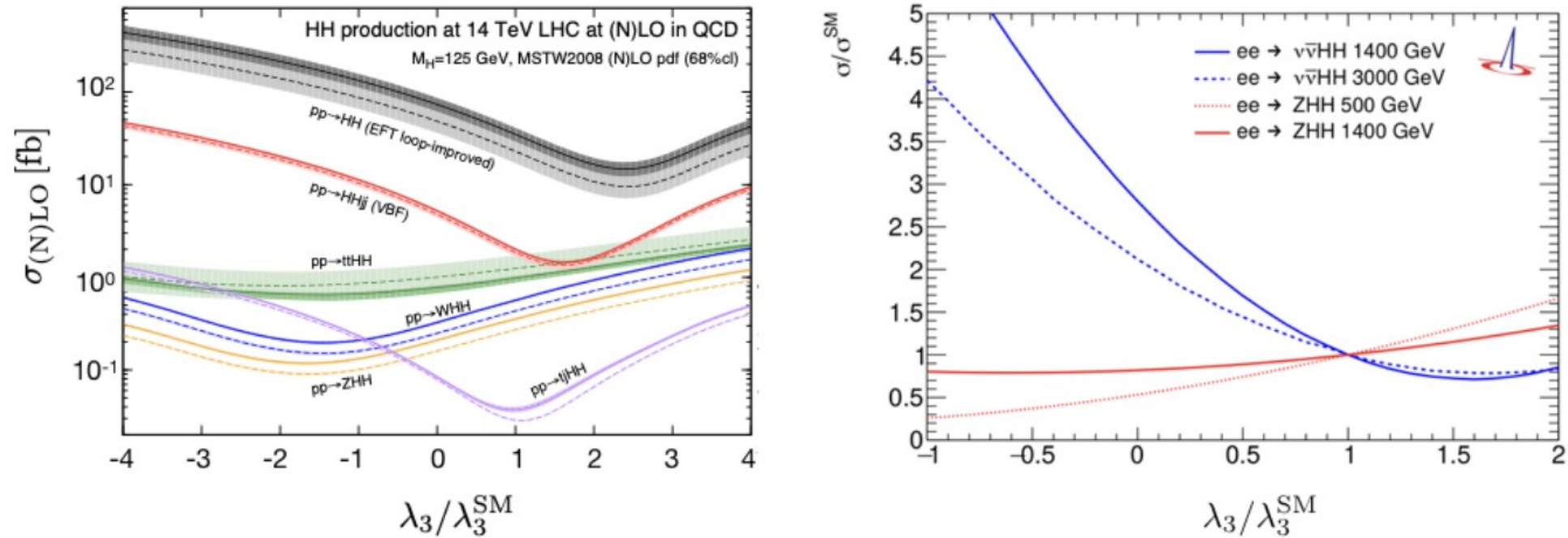
Plot taken from  
[de Blas et al., 1905.03764]

see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], etc.



# Future determination of $\lambda_{hhh}$

Higgs production cross-sections (here double Higgs production) depend on  $\lambda_{hhh}$

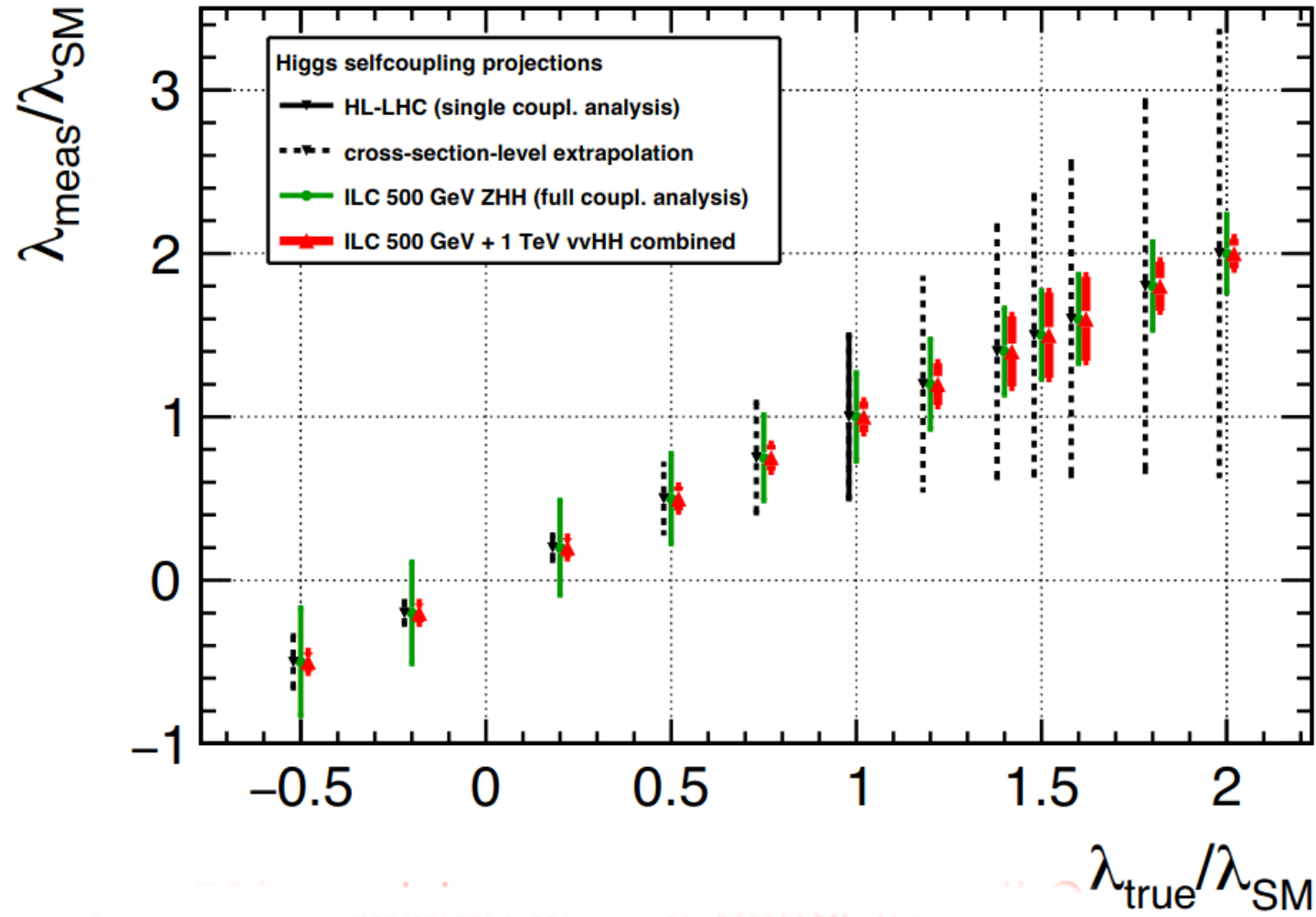


**Figure 10.** Double Higgs production at hadron (left) [65] and lepton (right) [66] colliders as a function of the modified Higgs cubic self-coupling. See Table 18 for the SM rates. At lepton colliders, the production cross sections do depend on the polarisation but this dependence drops out in the ratios to the SM rates (beam spectrum and QED ISR effects have been included).

Plots taken from  
[\[de Blas et al., 1905.03764\]](#)

# Future determination of $\lambda_{hhh}$

Achieved accuracy actually depends on the value of  $\lambda_{hhh}$



[J. List et al. '21],  
see also *talk* by  
G. Weiglein on  
Tuesday

See also [Dürrig, DESY-THESIS-2016-027]

# $\overline{\text{MS}}$ to OS scheme conversion

- $V_{\text{eff}}$ : we use expressions in MS scheme hence results for  $\lambda_{hhh}$  also in  $\overline{\text{MS}}$  scheme
- We include finite counterterms to express the Higgs trilinear coupling in terms of physical quantities

$$\underbrace{m_X^2}_{\overline{\text{MS}}} = \underbrace{M_X^2}_{\text{pole}} - \Re[\Pi_{XX}^{\text{fin.}}(p^2 = M_X^2)], \quad v^2 = \underbrace{(\sqrt{2}G_F)^{-1}}_{\equiv v_{\text{OS}}^2} + \frac{3M_t^2}{16\pi^2} \left(2 \log \frac{M_t^2}{Q^2} - 1\right) + \dots$$

- Also we include finite WFR effects  $\rightarrow$  OS scheme

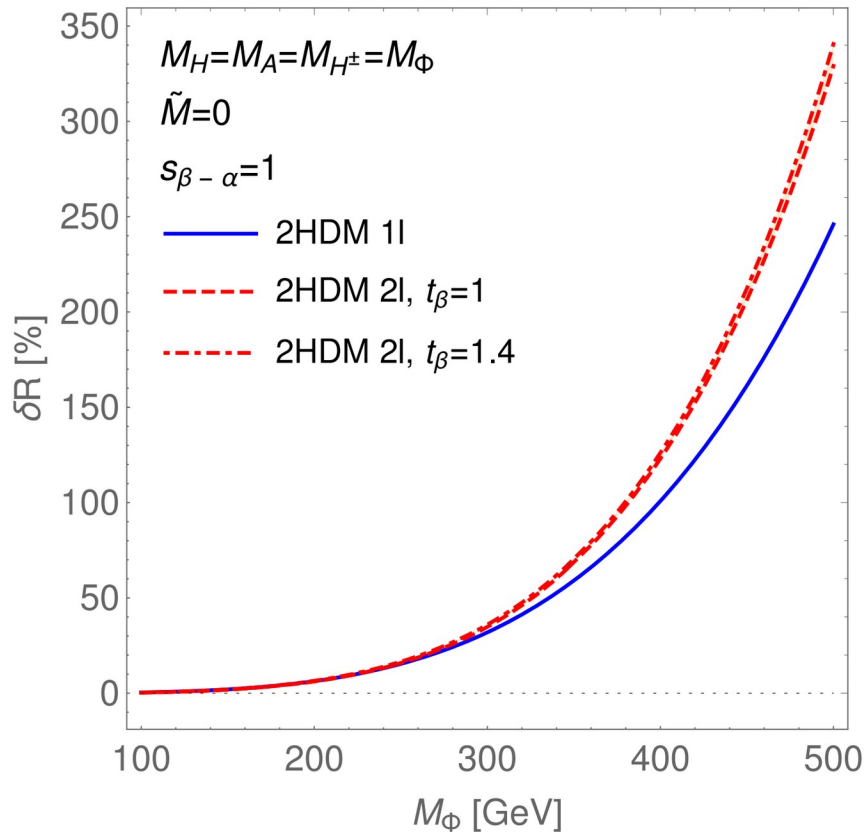
$$\underbrace{\hat{\lambda}_{hhh}}_{\text{OS}} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{finite WFR}} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}} = - \underbrace{\Gamma_{hhh}(0, 0, 0)}_{\text{3-pt. func.}}$$

# Possible enhancements at two loops

[JB, Kanemura 1911.11507]

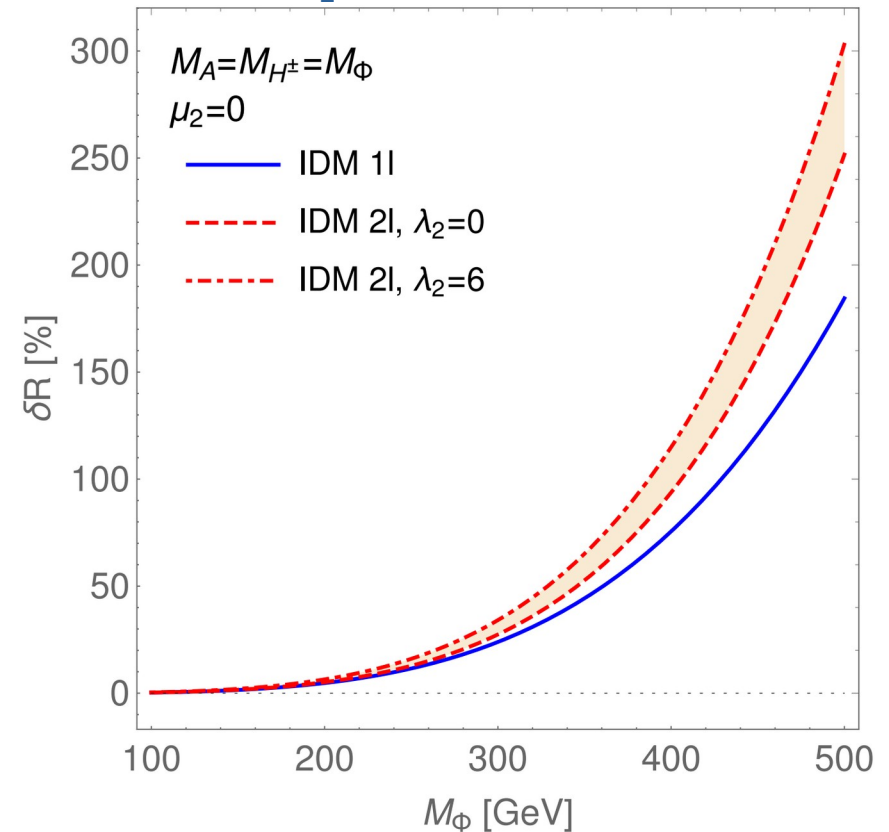
Each model exhibits a *new parameter* only entering  $\lambda_{\text{hhh}}$  at two loops:

## Aligned 2HDM: $\tan\beta$



$\tan\beta$  constrained by pert. unitarity  $\rightarrow$  no large effects

## IDM: $\lambda_2$ (inert quartic coupling)



$\lambda_2$  less constrained  $\rightarrow$  relatively large effects can appear!

# Classical scale invariance

- CSI: forbid mass-dimensional parameters at classical (= tree) level
  - tree-level potential:  $V^{(0)} = \Lambda_{ijkl} \varphi_i \varphi_j \varphi_k \varphi_l$
- However broken **explicitly** at loop level
- EW symmetry breaking: (c.f. [Coleman, Weinberg '73], [Gildener, Weinberg '76])
  - Must occur along a flat direction of  $V^{(0)}$  (= Higgs/scalon direction)
  - EW sym. broken à la Coleman-Weinberg along flat direction
  - EW scale generated by dimensional transmutation
- [JB, Kanemura, Shimoda '20]: **CSI assumed around EW scale, for phenomenology**
  - Higgs (scalon) automatically aligned at tree level → compatible with current exp. results
  - BSM states can't be decoupled (no BSM mass term!)
  - CSI scenarios: **alignment with decoupling**

# One-loop effective potential and $\lambda_{hhh}$ in CSI models

- Only source of mass = coupling to Higgs and its VEV:  $m_i^2(h) = m_i^2 \times \left(1 + \frac{h}{v}\right)^2$

- Greatly simplifies the one-loop potential along Higgs (scalon) direction:

$$V^{(1)} = A(v+h)^4 + B(v+h)^4 \log \frac{(v+h)^2}{Q^2}$$

with

$$A \equiv \frac{1}{64\pi^2 v^4} \left\{ \text{tr} \left[ M_S^4 \left( \log \frac{M_S^2}{v^2} - \frac{3}{2} \right) \right] - 4 \text{tr} \left[ M_f^4 \left( \log \frac{M_f^2}{v^2} - \frac{3}{2} \right) \right] + 3 \text{tr} \left[ M_V^4 \left( \log \frac{M_V^2}{v^2} - \frac{5}{6} \right) \right] \right\}$$

$$B \equiv \frac{1}{64\pi^2 v^4} (\text{tr} [M_S^4] - 4 \text{tr} [M_f^4] + 3 \text{tr} [M_V^4])$$

- Taking successive derivatives of the potential

- 1st derivative = tadpole equation → fix A in terms of v and B

- 2nd derivative = Higgs (effective potential) mass  $[M_h^2]_{V_{\text{eff}}}$  → fix B in terms of v and  $M_h$

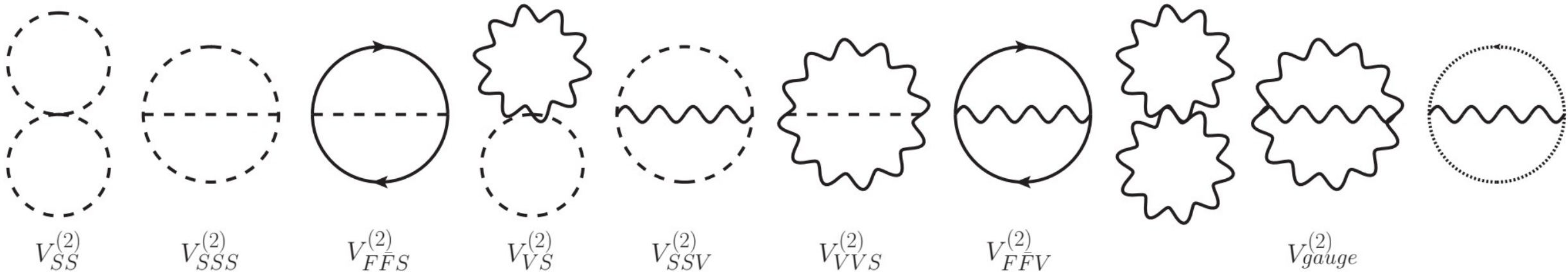
- 3rd derivative =  $\lambda_{hhh}$  but  $V^{(1)}$  is **entirely determined** by A, B →

$$\lambda_{hhh} = \frac{5[M_h^2]_{V_{\text{eff}}}}{v} = \frac{5}{3} \lambda_{hhh}^{\text{SM, tree}}$$

**Universal one-loop result in CSI theories!**

# Effective potential at two loops

- Form of  $V_{\text{eff}}$  changes at two loops:



- New type of contribution:

$$V_{\text{eff}} = A(v + h)^4 + B(v + h)^4 \log \frac{(v + h)^2}{Q^2} + \text{new log}^2 \text{ term!} + C(v + h)^4 \log^2 \frac{(v + h)^2}{Q^2}$$



# $\lambda_{hhh}$ at two loops in CSI models

[JB, Kanemura, Shimoda '20]

- Follow same procedure as at one loop:
  - Eliminate  $A$  with tadpole eq.,  $B$  with Higgs mass
  - Still,  **$C$  remains!**

- One finds: 
$$\lambda_{hhh} = \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min}} = \frac{5[M_h^2]V_{\text{eff}}}{v} + 32Cv$$

- Deviation in  $\lambda_{hhh}$  depends on  $\log^2$  term in  $V_{\text{eff}}$
- **Universality found at one loop is lost at two loops!**

# Theoretical and experimental constraints in [JB, Kanemura, Shimoda '20]

- **Perturbative unitarity**: we constrain parameters entering only at two loops  
→ tree-level perturbative unitarity suffices [Kanemura, Kubota, Takasugi '93]

- EW vacuum must be **true minimum of  $V_{\text{eff}}$** , i.e. check that

$$\underbrace{V_{\text{eff}}(v + h = 0)}_{=0} - V_{\text{eff}}(h = 0) > 0 \quad \Rightarrow \quad V_{\text{eff}}(h = 0) < 0$$

- $M_h$ , generated at loop level, must be **125 GeV**

→ imposes a relation between SM parameters,  $M_H$ ,  $M_A$ ,  $M_{H^\pm}$ ,  $\tan\beta$ , e.g. we can extract:

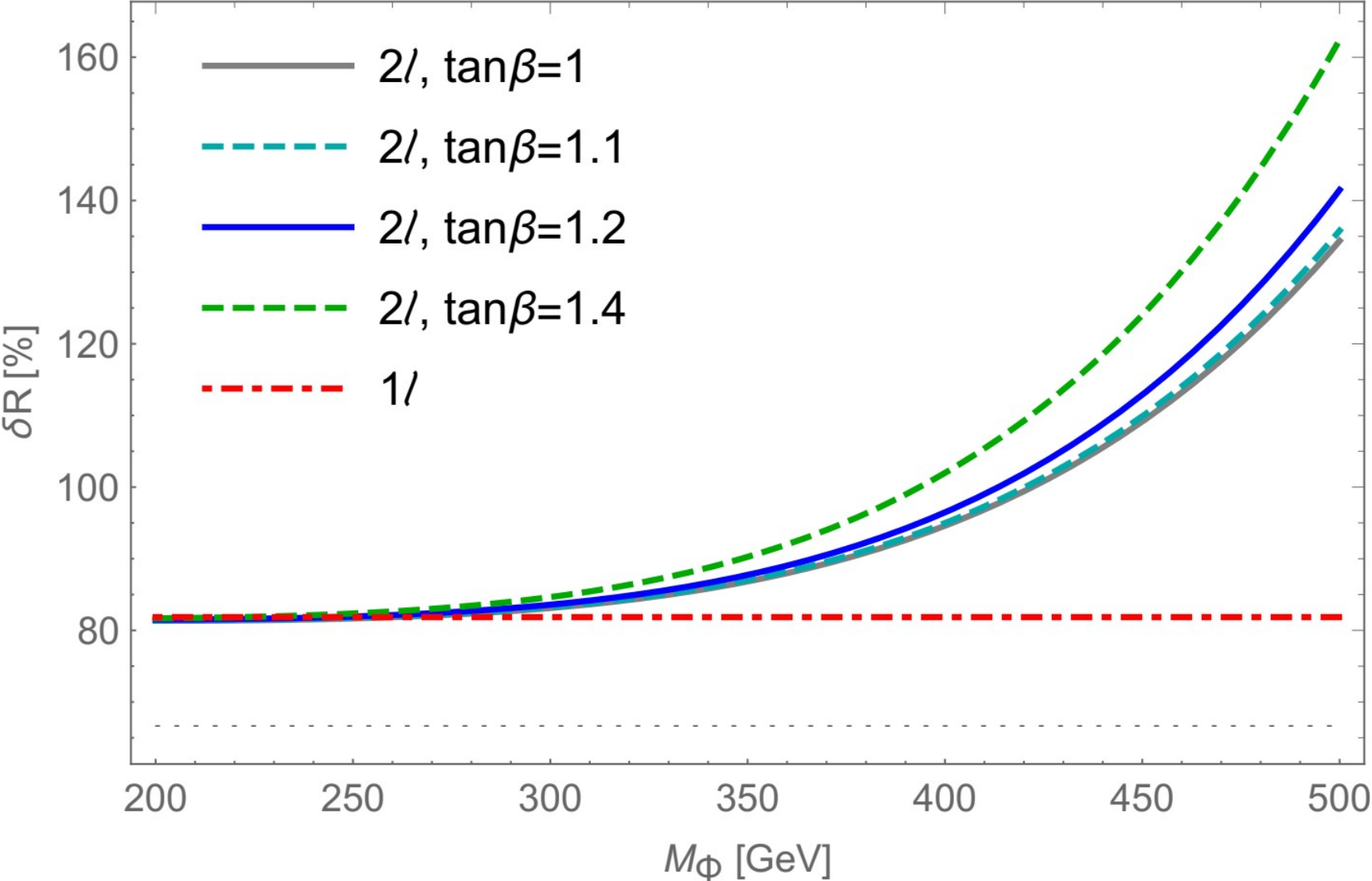
$$[M_h^2]_{V_{\text{eff}}} = \left. \frac{\partial^2 V_{\text{eff}}}{\partial h^2} \right|_{\text{min}} \quad \Rightarrow \quad \tan\beta = \tan\beta( \underbrace{M_h, M_t, \dots}_{\text{measured SM values}}, \underbrace{M_H, M_A, M_{H^\pm}}_{\text{BSM inputs}} )$$

- Limits from **collider searches** with HiggsBounds and HiggsSignals

# No constraints

Taking degenerate BSM masses:  $M_\phi = M_H = M_A = M_{H^\pm}$

[JB, Kanemura, Shimoda '20]



# Unitarity and constraint from $M_h$ in the CSI-2HDM

[JB, Kanemura, Shimoda '20]

