# Higgs Alignment and the Top Quark

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arXiv:2102.07242

Seminar “at” HPNP—Osaka University, March 25/26, 2021

# Higgs Alignment and the Top Quark: The Intro

There is a surprising connection between the top quark and Higgs alignment in Gildener-Weinberg multi-Higgs doublet models. Were it not for the top quark, its large mass, and the Glashow-Weinberg constraint on quark-Higgs couplings, the coupling of the 125 GeV Higgs to gauge bosons and fermions in such models would be indistinguishable from that of the Standard Model Higgs. The experimental consequence of this is that many popular searches for Beyond-Standard-Model Higgs bosons will remain fruitless.

## Outline:

1. Introduction to Gildener-Weinberg and alignment.
2. Influence of the top quark on GW Higgs models.
3. Some experimental consequences.

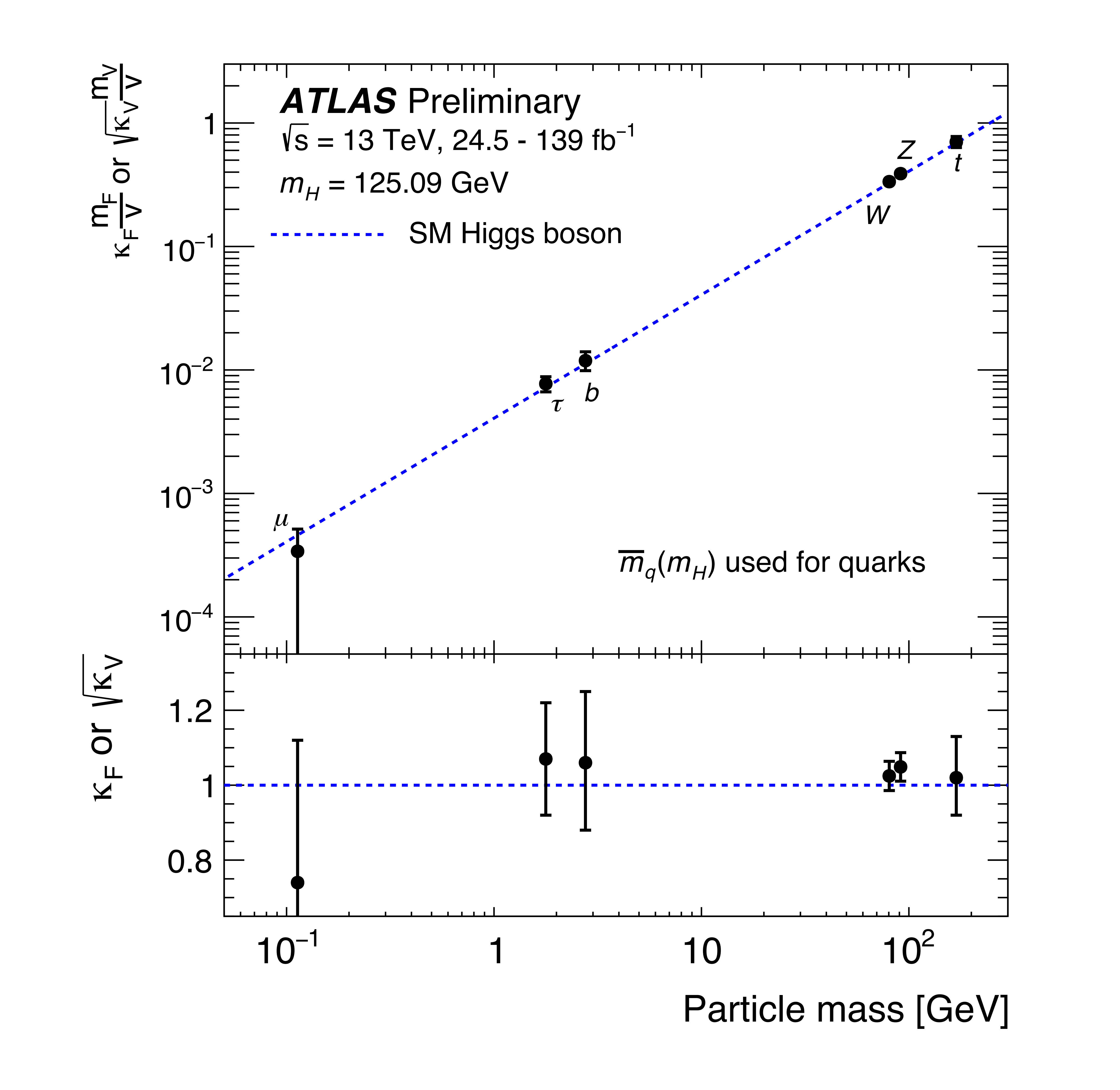
Every LHC measurement of H(125)'s couplings to gauge bosons:

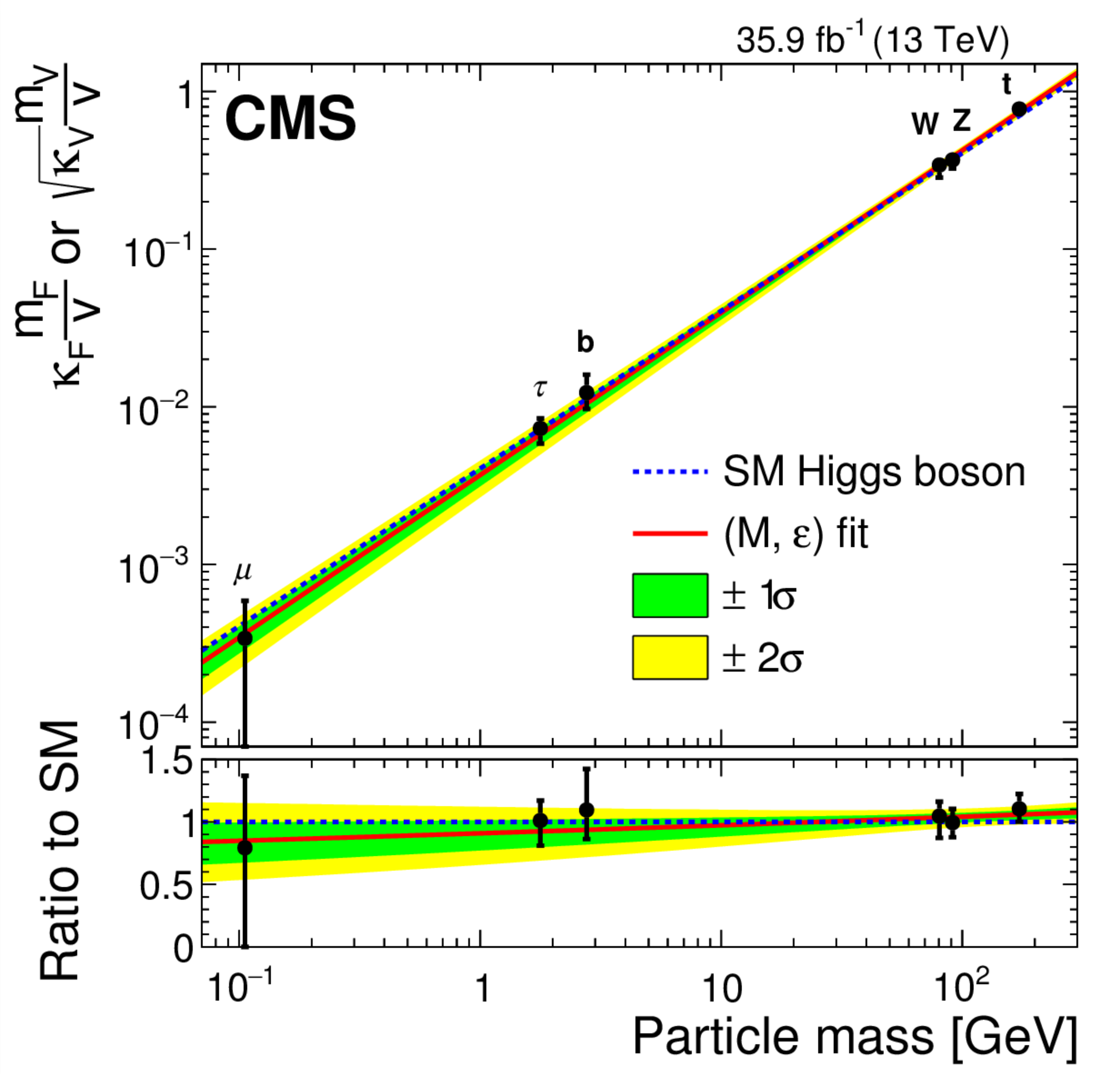
$$H \longleftrightarrow WW^*, ZZ^*, \gamma\gamma \text{ and } gg \to H$$

and to fermions:

$$H \to \bar{t}t,\ \bar{b}b,\ \tau^+\tau^-,\ \mu^+\mu^-$$

is consistent with its being the single Higgs boson of the SM:


ATLAS Preliminary
√s = 13 TeV, 24.5 - 139 fb⁻¹
m_H = 125.09 GeV
SM Higgs boson
m̄_q(m_H) used for quarks
Particle mass [GeV]
κ_F or √κ_V


35.9 fb⁻¹ (13 TeV)
CMS
SM Higgs boson
(M, ε) fit
± 1σ
± 2σ
Ratio to SM
Particle mass [GeV]

This is puzzling: practically all attempts to cure the ills of the Standard Model — most famously, naturalness — require two or more Higgs multiplets, usually doublets. But, why should one CP-even mass eigenstate have SM couplings?

The usual answer is Higgs alignment: Something, originally decoupling of heavier Higgses (Boudjema & Semenov, PRD **66**, 095007; Gunion & Haber, PRD **67**, 075019), causes the lightest CP-even H to be the linear combination

$$H \simeq \sum_i v_i \rho_i / v \quad \text{where} \quad v = \sqrt{\sum_i v_i^2}$$

(Note: I'm assuming N Higgs doublets b/c of the rho parameter.)

BUT — is decoupling natural? Is there a global symmetry to prevent large radiative corrections to alignment? There have been a few proposals, but the symmetries are rather elaborate and artificial, or related to supersymmetry.

BUT — is decoupling natural? Is there a global symmetry to prevent large radiative corrections to alignment? Yes: there have been a few proposals, but the symmetries are rather elaborate and artificial, or related to supersymmetry.

But there is one exception and, to me, it seems very attractive and very simple: The Higgs is a (pseudo-) Goldstone boson of spontaneously broken scale invariance. Then, Higgs alignment is automatic! (In tree approximation, it is exact.) And this has been in front of us since 1976:

**E. Gildener and S. Weinberg, PRD 13, 3333 (1976).**

Consider an N-Higgs-doublet model (NHDM):

$$\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_i^+ \\ \rho_i + i a_i \end{pmatrix}, \quad i = 1, 2, \ldots, N$$

The Goldstone bosons eaten by W and Z are

$$w^\pm = \sum_{i=1}^{N} v_i \phi_i^\pm / v, \;\; z = \sum_{i=1}^{N} v_i a_i / v \quad \text{where } v_i = \langle \rho_i \rangle \text{ and } v = \sqrt{\sum_i v_i^2}$$

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In GW-NHDM, H has the same form:

$$H = \sum_{i=1}^N v_i \rho_i / v \;\; \underline{\text{exactly}}, \text{ in tree approximation!}$$

This has profound consequences for BSM Higgs searches at LHC. Discussed later.

**It's aligned!** How is this arranged?

# The GW-2HDM

The key assumption of GW models is that the classical Lagrangian is scale-invariant: the Higgs potential $V_0$ is purely quartic **and** all fermion hard masses arise from their quartic Yukawa couplings to Higgs bosons. (Hence the need for complex Higgs doublets — SW, PRL **19**, 1264.— and no vectorlike quarks or leptons with electroweak interactions!)

# The GW-2HDM

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We use a simple N=2 HDM with a CP-invariant quartic potential (Lee & Pilaftsis, PRD **86**, 035004; also W. Shepherd & K.L., PRD **99**, 055015):

$$V_0 = \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ +\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \tfrac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right]$$

$$\text{where } \lambda_i = \lambda_i^*;\ \lambda_1 > 0, \lambda_2 > 0.$$

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$$+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \tfrac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right]$$

$$\text{where } \lambda_i = \lambda_i^*;\ \lambda_1 > 0, \lambda_2 > 0.$$

$V_0$ is a homogeneous polynomial of degree 4:

$$\Longrightarrow V_0 = \tfrac{1}{4}\sum_{i=1}^{2}\left[\Phi_i^\dagger\frac{\partial V_0}{\partial\Phi_i^\dagger} + \frac{\partial V_0}{\partial\Phi_i}\Phi_i\right] \Longrightarrow \mathbf{V_0 = 0 \text{ at } \underline{all} \text{ extrema.}}$$

(E. Pilon & K.L., PRD **101**, 055032)

$\Phi_1 = \Phi_2 = 0$ is the trivial "minimum" of $V_0$.

A deeper (-ve) minimum must occur in higher order of the loop expansion.

But a nontrivial minimum of $V_0$ can occur on the ray

$$\Phi_{1\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \cos\beta \end{pmatrix}, \quad \Phi_{2\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \sin\beta \end{pmatrix}$$

where $0 < \phi < \infty$ and $0 < \beta < \pi/2$.

The nontrivial extremal conditions are

$$\lambda_1 + \tfrac{1}{2}\lambda_{345} \tan^2\beta = \lambda_2 + \tfrac{1}{2}\lambda_{345} \cot^2\beta = 0;$$

$$\lambda_1, \lambda_2 > 0 \Longrightarrow \lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 < 0.$$

Use the "aligned basis"!

$$\Phi = \Phi_1 c_\beta + \Phi_2 s_\beta \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} w^+ \\ H + iz \end{pmatrix} \Longrightarrow \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}$$

$$\Phi' = -\Phi_1 s_\beta + \Phi_2 c_\beta \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H' + iA \end{pmatrix} \Longrightarrow \langle \Phi' \rangle = 0$$

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**N.B.:** $H = \rho_1 c_\beta + \rho_2 s_\beta \equiv (\rho_1 \phi_1 + \rho_2 \phi_2)/\phi$ is aligned!

Tree-level Higgs mass matrices are diagonal in this basis —

$$\mathcal{M}^2_{0-} = \begin{pmatrix} M_z^2 & 0 \\ 0 & M_A^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_5 \phi^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\pm} = \begin{pmatrix} M_{w^\pm}^2 & 0 \\ 0 & M_{H^\pm}^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{2}\lambda_{45} \phi^2 \end{pmatrix}$$

$$\mathcal{M}^2_{0+} = \begin{pmatrix} M_H^2 & 0 \\ 0 & M_{H'}^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_{345} \phi^2 \end{pmatrix}$$

The ray is a flat minimum if, like $\lambda_{345} = -2\lambda_1 \cot^2\beta < 0$, $\lambda_5 < 0$ and $\lambda_{45} = \lambda_4 + \lambda_5 < 0$. And $H$ is the *dilaton* of spontaneously broken scale invariance.

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$$\mathcal{M}^2_{0^+} = \begin{pmatrix} M_H^2 & 0 \\ 0 & M_{H'}^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_{345}\phi^2 \end{pmatrix}$$

— and they'll remain nearly diagonal thru 2nd order in the loop expansion of the effective potential of S. Coleman and E. Weinberg, PRD 7, 1888.

This means that H keeps its very nearly SM couplings thru $O(V_2)$ !

(E. Eichten and K.L., in preparation)

Spoiler alert: H acquires +ve mass-squared in $O(V_2)$

# Higgs Alignment and the Top Quark: The Result

To establish the top quark's role in Higgs alignment, it suffices to look at the one-loop effective potential of the GW-2HDM:

$$V_1 = \frac{1}{64\pi^2} \sum_n \alpha_n \overline{M}_n^4 \left( \ln \frac{\overline{M}_n^2}{\Lambda_{GW}^2} - k_n \right)$$ (S. Martin, PRD **65**,116003)

$$(\alpha_n, k_n) = (6, \tfrac{5}{6}),\ (3, \tfrac{5}{6}),\ (-12, \tfrac{3}{2}),\ (1, \tfrac{3}{2}),\ (1, \tfrac{3}{2}),\ (2, \tfrac{3}{2})$$

for $n = W^\pm,\ Z,\ t,\ H',\ A,\ H^\pm.$

N.B.: The renormalization scale $\Lambda_{\rm GW}$ explicitly breaks scale invariance.

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for $n = W^{\pm},\ Z,\ t,\ H',\ A,\ H^{\pm}$.

The field-dependent masses are (R. Jackiw, PRD 9, 1686; L.-P., ibid):

$$\overline{M}_n^2 = \begin{cases} M_n^2 \left(2\left(\Phi^\dagger\Phi + \Phi'^\dagger\Phi'\right)/\phi^2\right) = M_n^2\left((H^2 + H'^2 + \cdots)/\phi^2\right), & n \neq t \\ M_t^2 \left(2\Phi_1^\dagger\Phi_1/(\phi^2 c_\beta^2)\right) = M_t^2\left((Hc_\beta - H's_\beta)^2 + \cdots\right)/(\phi^2 c_\beta^2), & \end{cases}$$

where $M_W^2 = \frac{1}{4}g^2\phi^2,\ M_{H'}^2 = -\lambda_{345}\phi^2,\ M_t^2 = \frac{1}{2}\Gamma_t^2\phi^2 c_\beta^2,$ etc.

$$\overline{M}_n^2 = \begin{cases} M_n^2\left(2\left(\Phi^\dagger\Phi + \Phi'^\dagger\Phi'\right)/\phi^2\right) = M_n^2\left((H^2 + H'^2 + \cdots)/\phi^2\right), & n \neq t \\ M_t^2\left(2\Phi_1^\dagger\Phi_1/(\phi^2 c_\beta^2)\right) = M_t^2\left((Hc_\beta - H' s_\beta)^2 + \cdots\right)/(\phi^2 c_\beta^2), & \end{cases}$$

where $M_W^2 = \frac{1}{4}g^2\phi^2,\ M_{H'}^2 = -\lambda_{345}\phi^2,\ M_t^2 = \frac{1}{2}\Gamma_t^2\phi^2 c_\beta^2,$ etc.

The different top-mass term (“Type 1”) is dictated by the Glashow-Weinberg (PRD **15** 1958 (1977)) criterion for avoiding FCNC via neutral Higgs exchange. It makes all the difference between experimentally perfect and approximate alignment.

Following Gildener-Weinberg, the one-loop extremal conditions are (tree-level extremal conditions remain in force):

$$(1) \qquad 0 = \left.\frac{\partial(V_0 + V_1)}{\partial H}\right|_{\langle\,\rangle + \delta_1 H + \delta_1 H'} = \left.\frac{\partial V_1}{\partial H}\right|_{\langle\,\rangle} + \mathcal{O}(V_2),$$

$$(2) \qquad 0 = \left.\frac{\partial(V_0 + V_1)}{\partial H'}\right|_{\langle\,\rangle + \delta_1 H + \delta_1 H'} = \left.\frac{\partial^2 V_0}{\partial H'^2}\right|_{\langle\,\rangle} \delta_1 H' + \left.\frac{\partial V_1}{\partial H'}\right|_{\langle\,\rangle} + \mathcal{O}(V_2),$$

where $\langle\,\rangle$ means $\langle H \rangle = \phi$, $\langle H' \rangle = 0$.

(1) Determines $\Lambda_{\rm GW}$ in terms of $\phi = v = 246$ GeV at $\min(V_1)$.

(2) Determines the $\mathcal{O}(V_1)$ shift $\delta_1 H'$ from $\langle H' \rangle = 0$.

$$\frac{\partial \overline{M}_n^2}{\partial H'}\bigg|_{\langle\,\rangle} = \begin{cases} 2M_n^2 H'/\phi^2\big|_{\langle\,\rangle} = 0 & (n \neq t) \\ 2M_t^2(H's_\beta - Hc_\beta)s_\beta/(\phi c_\beta)^2\big|_{\langle\,\rangle} = -2M_t^2 \tan\beta/\phi \end{cases}$$

$$\Rightarrow \delta_1 H' \equiv -\frac{1}{M_{H'}^2}\,\frac{\partial V_1}{\partial H'}\bigg|_{\langle\,\rangle} = \frac{\alpha_t M_t^4 \tan\beta}{16\pi^2 M_{H'}^2 v}\left(\ln\frac{M_t^2}{\Lambda_{\mathrm{GW}}^2} + \frac{1}{2} - k_t\right) \quad (\text{at } \phi = v)$$

Typically, $\delta_1 H' = 1\text{-}3\,\mathrm{GeV}$, a tiny increase in $\sqrt{v^2 + (\delta_1 H')^2}$.

$\delta_1 H'$ establishes the connection of the top quark to Higgs alignment:

If $\delta_1 H' = 0$, $H = \rho_1 c_\beta + \rho_2 s_\beta$ is still a mass eigenstate.

$\rightarrow$ Large $M_t \Rightarrow$ its appearance in $V_1$.

$\rightarrow$ The Glashow-Weinberg no-FCNC $\Rightarrow \delta_1 H' \neq 0$ in $\mathcal{O}(V_1)$.

The $\mathcal{O}(V_1)$ elements of $\mathcal{M}^2_{0^+}$ further emphasize this connection:

$$\mathcal{M}^2_{HH} = \frac{\partial^2 V_1}{\partial H^2}\bigg|_{\langle\rangle} = \frac{\sum_n \alpha_n M_n^4}{8\pi^2 v^2} \equiv \frac{B}{8\pi v^2},$$

$\therefore$, $\mathcal{M}^2_{HH} \cong M_H^2 = (125\,\mathrm{GeV})^2$
$\Rightarrow V_1|_{\langle\rangle} = -16 v^2 M_H^2 < V_0|_{\langle\rangle} = 0$
(as promised)

$$\mathcal{M}^2_{HH'} = \frac{\partial^3 V_0}{\partial H \partial H'^2}\bigg|_{\langle\rangle} \delta_1 H' + \frac{\partial^2 V_1}{\partial H \partial H'}\bigg|_{\langle\rangle}$$

$$= -\frac{\alpha_t M_t^4 \tan\beta}{16\pi^2 v^2}\left(\ln\frac{M_t^2}{\Lambda^2_{\mathrm{GW}}} + \frac{5}{2} - k_t\right),$$

$\leftarrow$ Small and, strictly speaking, does not enter $M_H^2$ until $\mathcal{O}(V_2)$. ($\Rightarrow$ $H$-$H'$ mixing angle $\delta < 1\%$)

$$\mathcal{M}^2_{H'H'} = \frac{\partial^2 V_0}{\partial H'^2}\bigg|_{\langle\rangle} + \frac{\partial^3 V_0}{\partial H'^3}\bigg|_{\langle\rangle} \delta_1 H' + \frac{\partial^2 V_1}{\partial H'^2}\bigg|_{\langle\rangle}$$

$$= M^2_{H'} + \frac{\alpha_t M_t^4 \tan\beta}{8\pi^2 v^2}\left(\ln\frac{M_t^2}{\Lambda^2_{\mathrm{GW}}} + \frac{1}{2} - k_t + \tan^2\beta\right).$$

# Higgs Alignment: Experimental consequences

ATLAS & CMS discovered H(125) easily because of its rather strong coupling to WW and ZZ: production via WW and ZZ fusion and decay to WW* and ZZ*, (ZZ* —> 4 leptons was much more convincing than $\gamma\gamma$ ). gg fusion via a top quark loop is important because the Htt couping is "full strength" $M_t/v$.

Thus, many searches for new Beyond-Standard-Model (BSM) Higgs bosons rely on the lamp-post strategy: Assume VV -> H' and H' -> VV are important modes of the H' and that the H'tt coupling is full strength.

# For example (quickly):

**ATLAS:**

Search BSM H->ZZ->4l+llnunu

Search BSM H->WW->lvlv

VH all hadronic resonance search

Search BSM H->ZZ->4l and llvv

Search BSM h->2a->4b

Search BSM H -> ZZ

Search for heavy resonances decaying to VV in the semileptonic final states

Search BSM h(125)->Za, a->jet

VV/VH and ll/lv search combination 13 TeV 2016

**CMS:** Search for a heavy Higgs boson decaying to a pair of W bosons in proton-proton collisions at s√= 13 TeV

Search for a heavy pseudoscalar Higgs boson decaying into a 125 GeV Higgs boson and a Z boson in final states with two tau and two light leptons at s√= 13 TeV

Search for a new scalar resonance decaying to a pair of Z bosons in proton-proton collisions at s√= 13 TeV

Search for charged Higgs bosons produced via vector boson fusion and decaying into a pair of W and Z bosons using proton-proton collisions at s√= 13 TeV

## Drell-Yan and VV-fusion processes:

$$\mathcal{L}_{EW} = ieH^- \overleftrightarrow{\partial}_\mu H^+ (A^\mu + Z^\mu \cot 2\theta_W) + \frac{e}{\sin 2\theta_W} A \overleftrightarrow{\partial}_\mu (H_1 \sin\delta - H_2 \cos\delta) Z^\mu + \frac{ie}{2\sin\theta_W}\left(H^- \overleftrightarrow{\partial}_\mu (H_1 \sin\delta - H_2\cos\delta + iA) W^{+,\mu} - \text{h.c.}\right) + (H_1 \cos\delta + H_2 \sin\delta)\left(\frac{eM_W}{\sin\theta_W} W^{+,\mu} W^-_\mu + \frac{eM_Z}{\sin 2\theta_W} Z^\mu Z_\mu\right)$$

$$|\delta| \lesssim 1\%$$

## gb and gg-fusion processes:

$$\mathcal{L}_Y = \frac{\sqrt{2}\tan\beta}{v} \sum_{k,l=1}^{3} \left[H^+ (\bar{u}_{kL} V_{kl}\, m_{d_l} d_{lR} - \bar{u}_{kR}\, m_{u_k} V_{kl}\, d_{lL} + m_{\ell_k} \bar{\nu}_{kL} \ell_{kR}\, \delta_{kl}) + \text{h.c.}\right] - \left(\frac{v\cos\beta + H_1\cos\beta' - H_2 \sin\beta'}{v\cos\beta}\right) \sum_{k=1}^{3} (m_{u_k} \bar{u}_k u_k + m_{d_k} \bar{d}_k d_k + m_{\ell_k} \bar{\ell}_k \ell_k) - \frac{iA\tan\beta}{v} \sum_{k=1}^{3} (m_{u_k} \bar{u}_k \gamma_5 u_k - m_{d_k} \bar{d}_k \gamma_5 d_k - m_{\ell_k} \bar{\ell}_k \gamma_5 \ell_k)\,.$$

$\tan\beta \lesssim 0.50$

$\beta' = \beta - \delta \cong \beta$

That's it, folks. I hope you've enjoyed the talk or, at least, been intrigued by the results of the oh-so-simple (but oh-so-mysterious) Gildener-Weinberg scale-invariance hypothesis.

If there were time, there is one other result I'd like to show you that is very interesting and has important — and promising — experimental consequences. (See the extra slide.)

# The one-loop sum rule for the new Higgs boson masses

$$M_H^2 = \frac{B}{8\pi^2 v^2} = \frac{1}{8\pi^2 v^2}\left(6M_W^4 + 3M_Z^4 + 2M_{H^\pm}^4 + M_A^4 + M_{H'}^4 - 12M_t^4\right)$$

$$\Longrightarrow \left(M_{H'}^4 + M_A^4 + 2M_{H^\pm}^4\right)^{1/4} = 540\,\mathrm{GeV}$$

(W. Shepherd & K.L., ibid;
E. Pilon & K.L., ibid)

The significance of this sum rule is obvious: All the BSM Higgs bosons in the GW-2HDM must lie below about 400 GeV! ($M_A = M_{H^\pm}$ is assumed to eliminate the BSM Higgs contribution to the T-parameter.)

The limit $\tan\beta \lesssim 0.50$ comes from the search for $gb \to tH^- + c.c.$

It has not been improved upon since 2018 (Run 1!).
The reason is the large top-quark background at low masses.