



**TON DUC THANG UNIVERSITY**



# **Magnetic mass effect on the sphaleron energy**

**Eibun Senaha (Ton Duc Thang U)  
March 26, 2021 Online**

Ref. Koichi Funakubo (Saga U, Japan) and E.S., PRD102, 041701(R) (2020) [2003.13929]

# Outline

- Introduction
  - Electroweak baryogenesis (EWBG)
  - Sphaleron decoupling condition
- Magnetic mass effect on sphaleron in the SM
- Applications to 2 Higgs doublet model
- Summary

# Baryon Asymmetry of the Universe (BAU)

Our Universe is baryon-asymmetric.

$$\eta^{\text{BBN}} = \frac{n_B}{n_\gamma} = (5.8 - 6.5) \times 10^{-10},$$
$$\eta^{\text{CMB}} = \frac{n_B}{n_\gamma} = (6.105 - 0.055) \times 10^{-10}.$$

PDG2020

**Sakharov's conditions** [Sakharov, JETP Lett. 5 (1967) 24]

- (1) Baryon number violation
- (2) C and CP violation
- (3) Out of equilibrium

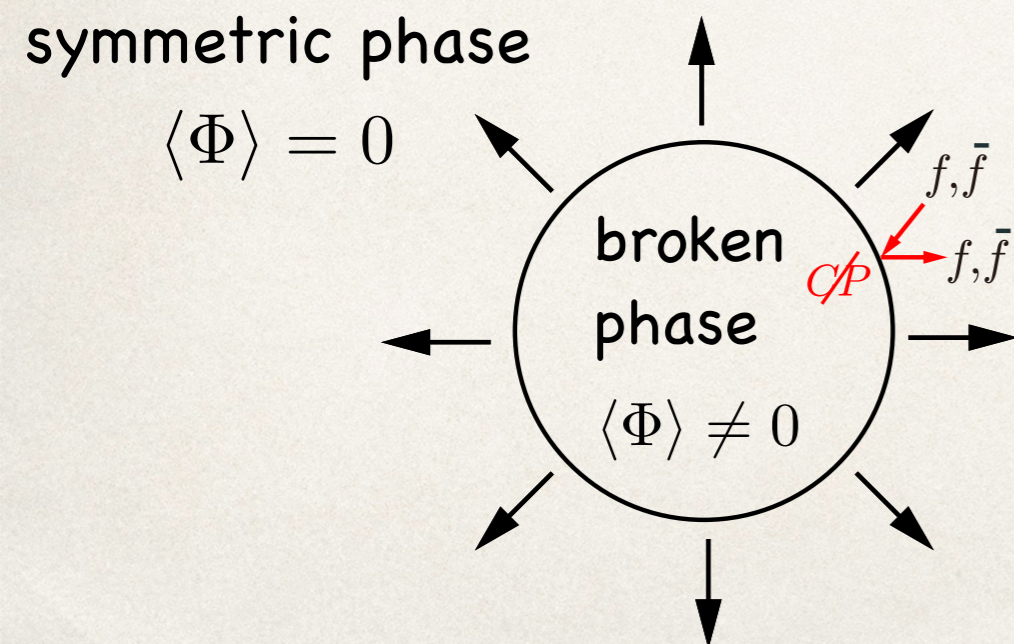
- ❑ after inflation (scale is model dependent)
- ❑ before Big-Bang Nucleosynthesis ( $T \approx O(1)$  MeV)

# EW baryogenesis (EWBG)

## Sakharov's conditions

[Kuzmin, Rubakov, Shaposhnikov, PLB155,36 ('85) ]

- \* **B violation**: anomalous (sphaleron) process
- \* **C violation**: chiral gauge interaction
- \* **CP violation**: KM phase and/or other sources in beyond the SM
- \* **Out of equilibrium**: 1<sup>st</sup>-order EW phase transition (EWPT) with expanding bubble walls



$n_B = 0 \rightarrow n_B \neq 0$  (sphaleron process)

baryogenesis occurs outside bubbles!

sphaleron decoupling condition

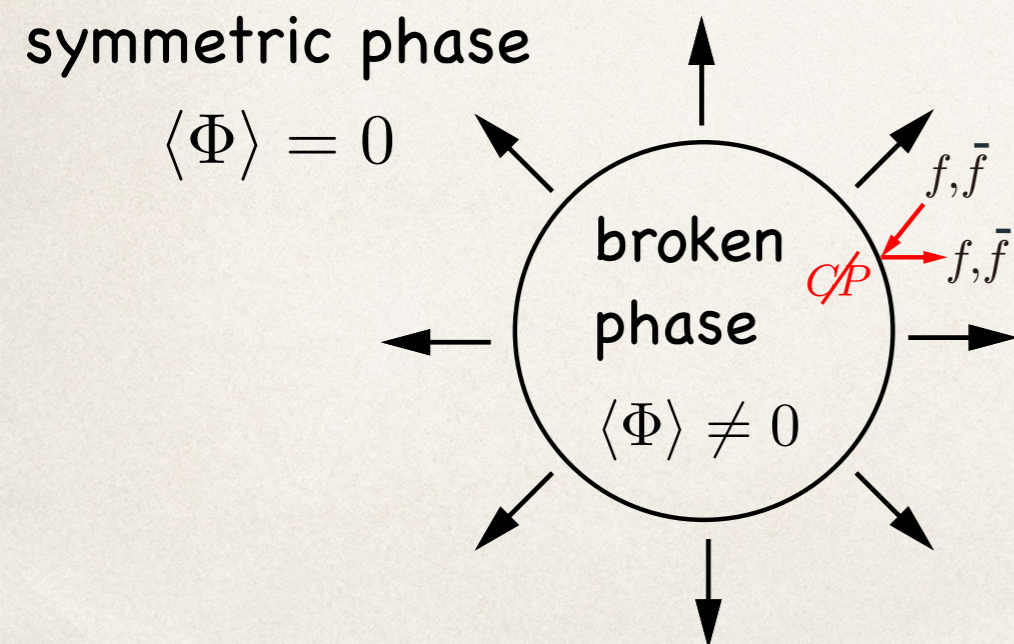
$$\Gamma_B^{(\text{br})}(T) < H(T)$$

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$$\Gamma_B^{(\text{br})}(T) < H(T)$$

# Sphaleron decoupling condition

To avoid washout of BAU, the sphaleron process must be suppressed after EWPT.

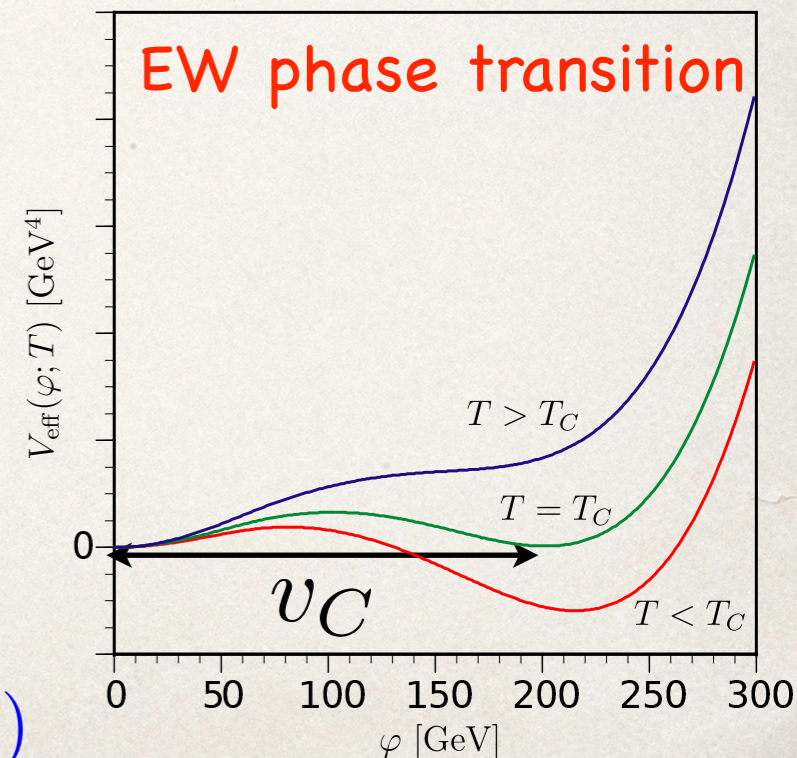
$$\Gamma_B^{(\text{br})}(T) \simeq (\text{prefactor}) e^{-E_{\text{sph}}(T)/T} < \boxed{H(T)} \simeq 1.66 \sqrt{g_*} T^2 / m_{\text{P}}^{\text{II}}$$

Hubble constant 1.22x10<sup>19</sup> GeV  
||  
dof of relativistic particles

Parametrizing  $E_{\text{sph}} = 4\pi v \mathcal{E}_{\text{sph}} / g$ ,

$$\frac{v}{T} > \frac{g}{4\pi \mathcal{E}_{\text{sph}}} \left[ 44.35 + \log \text{ corrections} \right] \equiv \zeta_{\text{sph}}(T)$$

- It is important to calculate  $\mathcal{E}_{\text{sph}}$  precisely.
- We evaluate the condition at  $T_c$ .  $\frac{v_C}{T_C} > \zeta_{\text{sph}}(T_C)$



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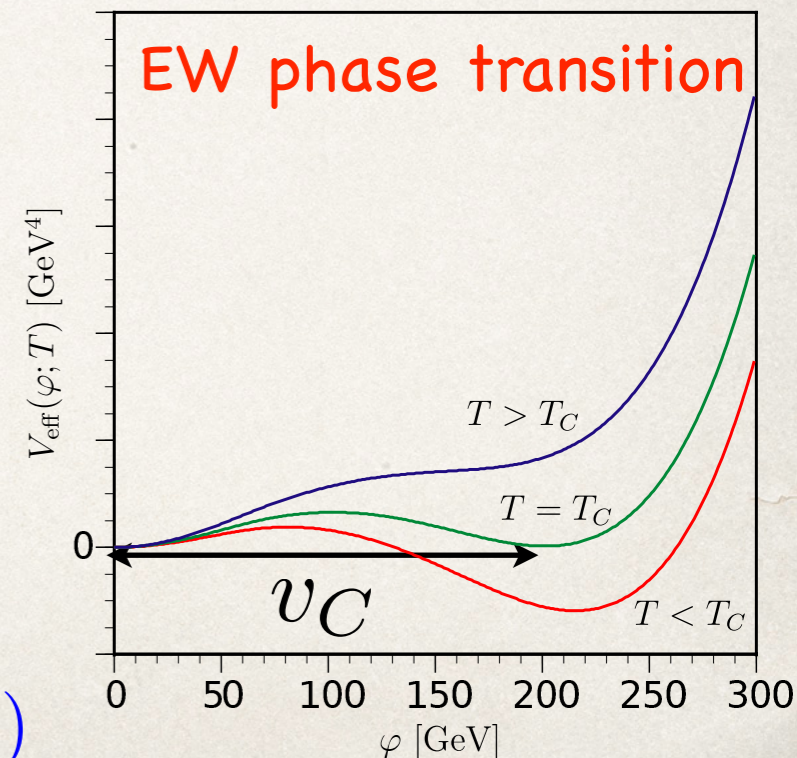
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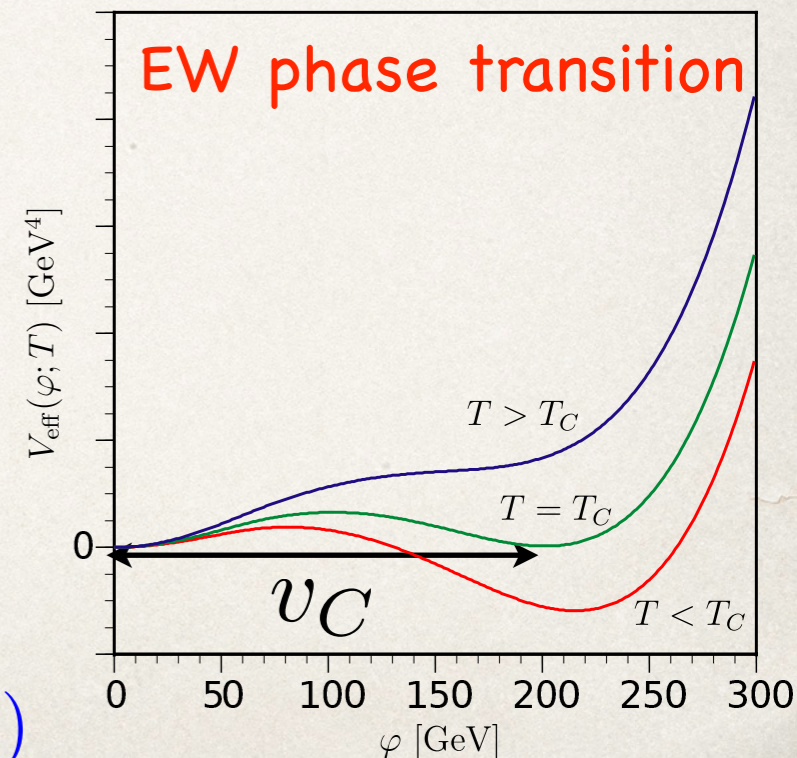
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↑  
next-to-leading

- It is important to calculate  $\mathcal{E}_{\text{sph}}$  precisely.

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# What is the problem?

- Effective potential with daisy resummation is often used to evaluate the sphaleron decoupling condition.
- In non-abelian gauge theories, transverse gauge bosons also have thermal corrections due to magnetic mass,  $m_T = O(g^2 T)$ .

$m_T$  effect on  $v_c/T_c$ : W. Buchmuller, Z. Fodor, T. Helbig, and D. Walliser, Ann.Phys.234,260 (1994).  
J. R. Espinosa, M. Quiros, and F. Zwirner, PLB314, 206 (1993).

But no detailed study of  $m_T$  effect on  $\zeta_{\text{sph}}$ !!

In this talk, we consider the  $m_T$  effect on both  $v_c/T_c$  and  $\zeta_{\text{sph}}$ .

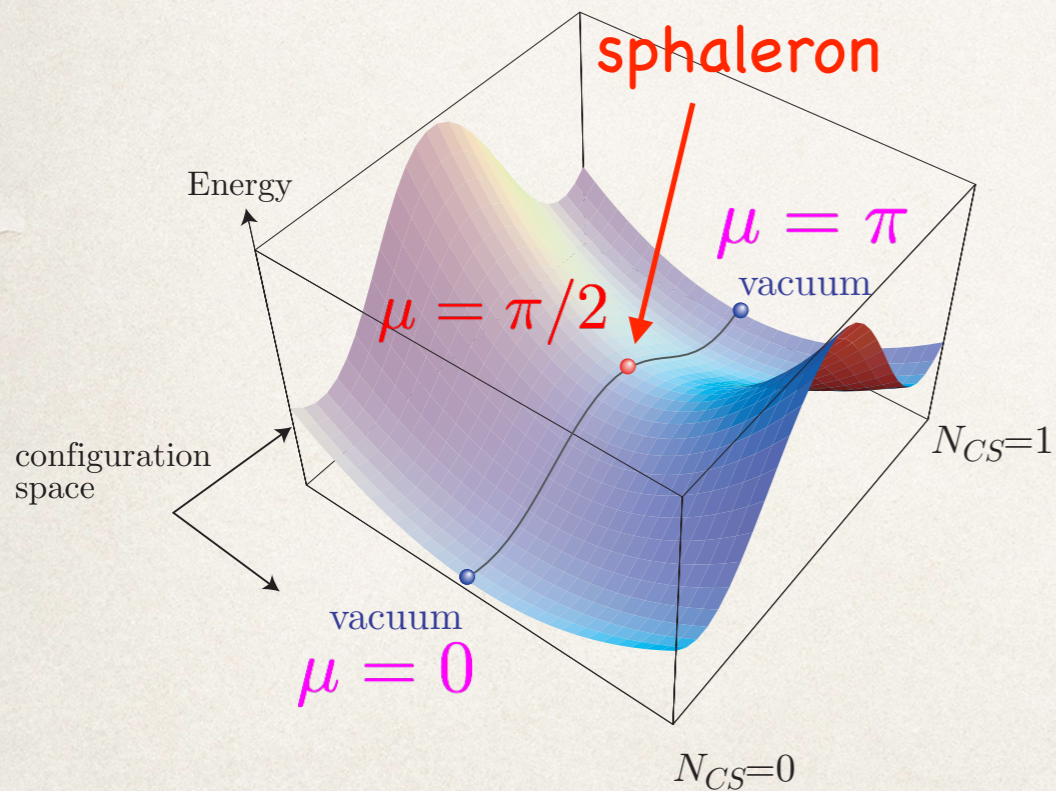
# Sphaleron in the SM

~ w/o  $U(1)_Y$  ~

$$\mathcal{L}_{\text{gauge+Higgs}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + (D_\mu \Phi)^\dagger D^\mu \Phi - V(\Phi),$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon^{abc} A_\mu^b A_\nu^c,$$

$$D_\mu \Phi = \left( \partial_\mu + ig A_\mu^a \frac{\tau^a}{2} \right), \quad V(\Phi) = \lambda \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^2.$$



Manton's ansatz.

To find a saddle point configuration, we use **noncontractible loop**.

$$\mu \in [0, \pi]$$

[N.S. Manton, PRD28 ('83) 2019]

# Sphaleron in the SM

~ w/o  $U(1)_Y$  ~

Energy functional  $A_0 = 0$

$$E_{\text{sph}} = \frac{4\pi v}{g} \int_0^\infty d\xi \left[ 4 \left( \frac{df}{d\xi} \right)^2 + \frac{8}{\xi^2} (f - f^2)^2 + \frac{\xi^2}{2} \left( \frac{dh}{d\xi} \right)^2 + h^2 (1 - f)^2 + \frac{\lambda}{4g^2} \xi^2 (h^2 - 1)^2 \right]$$
$$= \frac{4\pi v}{g} \mathcal{E}_{\text{sph}},$$

input:  $\frac{\lambda}{g^2} \simeq 0.3$  (SM)

Equations of motion for the sphaleron

$$\frac{d^2}{d\xi^2} f(\xi) = \frac{2}{\xi^2} f(\xi)(1 - f(\xi))(1 - 2f(\xi)) - \frac{1}{4} h^2(\xi)(1 - f(\xi)),$$
$$\frac{d}{d\xi} \left( \xi^2 \frac{dh(\xi)}{d\xi} \right) = 2h(\xi)(1 - f(\xi))^2 + \frac{\lambda}{g^2} (h^2(\xi) - 1)h(\xi)$$

with the boundary conditions:

$$\lim_{\xi \rightarrow 0} f(\xi) = 0, \quad \lim_{\xi \rightarrow 0} h(\xi) = 0,$$
$$\lim_{\xi \rightarrow \infty} f(\xi) = 1, \quad \lim_{\xi \rightarrow \infty} h(\xi) = 1.$$

$$\mathcal{E}_{\text{sph}} = 1.92,$$

$$(E_{\text{sph}} = 9.08 \text{ TeV}),$$

$$\zeta_{\text{sph}} = 1.17.$$

**[NOTE]**

$$\mathcal{E}_{\text{sph}}(T = 0) > \mathcal{E}_{\text{sph}}(T \neq 0); \quad \zeta_{\text{sph}}(T = 0) < \zeta_{\text{sph}}(T \neq 0)$$

Higher order corrections are important!!

# Higher-order corrections

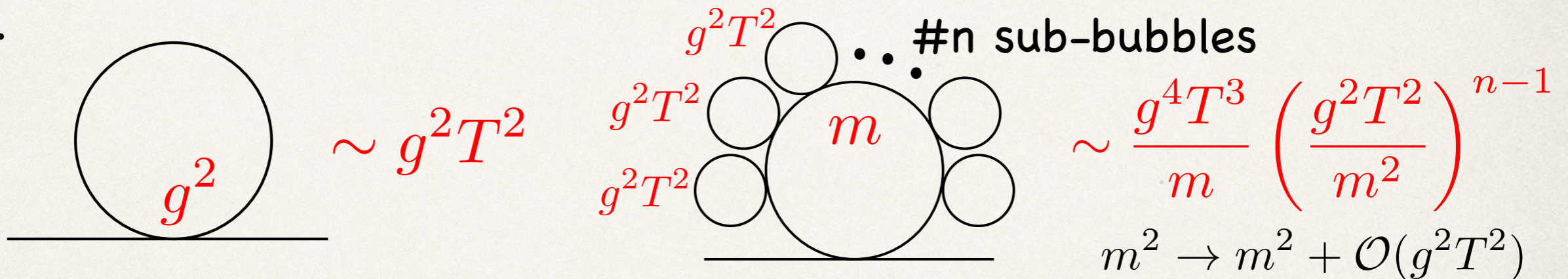
$$T=0: V_1^{T=0} = \sum_i \frac{\bar{m}_i^4}{64\pi^2} \left( \ln \frac{\bar{m}_i^2}{\bar{\mu}^2} - c_i \right)$$

$$T>0: V_1^{T \neq 0} = \sum_i \frac{T^4}{2\pi^2} I_{B,F} \left( \frac{\bar{m}_i^2}{T^2} \right), \quad I_{B,F}(a^2) = \int_0^\infty dx x^2 \ln \left[ 1 \mp e^{-\sqrt{x^2+a^2}} \right].$$

## Daisy resummation

Perturbative expansion breaks down at high T.

e.g.



$$\sim g^2 T^2 \quad \dots \quad \#n \text{ sub-bubbles} \quad \sim \frac{g^4 T^3}{m} \left( \frac{g^2 T^2}{m^2} \right)^{n-1}$$

$$m^2 \rightarrow m^2 + \mathcal{O}(g^2 T^2)$$

## Resummed Lagrangian

[Parwani (92), Buchmüller et al (93) etc.]

$$\mathcal{L} = \underbrace{(\mathcal{L}_R - \Delta m^2(T) \Psi^2)}_{\text{new 0th-order part}} + \underbrace{(\mathcal{L}_{\text{CT}} + \Delta m^2(T) \Psi^2)}_{\text{new counterterm}}$$

$\Psi$ : scalar & gauge fields  
 $\Delta m^2(T)$ : thermal masses

We evaluate sphaleron energy using the resummed Lagrangian.

# Magnetic mass corrections to $E_{\text{sph}}$

$$\mathcal{L}_{\text{eff}}^{(2)} = \text{Tr} [A^\mu \Pi_{\mu\nu} A^\nu] = \frac{1}{2} A^{a\mu} \Pi_{\mu\nu} A^{a\nu},$$

**Static limit**  $p^0 = 0$ ,  $\mathbf{p} \rightarrow 0$  with  $\partial_i A_i = 0$  (sphaleron ansatz)

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{1}{2} m_L^2(T) (A_0^a)^2 - \frac{1}{2} m_T^2(T) (A_i^a)^2, \quad m_{L,T}^2 = \lim_{p^0=0, \mathbf{p} \rightarrow 0} \Pi_{L,T}(p^0, \mathbf{p})$$

electric mass      color: red>magnetic mass

$E_{\text{sph}}$  is modified by  $\Delta E_{\text{sph}} = \frac{m_T^2}{2} \int d^3x A_i^a A_i^a.$

**Magnetic mass:**  $m_T = cg^2T$

$c = 0.11$  [Espinosa, Quiros, Zwirner, PLB314,206(93);  
Buchmuller, Fodor, Helbig, Walliser, AP234,260(94)]

$c = 0.28$  [Buchmuller, Philipsen, hep-ph/9411334]

$c = 0.38$  [Alexanian, Nair, hep-ph/9504256]

$c = 0.35$  [Patkos, Petreczky, Szep, hep-ph/9711263]

$c = 0.46$  [Heller, Karsch, Rank, hep-lat/9710033]

**gauge-dep. 1-loop gap eq.**

**gauge-inv. 1-loop gap eq.**

**Lattice**

Since there is no robust result, we regard  $c$  as the varying parameter.

# Sphaleron energy with $m_T$ .

## Energy functional

$$E_{\text{sph}} = \frac{4\pi v}{g} \int_0^\infty d\xi \left[ 4f'^2 + \frac{8}{\xi^2} (f - f^2)^2 + \frac{\xi^2}{2} h'^2 + (h^2 + r_m^2)(1 - f)^2 + \frac{\xi^2 V_0(h)}{g^2 v^4} \right] \equiv \frac{4\pi v}{g} \mathcal{E}_{\text{sph}}$$

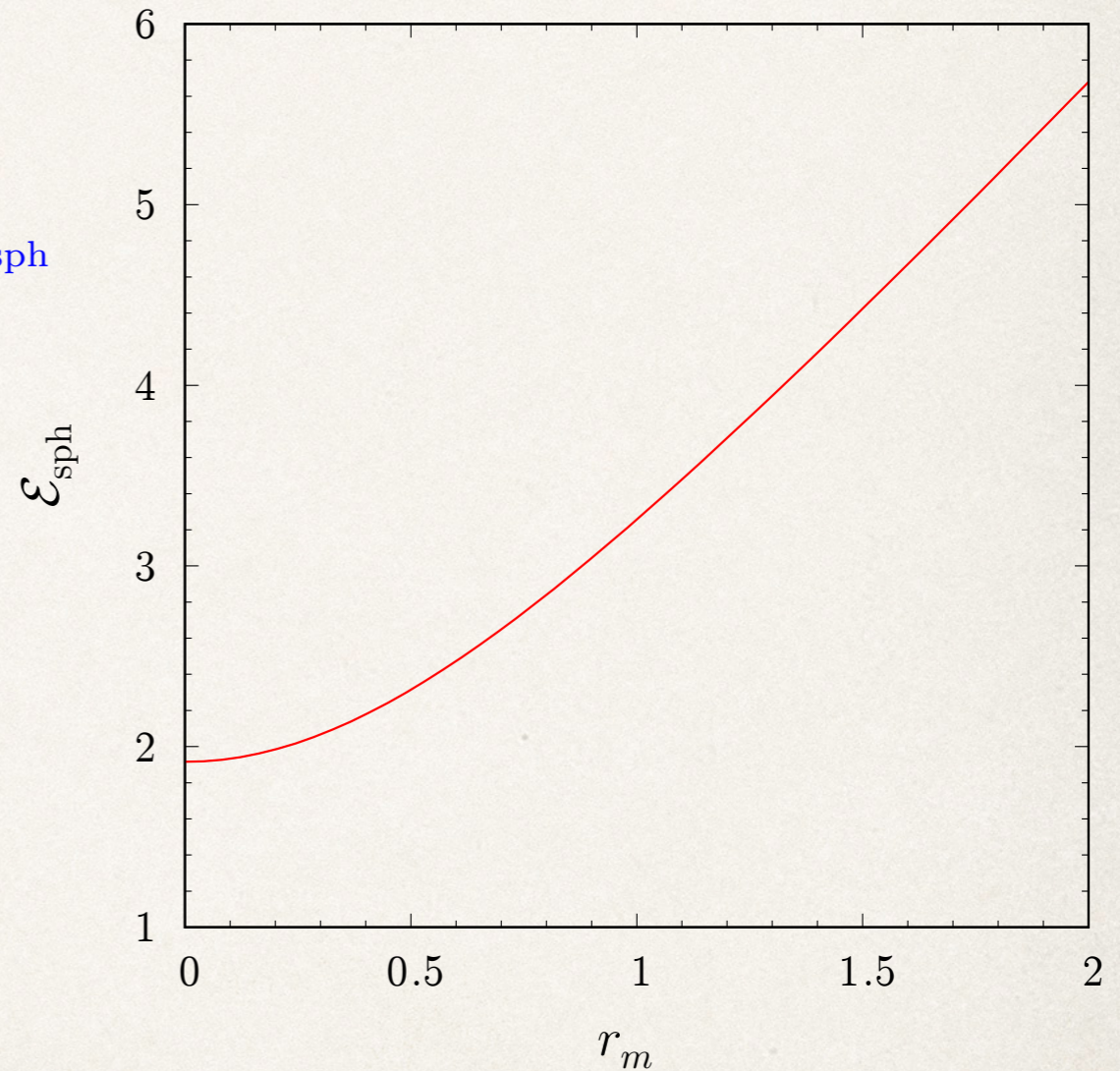
where  $\xi = gvr$ ,  $r_m = \frac{m_T}{m_W}$   $V_0(h) = \lambda v^4 (h^2 - 1)^2 / 4$

## Equations of motion

$$\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} (f - f^2)(1 - 2f) - \frac{1}{4} (h^2 + r_m^2)(1 - f),$$

$$\frac{d^2 h}{d\xi^2} = -\frac{2}{\xi} \frac{dh}{d\xi} + \frac{2}{\xi^2} h(1 - f)^2 + \frac{1}{g^2 v^4} \frac{\partial V_0}{\partial h},$$

w/ b.c.  $\lim_{\xi \rightarrow 0} f(\xi) = 0, \quad \lim_{\xi \rightarrow 0} h(\xi) = 0,$   
 $\lim_{\xi \rightarrow \infty} f(\xi) = 1, \quad \lim_{\xi \rightarrow \infty} h(\xi) = 1.$



Sphaleron energy gets larger as  $m_T$  increases.

w/o any symmetry, e.g.  $Z_2$



## General 2 Higgs doublet model (g2HDM)

Particle content: SM +  $\Phi_2$  ← 2nd Higgs doublet

Yukawa int.

$$\mathcal{L}_Y = \bar{q}_L (Y_1^{(d)} \Phi_1 + Y_2^{(d)} \Phi_2) d_R + \bar{q}_L (Y_1^{(u)} \tilde{\Phi}_1 + Y_2^{(u)} \tilde{\Phi}_2) u_R \\ + \bar{l}_L (Y_1^{(e)} \Phi_1 + Y_2^{(e)} \Phi_2) e_R + \text{h.c.} \quad \tilde{\Phi}_{1,2} = i\tau^2 \Phi_{1,2}^*$$

Higgs potential:

$$V_0(\Phi_1, \Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - (m_3^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) \\ + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \left\{ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right\} (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right],$$

**Assumption:** CP is **NOT** violated by the Higgs potential and VEVs.

**inputs:**  $\sin(\beta - \alpha)$ ,  $\tan \beta$ ,  $m_H$ ,  $m_A$ ,  $m_{H^\pm}$ ,  $M^2 = \frac{m_3^2}{\sin \beta \cos \beta}$ ,  $\lambda_{6,7}$   
 $v = 246$  GeV,  $m_h = 125$  GeV.

# Effective potential

EWPT is studied in the SM-like limit.  $\sin(\beta - \alpha) = \tan \beta = 1$

$$V_{\text{eff}}(\varphi; T) = V_0(\varphi) + V_1(\varphi; T),$$

where

$$V_0(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda_{\text{eff}}}{4}\varphi^4,$$

$$V_1(\varphi; T) = \sum_{\substack{i=h,H,A,H^\pm,G^0,G^\pm \\ W_{L,T}^\pm,Z_{L,T},\gamma_{L,T},t,b}} n_i \left[ \frac{\bar{m}_i^4}{64\pi^2} \left( \ln \frac{\bar{m}_i^2}{\bar{\mu}^2} - c_i \right) + \frac{T^4}{2\pi^2} I_{B,F} \left( \frac{\bar{m}_i^2}{T^2} \right) \right]$$

with  $I_{B,F}(a^2) = \int_0^\infty dx x^2 \log \left( 1 \mp e^{-\sqrt{x^2+a^2}} \right), \quad a^2 = m^2/T^2$

$\bar{m}_i^2$  are the thermally-corrected field dependent masses. (Parwani's method)

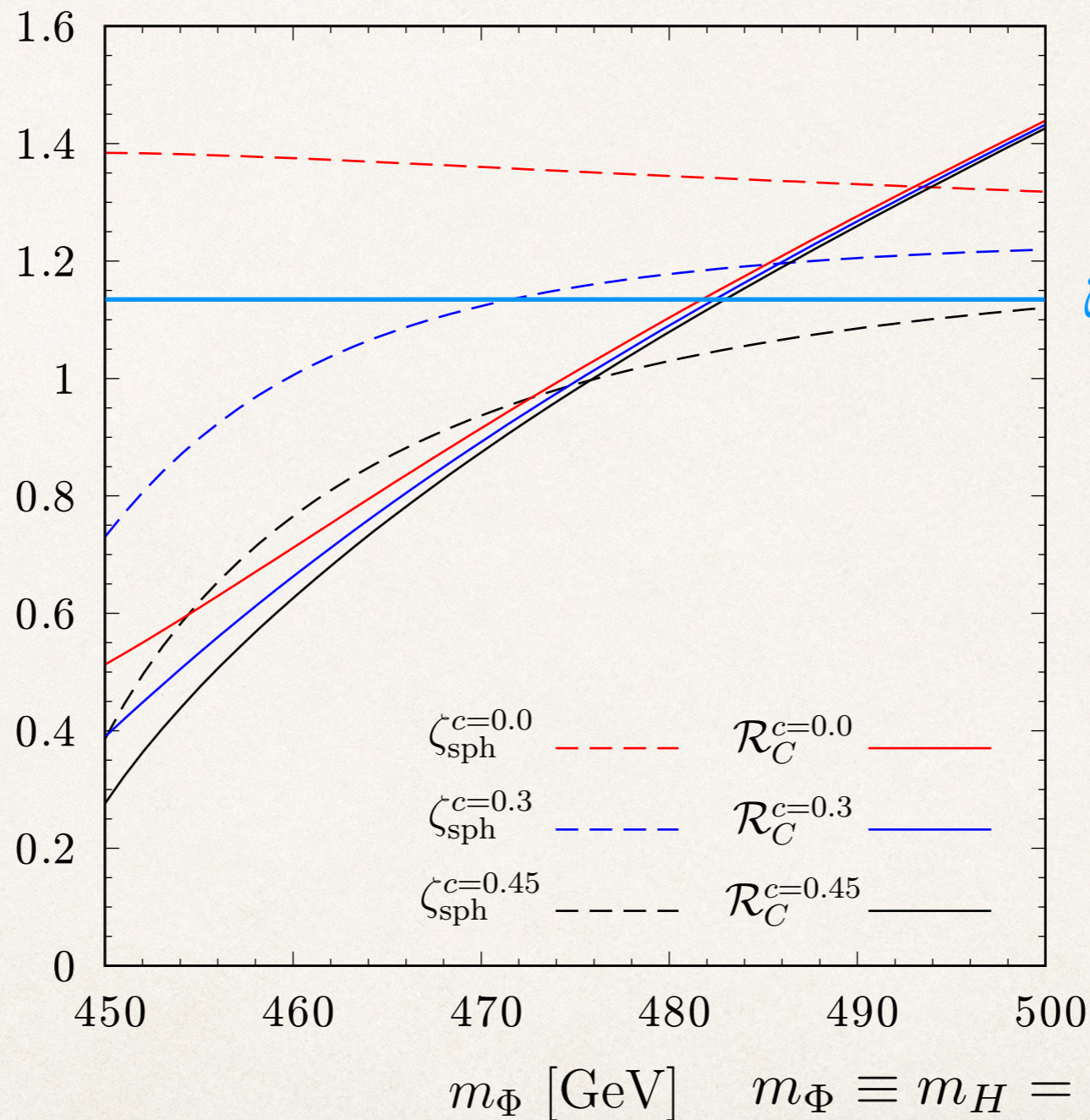
Using this potential, we evaluate  $v_c/T_c (\equiv R_c)$  and  $\zeta_{\text{sph}}(T_c)$ .



# $v_c/T_c > \zeta_{\text{sph}}(T_c)$ region

$$s_{\beta-\alpha} = t_\beta = 1, M = \sqrt{m_3^2/s_\beta c_\beta} = 300 \text{ GeV}, \lambda_{6,7} = 0.$$

$$v_c/T_c > \zeta_c \rightarrow \Gamma_{\text{sph}} < H$$



$$m_T = cg^2 T$$

$$\zeta_{\text{sph}} = 1.17 \text{ (@tree level)}$$

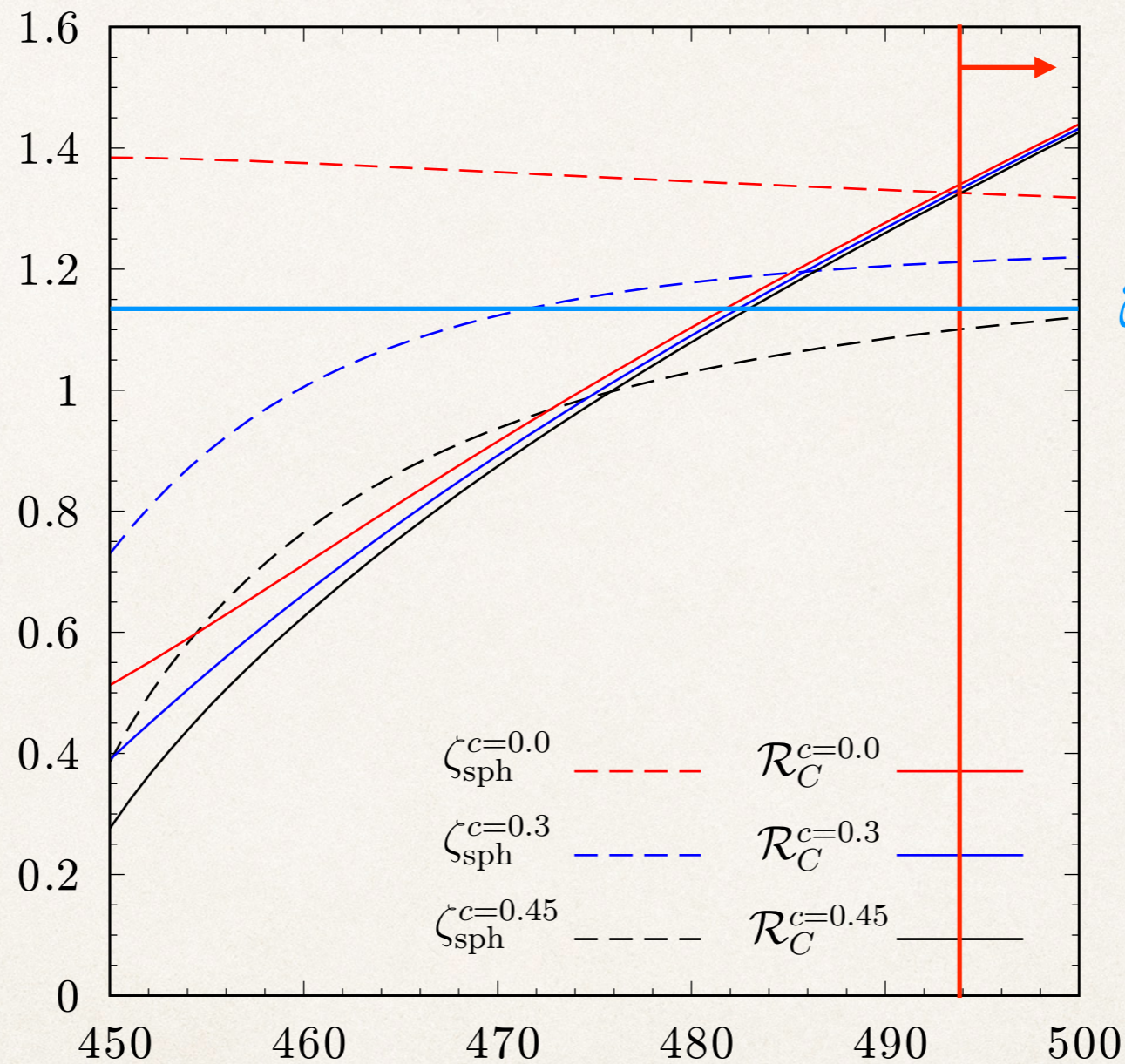
sphaleron decoupling region can be expanded due to the magnetic mass effect.

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$c=0.0$



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$$m_\Phi [\text{GeV}] \quad m_\Phi \equiv m_H = m_A = m_{H^\pm}$$

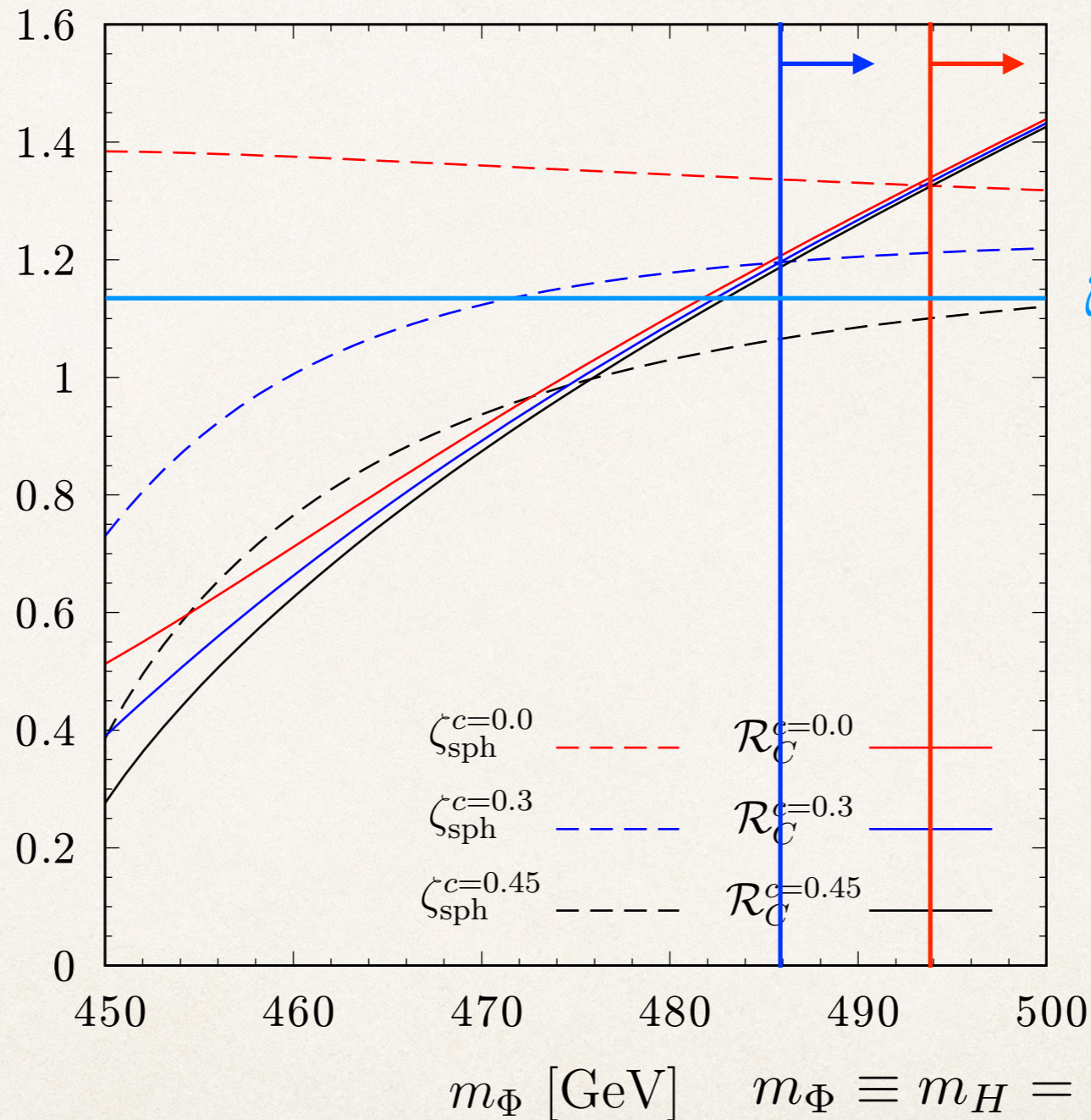
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$$c=0.3 \quad c=0.0$$



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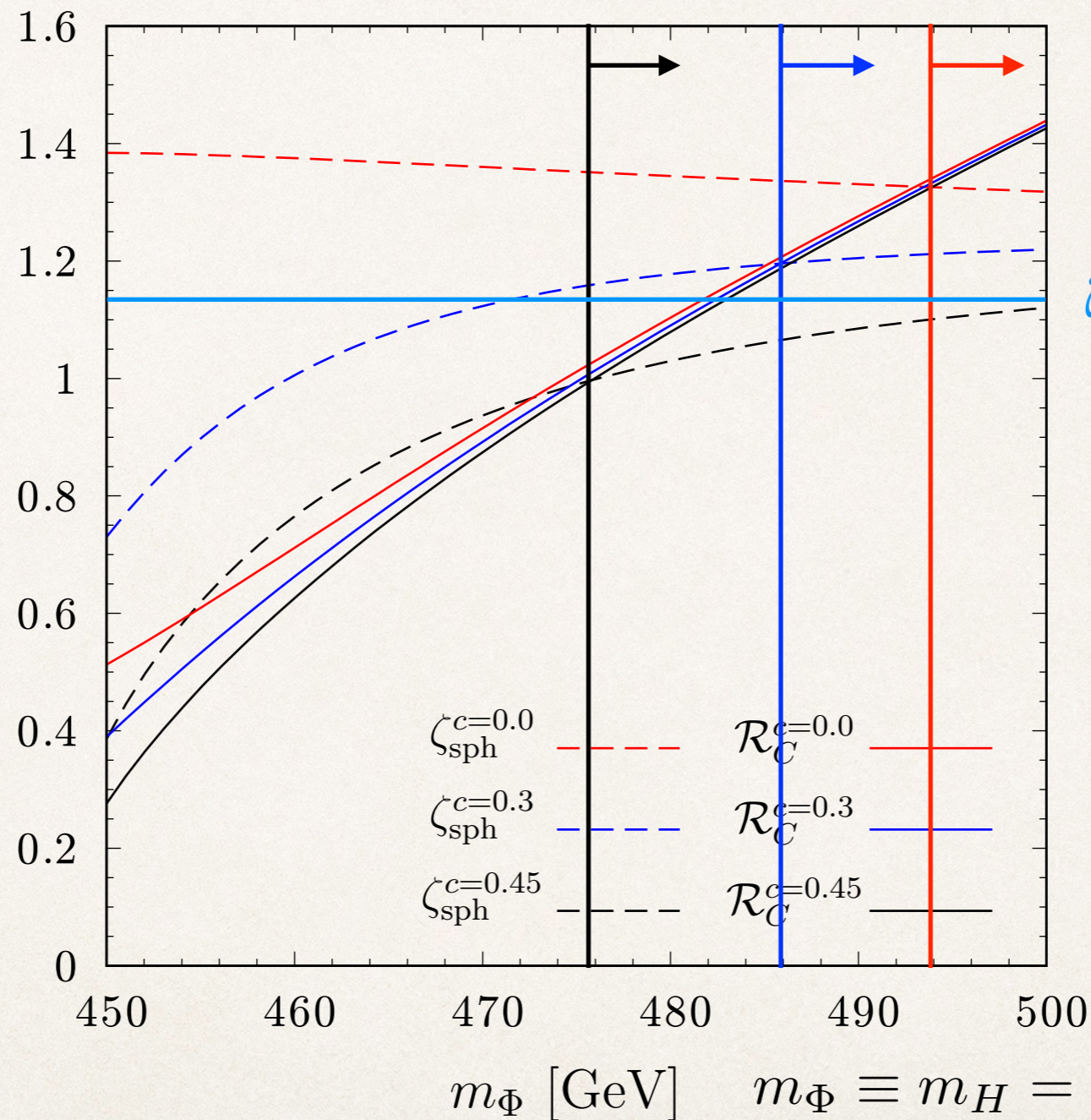
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$$v_c/T_c > \zeta_c \rightarrow \Gamma_{\text{sph}} < H$$

$$c=0.45 \quad c=0.3 \quad c=0.0$$



$$m_T = cg^2T$$

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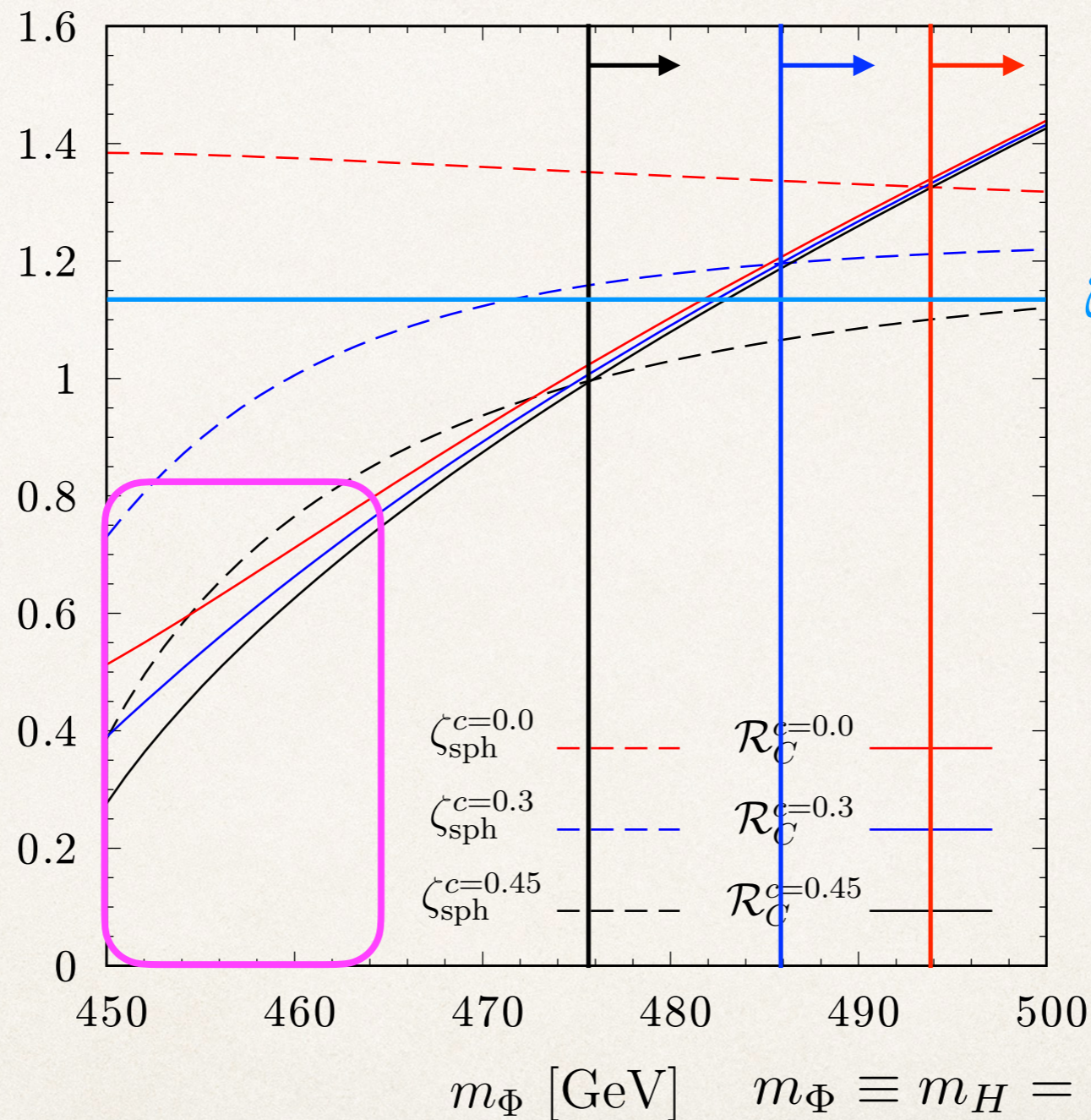
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$$c=0.45 \quad c=0.3 \quad c=0.0$$



$$m_T = c g^2 T$$

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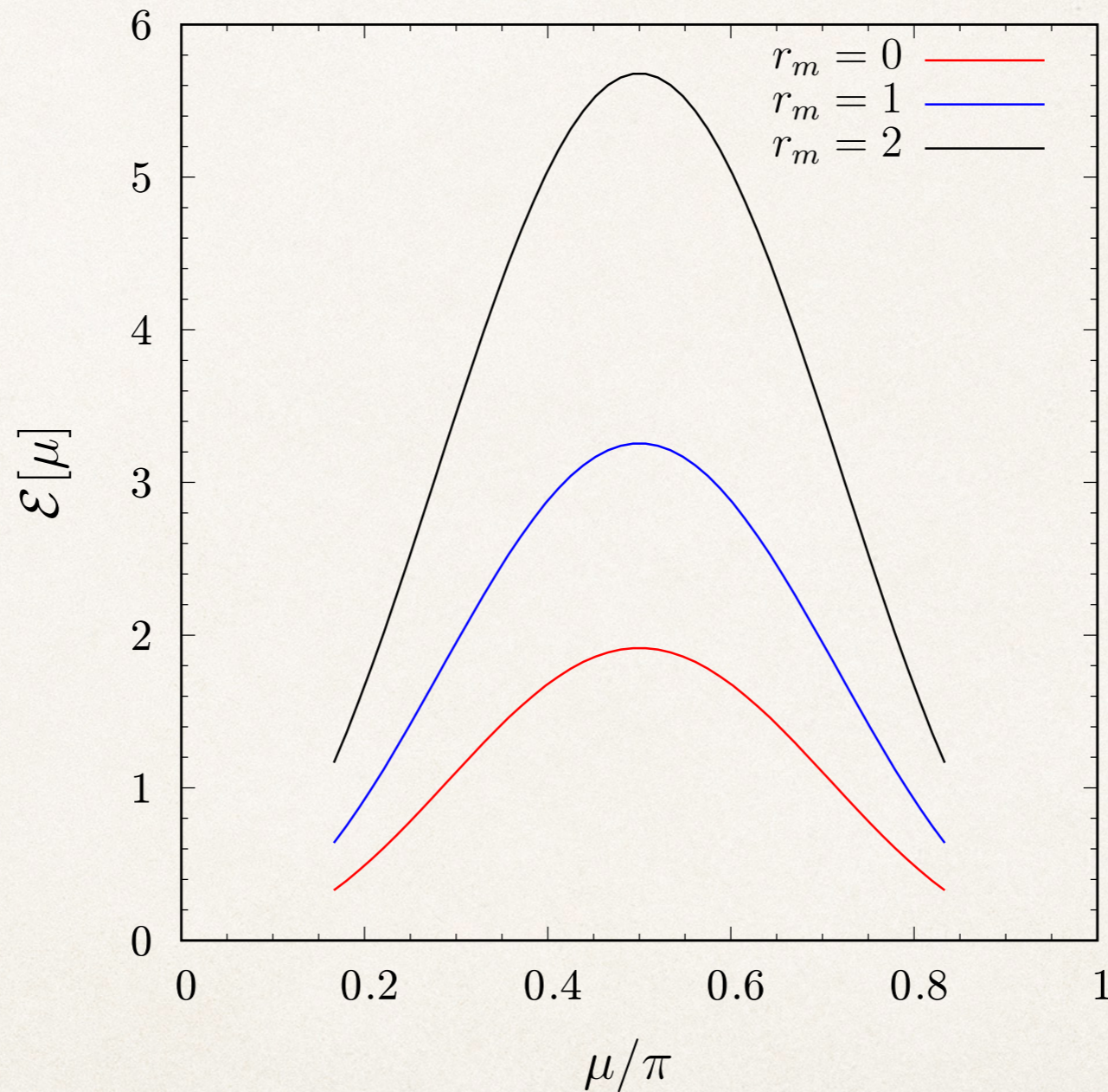
# Summary

- We have studied the sphaleron decoupling condition taking the magnetic mass into account.
- Nonzero magnetic mass can increase the sphaleron energy.
- We applied this to 2HDM and found that the sphaleron decoupling condition gets more relaxed, enlarging the domain of the successful EWBG regions.
- Our findings would hold in other BSM models as long as the gauge sector is common to the SM.

**Backup**

# Sphaleron energy in the SM

Cross-check of the results.



Obtained sphaleron energies become maximum at  $\mu = \pi/2$ .



# Magnetic mass corrections to $E_{\text{sph}}$

$$\mathcal{L}_{\text{eff}}^{(2)} = \text{Tr} [A^\mu \Pi_{\mu\nu} A^\nu] = \frac{1}{2} A^{a\mu} \Pi_{\mu\nu} A^{a\nu},$$

At  $T > 0$ , Lorentz sym. is broken by thermal bath specified by  $u^\mu$ .

$u^\mu = (1, \mathbf{0})$  in the rest frame of thermal bath

**Polarization tensor:**  $\{g_{\mu\nu}, p_\mu p_\nu, u_\mu u_\nu, p_\mu u_\nu + p_\nu u_\mu\}$

$$\Pi_{\mu\nu}(p^0, \mathbf{p}) = \Pi_L(p^0, \mathbf{p}) L_{\mu\nu}(p) + \Pi_T(p^0, \mathbf{p}) T_{\mu\nu}(p) + \Pi_G(p^0, \mathbf{p}) G_{\mu\nu}(p) + \Pi_S(p^0, \mathbf{p}) S_{\mu\nu}(p),$$

$$L_{\mu\nu}(p) = \frac{u_\mu^T u_\nu^T}{(u^T)^2}, \quad T_{\mu\nu}(p) = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} - L_{\mu\nu}(p), \quad u_\mu^T = u_\mu - (p \cdot u) p_\mu / p^2$$

$$G_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2}, \quad S_{\mu\nu}(p) = \frac{p_\mu u_\nu^T + p_\nu u_\mu^T}{\sqrt{(p \cdot u)^2 - p^2}}, \quad u_\mu^T p^\mu = 0$$

**Static limit**  $p^0 = 0, \mathbf{p} \rightarrow 0$  with  $\partial_i A_i = 0$  (sphaleron ansatz)

$$\mathcal{L}_{\text{eff}}^{(2)} = \frac{1}{2} m_L^2(T) (A_0^a)^2 - \frac{1}{2} m_T^2(T) (A_i^a)^2, \quad m_{L,T}^2 = \lim_{p^0=0, \mathbf{p} \rightarrow 0} \Pi_{L,T}(p^0, \mathbf{p})$$

electric mass      magnetic mass

$$A_0 = 0 \quad \Rightarrow \quad \Delta E_{\text{sph}} = \frac{m_T^2}{2} \int d^3 \mathbf{x} A_i^a A_i^a.$$

# Magnetic mass corrections to $E_{\text{sph}}$

Gauge-inv. dim.2 operator [D. Zwanziger, Nucl. Phys. B 345, 461 (1990)]

$$\int d^4x A_{\text{min}}^2 = \min_{\{U\}} \int d^4x \text{Tr}[(A_\mu^U)^2] \simeq \int d^4x \left[ F_{\mu\nu} \frac{1}{D^2} F^{\mu\nu} + \dots \right]$$

$$A_\mu^U = U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1}$$

expressed by infinite series of non-local gauge-inv. terms.

It is known that  $\int d^4x A_{\text{min}}^2 = \int d^4x \text{Tr}[A_\mu A^\mu]$  if  $\partial_\mu A^\mu = 0$

Since the sphaleron ansatz satisfies this condition, one has the same mass form as the previous case.

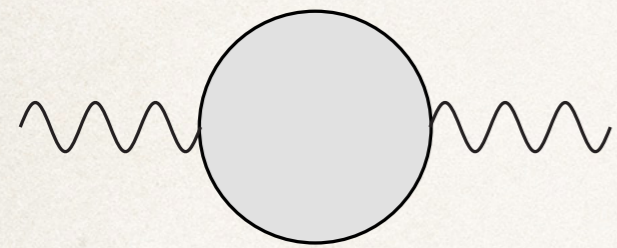
$$\Delta E_{\text{sph}} = \frac{m_T^2}{2} \int d^3x A_i^a A_i^a.$$

We regard this as the magnetic mass correction to  $E_{\text{sph}}$ .

# Magnetic mass

In SU(2) gauge Higgs model,

Espinosa, et al, PLB314, 206 (1993);  
Buchmuller, et al, AP234, 260 (1994).



$$\Rightarrow m_T^2 = \frac{g^2 T}{3\pi} m_T + \mathcal{O}(v)$$

1-loop gap eq. at high T.

$$m_T = c g^2 T, \quad c = \frac{1}{3\pi} \simeq 0.11$$

but, this is gauge dependent.

Other studies show that

methods	Refs.	c
gauge-inv. 1-loop gap eq.	Buchmuller, Philipsen, hep-ph/9411334	0.28
"	Alexanian, Nair, hep-ph/9504256	0.38
"	Patkos, Petreczky, Szep, hep-ph/9711263	0.35
Lattice	Heller, Karsch, Rank, hep-lat/9710033	0.46

Since there is no robust result, we regard c as the varying parameter.

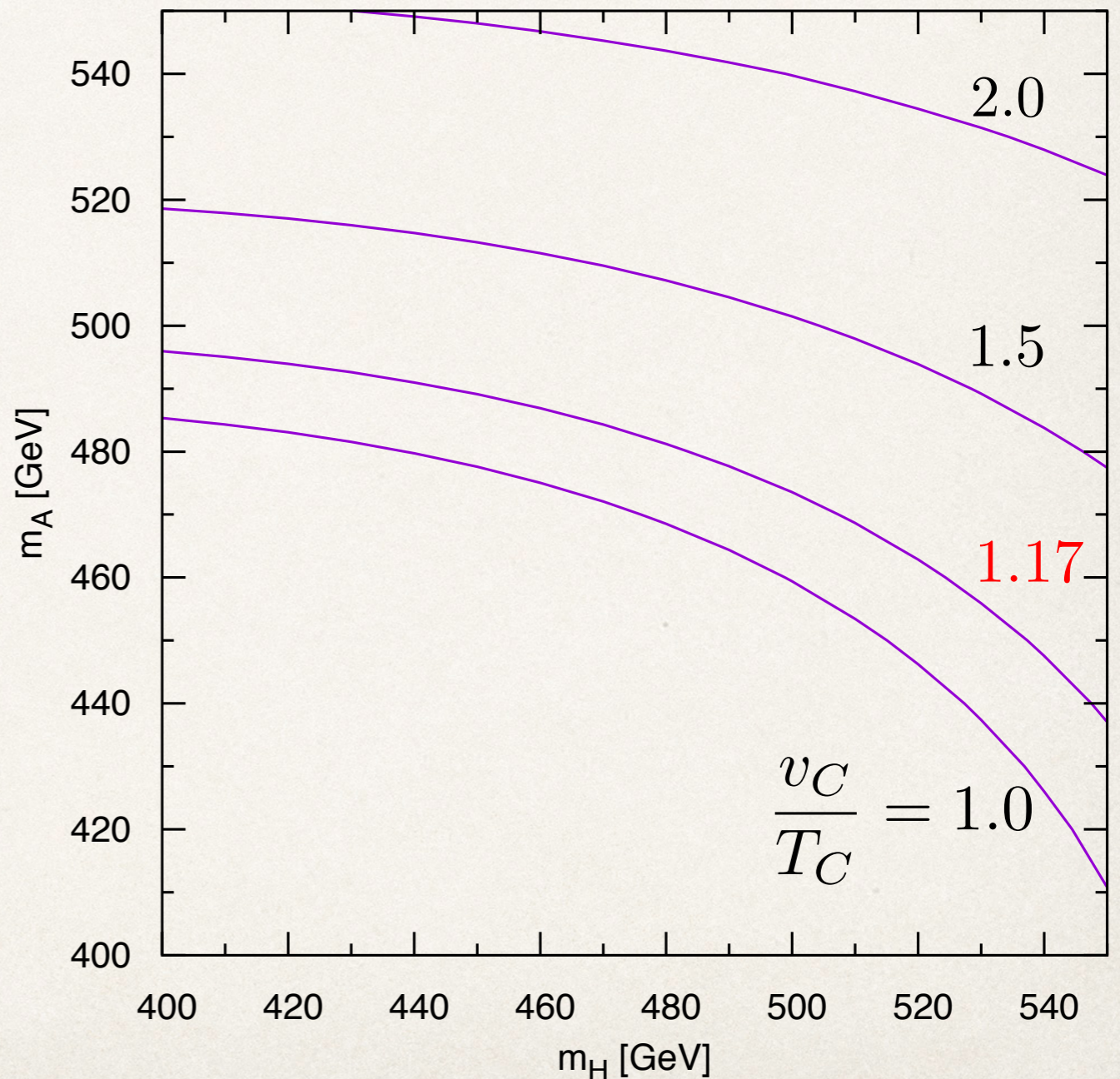
# 1<sup>st</sup>-order EWPT

$$s_{\beta-\alpha} = t_{\beta} = 1, m_{H^{\pm}} = m_A, M = \sqrt{m_3^2/s_{\beta}c_{\beta}} = 300 \text{ GeV}$$
$$\lambda_6 = \lambda_7 = 0$$

- Heavy Higgs w/ non-decoupling plays a role.
- Too heavy Higgs could violate perturbativity.
- EWBG-viable region

$$v_C/T_C > \zeta_{\text{sph}}$$

cf.  $\zeta_{\text{sph}}=1.17$  using  $V_0$  w/o  $m_T$ .



# 1<sup>st</sup>-order EWPT

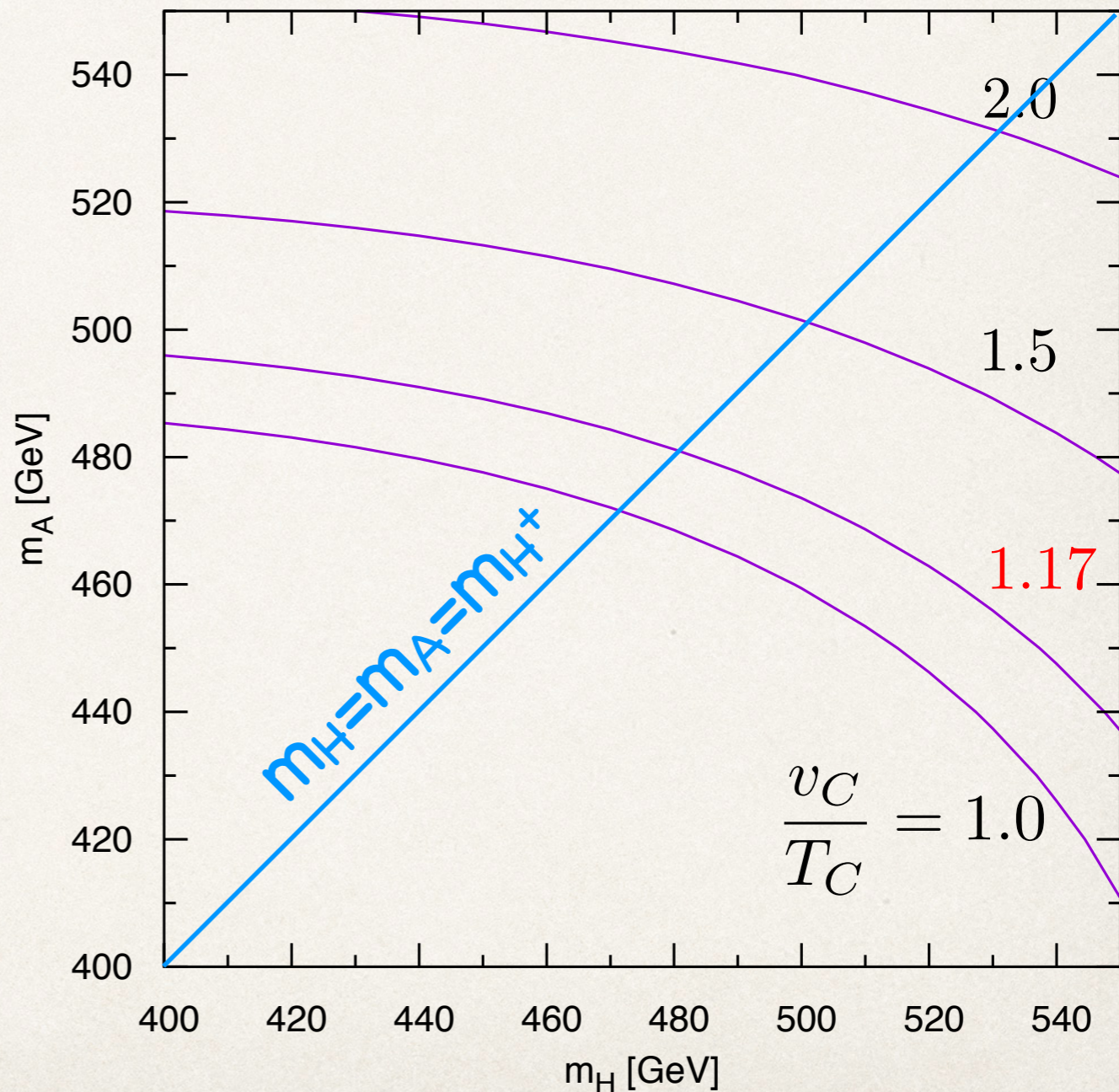
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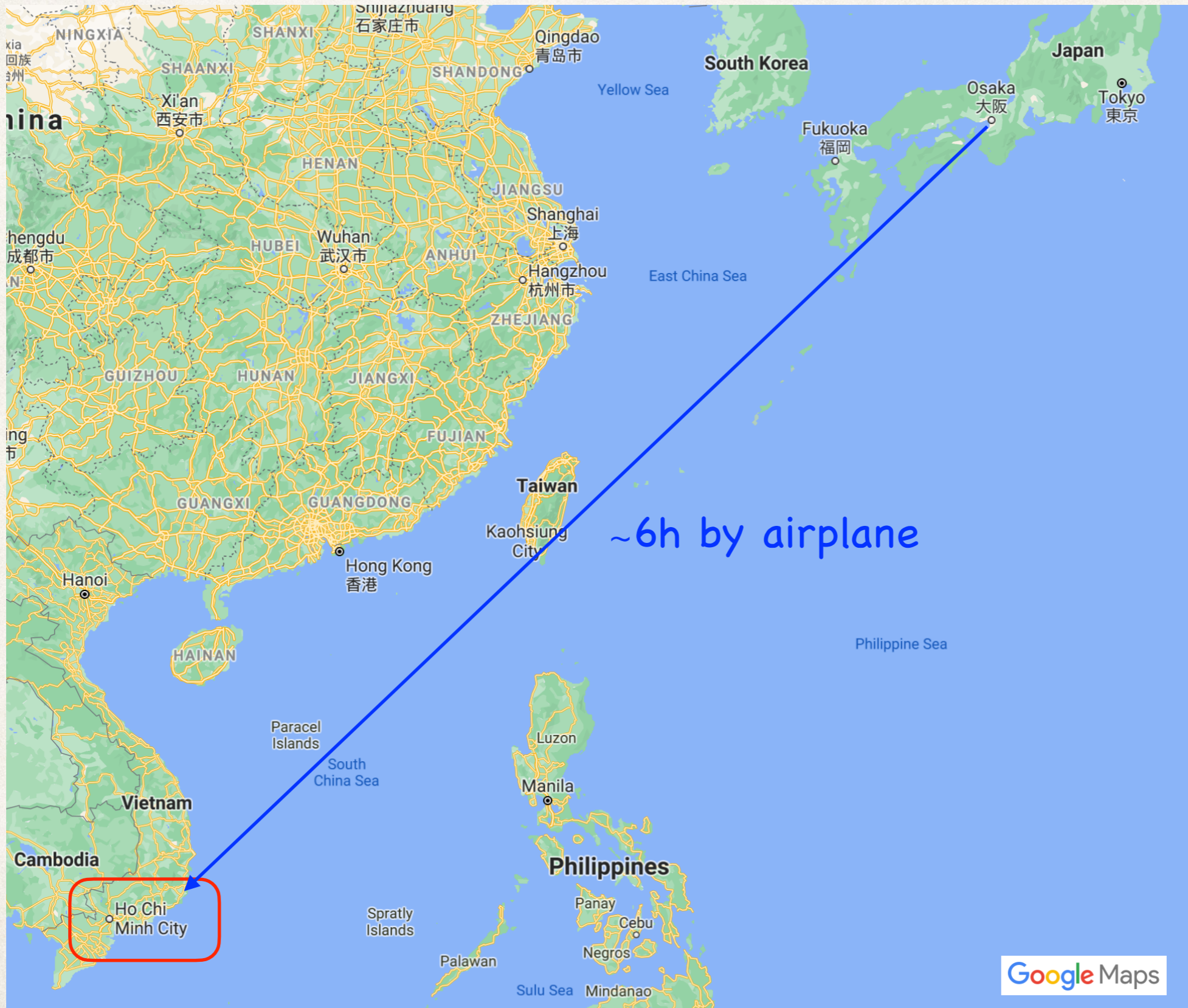
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# Where is Ton Duc Thang Univ.?



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I belong to Theoretical Particle Physics and Cosmology Research Group (TPPC) in Advanced Institute of Materials Science (AIMaS).