

# Affine Gravitational Scenario for Dark Matter Decays

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H. Azri, A. Jueid, C. Karahan, and S. Nasri,  
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# Gravity: Metrical or affine?

- ◆ GR is based on the metric tensor which is **only of secondary importance**: measurements of distances and angles...etc !
- ◆ Gravity is a “manifestation” of spacetime **curvature**, hence, what is needed is the property of parallel displacements of tensors. This is achieved by introducing the concept of **connection**.
- ◆ The **SM** interactions are written in the framework of gauge field theory where the **gauge** fields are described by **connections** (arising through gauge invariance...)
- ✓ ◆ Affine spacetime, with its geodesics and no prior concept of metric, provides a very simple and possibly viable description for gravity !

# Affine space structure

- Affine space is endowed with an affine connection from which we construct covariant derivatives:

$$\nabla_{\mu} A^{\alpha} = \partial_{\mu} A^{\alpha} + \Gamma_{\mu\beta}^{\alpha} A^{\beta}$$

- It vanishes locally,

$$\Gamma_{\mu\beta}^{\alpha}(P) = 0$$

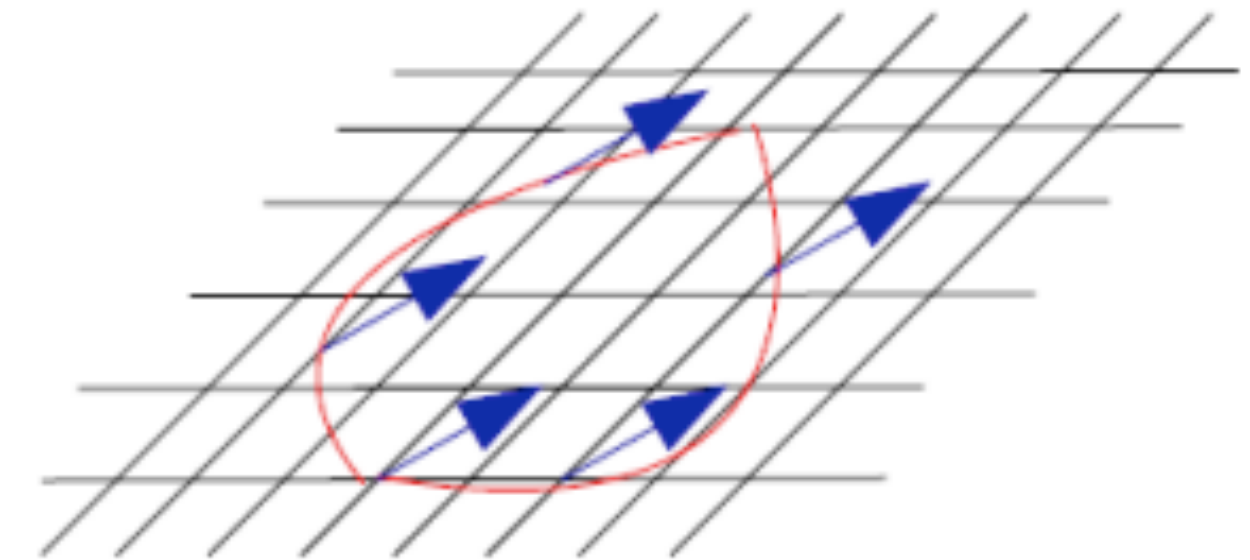
- It transforms as

$$\hat{\Gamma}_{\lambda\mu}^{\alpha} = \frac{\partial \hat{x}^{\alpha}}{\partial x^{\beta}} \frac{\partial x^{\sigma}}{\partial \hat{x}^{\lambda}} \frac{\partial x^{\tau}}{\partial \hat{x}^{\mu}} \Gamma_{\sigma\tau}^{\beta} + \frac{\partial^2 x^{\beta}}{\partial \hat{x}^{\lambda} \partial \hat{x}^{\mu}} \frac{\partial \hat{x}^{\alpha}}{\partial x^{\beta}}$$

- Curvature can be defined from the

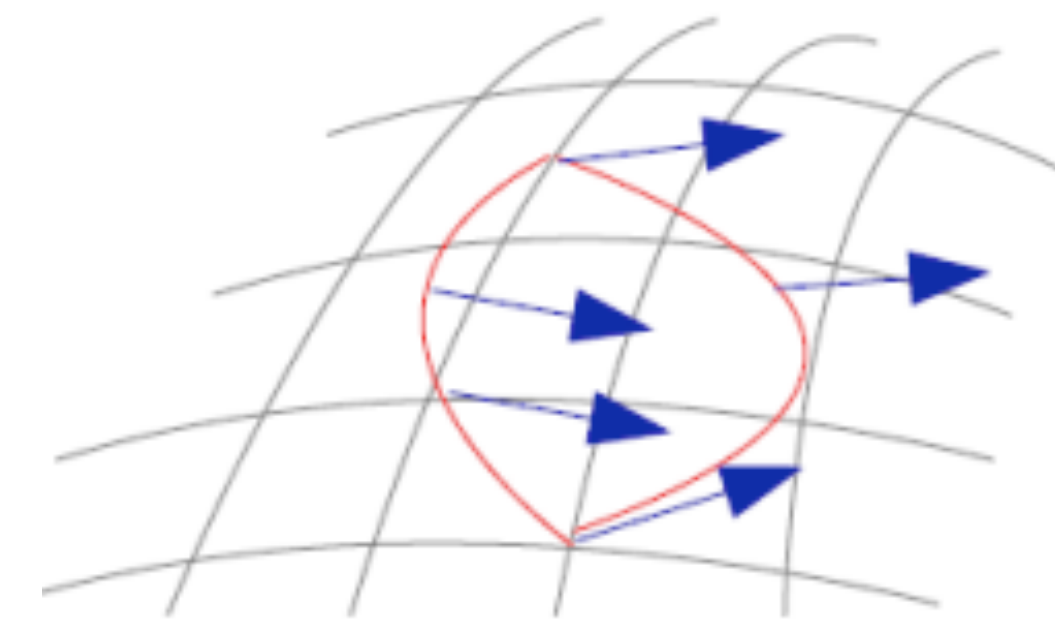
“failure” of the commutativity:  $[\nabla_{\mu}, \nabla_{\nu}]A^{\alpha} = R_{\beta\mu\nu}^{\alpha}(\Gamma)A^{\beta}$

When  $\Gamma$  is symmetric



$$\partial_{\mu} \partial_{\nu} A^{\alpha} - \partial_{\nu} \partial_{\mu} A^{\alpha} = 0$$

Flat background



$$\nabla_{\mu} \nabla_{\nu} A^{\alpha} - \nabla_{\nu} \nabla_{\mu} A^{\alpha} \neq 0$$

Curved background

◆ Gravitational action with the SM fields:

- This involves Einstein-Hilbert action and the SM Lagrangian

$$S = \int d^4x \sqrt{|g|} \left\{ \frac{M_{Pl}^2}{2} g^{\mu\nu} R_{\mu\nu}(g) + L[g, SM] \right\}$$

- The SM Lagrangian is written by simply replacing the flat metric by the general curved metric

$$L[g, SM] = \underbrace{-g^{\mu\nu} (D_\mu H)^\dagger (D_\nu H)}_{\text{Higgs}} - \underbrace{i\bar{\psi} g^{\mu\nu} \gamma_\mu \nabla_\nu \psi}_{\text{Fermions}} - \underbrace{\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha}^a F_{\nu\beta}^a}_{\text{Gauge bosons}}$$

◆ How about dark matter?

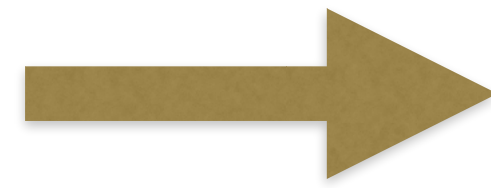
Its existence is revealed only through its gravitational influence!



The way it couples to gravity must be very crucial!

- ◆ Decoupling dark matter from the metric:

Unlike the SM particle fields, DM may couple only to the spacetime connection!



In this case, how to decouple it from the metric?

- ◆ From Palatini to purely affine action:

Take the following metric-affine model in which metric and connection are independent fields!

$$\mathcal{L}[\mathcal{F}] = \sqrt{|g|} \left\{ \frac{M^2}{2} g^{\mu\nu} R_{\mu\nu}(\Gamma) - \frac{1}{2} g^{\mu\nu} X_{\mu\nu} - U(\mathcal{F}) \right\}$$

Kinetic terms!

$$X_{\mu\nu}(\mathcal{I}) = \begin{cases} \nabla_\mu \phi \nabla_\nu \phi & \text{for scalar DM} \\ \bar{\chi} \gamma_\mu \nabla_\nu \chi & \text{for fermionic DM} \\ \frac{1}{2} g^{\alpha\beta} \mathbb{F}_{\alpha\mu} \mathbb{F}_{\beta\nu} & \text{for vector DM} \end{cases}$$

Fermions and vector fields involve metric in their kinetic terms. Scalars do not!

- ◆ From Palatini to purely affine action:

Field equations arise from variation with respect to the (non-dynamical) metric

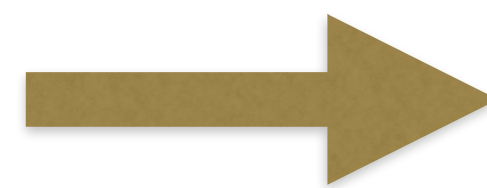
$$\mathcal{L}[\mathcal{F}] = \sqrt{|g|} \left\{ \frac{M^2}{2} g^{\mu\nu} R_{\mu\nu}(\Gamma) - \frac{1}{2} g^{\mu\nu} X_{\mu\nu} - U(\mathcal{F}) \right\}$$

For scalar DM   $M^2 R_{\mu\nu}(\Gamma) = \nabla_\mu \phi \nabla_\nu \phi + U(\mathcal{F}) g_{\mu\nu}$

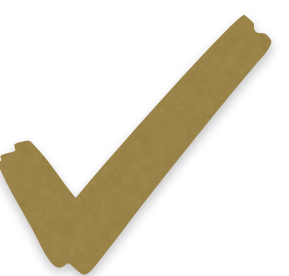
Metric arises only through the potential term!

Hence, for nonzero potential, the metric can be easily integrated out!

$$g_{\mu\nu} = \frac{M^2 R_{\mu\nu}(\Gamma) - X_{\mu\nu}(\phi)}{U(\phi)}$$



The Scalar DM Lagrangian can be written in terms of the connection and its curvature only!

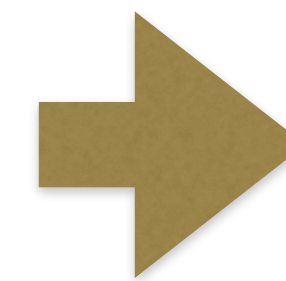


- ◆ If dark matter is to decouple totally from the metric, it can **simply be a scalar**

$$\mathcal{L}[\Gamma, \text{DM}] = \frac{\sqrt{|M^2 R_{\mu\nu}(\Gamma) - \nabla_\mu \phi \nabla_\nu \phi|}}{U(\phi)}$$

1. The model describes DM in curved spacetime interacting only with the connection (**notice the absence of metric!**)
2. Gravitational field equations are obtained by variation w.r.t the connection

$$\nabla_\alpha \left\{ \underbrace{\frac{\sqrt{|M^2 R_{\mu\nu}(\Gamma) - \nabla_\mu \phi \nabla_\nu \phi|}}{U(\phi)}}_{\sqrt{|g|} g^{\mu\nu}} ([M^2 R(\Gamma) - \nabla \phi \nabla \phi]^{-1})^{\mu\nu} \right\} = 0$$



$$M^2 R_{\mu\nu}(\Gamma) = \nabla_\mu \phi \nabla_\nu \phi + U(\mathcal{J}) g_{\mu\nu}$$

Einstein's field equations!

This action can also be constructed based only on general covariance consideration!

◆ Nonminimal couplings  
(DM\_Curvature interaction)

1. The  $Z_2$  symmetry enjoyed by the previous action is **broken** by an interaction between DM and the affine curvature like  $\xi\phi R_{\mu\nu}(\Gamma)$
2. Fields that **enjoy a complete structure in purely affine spacetime are scalars**, hence, the model can be extended by including the **SM-Higgs potential**

$$\mathcal{L}[\Gamma, \text{DM}] = \frac{\sqrt{\left| M^2 R_{\mu\nu}(\Gamma) - \nabla_\mu \phi \nabla_\nu \phi + \xi M \phi R_{\mu\nu}(\Gamma) \right|}}{V(\phi, H)}$$

Where  $V(\phi, H) \supset U(\phi) + m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2$ .

Notice that there is  
No DM-SM coupling!



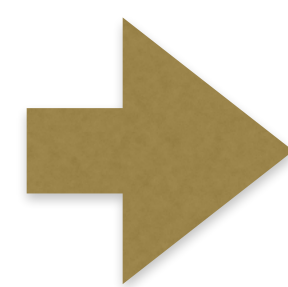
- ◆ Dark matter decays through the Higgs portal

- The affine gravitational sector is recast to its canonical form as

$$\mathcal{L}[\Gamma, \text{DM}] = \frac{\sqrt{\left| M^2 R_{\mu\nu}(\Gamma) - \alpha^{-1}(\phi) \nabla_\mu \phi \nabla_\nu \phi \right|}}{\alpha^{-2}(\phi) V(\phi, H)}$$

With the dimensionless parameter  $\alpha(\phi) = 1 + \xi\phi/M$

- As a result, the first order  $Z_2$ -symmetry breaking term implies DM decays to SM-Higgs through the induced interactions  $\alpha^{-2}(\phi) V(\phi, H)$



The interactions are a direct effects of the nonminimal coupling to affine gravity even in the absence of any explicit coupling of DM with SM particles within the potential !

◆ The resulting Decay modes

$$\phi \rightarrow hh$$

$$\phi \rightarrow hh^* \rightarrow hff\bar{f}, fVV, hhh \quad \text{Where } V = W^\pm/Z$$

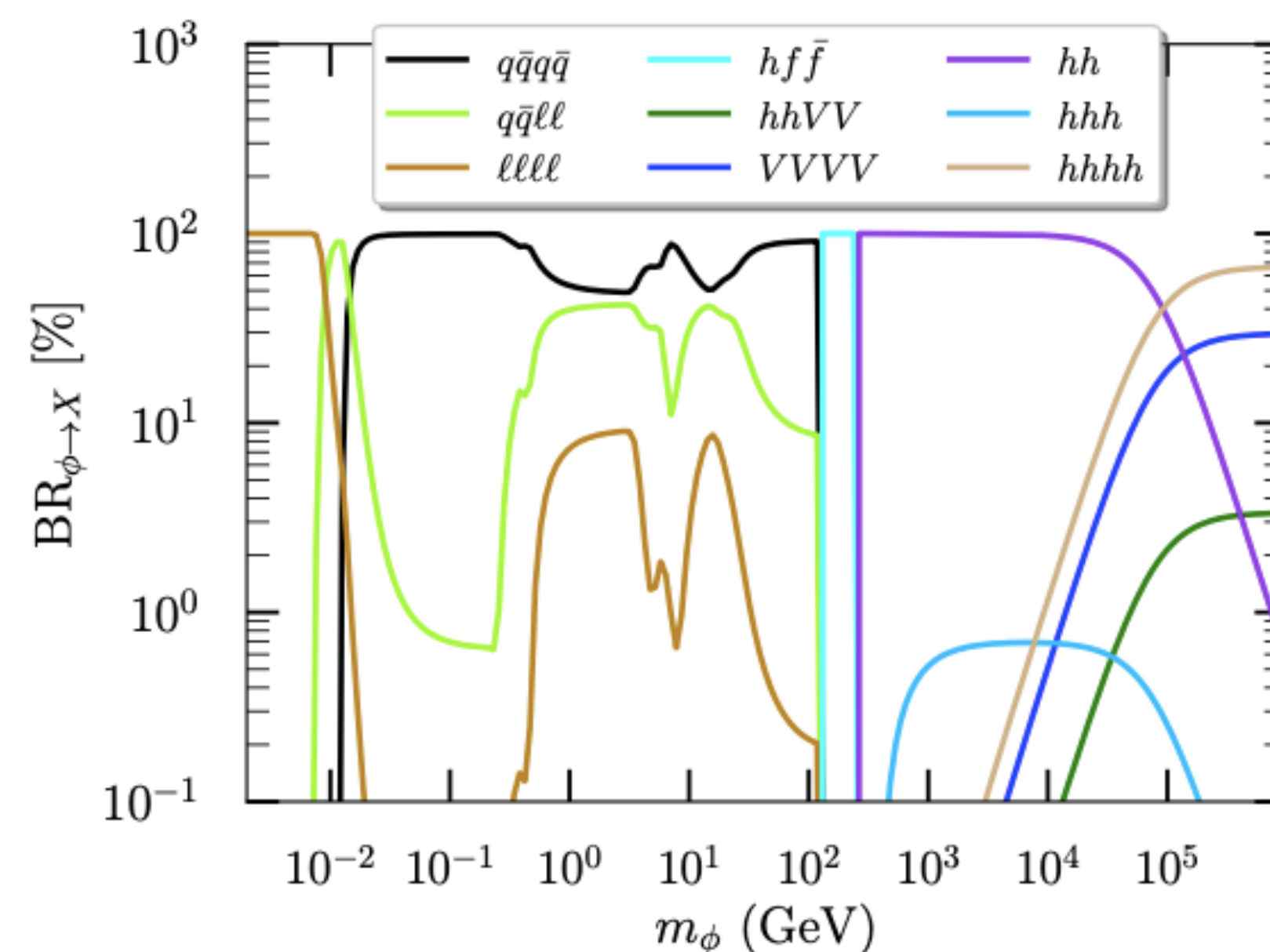
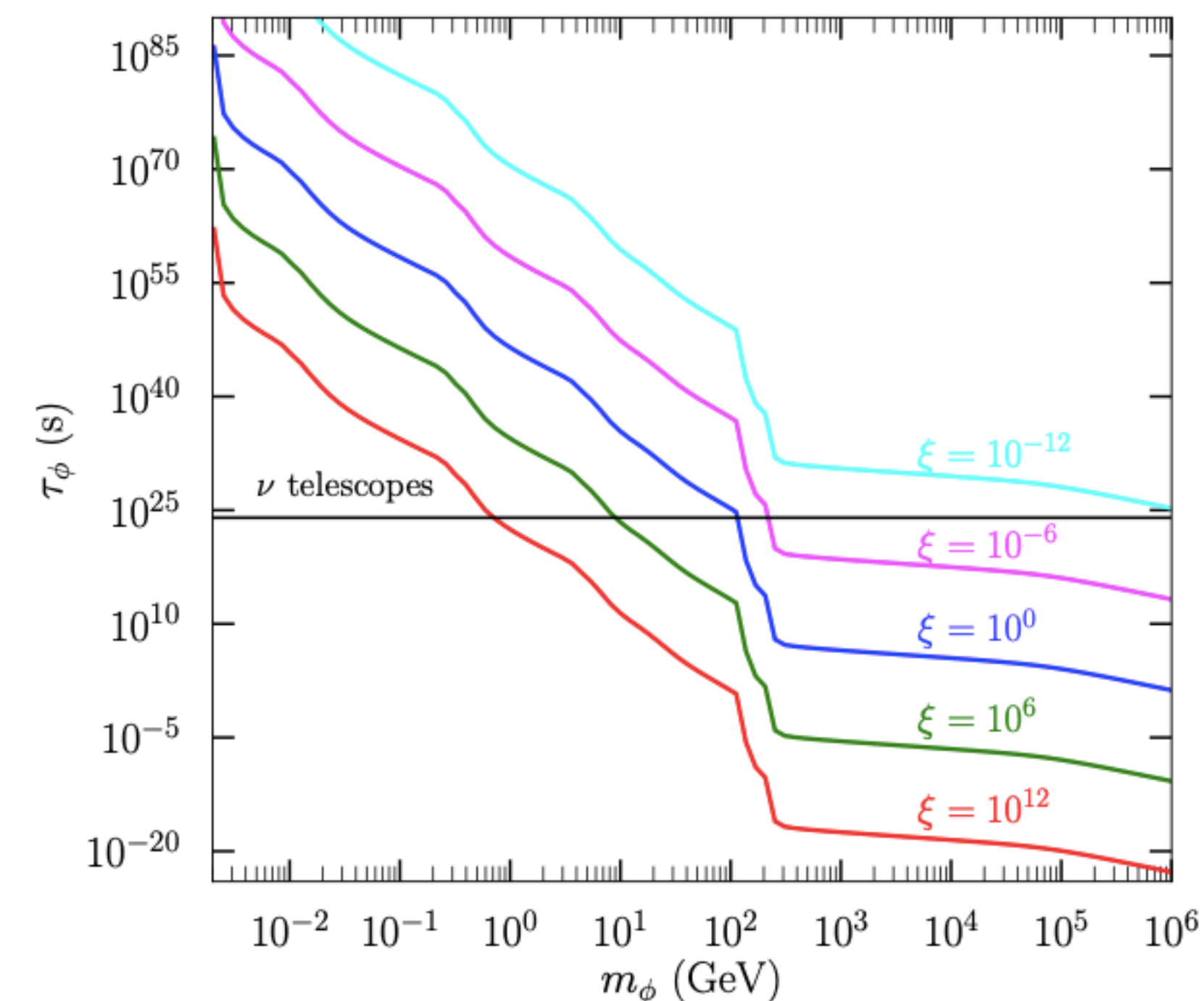
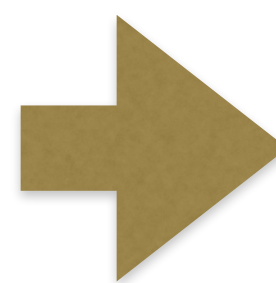
$$\phi \rightarrow h^*h^* \rightarrow VVVV, VVff\bar{f}, VVhh, f\bar{f}f'\bar{f}', f\bar{f}hh, hhhh$$

The decay rates are suppressed by the factor  $\xi^2/M^2$ , leading to a large enough life time when  $M = M_{Pl}$ !

◆ Relevant DM Mass

ranges:

- $m_\phi \sim 10^{-3} - 125 \text{ GeV}$
- $m_\phi \sim 125 - 250 \text{ GeV}$
- $m_\phi \sim 250 - 10^6 \text{ GeV}$



# Conclusion

- ◆ We have investigated the possibility that, unlike ordinary matter, DM interacts to gravity through only the affine connection of spacetime!
- ◆ We saw that DM decays, automatically, into the SM Higgs via a nonminimal coupling to gravity that breaks the  $Z_2$ -symmetry! This decay occurs without imposing any prior coupling between the two!
- ◆ The present scenario for DM decay is fully gravitational and it stands on a formulation of gravity, different from GR. It is expected to reveal more interesting features of the physics of DM!

Thank you