HPNP2021

25.-27. March 2021

Based on:
arXiv:2012.07740

Wilson is not anomalous: on gauge anomalies in SMEFT

Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul, and **Alejo N. Rossia**[1]

The puzzle to solve

Can chiral interactions at dim.-6 generate gauge anomalies? There were doubts about the operators on the right, which enter in triangle diagrams:

$$\mathcal{O}_{\varphi\psi_R} = i\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\overline{\psi}^i_R\gamma^\mu\psi^j_R$$

$$\mathcal{O}^{(1)}_{\varphi\psi_L} = i\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\overline{\psi}^i_L\gamma^\mu\psi^j_L$$

$$\mathcal{O}^{(3)}_{\varphi\psi_L} = i\left(H^\dagger \overleftrightarrow{D}^a_\mu H\right)\overline{\psi}^i_L\tau^a\gamma^\mu\psi^j_L$$

Current arguments

The following toy model:

$$\mathcal{L} = -\frac{F^2_{A,\mu\nu}}{4g^2_A} + |\partial\varphi|^2 - V(|\varphi|) + i\overline{\psi}^k\left(\partial\!\!\!/ + iq^A_k A\!\!\!/\right)\psi^k$$

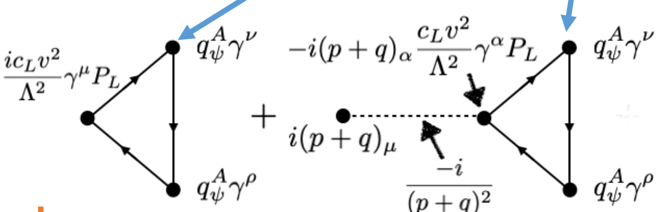
$$+\sum_{l=L,R}\sum_{k=1}^{2} i\frac{c_{l,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{\partial}_\mu\varphi\right)\overline{\psi}^k_l\gamma^\mu\psi^k_l \qquad \left(\varphi = \frac{v+h}{\sqrt{2}}e^{i\theta/v}\right)$$

has a chiral global $U(1)_B$ symmetry with this conserved current:

$$J^B_\mu = q^B_k\overline{\psi}^k\gamma_\mu\gamma_5\psi^k + q^B_\varphi v\left(\sum_{l=L,R}\frac{vc_{l,k}}{\Lambda^2}\overline{\psi}^k_l\gamma_\mu\psi^k_l - \partial_\mu\theta\right) + \mathcal{O}(h)$$

Leads to usual gauge anomaly cancellation conditions

No role here



They cancel each other. No dim-6 contribution!

Reasoning with Effective Field Theories (EFTs)

Toy model + gauged chiral $U(1)_B$ + Mass for the fermions.
Integrate the fermions out, keep only bosons.
The anomalies will be carried by Wess-Zumino terms:

Axionic: $\theta F_i\tilde{F}_j$ | Generalized Chern-Simons: $A_i\wedge A_j\wedge F_k$

In our toy-model, they are:

$$\mathcal{L}_{EFT}\supset -\frac{\left(q^A_k\right)^2}{16\pi^2}\frac{\theta}{v}F_A\tilde{F}_A - \frac{\left(q^B_k\right)^2}{24\pi^2}\frac{\theta}{v}F_B\tilde{F}_B - \frac{\left(q^A_k\right)^2 q^B_k}{6\pi^2}A_\mu B_\nu\tilde{F}^{\mu\nu}_A$$

Usual gauge anomaly cancellation conditions.
No dim-6 contribution!

EFT argument for SMEFT

Gauge anomalies are independent of Yukawa and gauge coupling values. Then, suppose all SM fermions much heavier than W, Z and integrate them all out of SMEFT!

$$\mathcal{L}_{EFT}\supset -\frac{C_{F_0W^3}}{32\pi^2}v\pi^3 B^{\mu\nu}\tilde{W}^3_{\mu\nu} + \frac{v^2E_{A_0BB}}{16\pi^2\Lambda^2}W^3_\mu B_\nu\tilde{B}^{\mu\nu} + \ldots$$

$$\delta\left(\mathcal{L}_{EFT}\right) = 0\,\frac{c_i}{\Lambda^2} + \left(\sum_{\psi_L,\psi_R} q^i_\psi q^j_\psi q^k_\psi\right)\epsilon^i A^j\tilde{A}^k$$

No dim-6 contribution | Usual conditions

SMEFT at dim-6 is gauge anomaly (from triangle diagrams) free

[1]: alejo.rossia@desy.de

Appendix A

On the conditions derived by Catà et al[1],…

They say that to make SMEFT gauge anomaly free, the following relations among dim-6 Wilson Coefficients (WCs) must be satisified:

$$c_{\varphi q}^{(3)} = c_{\varphi l}^{(3)} \qquad \frac{c_{\varphi q}^{(1)}}{y_q} = \frac{c_{\varphi l}^{(1)}}{y_l} = \frac{c_{\varphi u}}{y_u} = \frac{c_{\varphi d}}{y_d} = \frac{c_{\varphi e}}{y_e}$$

…their clear counterexample,…

Consider the SM and add a singlet Weyl fermion with a Majorana mass and interaction with the neutrinos:

$$\mathcal{L}_{BSM}^{\text{Int}} = -\left(\lambda_N\right)_i \bar{N}\tilde{H}^\dagger \ell_{L,i}$$

integrate it out and the only non-vanishing WCs are:

$$\frac{c_{\varphi l,ij}^{(3)}}{\Lambda^2} = -\frac{\lambda_{N,i}^*\lambda_{N,j}}{4M_N^2} \quad \frac{c_{\varphi l,ij}^{(1)}}{\Lambda^2} = \frac{\lambda_{N,i}^*\lambda_{N,j}}{4M_N^2} \quad \frac{c_{5,ij}}{\Lambda} = \frac{\lambda_{N,i}\lambda_{N,j}}{2M_N}$$

Both the SM and the NP sector are gauge anomaly free on their own.

…and what they missed

What they computed is equivalent to (in our toy model) have considered the classically non-conserved "current":

$$\tilde{J}_\mu^B = 2q_\varphi^B\left(\frac{c_{L,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,L}\gamma_\mu\psi_{k,L} + \frac{c_{R,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,R}\gamma_\mu\psi_{k,R}\right) + q_k^B\overline{\psi}_k\gamma_\mu\gamma_5\psi_k$$

Which misses the Goldstone contribution.

Notice that the constraints they found are trivially satisfied by models in the class of Universal Theories.

[1]: O. Catà, W. Killian, N. Kreher. *Gauge Anomalies in the Standard-Model Effective Field Theory*. arXiv:2011.09976

Details on our toy model and the current reasoning

The $U(1)_A$ is a vector-like gauge symm. Under which the scalar is neutral. The U(1)B transformation law is:

$$\varphi \to e^{iq_\varphi^B\epsilon_B}\varphi, \;\; \psi_k \to e^{iq_k^B\gamma_5\epsilon_B}\psi_k$$

And its Noether current in the unbroken phase is:

$$J_\mu^B = -\,iq_\varphi^B\left(\varphi^\dagger\overleftrightarrow{\partial}_\mu\varphi + 2i\frac{c_{L,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,L}\gamma_\mu\psi_{k,L} + 2i\frac{c_{R,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,R}\gamma_\mu\psi_{k,R}\right) + q_k^B\overline{\psi}_k\gamma_\mu\gamma_5\psi_k$$

If we want to gauge $U(1)_B$, we should cancel these anomalies:

$$U(1)_A^2\times U(1)_B:\;\left(q_k^A\right)^2 q_k^B = 0\,,\qquad U(1)_B^3:\;\left(q_k^B\right)^3 = 0$$

The Lagrangian with both gauge symmetries is:

$$\mathcal{L} = -\frac{1}{4g_A^2}F_{A,\mu\nu}^2 - \frac{1}{4g_B^2}F_{B,\mu\nu}^2 - \left|D\varphi\right|^2 - V(|\varphi|) + i\overline{\psi}_k \not{D}\psi_k + i\frac{c_{L,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{D}_\mu\varphi\right)\overline{\psi}_{k,L}\gamma^\mu\psi_{k,L} + i\frac{c_{R,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{D}_\mu\varphi\right)\overline{\psi}_{k,R}\gamma^\mu\psi_{k,R}$$

In the broken phase, the GB-fermion couplings are:

$$\mathcal{L} \supset -\frac{vc_{L,k}}{\Lambda^2}\partial_\mu\theta\overline{\psi}_{k,L}\gamma^\mu\psi_{k,L} - \frac{vc_{R,k}}{\Lambda^2}\partial_\mu\theta\overline{\psi}_{k,R}\gamma^\mu\psi_{k,R}$$

And to compute the anomalies, we compute:

$$\partial^\mu\langle 0|J_\mu^B(x)J_\nu^A(y)J_\rho^A(z)|0\rangle$$

The Feynman diagrams cancel only after taking the derivative

For completeness, the EOMs:

$$\Box\varphi + V'(|\varphi|) + i\frac{c_{L,k}}{\Lambda^2}\partial\varphi\overline{\psi}_{k,L}\gamma\psi_{k,L} + i\frac{c_{L,k}}{\Lambda^2}\partial\left(\varphi\overline{\psi}_{k,L}\gamma\psi_{k,L}\right) + (L\leftrightarrow R) = 0$$

$$\not{D}\psi_{k,L/R} + \frac{c_{L/R,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{\partial}_\mu\varphi\right)\gamma^\mu\psi_{k,L/R} = 0\;,$$

Appendix B

More on the EFT reasoning for our toy model

The Yukawa term that we add to the model is:

$$\delta\mathcal{L} = -y_k \varphi \overline{\psi}_{k,L} \psi_{k,R} + h.c. \qquad \left(m_k = \frac{y_k v}{\sqrt{2}} \right)$$

And gauge invariance requires:

$$2\, q_k^B = q_\varphi^B$$

The anomalous Ward identities do not change.

The anomalous gauge variation of the EFT is:

$$\delta\mathcal{L}_{\text{eff}} = -\frac{\epsilon_A}{24\pi^2} 2 \left(q_k^A\right)^2 q_k^B F_A \tilde{F}_B - \frac{\epsilon_B}{24\pi^2} \left[\left(q_k^A\right)^2 q_k^B F_A \tilde{F}_A + 2 \left(q_k^B\right)^3 F_B \tilde{F}_B \right]$$

And the gauge transformation rules are:

$$\delta A_\mu = -\partial_\mu \epsilon_A, \quad \delta B_\mu = -\partial_\mu \epsilon_B, \quad \delta\theta = v q_\varphi^B \epsilon_B$$

And the details on our SMEFT computation

We parametrize the Higgs as:

$$\varphi = e^{i\frac{\pi^a}{v}\sigma^a} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad \delta\pi^a = \frac{v}{2}\epsilon^a - v\delta^{a3} y_\varphi \epsilon_Y$$

Modifications to the LH gauge couplings:

$$-\overline{\psi}_{L,i} \Bigg(\delta_{ij} \left[y_\psi \not{B} + T^a \not{W}^a \right] + \left[v^2 \left(y_\varphi \not{B} - \frac{\not{W}^3}{2} \right) - v \not{\partial} \pi^3 \right] \left[\frac{c^{(1)}_{\varphi\psi,ij}}{\Lambda^2} - 2T^3 \frac{c^{(3)}_{\varphi\psi,ij}}{\Lambda^2} \right] + 2T^{a=1,2} \frac{c^{(3)}_{\varphi\psi,ij}}{\Lambda^2} \left[\frac{v^2}{2} \not{W}^a + v \not{\partial} \pi^a \right] \Bigg) \psi_{L,j}$$

Yukawa couplings:

$$\mathcal{L} \supset -\frac{1}{\sqrt{2}} \left((v + i\pi^3) \overline{u}_L + i(\pi^1 + i\pi^2) \overline{d}_L \right) Y_u u_R - \frac{1}{\sqrt{2}} \left((v - i\pi^3) \overline{d}_L + i(\pi^1 - i\pi^2) \overline{u}_L \right) Y_d d_R - \frac{1}{\sqrt{2}} \left((v - i\pi^3) \overline{e}_L + i(\pi^1 - i\pi^2) \overline{\nu}_L \right) Y_e e_R + h.c.$$

EFT from dim-4 couplings:

$$\begin{aligned}
\mathcal{L}_{\text{EFT}} \supset &- \frac{1}{16\pi^2} \frac{\pi^3}{v} B\tilde{B} \left(3 \left[y_u^2 + y_Q y_u - y_d^2 - y_Q y_d \right] + y_\nu^2 + y_L y_\nu - y_e^2 - y_e y_L \right) \\
&- \frac{1}{16\pi^2} \frac{\pi^3}{v} B\tilde{W}^3 \left(\frac{3(y_d + 4y_Q + y_u)}{2} + \frac{y_e + 4y_L + y_\nu}{2} \right) \\
&- \frac{1}{8\pi^2} B_\mu W^3_\nu \tilde{B}^{\mu\nu} \frac{(y_\nu + y_e)(y_e + y_L + y_\nu) + 3(y_u - y_d)(y_d + y_Q + y_u)}{2} \\
&- \frac{1}{8\pi^2} B_\mu W^3_\nu \tilde{W}^{3,\mu\nu} \frac{3(y_u + y_d) + y_e + y_\nu}{4}
\end{aligned}$$

Gauge variation:

$$\delta\mathcal{L}_{\text{EFT}} = -\frac{\epsilon_Y}{16\pi^2} \left[\left(6y_Q^3 + 2y_L^3 - 3y_u^3 - 3y_d^3 - y_e^3 - y_\nu^3 \right) B\tilde{B} + \frac{3y_Q + y_L}{2} W^3 \tilde{W}^3 \right] - \frac{\epsilon_3}{16\pi^2} \left(3y_Q + y_L \right) B\tilde{W}^3$$

WCs of the bosonic EFT:

$$\mathcal{C}_{F_0 W^3} = \frac{1}{6\Lambda^2} \left[3c^{(1)}_{\varphi d} + 3c^{(1)}_{\varphi u} + 12 c^{(1)}_{\varphi Q} + c^{(1)}_{\varphi e} + 4c^{(1)}_{\varphi L} \right]$$

$$\begin{aligned}
E_{A_0 BB} = &\, c^{(1)}_{\varphi u} (y_Q - y_u)(y_Q + 2y_u) - c^{(1)}_{\varphi d} (y_d - y_Q)(2y_d + y_Q) \\
&- c^{(1)}_{\varphi Q} \left(y_d^2 + y_d y_Q - 4y_Q^2 + y_Q y_u + y_u^2 \right) \\
&- c^{(3)}_{\varphi Q} (y_d - y_u)(y_d + y_Q + y_u) - \frac{1}{3} c^{(1)}_{\varphi e} (y_e - y_L)(2y_e + y_L) \\
&- \frac{1}{3} c^{(1)}_{\varphi L} \left(y_e^2 + y_e y_L - 4y_L^2 + y_L y_\nu + y_\nu^2 \right) \\
&- \frac{1}{3} c^{(3)}_{\varphi L} \left(y_e^2 + y_L y_e + y_\nu^2 + y_\nu y_e \right)
\end{aligned}$$

Further reading:

J. Preskill, *Gauge anomalies in an effective field theory*, Annals Phys. 210 (1991) 323–379

F. Feruglio, *A Note on Gauge Anomaly Cancellation in Effective Field Theories*, arXiv: 2012.13989 [hep-ph]