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Positivity bounds on Minimal Flavor Violation

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Question: is Minimal Flavor Violation compatible with Positivity bounds?

Ingredients

Positivity bounds from analyticity in the UV

$$\frac{1}{2}\frac{\mathrm{d}^2\mathcal{A}(s)}{\mathrm{d}s^2}\bigg|_{s=0} = \frac{1}{2\pi i}\oint_{\mathcal{C}}\frac{\mathrm{d}s}{s^3}\mathcal{A}(s) > 0$$

SM Effective Field Theory

$$\mathcal{L}_{SMEFT} = \mathcal{L}^{(4)}_{SM} + \sum_{n\geq 5}\frac{c_n}{\Lambda^{n-4}}\mathcal{O}^{(n)}$$

Minimal Flavor Violation (MFV)

$$Y_u, Y_d, Y_e \to \text{spurions of } SU(3)^5_{\text{flavor}}$$

$$\downarrow$$

$$\mathcal{L}_{SMEFT} \in \mathbf{1} \text{ of } SU(3)^5$$

4-Fermi operators: $c_{mnpq}\partial^2(\bar\psi_m\Gamma\psi_n)(\bar\psi_p\Gamma\psi_q)$

$$M \equiv Y_u Y_u^\dagger \in \mathbf{1}_Q \oplus \mathbf{8}_Q, \quad \tilde M \equiv Y_u^\dagger Y_u \in \mathbf{1}_u \oplus \mathbf{8}_u$$

$\downarrow$ MFV

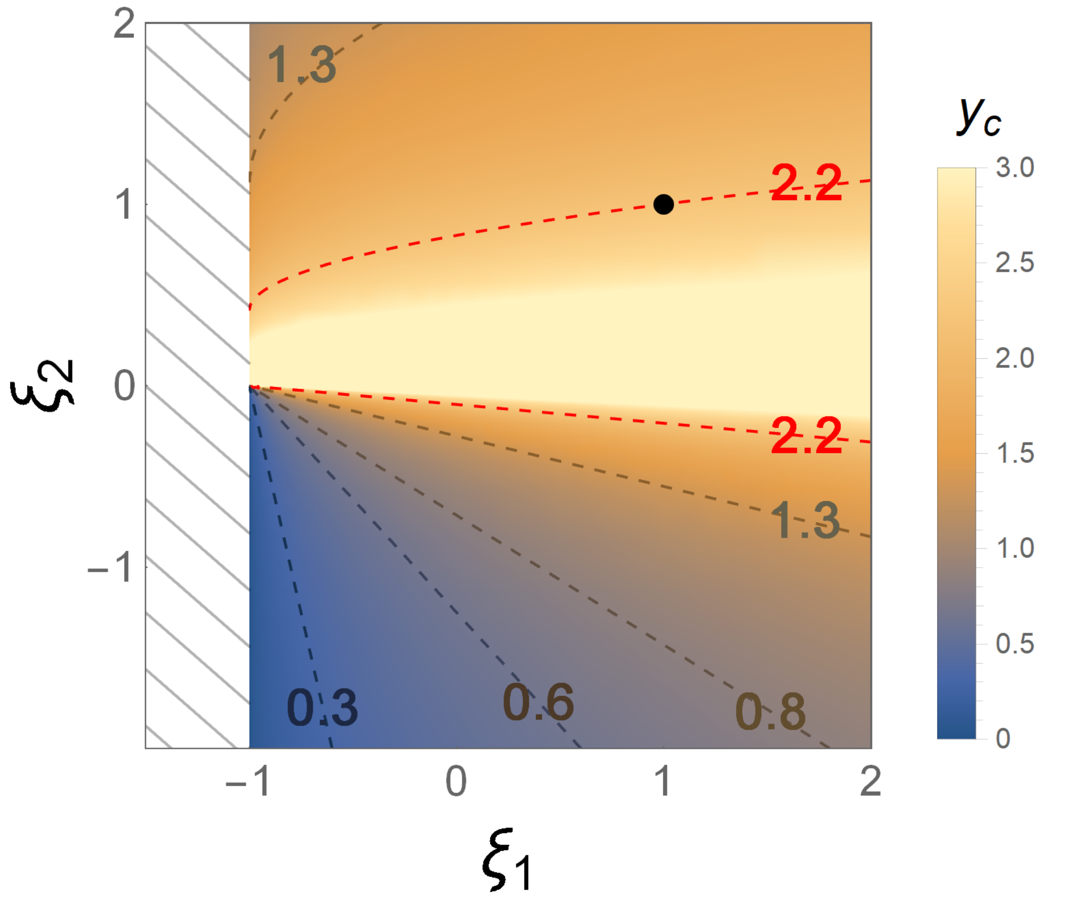
$$c^{u,i}_{mnpq} = \xi_1^{u,i}(\delta_{mn}\delta_{pq}) + \xi_2^{u,i}(\tilde M_{mn}\delta_{pq} + \delta_{mn}\tilde M_{pq}) + \xi_3^{u,i}(\delta_{mq}\delta_{pn}) + \xi_4^{u,i}(\tilde M_{mq}\delta_{pn} + \delta_{mq}\tilde M_{pn})$$

Studying the bounds

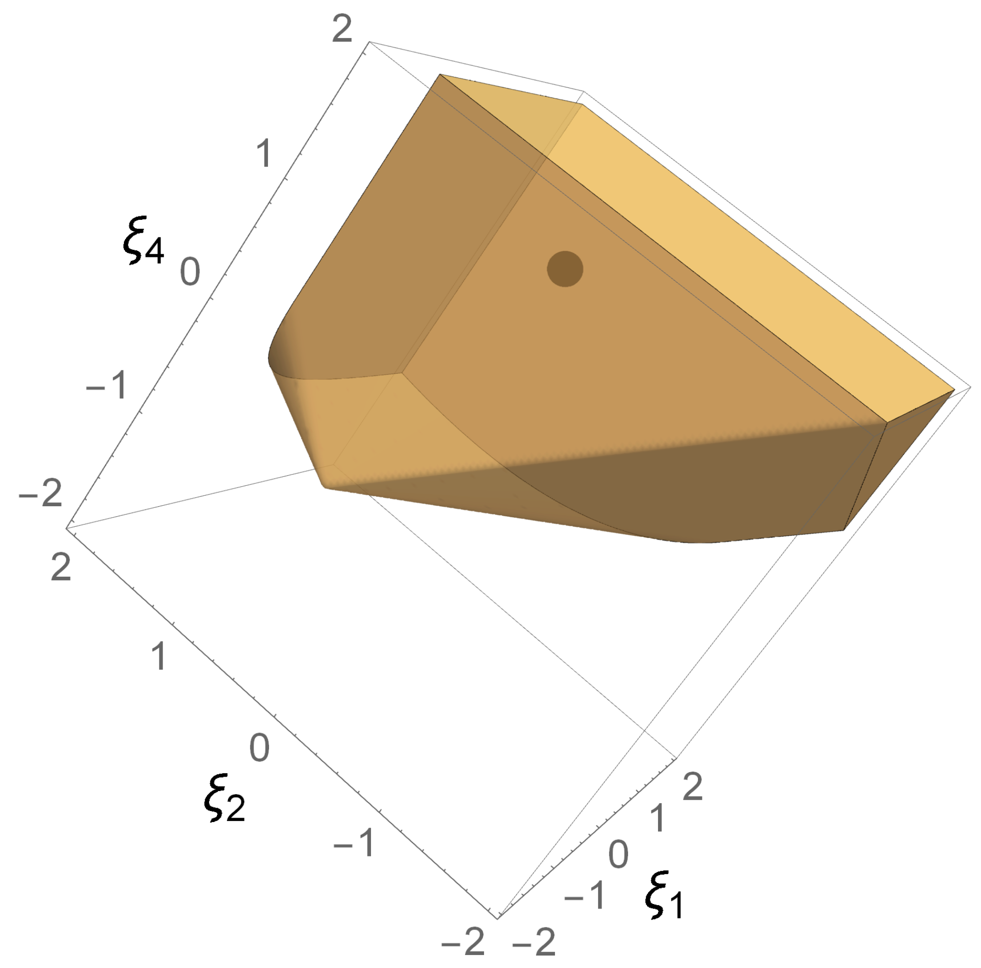
Generically of the form $\alpha_m\alpha_q^*\beta_n\beta_p^* c(\xi, Y)_{mnpq} > 0$, α_n, β_n are arbitrary N_f-dimensional complex vectors, we need to remove them to get bounds on the ξ flavor-blind coefficients, depending on the biggest Yukawas (smaller Y's and CKM elements are subleading contributions)

Define $C(\beta)_{mq} = c_{mnpq}\beta_n\beta_p^*$, the bounds require that it is positive definite $\longrightarrow$ The bounds are a condition on its eigenvalues, depending only on β_n

$N_f = 2$



$N_f = 3, \quad y_t = 1$



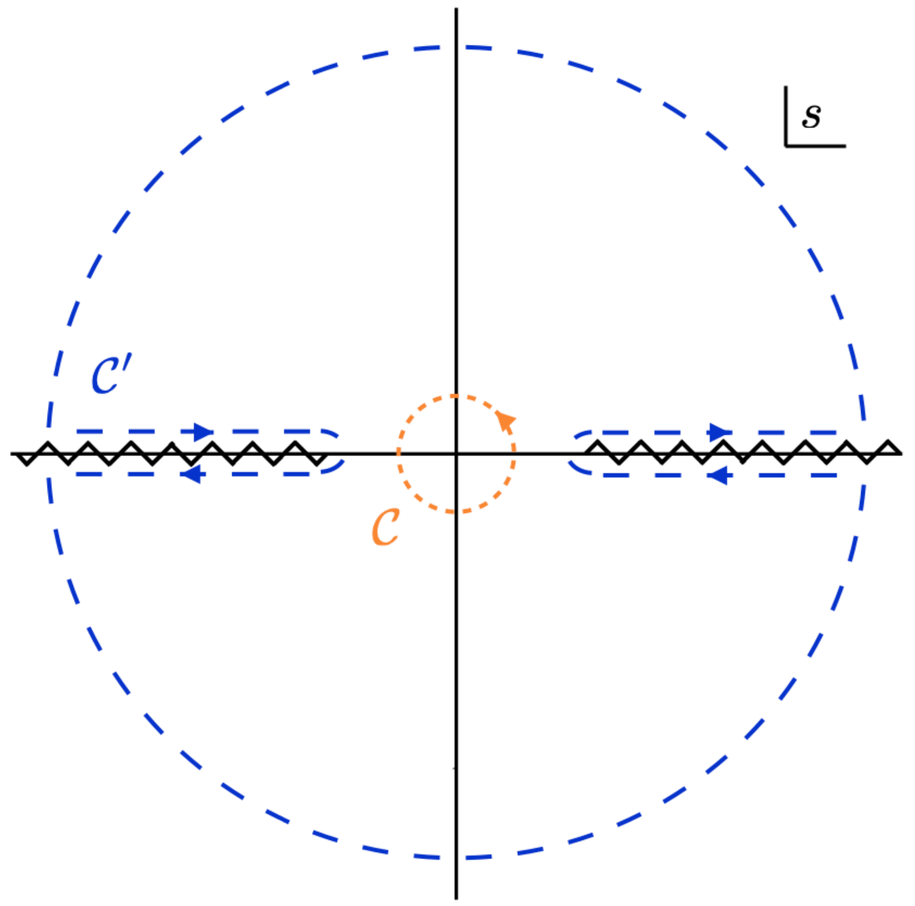
Positivity bounds and Minimal Flavor Violation are compatible!

Backup I

Positivity bounds: example

Scalar with shift symmetry: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{c}{M^4}(\partial_\mu \phi)^4$

$2 \to 2$ scattering in the forward limit $t \to 0$ $\longrightarrow$ $\mathcal{A}(s) \equiv \mathcal{M}(s, t=0) = \frac{4cs^2}{M^4}$



$$\mathcal{A}(s) = \sum_{n=0}^{\infty} \lambda_n s^n$$

$$\lambda_2 = \frac{1}{2\pi i}\oint_C \frac{\mathcal{A}(s)}{s^3}\, ds = \frac{4c}{M^4} \longrightarrow c > 0$$

$$\lambda_2 = C_R + \frac{1}{\pi i}\int_{s_d}^{+\infty} \left[\mathcal{A}(s+i\varepsilon) - \mathcal{A}(s-i\varepsilon)\right] ds \longrightarrow c > 0$$

$$= \frac{2}{\pi}\int_{s_d}^{+\infty} \frac{ds}{s^2}\sigma(s) > 0$$

Full list of dimension 8 4-Fermi operators

Type	Content	Operator	Symmetry
self-quartic	(4-u)	$\mathcal{O}_1[u] = c^{u,1}_{mnpq}\partial_\mu(\bar{u}_m\gamma_\nu u_n)\partial^\mu(\bar{u}_p\gamma^\nu u_q)$	$c_{mnpq} = c_{pqmn}$, $c_{mnpq} = c^*_{nmqp}$
		$\mathcal{O}_3[u] = c^{u,3}_{mnpq}\partial_\mu(\bar{u}_m T^a\gamma_\nu u_n)\partial^\mu(\bar{u}_p T^a\gamma^\nu u_q)$	
	(4-Q)	$\mathcal{O}_1[Q] = c^{Q,1}_{mnpq}\partial_\mu(\bar{Q}_m\gamma_\nu Q_n)\partial^\mu(\bar{Q}_p\gamma^\nu Q_q)$	
		$\mathcal{O}_2[Q] = c^{Q,2}_{mnpq}\partial_\mu(\bar{Q}_m\tau^I\gamma_\nu Q_n)\partial^\mu(\bar{Q}_p\tau^I\gamma^\nu Q_q)$	
		$\mathcal{O}_3[Q] = c^{Q,3}_{mnpq}\partial_\mu(\bar{Q}_m T^a\gamma_\nu Q_n)\partial^\mu(\bar{Q}_p T^a\gamma^\nu Q_q)$	
		$\mathcal{O}_4[Q] = c^{Q,4}_{mnpq}\partial_\mu(\bar{Q}_m T^a\tau^I\gamma_\nu Q_n)\partial^\mu(\bar{Q}_p T^a\tau^I\gamma^\nu Q_q)$	
	(4-d)	$\mathcal{O}_1[d] = c^{d,1}_{mnpq}\partial_\mu(\bar{d}_m\gamma_\nu d_n)\partial^\mu(\bar{d}_p\gamma^\nu d_q)$	
		$\mathcal{O}_3[d] = c^{d,3}_{mnpq}\partial_\mu(\bar{d}_m T^a\gamma_\nu d_n)\partial^\mu(\bar{d}_p T^a\gamma^\nu d_q)$	
cross-quartic	(2-u)(2-Q)	$\mathcal{O}_{K1}[u,Q] = -a^{uQ,1}_{mnpq}(\bar{u}_m\gamma_\mu\partial_\nu u_q)(\bar{Q}_n\gamma^\nu\partial^\mu Q_p)$	$a^{\psi\chi}_{mnpq} = a^{\chi\psi}_{nmqp}$, $a_{mnpq} = a^*_{qpnm}$
		$\mathcal{O}_{K3}[u,Q] = -a^{uQ,3}_{mnpq}(\bar{u}_m T^a\gamma_\mu\partial_\nu u_q)(\bar{Q}_n T^a\gamma^\nu\partial^\mu Q_p)$	
	(2-d)(2-Q)	$\mathcal{O}_{K1}[d,Q] = -a^{dQ,1}_{mnpq}(\bar{d}_m\gamma_\mu\partial_\nu d_q)(\bar{Q}_n\gamma^\nu\partial^\mu Q_p)$	
		$\mathcal{O}_{K3}[d,Q] = -a^{dQ,3}_{mnpq}(\bar{d}_m T^a\gamma_\mu\partial_\nu d_q)(\bar{Q}_n T^a\gamma^\nu\partial^\mu Q_p)$	
	(2-d)(2-u)	$\mathcal{O}_{K1}[d,u] = -a^{du,1}_{mnpq}(\bar{d}_m\gamma_\mu\partial_\nu d_q)(\bar{u}_n\gamma^\nu\partial^\mu u_p)$	
		$\mathcal{O}_{K3}[d,u] = -a^{du,3}_{mnpq}(\bar{d}_m T^a\gamma_\mu\partial_\nu d_q)(\bar{u}_n T^a\gamma^\nu\partial^\mu u_p)$	

There are other operators of the form

$$\mathcal{O} = \partial_\mu(\bar{\psi}_m\gamma_\nu\psi_n)\partial^\mu(\bar{\chi}_p\gamma^\nu\chi_q), \quad \psi \neq \chi$$

However their contribution to the amplitude vanishes in the forward limit $t \to 0$, so they are not subject to these bounds

Backup II

How do we get the bounds?

Scatter flavor superpositions as initial and final states. E.g. scattering right-handed up-type quarks:

$$|\psi_1\rangle = \alpha_{mi}\,|\bar{u}_{mi}\rangle \qquad |\psi_2\rangle = \beta_{mi}\,|u_{mi}\rangle$$
$$|\psi_3\rangle = \beta^*_{mi}\,|\bar{u}_{mi}\rangle \qquad |\psi_4\rangle = \alpha^*_{mi}\,|u_{mi}\rangle$$

We get

$$\mathcal{A} = 4s^2\left[\left(c^{u,1}_{mnpq} - \frac{1}{6}c^{u,3}_{mnpq}\right)\alpha^*_{mi}\beta_{ni}\beta^*_{pj}\alpha_{qj} + \frac{1}{2}c^{u,3}_{mnpq}\alpha^*_{mi}\beta_{nj}\beta^*_{pj}\alpha_{qi}\right].$$

producing the two bounds:

$$\alpha_m\alpha^*_q\beta_n\beta^*_p\left(c^{u,1}_{mnpq} + \frac{1}{3}c^{u,3}_{mnpq}\right) > 0$$
$$\alpha_m\alpha^*_q\beta_n\beta^*_p c^{u,3}_{mnpq} > 0$$

Then performing a linear redefinition on the flavory-blind coefficients ξ we can take them both into the form

$$\alpha_m\alpha^*_q\beta_n\beta^*_p c(\xi)^{u,i}_{mnpq} > 0, \quad i = 1,3$$

Approximations

We take only the largest up Yukawa to be non-zero and study the bounds as a function of it, both for $N_f = 2$ and $N_f = 3$. The down Yukawa matrix is set to zero.

$$Y_u = \begin{pmatrix} \cancel{y_u} & 0 & 0 \\ 0 & \cancel{y_c} & 0 \\ 0 & 0 & y_t \end{pmatrix} \qquad Y_d \to 0$$

Some remarks:

- To account for y_t being of $\mathcal{O}(1)$, one needs to resum and set $y_t = 1$ in the end
- As we can always rotate in the Lagrangian to have diagonal Y_u and the CKM matrix entirely in Y_d, the latter does not show at leading order in the bounds. To observe it we need to include both Y_u and Y_d contributions
- Alternatively, we can scatter initial and final states limited to two flavors. In this case the bounds are as in $N_f = 2$ but depend on the CKM entries through the combination $\sigma = A^2\lambda^4$ (Wolfenstein parametrisation)