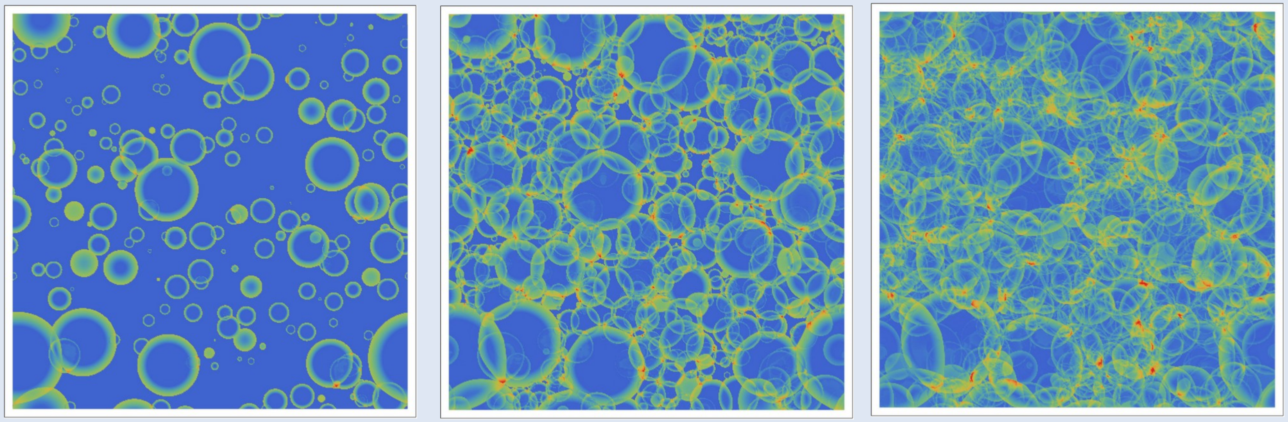
Gravitational waves from cosmological phase transitions

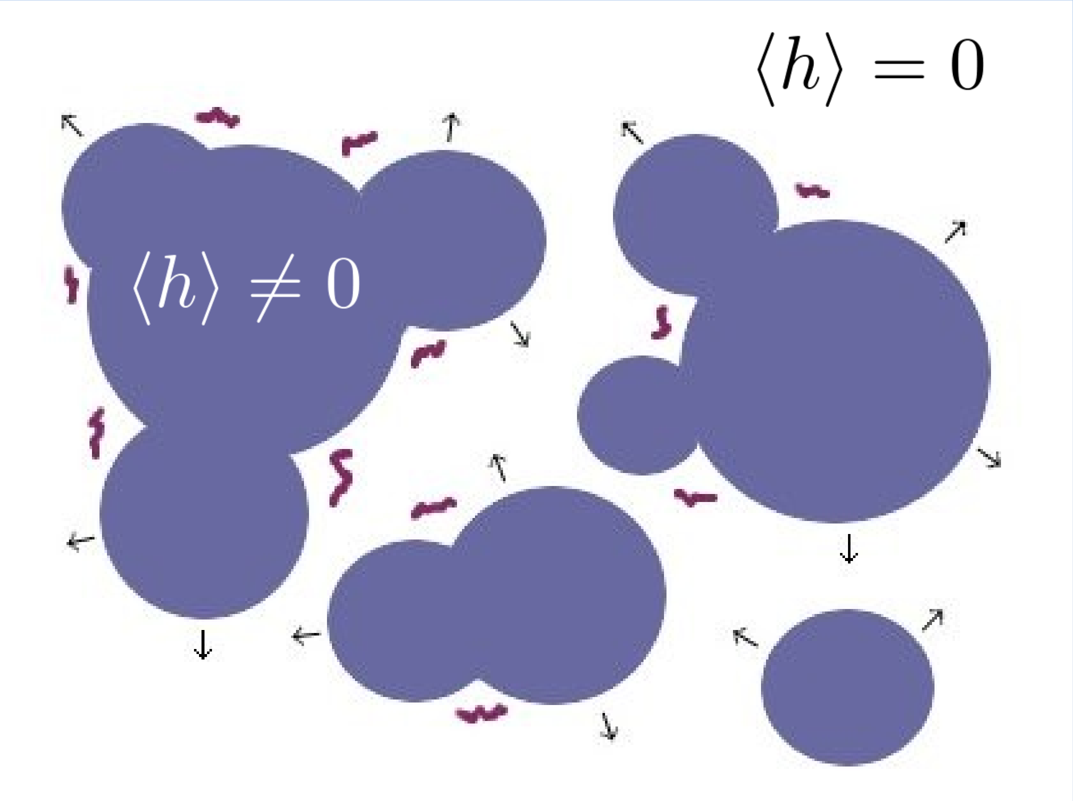
Thomas Konstandin

HPNP, March 27, 2021



in collaboration with: R. Jinno, H. Rubira, F. Giese, K. Schmitz, J. v. De Vis

Gravitational waves

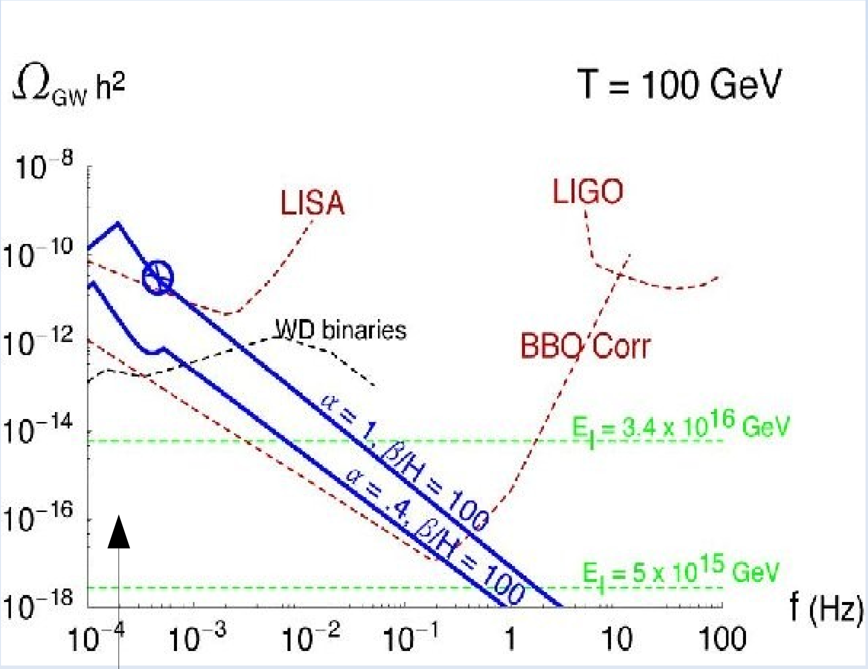


During the first-order phase transitions, nucleated bubbles of the new phase expand. Finally, the colliding bubbles generate stochastic gravitational waves.

Observation

[Grojean&Servant '06]

The produced gravitational waves can be observed with laser interferometers in space



redshifted **Hubble horizon** during a phase transition at T ~ 100 GeV

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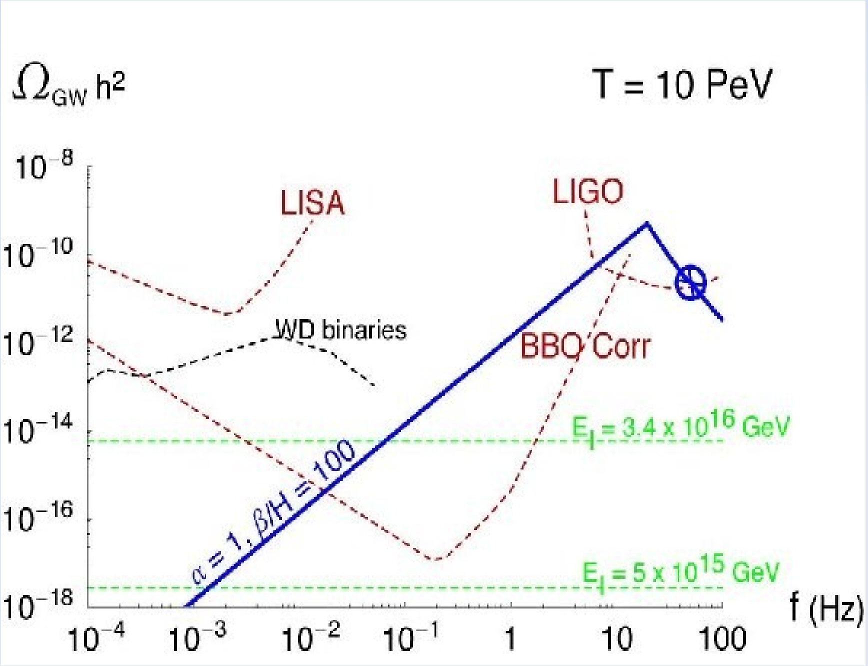


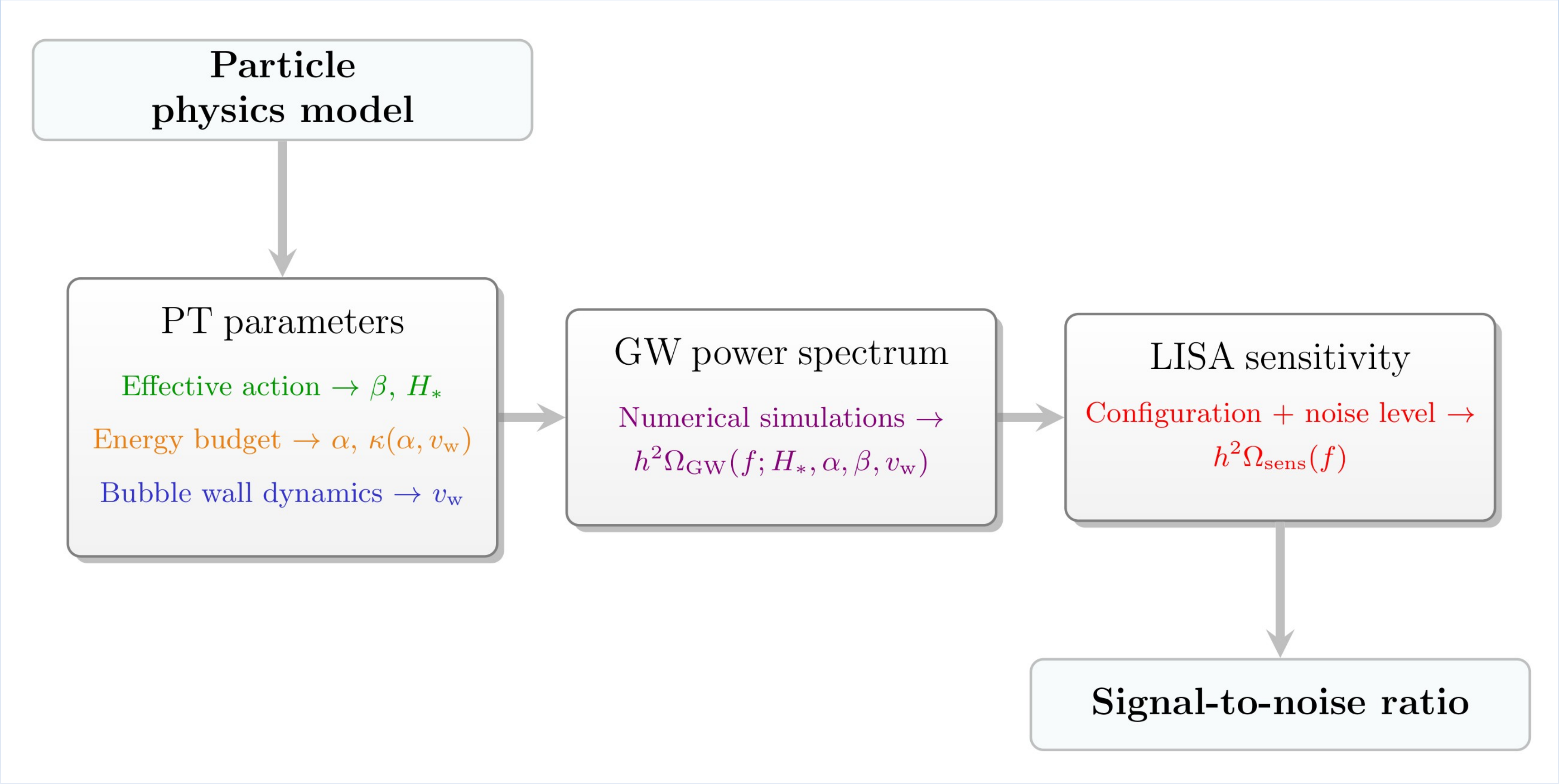
Strong phase transition at larger temperatures produce the same energy fraction of gravitational waves but at higher frequencies.

Future space telescopes

Space based experiments are sensitive to smaller frequencies where stochastic backgrounds GWs can provide a link to EW physics.

Connecting models with signals

[LISA Cosmo WG '19]



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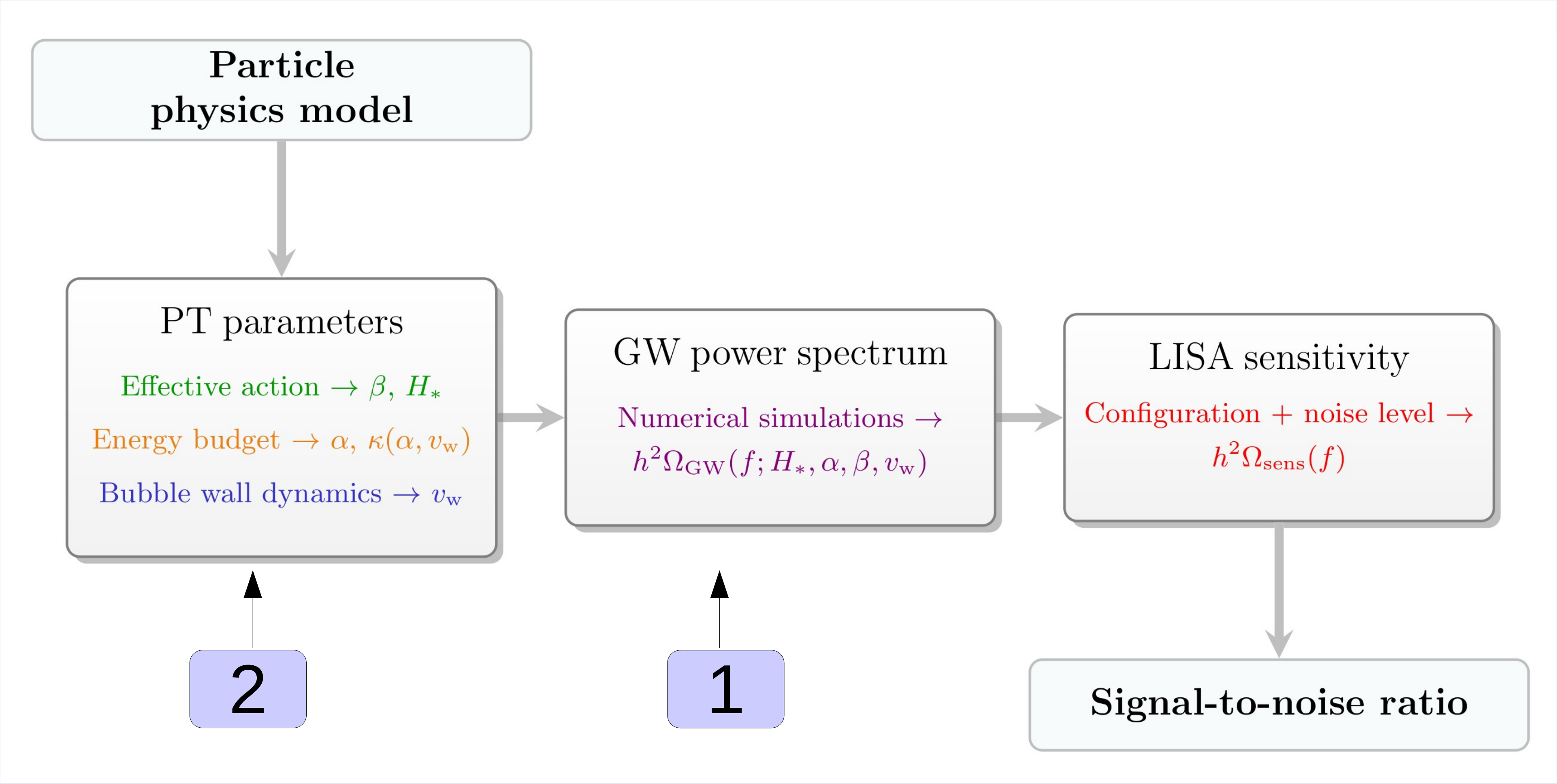


Sources of GWs from PTs

During and after the phase transition, several sources of GWs are active

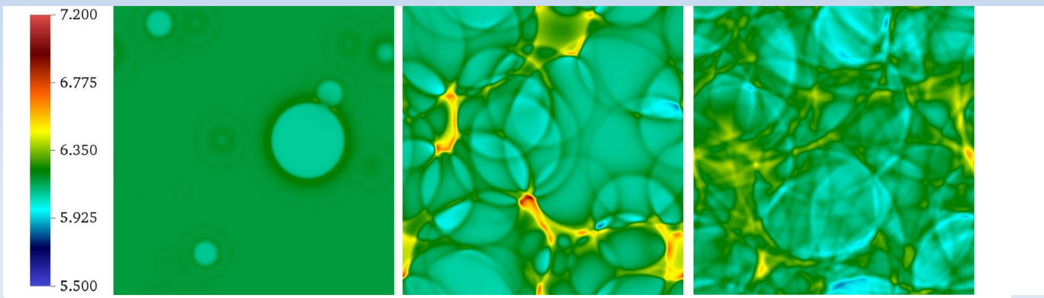
- Collisions of the scalar field configurations / initial fluid shells
- Sound waves after the phase transition (long-lasting → dominant source)
- Turbulence
- Magnetic fields

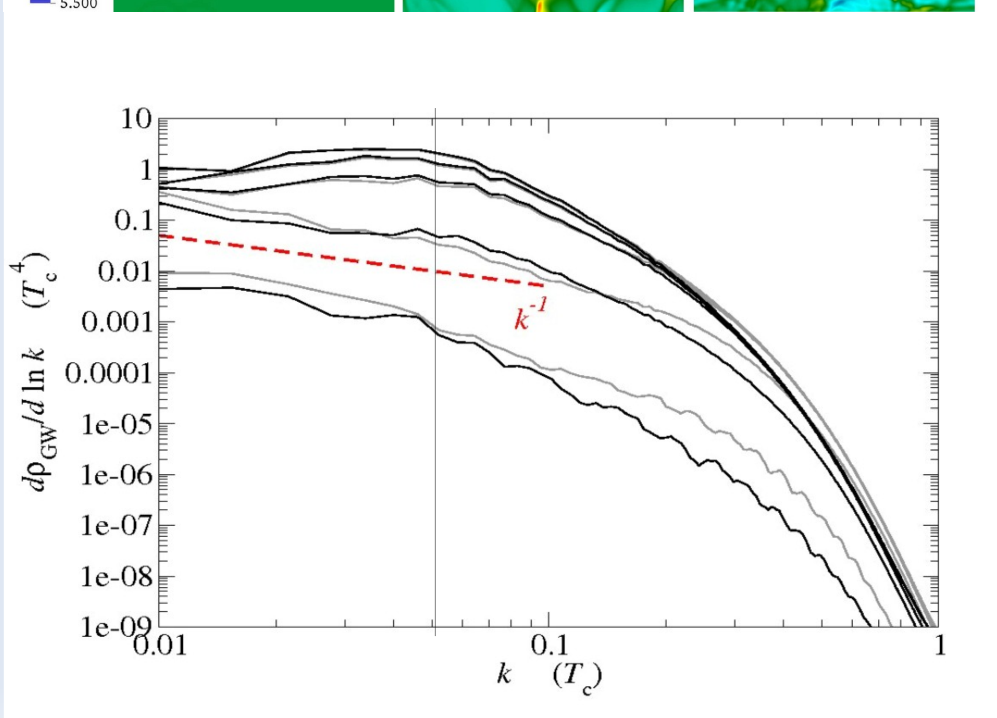
Which source dominates depends on the characteristics of the PT

State-of-the-art

[Hindmarsh, Huber , Rummukainen, Weir '13, '15, '17]

After the PT, the system can be descibed using hydrodynamics (fluid + Higgs).

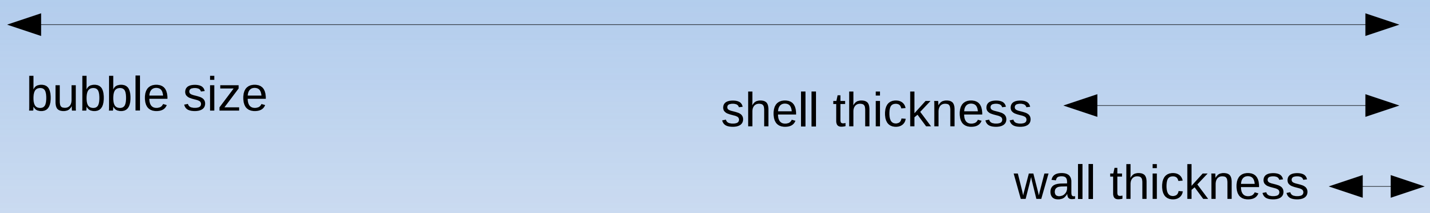




The produced GW spectrum can be read off from the simulation.

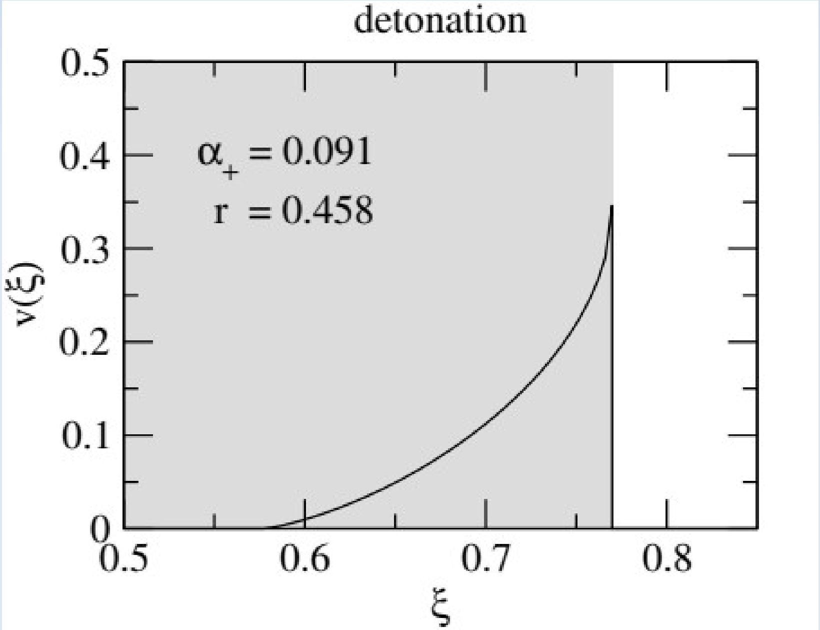
Really robust results but how to extrapolate to other models and realistic wall thickness?

Length scales



One technical main problem of the simulations is that they have to resolve differnt length scales: the bubble size, the sound shell thickness and the bubble wall thickness.

In particular, the bubble wall thickness is many orders smaller than the bubble size, so extrapolations to the physical point have to be used.

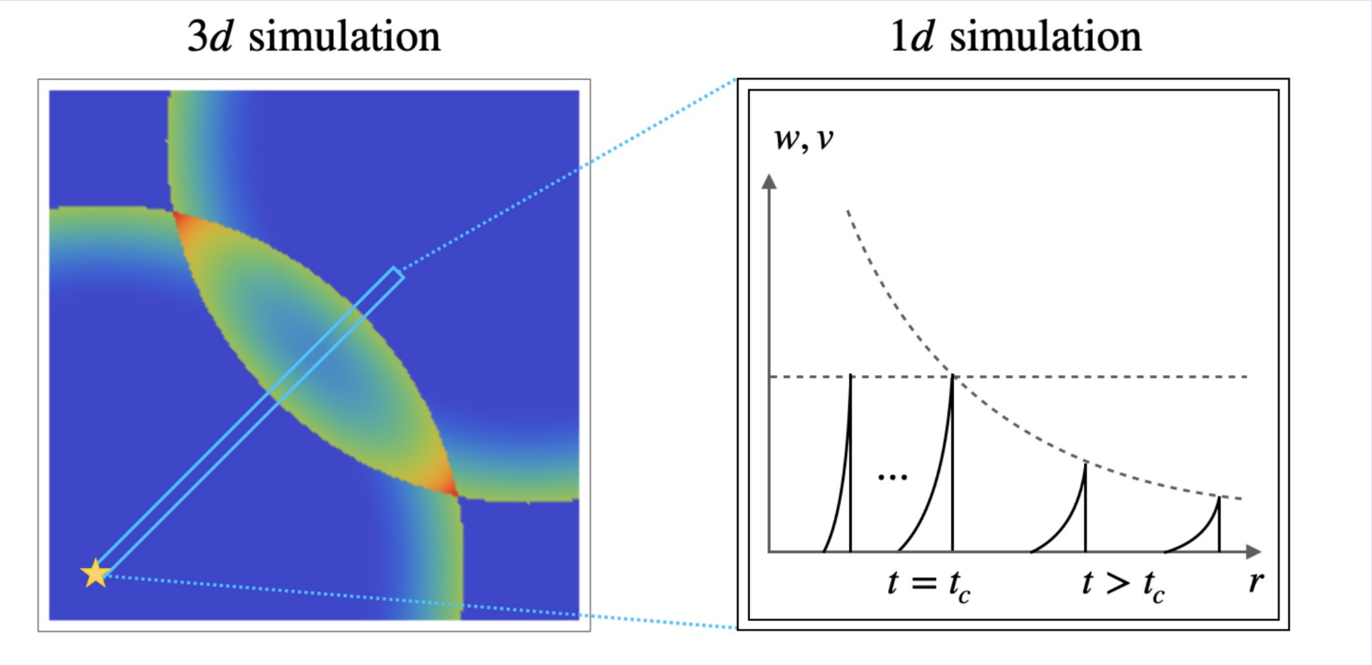


$$\xi = \frac{r}{t}$$

Novel simulations

We conceived new simulations where the bubble wall thickness only enters through the boundary conditions of the simulation. We achieve this by doing simulations of colliding 1D bubbles and then embed these bubbles into a 3D grid.

This assumes relatively weak phase transitions (and linear superposition of sound waves) but is valid for very thin shells and relativistic bubble wall velocities.



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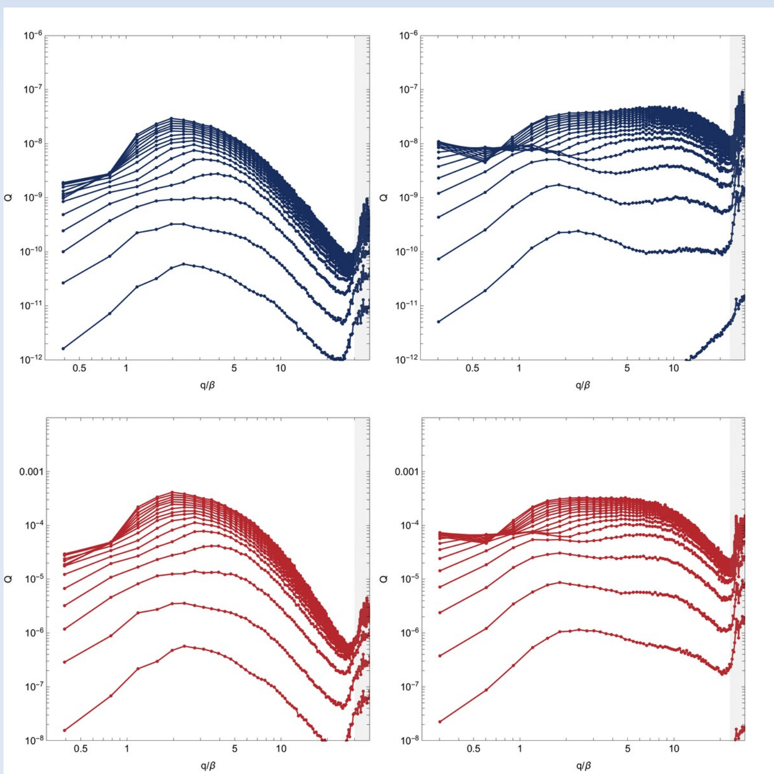
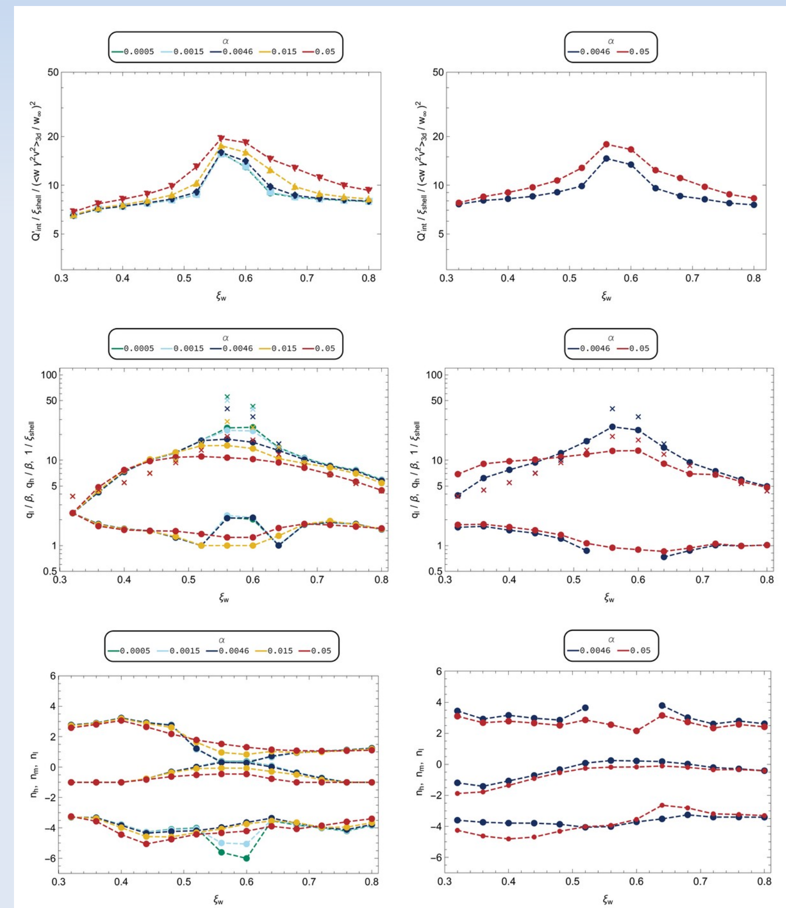
Final spectra

Many of these lightweight simulations can be performed and the relevant parameters of the GW spectra can be extracted. This also – for the first time – gives access to phase transitions with thin shells and large wall velocities.





[Jinno, TK, Rubira '20]

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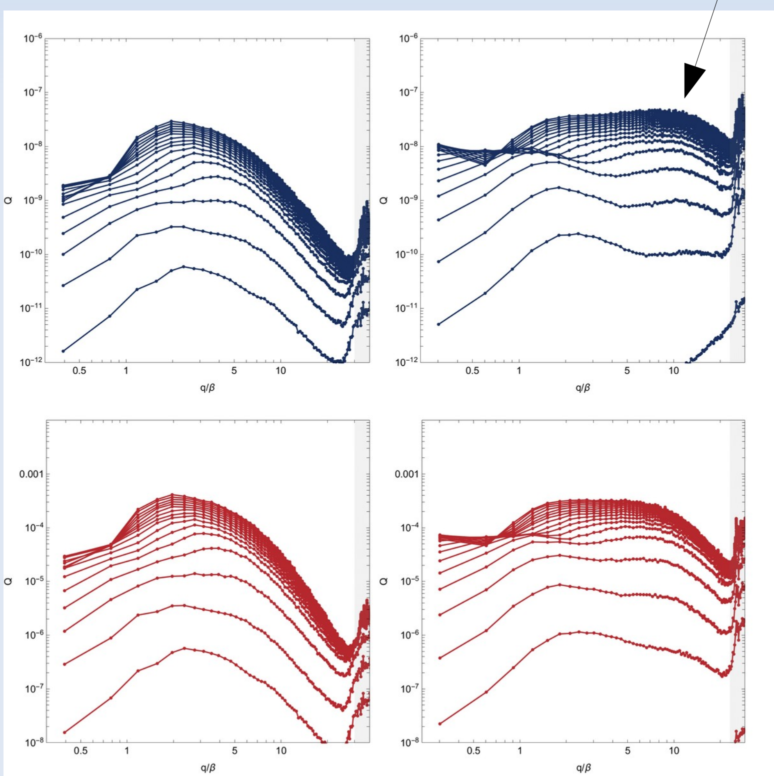
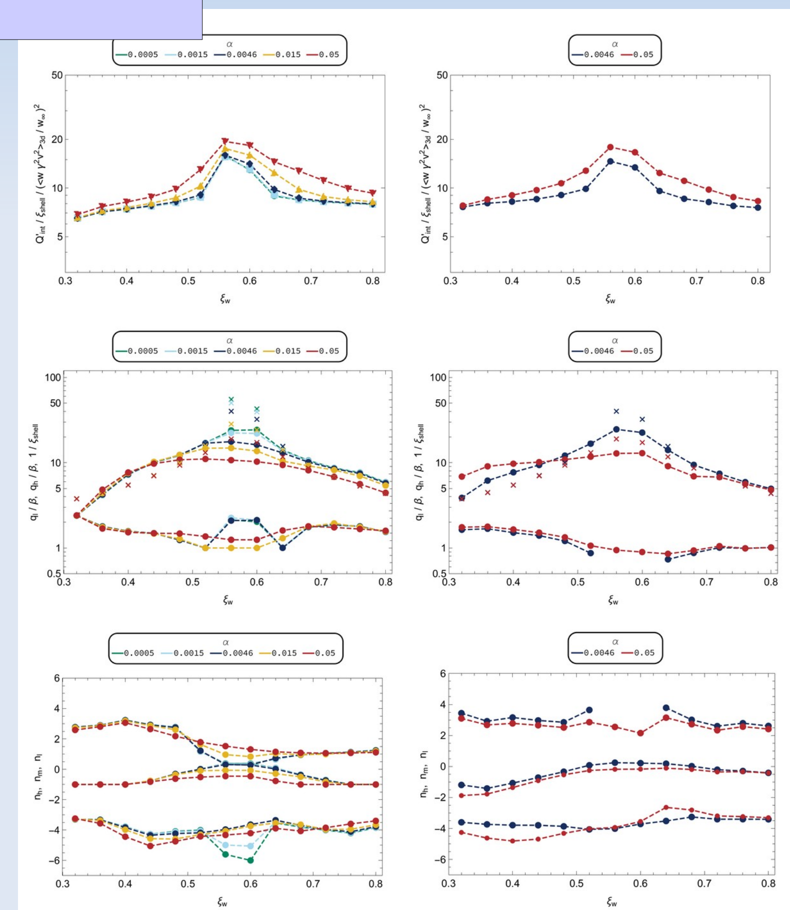
An additional scale emerges for thin shells





[Jinno, TK, Rubira ’20]

Summary

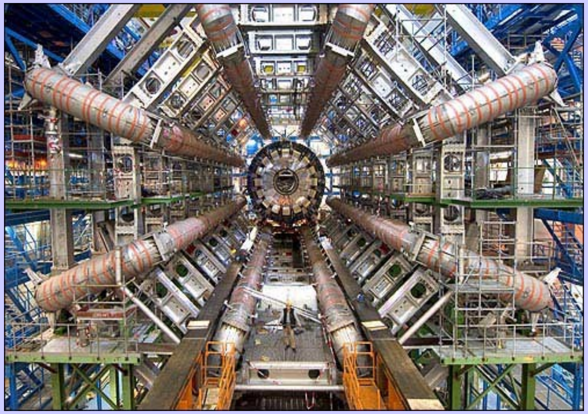
Our novel approach to simulating the fluid during cosmological phase transitions is lightweight and allows to study parametric dependences in detail.

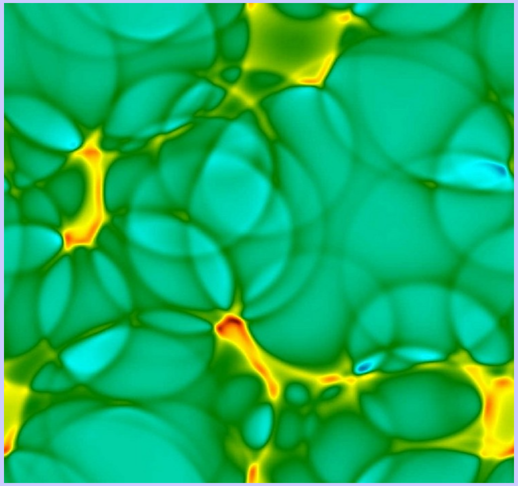
It is based on ’integrating out’ the scale of the Higgs wall separating the two phases.

This simulation also works in cases where the full hydrodynamic simulations are prohibitively expensive (e.g. fast walls, many bubbles).

Lightweight simulations provide a toolbox for further studies: non-Gaussianity / power spectrum of the fluid velocity field, impact of initial fluctutions....

How to connect models and simulations?





Model-dependence

The Weinberg master formula determines how stochastic gravitational waves are produced

$$\frac{dE_{GW}}{d\omega d\Omega} = 2G\omega^2 \Lambda_{ij,lm}(\hat{\mathbf{k}}) T^*_{ij}(\hat{\mathbf{k}}, \omega) T_{lm}(\hat{\mathbf{k}}, \omega),$$

And generally the energy fraction in GWs scales as

$$\Omega_{GW*}(f) \propto K^2$$

where **K** denotes the kinetic energy fraction in the fluid after the phase transition that is where the model-dependence will enter for most parts.

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rest of the talk

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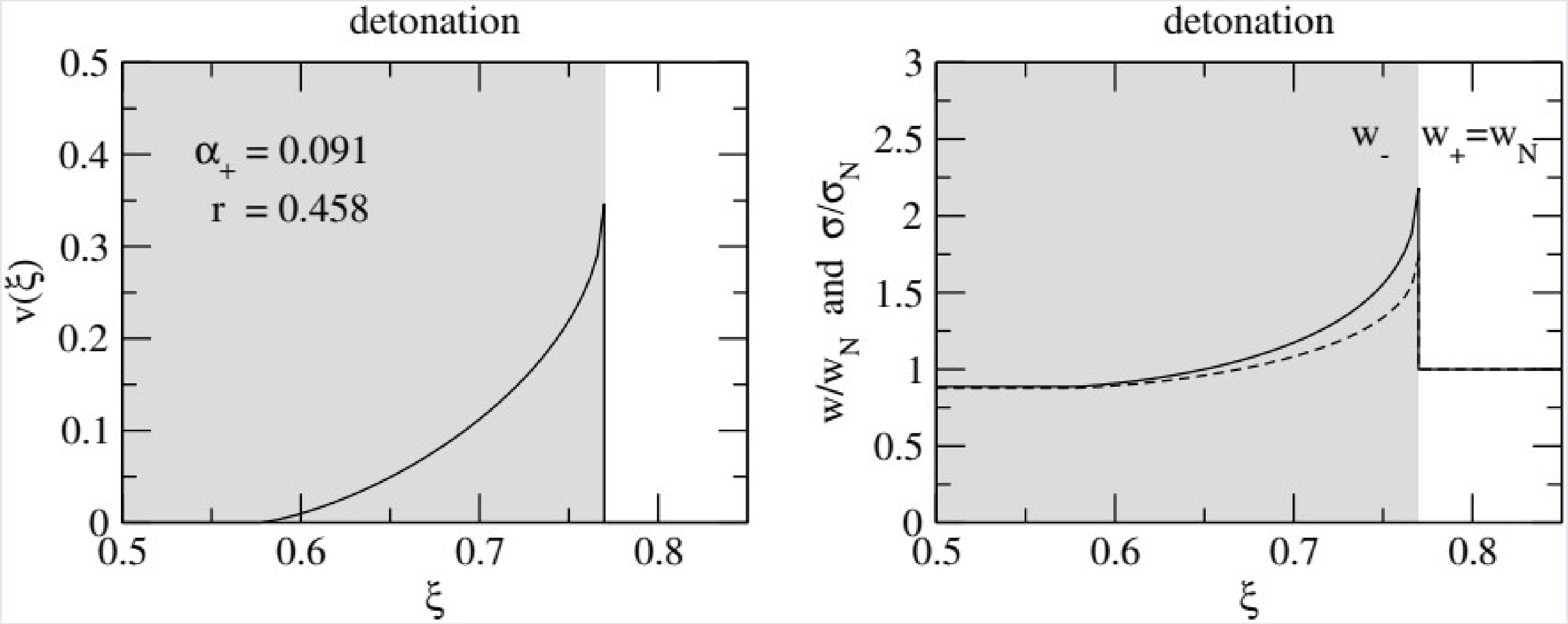
Kinetic energy with spherical symmetry

The bulk kinetic energy depends on the enthalpy *w* and the fluid velocity *v* and can be determined from an isolated spherical bubble before collision

$$K \equiv \frac{\rho_{kin}}{e}, \qquad \rho_{kin} = \frac{1}{V}\int dV\, v^2\gamma^2\, w\,.$$



$$\xi = r/t$$

Bag model

[Kosowsky, Turner , Watkins, '92]
[Espinosa, TK, No, Servant '10]

The kinetic enregy fraction has been calculated in the bag model

$$p_s = \frac{1}{3} a_+ T^4 - \epsilon, \quad p_b = \frac{1}{3} a_- T^4 \, .$$

$$e_s = a_+ T^4 + \epsilon, \quad e_b = a_- T^4 \, ,$$

bag constant

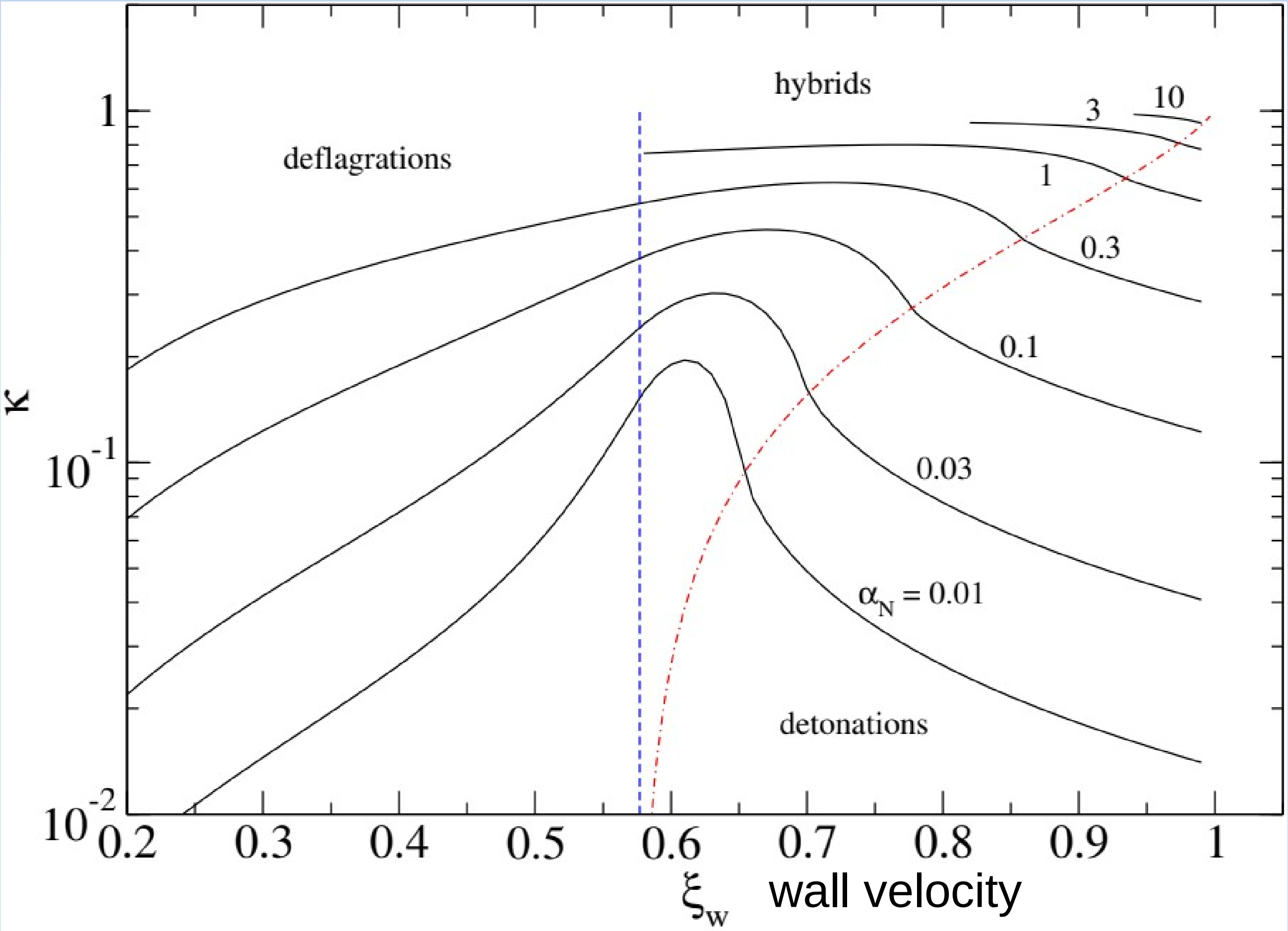
The strength of the phase transition is characterized by

$$\alpha = \frac{\epsilon}{a_+ T^4}$$

Kinetic energy fraction and efficiency coefficient

$$K = \frac{\alpha}{\alpha + 1}\kappa$$

[Espinosa, TK, No, Servant ’10]



Fits to this numerical result are used in pheno studies.

How to match to other models?

Fitting functions of these results are used in phenomenological analysis but what is the strength parameter in a general models? In particular if only quantities at nucleation temperature are used?

$$DX = (X_s(T_n) - X_b(T_n))$$

$\alpha \propto Dp$

If the pressure difference vanishes, the bubble becomes static

$\alpha \propto De$

The energy difference fuels the kinetic motion of the bulk fluid

$\alpha \propto D\theta \propto (De - 3Dp)$

The trace difference is the bag constant in the bag model and also comes about naturally in lattice simluations

A model comparison

[Giese, TK, van de Vis '20]

model/method	M1	M2	M3	M4	M5	M6
SM_1	0.00143		4.99 %	3.55 %	-88.45 %	713.34 %
SM_2	0.00401		1.70 %	-0.72 %	-66.69 %	351.90 %
SM_3	0.00014		1.37 %	0.94 %	-89.16 %	779.35 %
SM_4	0.00039		0.42 %	-0.32 %	-67.85 %	405.11 %
$2step_1$	0.00036		13.61 %	17.39 %	-89.52 %	945.17 %
$2step_2$	0.00563		15.68 %	21.90 %	-50.01 %	366.20 %
$2step_3$	0.00070		35.97 %	47.28 %	-89.85 %	1235.34 %
$2step_4$	0.01576		40.05 %	58.29 %	-41.80 %	485.16 %

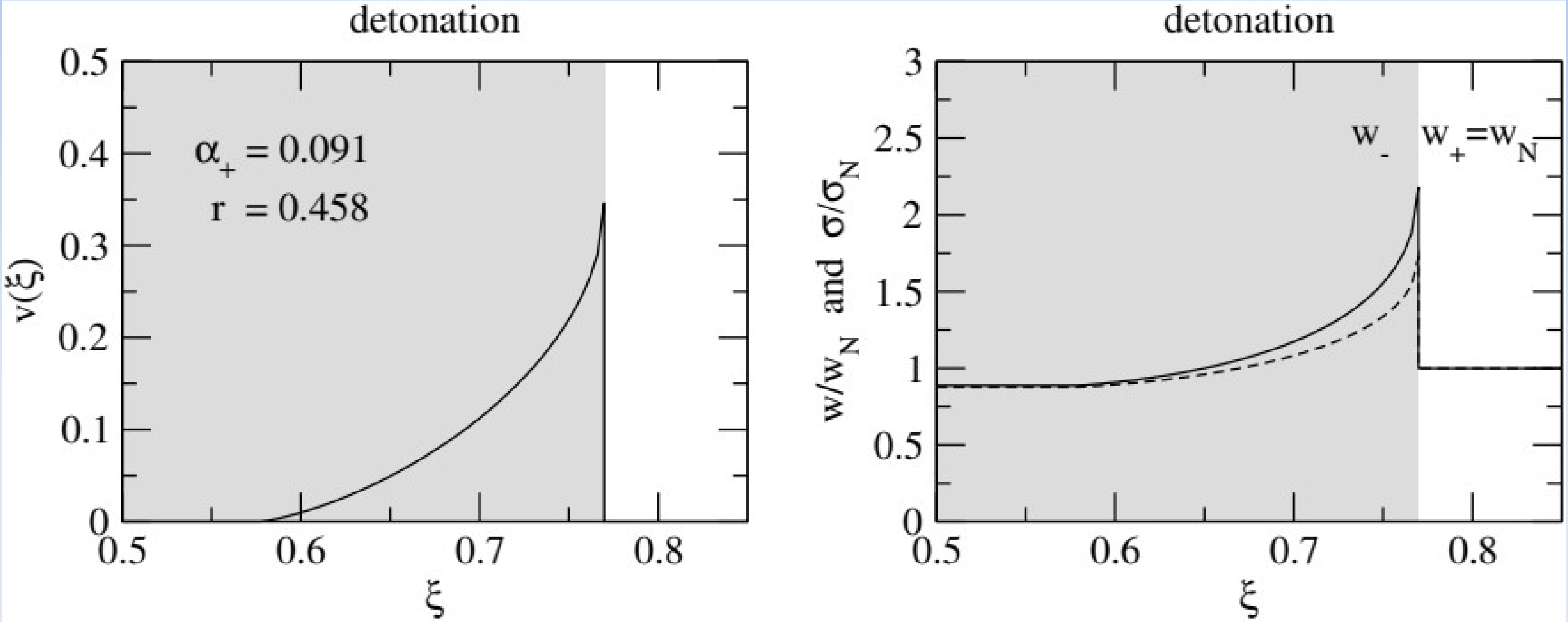
Table 4: Relative errors of the methods M2-M6 compared to the fully numerical result M1. The model parameters are given in Table 1 and 2 and a wall velocity of $\xi_w = 0.9$ was used.

new approach

$\Delta\theta$ $\Delta\theta$ Δp Δe

methods used in the literature

The matching equation



$$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)},$$

These equations determine T_- and v_- as functions of $v_+ = v_w$ and $T_+ = T_{nucleation}$

The matching equation

[Giese, TK, van de Vis '20]

The temperature T- can be eliminated using

$$\frac{p_b(T_+) - p_b(T_-)}{e_b(T_+) - e_b(T_-)} \simeq \left.\frac{dp_b/dT}{de_b/dT}\right|_{T_n} \equiv c_s^2 \,.$$

This then leads to

$$\frac{v_+}{v_-} \simeq \frac{(v_+ v_-/c_s^2 - 1) + (De - Dp/c_s^2)/w_+}{(v_+ v_-/c_s^2 - 1) + v_+ v_-(De - Dp/c_s^2)/w_+} \,.$$

This motivates the following definition of the strength parameter in terms of the *pseudotrace*

$$\bar{\theta} \equiv e - p/c_s^2 \,, \quad \alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3w_+} \,,$$

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K should only depend on these two quantities!

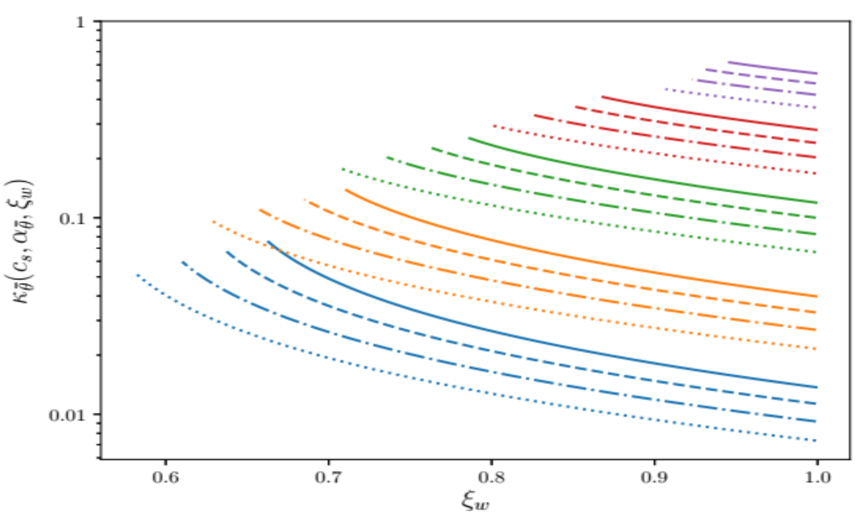
A sound argument to go beyond the bag model

[Leitao and Megevand '14] ν-model

$$p_s = \frac{1}{3} a_+ T^4 - \epsilon\,, \qquad e_s = a_+ T^4 + \epsilon\,, \quad c_s^2 = \frac{1}{\nu - 1}$$

$$p_b = \frac{1}{3} a_- T^\nu\,, \qquad e_b = \frac{1}{3} a_- (\nu - 1) T^\nu\,,$$

model/method	M1	M2
SM_1	0.00143	0.45 %
SM_2	0.00401	0.43 %
SM_3	0.00014	0.04 %
SM_4	0.00039	0.04 %
$2step_1$	0.00036	-0.21 %
$2step_2$	0.00563	-0.80 %
$2step_3$	0.00070	-0.77 %
$2step_4$	0.01576	-3.52 %



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Coding the kinetic energy fraction

```python
import numpy as np
from scipy.integrate import odeint
from scipy.integrate import simps

def kappaNuModel(cs2,al,vp):
  nu = 1./cs2+1.
  tmp = 1.-3.*al+vp**2*(1./cs2+3.*al)
  disc = 4*vp**2*(1.-nu)+tmp**2
  if disc<0:
    print("vp too small for detonation")
    return 0
  vm = (tmp+np.sqrt(disc))/2/(nu-1.)/vp
  wm = (-1.+3.*al+(vp/vm)*(-1.+nu+3.*al))
  wm /= (-1.+nu-vp/vm)

  def dfdv(xiw, v, nu):
    xi, w = xiw
    dxidv = (((xi-v)/(1.-xi*v))**2*(nu-1.)-1.)
    dxidv *= (1.-v*xi)*xi/2./v/(1.-v**2)
    dwdv = nu*(xi-v)/(1.-xi*v)*w/(1.-v**2)
    return [dxidv,dwdv]

  n = 501 # change accuracy here
  vs = np.linspace((vp-vm)/(1.-vp*vm), 0, n)
  sol = odeint(dfdv, [vp,1.], vs, args=(nu,))
  xis, ws = (sol[:,0],-sol[:,1]*wm/al*4./vp**3)

  return simps(ws*(xis*vs)**2/(1.-vs**2), xis)
```

Table 5: Python code to calculate $\kappa_{\bar{\theta}}$ in the ν-model as a function of the speed of sound squared c_s^2, the strength of the phase transition $\alpha_{\bar{\theta}}$ and the wall velocity ξ_w.

Summary

To extrapolate the results from hydrodynamic simulations to other models one needs the energy fraction of a single expanding bubble.

In the literature this is typically done by matching the bag model where the energy fraction is known (as a fit).

This leads to errors of order O(1) or O(10).

A model-independent approach suggests to use the *speed-of-sound* in the broken phase and the *pseudo-trace* as the strength parameter of the matching.

This reduces the error to O(few %) using the Python code snippet.

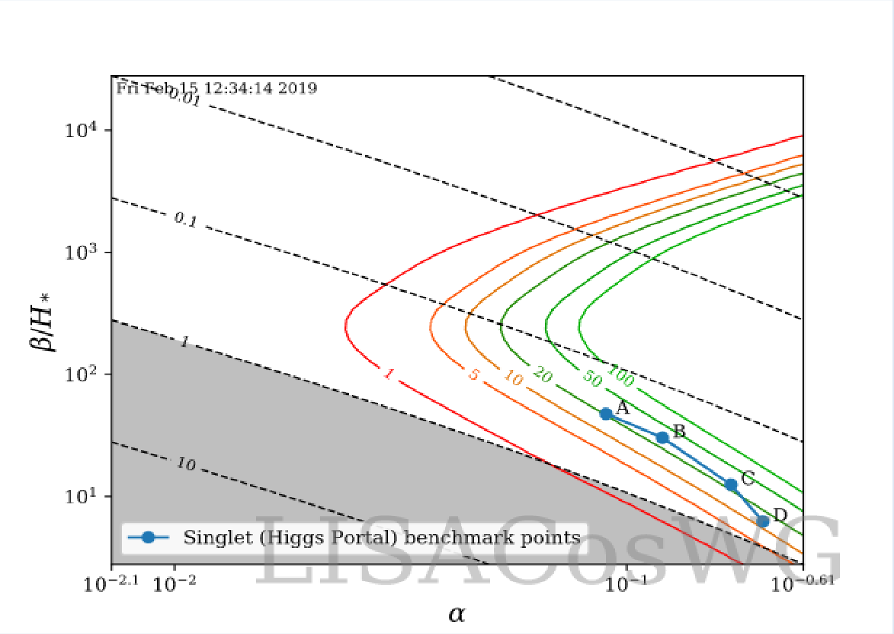
Putting it all together

The different sources and the relation to particlue physics model building is discussed in publications by the LISA cosmology working group on GWs from cosmological phase transitions:

Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions
Caprini et al.
arxiv/1512.06239

Detecting ravitational waves from cosmological phase transitions with LISA: an update
Caprini et al.
arxiv/1910.13125

web-tool by *David Weir*
http://www.ptplot.org



Conclusions

The observation of Gravitational Waves started a new era in astro physics.

The main appeal of these observations is that one can probe the era before electromagnetic decoupling.

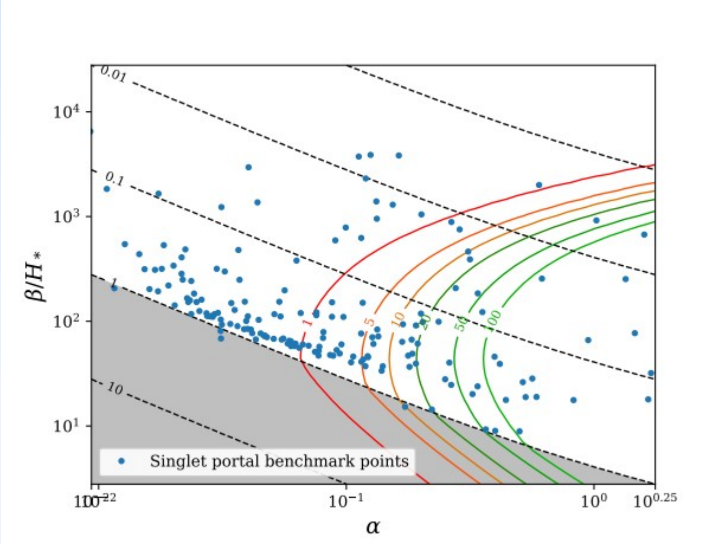
In principle, experiments as LISA/LIGO/DECIGO allow to test phase transitions (and hence particle physics) from EW scales up to very high scales $\sim 10^6$ GeV.

KAGRA will join the LIGO/VIRGO network soon.

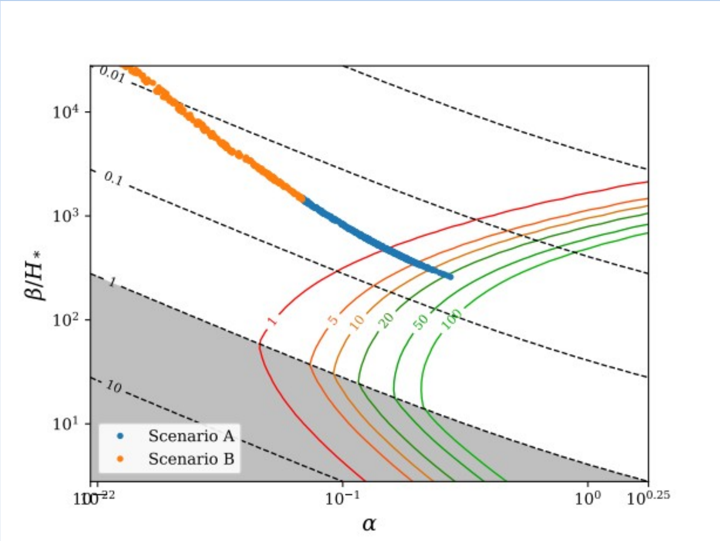
LISA will fly in the 2030s and cover a large range of cosmological phase transitions in terms of strength and temperatures close to electroweak scales.

Thank you

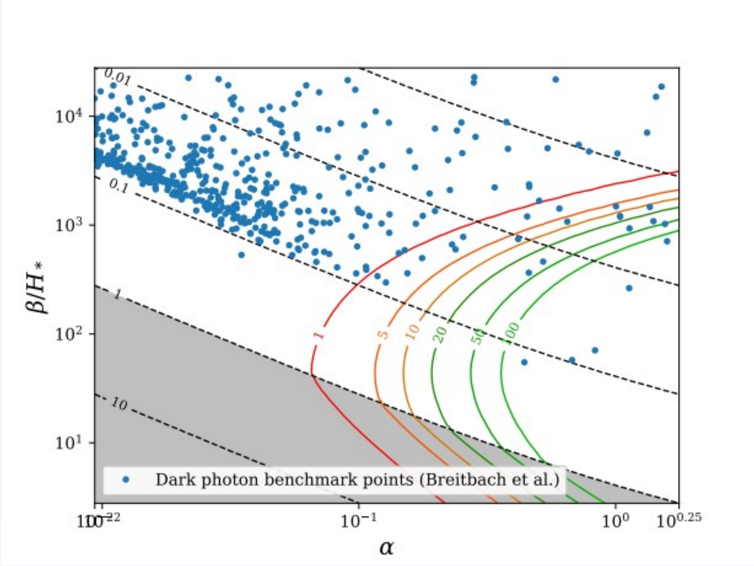
singlet portal model



SM EFT



dark photon



THD